Simulation of dielectric and resonance and anti-resonance data using modified Lorentz equation (T and ω simultaneously) of relaxor ferroelectric and piezoelectric ceramics

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Abstract. Dielectric data of new $[Ba(Nd_xTi_{1-2x}Nb_x)O_3]_{0.30}[Na_{0.5}Bi_{0.5}TiO_3]_{0.70}$ ($x = 0.075$) relaxor ceramic was modelled using a new modified Lorentz equation (T and ω simultaneously) as proposed by us. The activation energy for thermally activated orientation of dipoles and relaxation times were estimated. Dielectric resonance and anti-resonance data as a function of temperature and angular frequency of other piezoelectric compound $[Ba(Nd_{0.1}Ti_{0.8}Nb_{0.1})O_3]_{0.35}$ $[(Na_{0.5}Bi_{0.5})TiO_3]_{0.65}$ was also modelled using the modified Lorentz equation as proposed by us. It is shown that using this equation, it is possible to obtain the polarizability, piezoelectric charge constant, piezoelectric voltage constant and activation energy for resonance and anti-resonance.

Keywords. Dielectric relaxor; Lorentz equation; modified Lorentz equation; electromechanical coupling; piezoelectric charge constant.

1. Introduction

In relaxor ferroelectrics, the temperature dependence of dielectric constant is usually indirectly characterized from ω and T by the Arrhenius law ($\omega = \omega_0 \exp(-[E_a/(K_BT)])$) and modified Arrhenius law ($\omega = \omega_0 \exp(-[E_a/(K_BT)]^p)$) [1] and also by Vogel–Fulcher's relation ($\omega = \omega_0 \exp(-[E_a/(K_B*(T_m - T_f))])$) [2,3]. These equations give information about the activation energy for relaxation and long range inter-actions and dipolar freezing temperature. It would be better if one can describe the temperature dependence of dielectric constant at different constant frequencies directly. This has prompted us to propose a modified Lorentz equation to describe the temperature dependence of dielectric constant of normal [4] and relaxor [5,6] ferroelectrics in our previous studies. The proposed modified Lorentz equation describes the dielectric constant as a function of temperature only and not with frequency. Hence, in the present study, we have re-modified the Lorentz equation to include the term $((\omega\tau)^2 + 1)$ along with T (as shown in equation (3)) as described in results and discussion section and were successful in fitting the experimental dielectric data with this new equation. By doing so, we were successful in directly obtaining the activation energy and relaxation time simultaneously.

In piezoelectric materials, estimation of piezoelectric charge constant and piezoelectric voltage coefficient is generally carried out using sophisticated and costly equipment, for example, like d_{33} meter. These studies give information

about how effectively a piezoelectric material produces strain with the applied electric field [7–10]. This information is useful in designing piezoelectric transducers and actuators. It is also known that piezoelectric materials usually exhibit electromechanical coupling and these are estimated experimentally from the resonance and anti-resonance studies using an impedance analyser. From the resonance and anti-resonance frequency values and capacitance at 1 kHz, the piezoelectric charge constant is also estimated using the standard piezoelectric formula [11], but this formula is arbitrary, even though piezoelectric standards are followed. In view of the above, in our previous studies, we have proposed a modified Lorentz equation to simulate the dielectric resonance and anti-resonance data to obtain the polarizability, piezoelectric charge constant and piezoelectric voltage constant and were successful in fitting the experimental data using the modified Lorentz equation very well and obtained the above parameters indirectly from the fitting [12–14]. But, this modified Lorentz equation describes the variation of dielectric constant with frequency only, but not with temperature. Hence, in the present study, the Lorentz equation was re-modified to describe the variation of dielectric constant with frequency and temperature simultaneously and was successful in fitting the experimental resonance and anti-resonance data. Inclusion of T in equation (6) has made it possible to obtain the activation energy for resonance and anti-resonance.

In the present study, new ceramic composition $[Ba(Nd_x Ti_{1-2x}Nb_x)O_3]_{0.30}[Na_{0.5}Bi_{0.5}TiO_3]_{0.70}$ ($x = 0.075$) is

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chosen for dielectric investigation. The experimental dielectric resonance and anti-resonance data of another $[Ba(Nd_{0.1}Ti_{0.8}Nb_{0.1})O_3]_{0.35}[(Na_{0.5}Bi_{0.5})TiO_3]_{0.65}$ piezoelectric ceramic was used for electromechanical coupling studies [15]. The results on the modelling of dielectric and resonance and anti-resonance data as a function of temperature and angular frequency using equations (3 and 6), respectively, are presented and discussed in this paper.

2. Experimental

A new $[Ba(Nd_xTi_{1-2x}Nb_x)O_3]_{0.30}[Na_{0.5}Bi_{0.5}TiO_3]_{0.70}$ dielectric relaxor ceramic with $x = 0.075$ was prepared through a solid state sintering route. The calcination of powder and final sintering temperature of prepared pellets are 900 (5 h) and 1175°C (2 h), respectively. The dielectric measurements as a function of temperature at different constant frequencies were carried out using a HP4192A impedance analyser on the above dielectric relaxor ceramic.

For electromechanical studies; capacitance resonance and anti-resonance studies at different constant temperatures were also carried out using the HP4192A impedance analyser on another piezoelectric compound: $Ba(Nd_{0.1}Ti_{0.8}Nb_{0.1})O_3]_{0.35}$ $[(Na_{0.5}Bi_{0.5})TiO_3]_{0.65}$ which is under investigation. Preparation of $[Ba(Nd_{0.1}Ti_{0.8}Nb_{0.1})O_3]_{0.35}[(Na_{0.5}Bi_{0.5}) TiO_3]_{0.65}$ piezoelectric ceramic, its X-ray, dielectric and ferroelectric data were reported earlier [16].

3. Results and discussion

Figure 1a and b shows the variation of dielectric constant with temperature at different constant frequencies for $[Ba(Nd_xTi_{1-2x}Nb_x)O_3]_{0.30}[Na_{0.5}Bi_{0.5}TiO_3]_{0.70}$ ceramic with $(x = 0.075)$. Dielectric relaxor behaviour is generally characterized by dispersion of dielectric constant with a frequency below T_{max} and shifting of T_{max} towards a higher temperature side with the increase in frequency. It can be seen from the figures that the present ceramic material exhibits dielectric relaxor behaviour. Below T_{max}, the dielectric constant decreased with the increase in frequency in the range of 10 kHz–1 MHz; whereas the dielectric constant increased with the increase in frequency in the range of 1.5–3.5 MHz. Different dielectric parameters obtained from the dielectric data are tabulated in table 1. The dielectric loss increased with the increase in frequency.

The Lorentz equation (equation (1)) is generally used to characterize the dielectric data as shown in figure 2a and b. The obtained parameters are tabulated in table 2. This
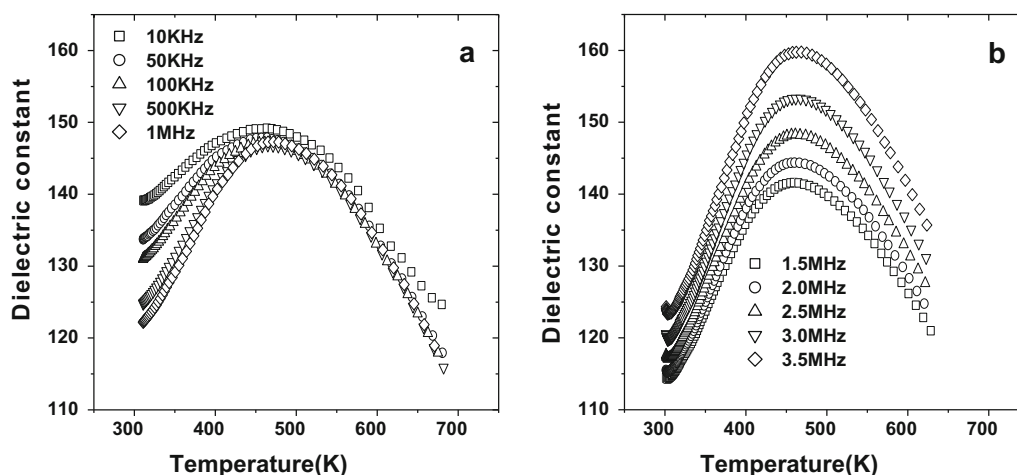

Figure 1. (**a** and **b**) Dielectric constant *vs*. temperature at different constant frequencies for $[Ba(Nd_xTi_{1-2x}Nb_x)O_3]_{0.30}[Na_{0.5}Bi_{0.5}TiO_3]_{0.70}$ with $x = 0.075$.

Table 1. Parameters obtained from dielectric data for $[Ba(Nd_{0.075}Ti_{0.85}Nb_{0.075})O_3]_{0.30}[(Na_{0.5}Bi_{0.5})TiO_3]_{0.70}$ ceramic.

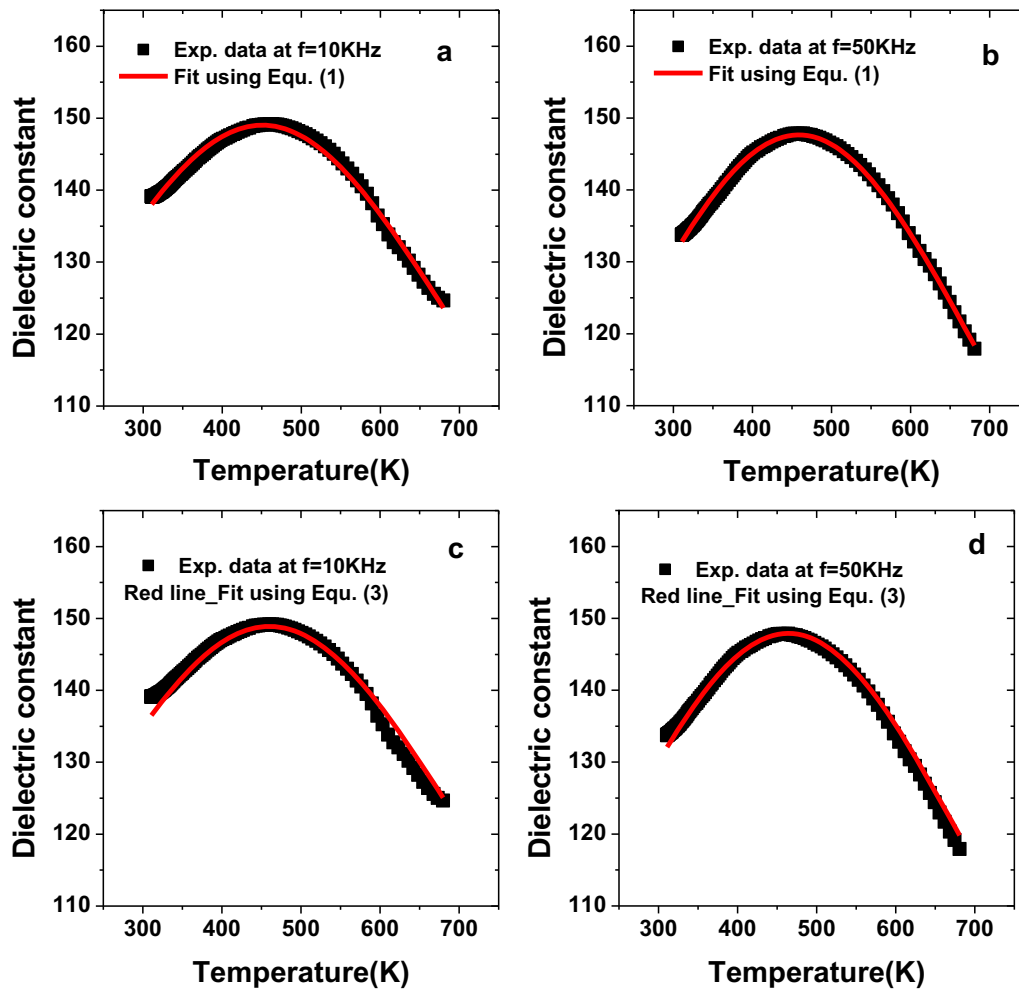| Parameter | Frequency | | | | |
	$f = 10\,kHz$	$f = 50\,kHz$	$f = 100\,kHz$	$f = 500\,kHz$	$f = 1\,MHz$
ε'_{RT}	139.15	133.74	130.98	124.78	122.18
T_{max}	460.00	464.10	464.66	464.92	471.76
$\tan(\delta)$	0.033	0.039	0.042	0.044	0.040
ε'_{max}	149.17	147.84	147.19	146.80	147.31

Figure 2. (**a** and **b**) Dielectric constant *vs.* temperature at different constant frequencies (experimental and Lorentz equation fit (equation (1)) and (**c** and **d**) dielectric constant *vs.* temperature at different constant frequencies (experimental and fitting using equation (3)) for $[Ba(Nd_xTi_{1-2x}Nb_x)O_3]_{0.30}[Na_{0.5}Bi_{0.5}TiO_3]_{0.70}$ with $x = 0.075$.

Table 2. Parameters obtained from Lorentz equation (equation (1)) fitting to experimental dielectric data for $[Ba(Nd_{0.075}Ti_{0.85}Nb_{0.075})O_3]_{0.30}[(Na_{0.5}Bi_{0.5})TiO_3]_{0.70}$ ceramic.

	Frequency				
Parameters	$f = 10\,\mathrm{kHz}$	$f = 50\,\mathrm{kHz}$	$f = 100\,\mathrm{kHz}$	$f = 500\,\mathrm{kHz}$	$f = 1\,\mathrm{MHz}$
y_0	36.03	22.36	47.89	67.54	69.59
x_c(K)	450.60	458.38	463.11	477.08	484.03
w (K)	850.85	804.89	671.29	524.83	491.68
A (K)	151026.10	158407.72	104617.17	65444.61	60222.84
R^2	0.98992	0.99644	0.99698	0.99748	0.99680

equation does not give much information except for area under, the curve, width and peak positions. Hence, in our earlier studies, we proposed a modified Lorentz equation (equation (2)) [4–6] and were successful in fitting the dielectric data. Here, ε_s is the static dielectric constant, ε_∞ is the infinite dielectric constant and E_{act} is the activation energy for thermally activated orientation of dipoles. But, this equation describes the variation of dielectric constant with temperature only. It is desirable to propose a new equation which describes the temperature dependence of dielectric constant with frequency also. Hence, in the present case, the term $((\omega\tau)^2 + 1)$ is incorporated into equation (2) and is shown in

Table 3. Parameters obtained from the modified Lorentz equation (equation (3)) fitting to experimental dielectric data for [Ba (Nd$_{0.075}$ Ti$_{0.85}$Nb$_{0.075}$)O$_3$]$_{0.30}$[(Na$_{0.5}$Bi$_{0.5}$)TiO$_3$]$_{0.70}$ ceramic.

Parameters	Frequency				
	$f = 10\,\text{kHz}$	$f = 50\,\text{kHz}$	$f = 100\,\text{kHz}$	$f = 500\,\text{kHz}$	$f = 1\,\text{MHz}$
ε_s	165.04	178.82	212.17	198.20	190.11
ε_∞	129.20	156.28	164.49	130.71	120.96
E_{act} (meV)	25.25	38.38	96.21	200.74	132.70
ω (Hz)	62857.14	314285.71	628571.42	3142857.14	6285714.28
τ (ms)	4.66	1.41	1.77	0.76	0.25
A (K)	151026	158407.72	104617.17	65444.61	60222.84
w (K)	850.85	804.89	671.29	524.83	491.68
R^2	0.95298	0.98468	0.99543	0.95033	0.95625

equation (3). This modification allows us to describe the temperature dependence of dielectric constant at different frequencies and also helps us to estimate the activation energy for thermally activated orientation of dipoles and relaxation times simultaneously.

$$y = y_0 + \frac{2A}{\pi} \frac{w}{4(x - x_c)^2 + w^2}, \tag{1}$$

$$\varepsilon' = (\varepsilon_s - \varepsilon_\infty) + \frac{2A}{\pi} \frac{w}{4((T - T_c)K_B/E_a)^2 + w^2}, \tag{2}$$

$$\varepsilon' = (\varepsilon_s - \varepsilon_\infty) + \frac{2A}{\pi} \frac{w}{4((T - T_c)K_B/E_a)^2 * ((\omega * \tau)^2 + 1) + w^2}. \tag{3}$$

Figure 2c and d shows the temperature-dependent dielectric plots at different frequencies and the fitted curves using equation (3). Equation (3) is found to be best fit to the experimental dielectric data. The regression values for the fitting and other parameters obtained are tabulated in table 3. It can be seen from table 3 that the activation energy for dipolar relaxation is 25.25 meV for dielectric data at $f = 10$ kHz and with the increase in frequency, the activation energy is increased showing a value of 132.70 meV at $f = 1$ MHz. It can also be seen from table 3 that relaxation time for dielectric data at $f = 10$ kHz is 4.66 ms and with the increase in frequency the relaxation time decreases and the value of relaxation time is 0.25 ms at $f = 1$ MHz.

Figure 3a shows the variation of dielectric constant as a function of angular frequency at different constant temperatures studied under clamped state (in jig with unknown, but constant spring constant) for [Ba(Nd$_{0.1}$Ti$_{0.8}$Nb$_{0.1}$)O$_3$]$_{0.35}$ [(Na$_{0.5}$Bi$_{0.5}$)TiO$_3$]$_{0.65}$ ceramic. It can be seen from this curve that broad resonance and anti-resonance behaviour was observed. It can also be seen from figure 3b that resonance frequency decreased with the increase in temperature up to 398 K; there after it increases slightly with the increase in temperature; whereas anti-resonance frequency decreased

with the increase in temperature. From the resonance and anti-resonance frequencies, the thickness mode of electromechanical coupling factor (K_t) was estimated using the standard formula on piezoelectricity [11] and the values are tabulated in table 4. Figure 3c shows that the value of K_t decreases with the increase in temperature from 323 to 498 K. The high values of electromechanical coupling factor may be due to the stress-induced stretched relaxations. The decrease of electromechanical coupling factor with the increase in temperature may be ascribed to the decrease of stress in the ceramic sample, which in turn is due to clamping of ceramic sample in jig.

The Lorentz equation for two peaks (equation (4)) is generally used to characterize the dielectric resonance and anti-resonance data as shown in figure 4a and b. The obtained parameters are tabulated in table 5. This equation does not give much information except for area under the curve, width and peak positions. Hence, in our earlier studies, we have proposed modified Lorentz equation (equation (5)) [12–14] and were successful in fitting the dielectric resonance and anti-resonance data. Here, ε_s is the static dielectric constant, ε_∞ is the infinite dielectric constant, N_A is the Avogadro number, ρ is the experimental density, α is the polarizability, M is the molecular weight, d_{33} is the piezoelectric charge constant, g_{33} is the piezoelectric voltage constant, ω is the angular frequency, ω_{c1} is the angular resonance frequency, ω_{c2} is the angular anti-resonance frequency and τ, τ_{c1} and τ_{c2} are the relaxation times. By fitting equation (5) to the experimental data, it is possible to obtain the polarizability, d_{33} and g_{33} indirectly. However, equation (5) is a function of ω only and does not incorporate the temperature. Since the dielectric constant is frequency- and temperature-dependent, it is better to modify equation (5) to describe the dielectric dependence on ω and T simultaneously. Hence, in the present study, the terms $((T * K_B/E_{\text{act\_R}})^2) + 1)$ and $((T * K_B/E_{\text{act\_AR}})^2) + 1)$ are incorporated into equation (5) and is shown as equation (6). The incorporation of these terms in equation (5) allow us to determine the activation energy for resonance ($E_{\text{act\_R}}$) and activation energy for anti-resonance ($E_{\text{act\_AR}}$) along with the previously obtained parameters like d_{33} and g_{33}.

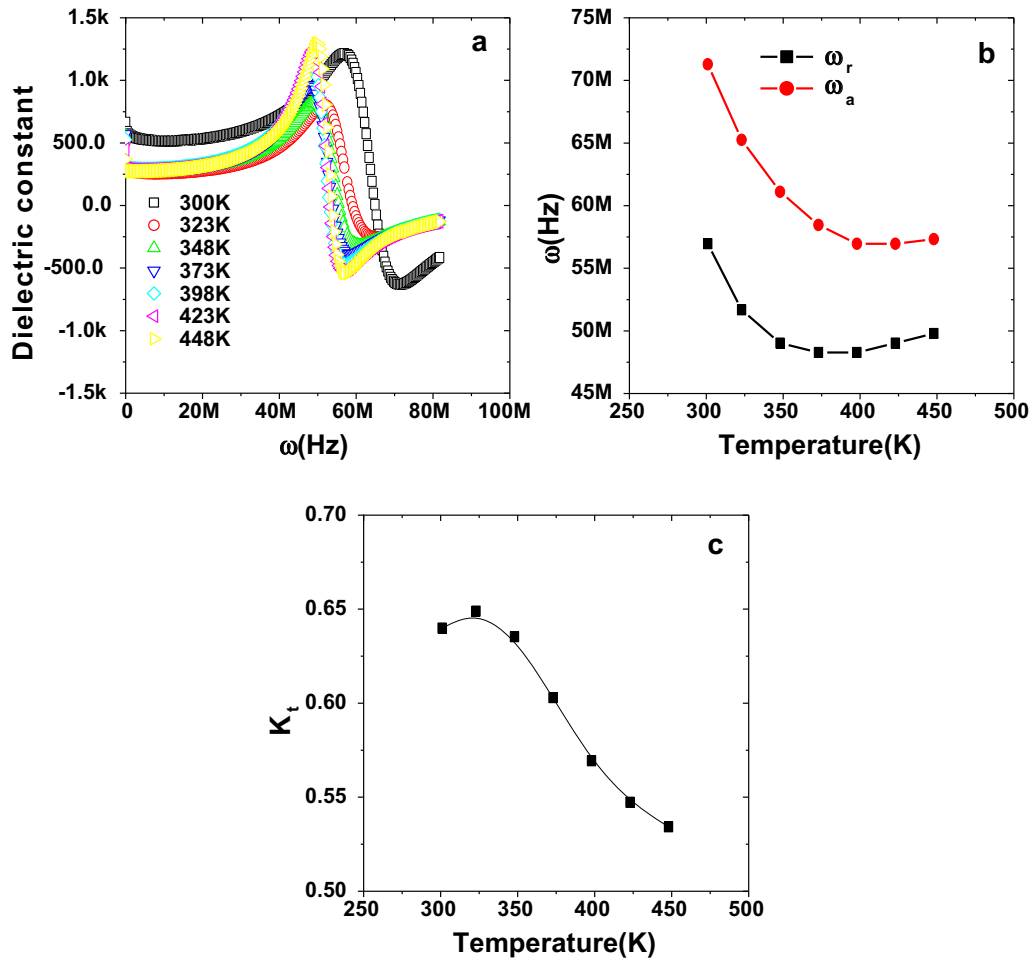**Figure 3.** (**a**) Dielectric constant *vs.* angular frequency at different constant temperatures plots, (**b**) resonance and anti-resonance angular frequency *vs.* temperature plot and (**c**) thickness mode of electromechanical coupling factor *vs.* temperature plot for $[Ba(Nd_{0.1}Ti_{0.8}Nb_{0.1})O_3]_{0.35}[(Na_{0.5}Bi_{0.5})TiO_3]_{0.65}$ ceramic.

Table 4. Resonance and anti-resonance angular frequency and electromechanical coupling factor values for $[Ba(Nd_{0.1}Ti_{0.8}Nb_{0.1})O_3]_{0.35}$ $[(Na_{0.5}Bi_{0.5})TiO_3]_{0.65}$ ceramic in clamped state.

Parameters	Temperature (K)						
	300	323	348	373	398	423	448
ω_r (Hz)	5.69×10^7	5.16×10^7	4.90×10^7	4.82×10^7	4.82×10^7	4.90×10^7	4.97×10^7
ω_a (Hz)	7.12×10^7	6.52×10^7	6.11×10^7	5.84×10^7	5.69×10^7	5.69×10^7	5.73×10^7
K_t	0.63	0.64	0.63	0.60	0.56	0.54	0.53

$$y = y_0 + \frac{2A_1}{\pi} \frac{w_1}{4(x - x_{c1})^2 + w_1^2} + \frac{2A_2}{\pi} \frac{w_2}{4(x - x_{c2})^2 + w_2^2}, \tag{4}$$

$$\varepsilon' = (\varepsilon_s - \varepsilon_\infty) + \frac{N_A \rho \alpha}{3\varepsilon_0 M} + \frac{d_{33}}{g_{33}} + \frac{2A_1}{\pi} \frac{w_1}{4(\tau\omega - \tau_{c1}\omega_{c_1})^2 + w_1^2} + \frac{2A_2}{\pi} \frac{w_2}{4(\tau\omega - \tau_{c2}\omega_{c_2})^2 + w_2^2}, \tag{5}$$

$$\varepsilon' = (\varepsilon_s - \varepsilon_\infty) + \frac{N_A \rho \alpha}{3\varepsilon_0 M} + \frac{d_{33}}{g_{33}}$$
$$+ \frac{2A_1}{\pi} \frac{w_1}{4(\tau\omega - \tau_{c1}\omega_{c_1})^2 * ((T * K_B/E_{act\_R})^2 + 1) + w_1^2}$$
$$+ \frac{2A_2}{\pi} \frac{w_2}{4(\tau\omega - \tau_{c2}\omega_{c_2})^2 * ((T * K_B/E_{act\_AR})^2 + 1) + w_2^2}. \tag{6}$$

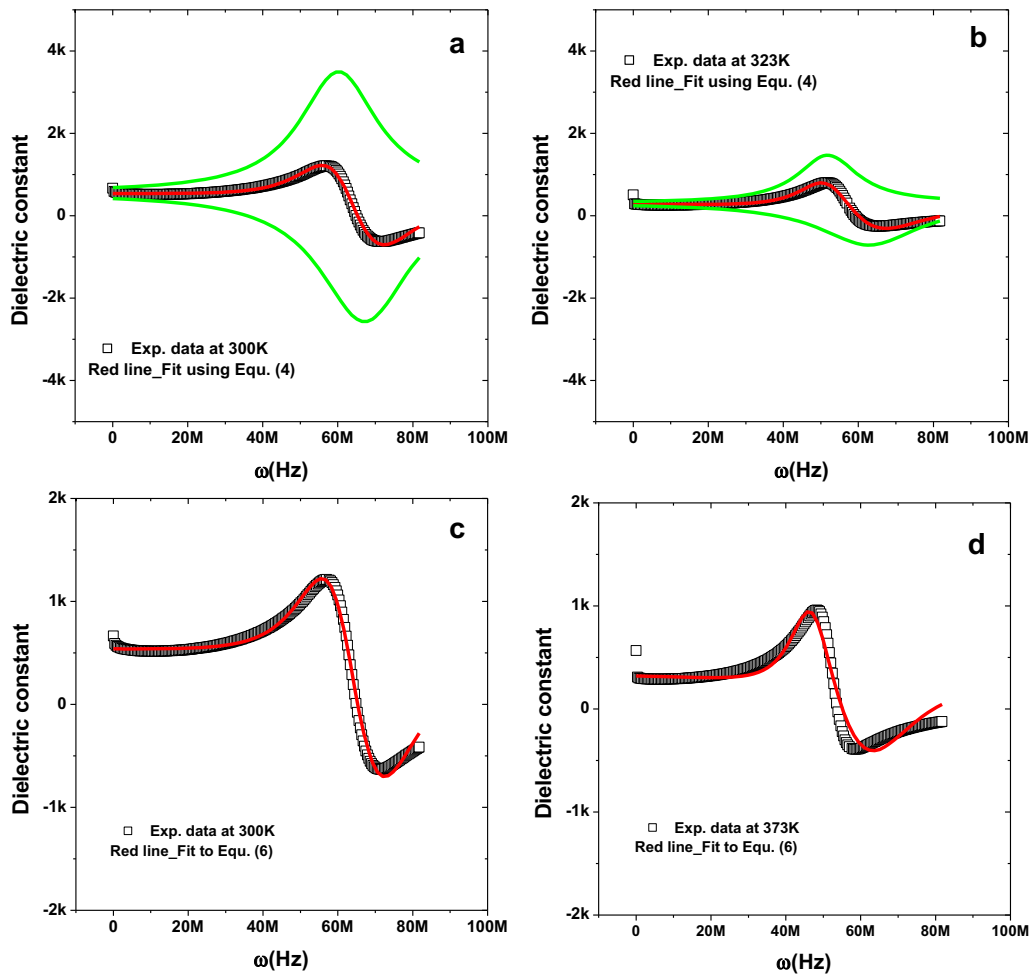**Figure 4.** (**a** and **b**) Dielectric constant *vs.* angular frequency at different constant temperatures (experimental and Lorentz equation fit using equation (4)) and (**c** and **d**) dielectric constant *vs.* angular frequency at different constant temperatures (experimental and modified Lorentz equation fit using equation (6)) for $[Ba(Nd_{0.1}Ti_{0.8}Nb_{0.1})O_3]_{0.35}[(Na_{0.5}Bi_{0.5})TiO_3]_{0.65}$ ceramic.

Table 5. Parameters obtained from Lorentz equation (equation (4)) fitting to experimental data of $[Ba(Nd_{0.1}Ti_{0.8}Nb_{0.1})O_3]_{0.35}[(Na_{0.5}Bi_{0.5})TiO_3]_{0.65}$ ceramic in clamped state.

Parameter	Temperature (K)						
	300	323	348	373	398	423	448
y_0	558.85	307.64	331.42	354.81	334.20	310.99	298.53
x_{c1} (Hz)	6.02×10^7	5.18×10^7	4.82×10^7	4.71×10^7	4.80×10^7	4.87×10^7	4.97×10^7
w_1 (Hz)	2.51×10^7	1.96×10^7	1.51×10^7	1.40×10^7	1.69×10^7	1.61×10^7	1.55×10^7
A_1 (Hz)	1.1×10^{11}	3.5×10^{10}	2.2×10^{10}	2.5×10^{10}	5.2×10^{10}	5.7×10^{10}	5.7×10^{10}
x_{c2} (Hz)	6.71×10^7	6.28×10^7	6.28×10^7	6.06×10^7	5.57×10^7	5.54×10^7	5.59×10^7
w_2 (Hz)	2.92×10^7	3.27×10^7	3.06×10^7	3.02×10^7	2.69×10^7	2.39×10^7	2.26×10^7
$-A_2$ (Hz)	1.4×10^{11}	5.2×10^{10}	4.2×10^{10}	4.7×10^{10}	7.0×10^{10}	7.2×10^{10}	7.1×10^{10}
R^2	0.99277	0.97804	0.96014	0.94758	0.94184	0.93576	0.93346

Figure 4c and d shows the experimental and simulated data using equation (6). Equation (6) is found to be best fit to the experimental data as evidenced by regression value as shown in table 6. The other parameters obtained from the simulation are tabulated in table 6. Figure 5a shows the variation of static and infinity dielectric constant
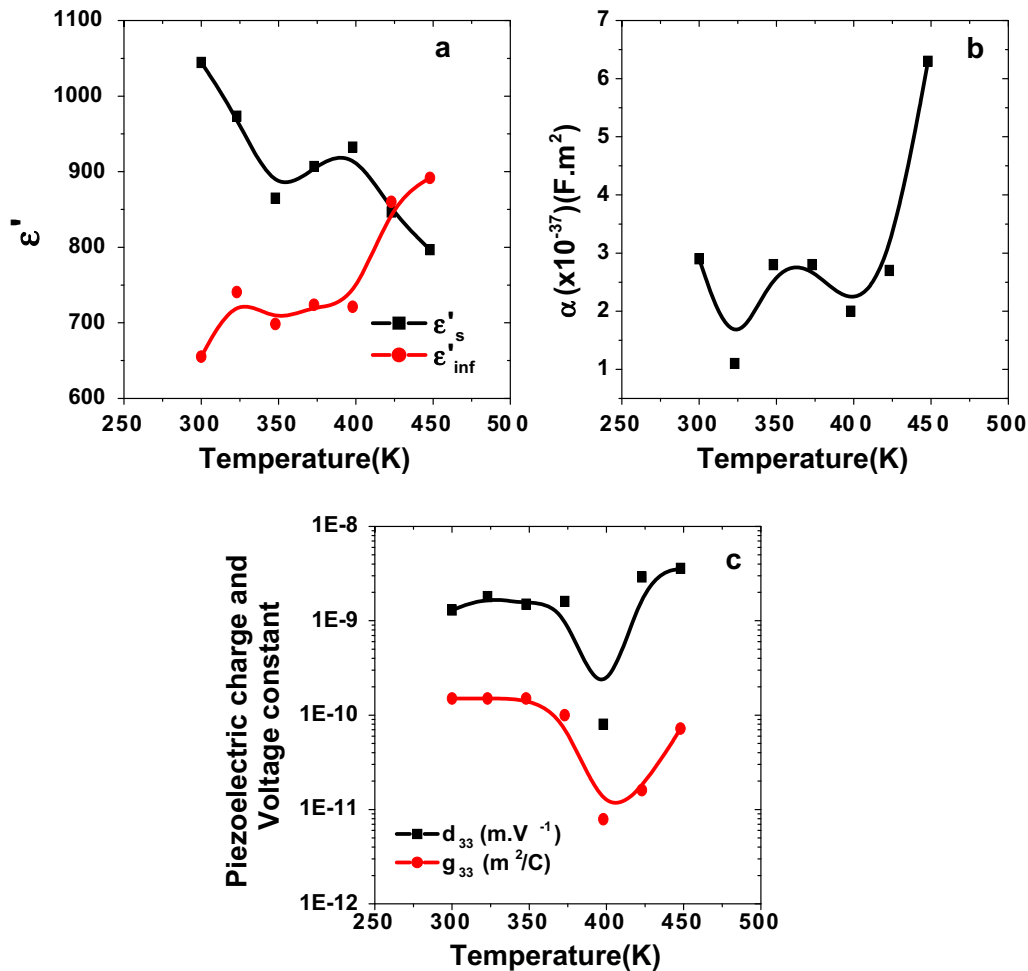
Figure 5. (**a**) Static and infinity dielectric constants *vs.* temperatures plots, (**b**) polarizability *vs.* temperature plot and (**c**) piezoelectric charge and voltage constant *vs.* temperature plot for $[Ba(Nd_{0.1}Ti_{0.8}Nb_{0.1})O_3]_{0.35}[(Na_{0.5}Bi_{0.5})TiO_3]_{0.65}$ ceramic.

as a function of temperature. At lower temperatures, the static dielectric constant is higher than the infinity dielectric constant indicating frequency-dependence relaxation behaviour. At higher temperatures, both the static and infinity dielectric constants approach each other indicating disappearance of space charge polarization. Figure 5b shows the temperature dependence of polarizability. There is no systematic variation with temperature. The change in slopes in polarizability *vs.* temperature plot is due to the contribution of different polarizable species present in the ceramic sample which gets activated at different temperature regions with different activation energies. Figure 5c shows the variation in d_{33} and g_{33} with temperature. The d_{33} values at 300 and 448 K are 1.3×10^{-9} and 3.9×10^{-9} m V^{-1}, respectively. The d_{33} value varies a little with temperature from 300 to 373 K and thereafter it shows a dip and with further increase in temperature, it increases. Logarithm of d_{33} is plotted as a function of $1000/T$

(figure is not shown here). From the two slopes, the activation energy is estimated by a linear fit to the data. The activation energy is found to be 0.20 and 1.19 eV in the temperature range of 300–373 and 398–448 K, respectively. The activation energy obtained from the $\log(d_{33})$ *vs.* $1000/T$ plot may be ascribed to the switching of polarization in different directionally polarized domain regions separated by domain walls. It is known that 1800 domains switching requires higher activation energy when compared to 900 domain switching. The activation energy of 1.19 eV in the temperature region 398–448 K may be ascribed to the switching of domains separated by domain wall having polarization in the opposite direction i.e., 1800 to be in unidirectional. Table 6 shows the other useful parameters obtained from the modelling of dielectric resonance and anti-resonance data using equation (6). The activation energy for resonance and anti-resonance at 300 K is estimated to be 0.03 and 0.03 eV, respectively.

Table 6. Parameters obtained from modified Lorentz equation (equation 6) fitting to experimental data of $[Ba(Nd_{0.1}Ti_{0.8}Nb_{0.1})O_3]_{0.35}$ $[(Na_{0.5}Bi_{0.5})TiO_3]_{0.65}$ ceramic in clamped state.

Parameter	Temperature (K)						
	300	323	348	373	398	423	448
ε_s	1044.40	973.46	864.91	906.97	932.51	846.48	796.66
ε_∞	655.28	740.52	698.53	723.90	721.21	860.04	891.81
α (F m^2)	2.9×10^{-37}	1.1×10^{-37}	2.8×10^{-37}	2.8×10^{-37}	2.0×10^{-37}	2.7×10^{-37}	6.3×10^{-37}
d_{33} (m V^{-1})	1.3×10^{-9}	1.8×10^{-9}	1.5×10^{-9}	1.6×10^{-9}	8.0×10^{-11}	2.9×10^{-9}	3.6×10^{-9}
g_{33} (m^2 C^{-1})	1.5×10^{-10}	1.5×10^{-10}	1.5×10^{-10}	1.0×10^{-10}	7.9×10^{-12}	1.6×10^{-11}	7.2×10^{-11}
τ (s)	0.75	0.65	0.74	0.77	0.75	0.42	0.59
τ_{c1} (s)	0.79	0.66	0.73	0.75	0.75	0.42	0.59
τ_{c2} (s)	0.71	0.63	0.76	0.80	0.73	0.41	0.57
E_{act_R} (eV)	0.03	0.02	0.04	0.04	0.04	0.02	0.03
E_{act_AR} (eV)	0.03	0.02	0.04	0.04	0.04	0.02	0.03
A_1 (Hz)	1.1×10^{11}	3.5×10^{10}	2.2×10^{10}	2.5×10^{10}	5.2×10^{10}	5.7×10^{10}	5.7×10^{10}
w_1 (Hz)	2.51×10^7	1.96×10^7	1.51×10^7	1.40×10^7	1.69×10^7	1.61×10^7	1.55×10^7
$-A_2$ (Hz)	1.4×10^{11}	5.2×10^{10}	4.2×10^{10}	4.7×10^{10}	7.0×10^{10}	7.2×10^{10}	7.1×10^{10}
w_2 (Hz)	2.92×10^7	3.27×10^7	3.06×10^7	3.02×10^7	2.69×10^7	2.39×10^7	2.26×10^7
ω_{c1} (Hz)	5.69×10^7	5.16×10^7	4.90×10^7	4.82×10^7	4.82×10^7	4.90×10^7	4.97×10^7
ω_{c2} (Hz)	7.12×10^7	6.52×10^7	6.11×10^7	5.84×10^7	5.69×10^7	5.69×10^7	5.73×10^7
R^2	0.99271	0.97807	0.96099	0.94837	0.94198	0.93535	0.93302

4. Conclusions

$[Ba(Nd_xTi_{1-2x}Nb_x)O_3]_{0.30}[Na_{0.5}Bi_{0.5}TiO_3]_{0.70}$ with ($x = 0.075$) piezoelectric ceramic was prepared through a solid-state sintering route. Dielectric relaxor behaviour was observed in the temperature dependence of dielectric plots. The new modified Lorentz equation (equation (3)) as proposed by us, was used to describe the dielectric data with temperature and frequency simultaneously to obtain activation energy for thermally activated orientation of dipoles and relaxation times. The activation energy for dipolar relaxation is 25.25 meV for dielectric data at $f = 10$ KHz and with the increase in frequency, the activation energy is increased showing a value of 132.70 meV at $f = 1$ MHz. A new modified Lorentz equation (equation (6)) is proposed to describe/model the dielectric resonance and anti-resonance data as a function of temperature and angular frequency simultaneously and this equation is found to be best fit to the experimental data of $[Ba(Nd_{0.1}Ti_{0.8}Nb_{0.1})O_3]_{0.35}[(Na_{0.5}Bi_{0.5})TiO_3]_{0.65}$ piezoelectric ceramic. Using this equation, it is shown that it is possible to obtain polarizability, piezoelectric charge constant, piezoelectric voltage constant and activation energy for resonance and anti-resonance. From the above studies, it is concluded that equation (6) proposed by us may also be used to model the dielectric resonance and anti-resonance data with T and ω for other piezoelectric materials and this equation will be helpful in obtaining useful parameters which in turn would be helpful in designing a piezoelectric transducer.

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