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Application of Chebyshev collocation method for relocating of spacecrafts in Hill's frame

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Abstract. In this study, the Chebyshev collocation method is used for solving the spacecraft relative motion of equations in Hill's frame. Three different models of governing equations of relative motion (M1, M2, and M3) are considered and the maneuver cost required moving the spacecraft from one state to another is computed in the form of delta velocity at the first terminal point as a function of time of flight (TOF) and inter-satellite distance (ISD). A quantitative as well as qualitative difference is observed in the maneuver cost with the inclusion of radial and/or out of plane separation in along track separation of chaser. Also, a relative comparison of path profiles is made by considering M1, M2 and M3 models. Path profiles for M3 model are found close to M2 model for short intervals for a fixed ISD, whereas path profiles for M2 and M3 do not match even for small values of ISD for a fixed but long TOF. Path profiles for M1 models match to M2 model for very low values of target orbit eccentricities.

Keywords. Docking—TPBVP—Chebyshev collocation method—CW equations—relative motion.

1. Introduction

Spacecrafts docking consists of combining two or more spacecraft in space. This area has a lot of applications like to transport human to international space station, for maintenance of existing satellites or to add few more payload or to replace a subsystem in an existing satellite. Satellite docking requires technology drawn from various research fields such as relative orbit determination, position reconfiguration, relative attitude determination, relative attitude control, etc. Among these technologies, the present work focuses on position reconfiguration which is to relocate satellites into the desired relative position between the satellites. The reconfiguration/rendezvous of satellites can be achieved by optimizing the thrust accelerations required or by using an impulsive maneuver strategy.

The relative motion of the moon with respect to the Sun–Earth system was first studied by Hill (1878). Based on Hill's lunar theory, relative motion of spacecraft in Hill's frame was studied by Clohessy and Wiltshire (1960) and consequently extended by Tschauner and Hempel (1965). Lawden (1963) found an improved form for relative motion by including reference orbit eccentricity, and Carter (1990) later extended Lawden's solution. All these researchers mainly focused on the modeling of the relative dynamics in Hill's frame.

The two-point boundary value problem (TPBVP) of relative motion, where the initial and final relative positions, times and the orbit of the target satellite are known, and the orbit of the chaser is going to be determined, has not yet been sufficiently studied, although it plays a key role in formation reconfiguration and orbit transfer of spacecrafts.

Deriving from the non-periodic solution of the homogeneous Clohessy-Wiltshire equations, Mullins (1992) obtained a set of solutions to TPBVP in circular reference orbits. Carter (1998) and Yamanaka and Ankersen (2002) obtained a state transition matrix for propagating initial value problems involving the differential equations of relative motion. Hamiltonian canonical transformation and generating function are used by Guibout and Scheeres (2004) for solving the two-point boundary value problems and consequently by Park *et al.* (2006). Their method has few merits like fast convergence; having explicit physical significance; and does not need guess of initial value of iteration as compared to the classical methods (shooting algorithm, finite differences). A high-order expansion method was proposed by Lizia *et al.* (2008) for solving two-point boundary value problems in astrodynamics. A TPBVP of spacecraft formation flying is addressed by Jiang *et al.* (2009) by considering an unperturbed elliptical reference orbit. They reformed the problem into a relative Lambert's problem and pointed out that it can be solved similarly to the classical Lambert problem. However, significant perturbations, such as the J_2 term, were not considered in their work. Chen and Dai (2011) presented a canonical method to solve the TPBVPs of Hamiltonian systems with a primary integrable component. Recently, few researchers Cheng *et al.* (2017a; 2017b) used the Chebyshev collocation method for solution of astrodynamics problems.

Dynamics of rendezvous of two spacecraft is also addressed by some researchers. Among them, Campbell (2003) considered a reconfiguration problem between two periodic solutions and formulated a minimum time problem as well as a minimum fuel problem, in which the time interval is finite and the cost function is the absolute integral of the thrust. This work is further extended by Zanon and Campbell (2006). Vaddi *et al.* (2005) considered formation and reconfiguration problems using orbit element differences and obtained a two-impulse solution. Some amount of work is done in the direction of optimal methods for satellite formations re-configurations. Like in Zhang *et al.* (2011), mixed integer programming is used for trajectory planning with constraint avoidance, and more recently, in Kim and Spencer (2002) and Ichimura and Ichikawa (2008), studies of optimal multi-objective impulsive rendezvous are reported.

In this work, author developed a mathematical tool to design the maneuver strategy for relocating satellites in Hill's frame under three different assumptions. For this Newton–Kantorovich/Chebyshev pseudospectral (NK/CPS) approach is used which is introduced by Boyd (2000). Traditionally, pseudospectral methods are extensively used in fluid dynamics (Kumar *et al.* 2011; Bhowmik *et al.* 2015). The objective of the work is to design the maneuver strategy so that the cost can be minimized by choosing a proper time of flight with mission constraints.

2. Mathematical formulation

Here relative motion of Chaser spacecraft with respect to target spacecraft is considered without any perturbation like J_2, drag force, solar radiation pressure and third body perturbations etc. Three different existing models are chosen. A detailed description is provided about all models follows:

Model-1 (M1):

In this case, a generalized model for relative motion of two satellite is considered under the following constraints taken into consideration: (i) both satellite are moving in elliptical orbits, (ii) satellites can have any relative separation.

The governing equations are

$$\ddot{x} - 2\dot{\theta}_0\dot{y} - \ddot{\theta}_0 y - \dot{\theta}_0^2 x = -\frac{\mu\,(r_0 + x)}{\left[(r_0+x)^2 + y^2 + z^2\right]^{\frac{3}{2}}} + \frac{\mu}{r_0^2} \tag{1}$$

$$\ddot{y} + 2\dot{\theta}_0\dot{x} + \ddot{\theta}_0 x - \dot{\theta}_0^2 y = -\frac{\mu y}{\left[(r_0+x)^2 + y^2 + z^2\right]^{\frac{3}{2}}} \tag{2}$$

$$\ddot{z} = -\frac{\mu z}{\left[(r_0+x)^2 + y^2 + z^2\right]^{\frac{3}{2}}} \tag{3}$$

With

$$\ddot{r}_0 = r_0\dot{\theta}_0^2 - \frac{\mu}{r_0^2} \tag{4}$$

$$\ddot{\theta}_0 = -\frac{2\dot{r}_0\dot{\theta}_0}{r_0} \tag{5}$$

Model-2 (M2):

In this case, a model for relative motion of two satellite is considered under the following constraints taken into consideration: (i) target satellite has a circular orbit and chaser may have circular or elliptical orbit, (ii) satellites can have any relative separation.

Under these assumptions, the governing equations are

$$\ddot{x} - 2n_0\dot{y} - n_0^2 x = -\frac{\mu\,(a_0 + x)}{\left[(a_0+x)^2 + y^2 + z^2\right]^{\frac{3}{2}}} + \frac{\mu}{a_0^2} \tag{6}$$

$$\ddot{y} + 2n_0\dot{x} - n_0^2 y = -\frac{\mu y}{\left[(a_0+x)^2 + y^2 + z^2\right]^{\frac{3}{2}}} \tag{7}$$

$$\ddot{z} = -\frac{\mu z}{\left[(a_0+x)^2 + y^2 + z^2\right]^{\frac{3}{2}}} \tag{8}$$

With

$$n_0 = \sqrt{\frac{\mu}{a^3}} \tag{9}$$

Model-3 (M3):

In this case, a model for relative motion of two satellite is considered under the following constraints taken into consideration: (i) target satellite has a circular orbit and chaser may have circular or elliptical orbit, (ii) satellites are very close to each other.

The governing equations are

$$\ddot{x} - 2n_0\dot{y} - 3n_0^2 x = 0 \tag{10}$$
$$\ddot{y} + 2n_0\dot{x} = 0 \tag{11}$$
$$\ddot{z} + n_0^2 z = 0 \tag{12}$$

With

$$n_0 = \sqrt{\frac{\mu}{a^3}} \tag{13}$$

2.1 *Newton–Kantorovich/Chebyshev pseudospectral method*

In the NK/CPS method, nonlinear differential equations are linearized about the nominal solution and the successively improved solutions. These linearized differential equations are solved using Chebyshev pseudospectral collocation with entirely linear operations.

2.1.1 Chebyshev pseudospectral method In the Chebyshev pseudospectral method, a function is approximated using Lagrange interpolating polynomials, and the interpolation coefficients are the values of the function at the node points. Let $x(t) = [x_1(t), x_2(t), ..., x_p(t)]^T$ denote the target function vector so that the polynomial approximation is in the following form:

$$x_n(t) = \sum_{\mathrm{j=0}}^{\mathrm{N}} x_j \phi_j(t) \tag{14}$$

$$\phi_j(t) = \prod_{\substack{\mathrm{m=0}\\ \mathrm{m\neq j}}}^{\mathrm{N}} \frac{(t - t_m)}{(t_j - t_m)}, \quad \mathrm{j = 0(1)N} \tag{15}$$

where $x^N(t)$ denotes the Nth-order polynomial approximation of $x(t)$, x_j are the interpolation coefficient vectors, $\phi_j(t)$ are the Lagrange interpolating polynomials of order N, and t_m and t_j are the CGL interpolation points, which are given in closed form:

$$\mathrm{t_k} = -\cos\left(\frac{\pi \mathrm{k}}{\mathrm{N}}\right), \quad \mathrm{k = 0(1)N} \tag{16}$$

To solve the differential equation,

$$\dot{x}(t) = \mathrm{f}(x, t) \tag{17}$$

the method of "collocation" is used, in which the residuals of the differential equation are set to zero at a set of interpolation points equal in number to the undetermined coefficients:

$$\dot{x}^N(t_k) = \mathrm{f}(x_k, t_k) \tag{18}$$

At the CGL points, the derivative of the interpolating polynomial approximation of Eq. (14) can be expressed in a matrix form using the interpolation coefficients:

$$\dot{x}^N(t_k) = \sum_{\mathrm{j=0}}^{\mathrm{N}} x_j \dot{\phi}_j(t_k) = \sum_{\mathrm{j=0}}^{\mathrm{N}} A_{kj} x_j \tag{19}$$

where A_{kj} is the $N+1 \times N+1$ differentiation matrix, with

$$A_{jk} = \begin{cases} \frac{c_j(-1)^{k+j}}{c_k(\zeta_j - \zeta_k)} & j \neq k \\ \frac{\zeta_j}{2\left(1-\zeta_j^2\right)} & 1 \le j = k \le N-1 \\ \frac{2N^2+1}{6} & j = k = 0 \\ -\frac{2N^2+1}{6} & j = k = N \end{cases} \tag{20}$$

$$B_{jk} = A_{jm} \cdot A_{mk} \tag{21}$$

with

$$c_j = \begin{cases} 2 & j = 0, N \\ 1 & 1 \le j \le N-1 \end{cases} \tag{22}$$

and

Thus, Eq. (18) becomes a set of algebraic equations of the interpolation coefficients x_k $(k = 0(1)N)$. It is worth noting that, if the right hand-side function of Eq. (17), $f(x, t)$, is a linear function of x, then Eq. (18) will be linear algebraic equations in x_k.

2.1.2 Newton–Kantorovich iterations The differential equations for practical problems are generally nonlinear. The Chebyshev pseudo spectral collocation can be applied after linearization of the nonlinear differential equations around the nominal solution. To improve the accuracy, the linearization around the updated solution and the Chebyshev pseudo spectral collocation are iterated. Given a nominal solution $x^0(t)$, the nonlinear differential equations of Eq. (17) are linearized by Taylor expansion:

$$\dot{x}(t) = f(x^{(0)}, \mathrm{t}) + \left.\frac{\partial f}{\partial x^T}\right|_{x^{(0)}} \left(x - x^{(0)}\right) + o\left(\left|x - x^{(0)}\right|\right) \tag{23}$$

After dropping the $o\left(\left|x - x^{(0)}\right|\right)$ term, Eq. (23) becomes a linear differential equation in $x(t)$. By solving this linear differential equation, the first-order solution $x^{(1)}(t)$ is obtained. Then, this linearization is repeated by updating $x^{(0)}(t)$ with $x^{(1)}(t)$. The general iteratively linearized differential equations are

$$\dot{x}^{(k)}(t) = f(x^{(k-1)}, \mathrm{t}) + \left.\frac{\partial f}{\partial x^T}\right|_{x^{(0)}} \left(x^{(k)} - x^{(k-1)}\right) \quad k = 1, 2, ... \tag{24}$$

By iteratively solving Eq. (24), the approximate solutions to Eq. (17) with high-order accuracy can be obtained.

2.2 *Computation of maneuver cost*

Maneuver cost in terms of delta velocity is computed by using the formula

$$\text{Maneuver Cost} = \sqrt{(x_{cheb} - x_{init})^2 + (y_{cheb} - y_{init})^2 + (z_{cheb} - z_{init})^2} \tag{25}$$

where $(x_{cheb}, y_{cheb}, z_{cheb})$ is the velocity vector required to transfer the satellite form one state to another obtained after applying Chebyshev collocation method and $(x_{init}, y_{init}, z_{init})$ is the initial velocity vector.

3. Validation of numerical solution

The validation of the numerical method is carried out by two ways: (i) the response of solution to the order of polynomials considered in Chebyshev approximation for M1 model (Table 1), and (ii) the final relative state of the chaser satellite is computed using Matlab function ode113() for initial conditions obtained by collocation method and compared with the desired state (Table 2). From Table 1, it can be seen that terminal velocities obtained using collocation method converges as order of polynomial increases. Accuracy improves up to 10 digits for 41 orders of polynomials. Further by using the initial state of the satellite, its state after the desired time is computed and compared with the possible solution.

From Table 2, it can be seen that positions and velocities after a desired time are accurate of the order of millimeter and millimeter/sec, respectively. In Table 2, the desired final position is (0, 4, 0) km.

4. Results and discussion

In this work, author solved three different models of the relative equations of motion in Hill's frame without considering any perturbations on the satellites by using Chebyshev collocation method. The considered three different models are (i) CW equations (M3), (ii) non-linear model by considering circular motion of target satellite (M2), and (iii) generalized non-linear equations (M1). The required maneuver cost, to transfer chaser from one location to the desired location in Hill's frame, in terms of delta velocity is computed as a function of time of flight and inter-satellite distances. The relative equations of motions are solved for when (i) to change only along track separation, (ii) to change along track and radial separation together, and (iii) to change along, across and radial separation together. Results are presented in two sections. In first section a relative comparisons of M1, M2 and M3 models are made as a function of ISD, time of flight and eccentricities. In second section a maneuver strategy is discussed by consider (a) only along track separation, (b) along-track as well as radial separation and (c) along-track, radial & cross-track separations, respectively.

Table 1. Comparison of final position of chaser obtained by using ode113() function of Matlab by using initial conditions obtained by (NK/CPS) and actual position of chaser with a time of flight of 6000s.

Model used	Initial velocity (NK/CPS)	Final velocity (NK/CPS)	Final position obtained by using ode113()	Final velocity obtained by using ode113()
M1	(−0.135094, 0.321104, 0)	(−0.107482, 0.333981, 0)	(2.4e−07, 3.999997, 0)	(−0.107482, 0.333981, 0)
M2	(−0.141383, 0.321807, 0)	(−0.107413, 0.334439, 0)	(8.0e−11, 3.999999, 0)	(−0.107413, 0.334439, 0)
M3	(−0.017250, 0.336964, 0)	(0.017250, 0.336964, 0)	(3.0e−10, 3.999999, 0)	(0.017250, 0.336964, 0)

Table 2. Dependency of the radial and along-track velocities (m/s) at the terminal points for different order of base polynomials for a time of flight of 6000 s for only along track traveling.

Order of polynomials	$x_0 = (0, 10, 0)$ (Radial, alongTrack, crossTrack)	$x_f = (0, 4, 0)$ (Radial, alongTrack, crossTrack)
11	(−0.1224950840, 0.3197420584, 0)	(−0.1076356327, 0.3331225083, 0)
21	(−0.1224776252, 0.3197017214, 0)	(−0.1076198094, 0.3330733002, 0)
31	(−0.1224776001, 0.3197017214, 0)	(−0.1076197843, 0.3330732998, 0)
41	(−0.1224776001, 0.3197017214, 0)	(−0.1076197843, 0.3330732998, 0)

4.1 *Relative comparisons of M1, M2 and M3 models*

In this section an attempt has been made to establish the result that which model (among M1, M2 and M3) can be used for relocating the satellite from one position to another in Hill's frame. As M3 model is derived from M2 by the assumption that relative distance between target and chaser is very small and due to this linear assumption is assumed to be valid. Due to this fact, models M2 and M3 are compared as a function of time of flight and ISD & plotted in Figs 1 and 2 by using initial conditions obtained for M3 model.

It can be seen that in Fig. 1, initially for time of flight of 6000 seconds there is a significant difference in the path profile of chaser for M2 and M3 models but as time of flight reduces the difference in the path profile also reduces and both models perform same for time of flight of 1000 seconds for relocating a satellite from (0, 10, 0) km to (0, 1, 0) km. But this time of flight for which M2 and M3 models path profile overlaps in this case is a function of ISD also (results are not shown here). In Fig. 2 path profiles of M2 and M3 models are shown for fixed time of flight but varying ISD to relocate the chaser from (0, 10, 0) km to (0, 9, 0) km. It can be seen that in this case path profiles not matched even for moving chaser satellite around 100 m (Fig. 2(d)).

Further as in M2 model assumption is that target is moving on a circular orbit hence here comparison of M1 and M2 models is performed for different values of target satellite eccentricity. The relative motion of chaser w.r.t. target are plotted in Fig. 3(a)–(c) for three different values of eccentricities viz. $e = 0.001, 0.0005$, and 0.0001. From these plots, it can be clearly seen that impact of target satellite eccentricity cannot be ignored for motion of chaser w.r.t. target as even for $e = 0.001$ chaser path profiles are deviates from each other by using M1 and M2 models. These path profiles converges for very small values of target eccentricities like $e = 0.0001$.

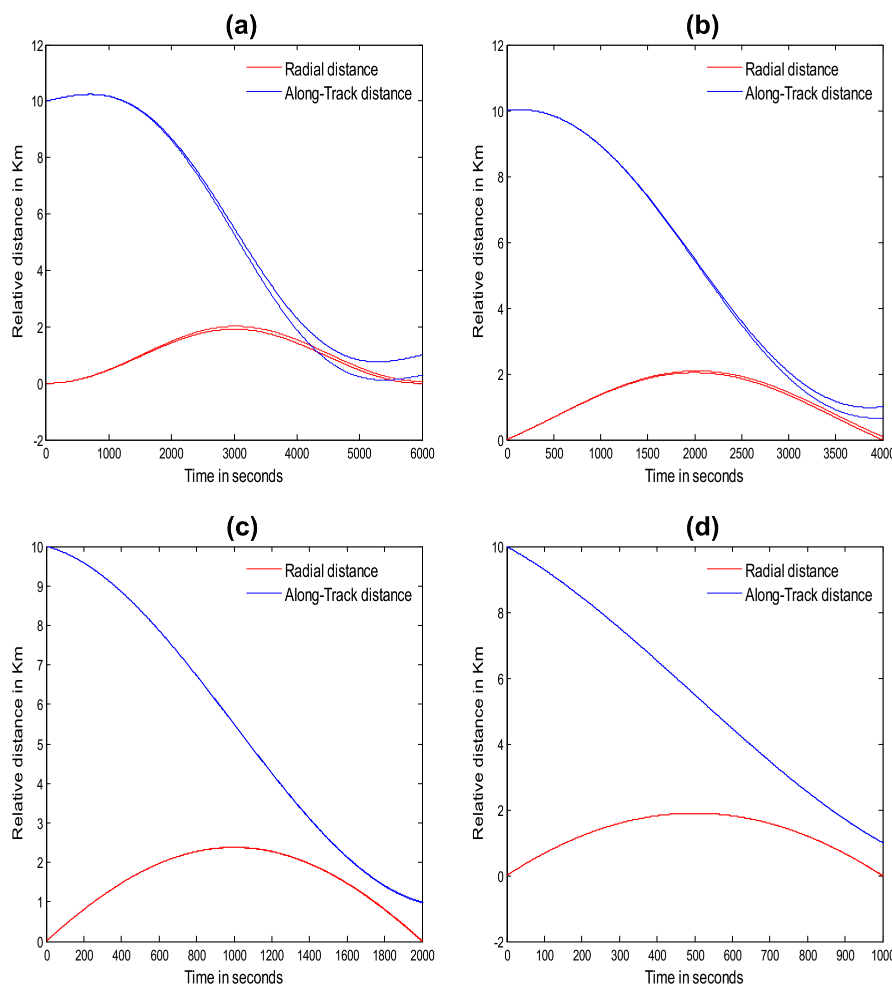

Figure 1. Comparisons of M2 (solid lines) and M3 (dotted lines) model as a function of time of flight for a given position relocation of satellites from (0, 10, 0) km to (0, 1, 0) km as a function of time of flight.

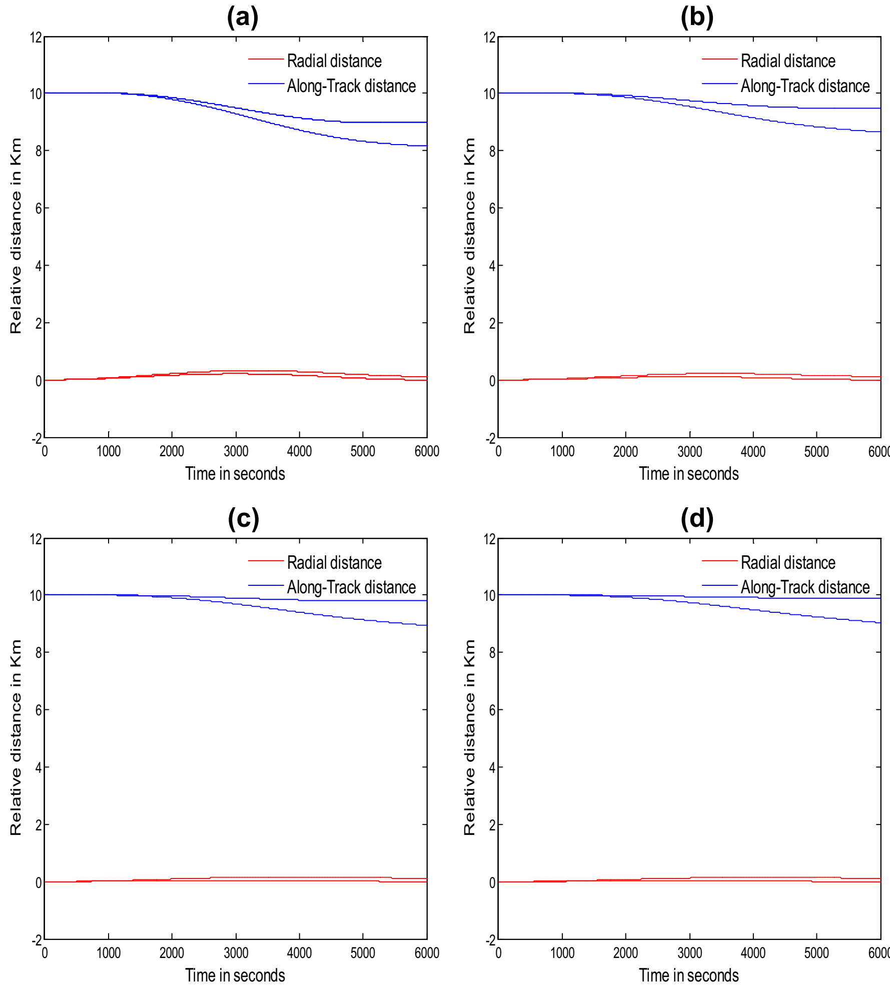

Figure 2. Motion of Chaser satellite with respect to target with M2 (— solid lines) and M3 models (…… dotted lines) for a change of ISD form (a) (0, 10,0) km to (0, 9, 0) km (b) (0, 10, 0) km to (0, 9.5, 0) km (c) (0, 10, 0) km to (0, 9.8, 0) km (d) (0, 10, 0) km to (0, 9.9, 0) km.

Hence from the above study it can be concluded that M1 model (generalized model) deviates from M2 and M3 significantly as a function of time of flight, ISD and eccentricity so in the further studies only M1 model is considered.

4.2 *Computation of maneuver cost as a function of time of flight using M1 model*

4.2.1 Only along-track separation is considered In this section only along-track separation is assumed. Table-1 shows the cost function for M1 models for three different inter-satellite distances travels in along track direction, viz (a) (0, 10, 0) to (0, 1, 0), (b) (0, 10, 0) to (0, 2, 0), and (iii) (0, 10, 0) to (0, 5, 0) as a function of time of flight (TOF). It can be seen that maneuver cost is an increasing function of inter satellite distance for a fixed TOF. But as a function of TOF first it decreases, then increases and again decreases for the considered duration and for a fixed ISD. Also maneuver cost is minimal for TOF of one orbital period of target. Minimum maneuver costs are 1.035, 0.924, and 0.584 m/s for transferring satellite from 10 km to 1, 2, and 5 km, respectively (Table 3).

4.2.2 Only along-track and radial separations are considered In this section, both along track and radial separations are considered. First, chaser is moved from position (1, 10, 0) to (0, 1, 0) km and maneuver cost are tabulated in Table 4. Further, study is extended by taking radial separation as −1 km but for same along-track separation and results are tabulated in Table 5. Results show that there is significant quantitative and qualitative change in transfer cost with the inclusion

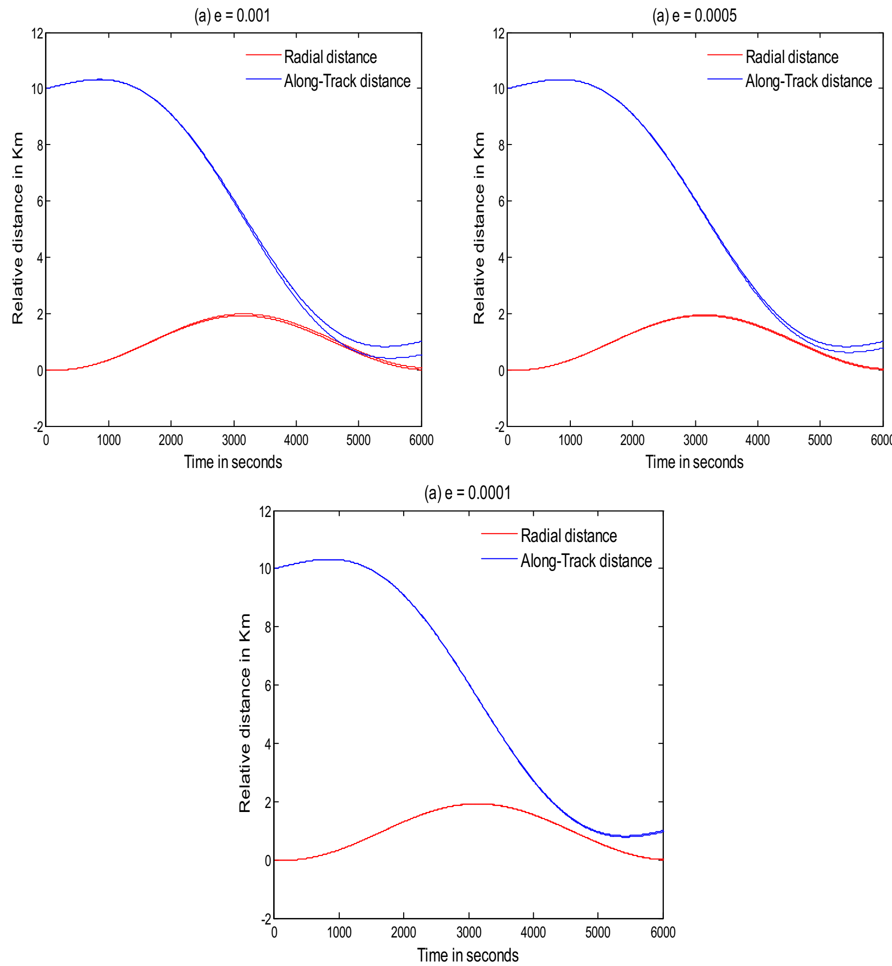

Figure 3. Motion of Chaser satellite with respect to target with M2 (solid lines) and M3 models (dotted lines) for a change of ISD form (0, 10,0) km to (0, 1, 0) km for three different values of eccentricities.

Table 3. Maneuver cost in m/s as a function of time of flight for three different along track travels, viz. (a) (0, 10, 0) km to (0, 1, 0) km, (b) (0, 10, 0) km to (0, 2, 0) km, and (c) (0, 10, 0) km to (0, 5, 0) km for M1 models.

Time of Flight	Cost Function in case (a)	Cost Function in case (b)	Cost Function in case (c)
100	180.333	160.296	100.185
200	90.649	80.576	50.360
500	37.357	33.206	20.753
1000	19.499	17.332	10.831
2000	8.945	7.951	4.969
3000	4.679	4.158	2.597
4000	2.541	2.258	1.406
5000	1.389	1.233	0.761
6000	1.035	0.924	0.584
7000	1.727	1.534	0.953
8000	7.057	6.273	3.911
9000	4.167	3.701	2.302
10000	1.414	1.255	0.777

Table 4. Maneuver cost as a function of TOF for three different along track and radial travels, viz. (a) (1, 10, 0) km to (0, 1, 0) km, (b) (1, 10, 0) km to (0, 2, 0) km, and (c) (1, 10, 0) km to (0, 5, 0) km for M1 model.

Time of Flight	Cost Function in case (a)	Cost Function in case (b)	Cost Function in case (c)
100	181.437	161.539	102.168
200	91.174	81.6154	51.336
500	37.379	33.262	20.992
1000	19.056	16.920	10.589
2000	8.241	7.304	4.618
3000	4.269	3.844	2.708
4000	2.657	2.500	2.217
5000	2.899	2.922	3.042
5500	4.943	4.966	5.048
5700	8.277	8.293	8.345
5900	38.275	38.366	38.611
6000	42.643	42.518	42.116
6100	13.846	13.842	13.823
6300	6.192	6.211	6.278
6500	4.247	4.277	4.387
7000	2.761	2.800	2.998
8000	2.935	2.465	2.846
9000	2.396	2.191	2.519
10000	2.277	2.323	2.529

of out-of-plane separation. Due to inclusion of radial separation, transfer cost attains its one local maximal around the period of the target orbit (6000 s) whereas for only along track separation transfer cost attains its one local minimal around the period of the target orbit. Transfer cost is minimal for time of flight of 4000 and 7000 seconds. This quantitative result is very important from mission design point of view. Similar results are valid for transferring chaser from the state (−1, 10, 0) km to (0, 1, 0), (0, 2, 0) and (0, 5, 0) km (Table 5)

4.2.3 Along-track, radial and cross-track separations are considered In this section, transfer cost is computed for three-dimensional initial separation. All three separations viz (i) radial, (ii) along-track, and (iii) cross-track are assumed. Out-of-plane separation is taken as 1 km and like in earlier section, radial and along track separations are taken as 1 and 10 km, respectively. Results are tabulated in Table 6. Results shows that comparatively amount of transfer cost for all three models not varies much but there is a significant quantitative and qualitative change in transfer cost with the inclusion of out of plane separation. Due to out of plane separation, transfer cost attains its one local maximal around the half period of the target (3000 s) unlike the radial separation where transfer cost attains its one local minimal around the period of the target orbit. Transfer cost is minimal for time of flight of 4000 and 7000 s. This quantitative result is very important from mission point of view. Another important observation is that in this case two local maximal are obtained, one at half period and another at full period of the target orbit.

5. Conclusions

In this work, author solved three different models of the relative equation of motions in Hill's frame without considering any perturbations on the satellites by using Chebyshev collocation method. The considered three different models are (i) CW equations, (ii) non-linear model by taking circular motion of target satellite, (iii) generalized equations. Following conclusions can be made from this study

1. Maneuver cost of a transfer is a monotonic increasing function of inter-satellite distance.
2. Maneuver cost of transfer first reduces, then increases and again reduces as a function of time of flight for only along-track separation.

Table 5. Maneuver cost in m/s as a function of time of flight for three different along track and radial travels, viz. (a) (−1, 10, 0) km to (0, 1, 0) km, (b) (−1, 10, 0) km to (0, 2, 0) km, and (c) (−1, 10, 0) km to (0, 5, 0) km for M1 model.

Time of Flight	Cost Function in case (a)	Cost Function in case (b)	Cost Function in case (c)
100	181.460	161.562	102.191
200	91.265	81.263	51.425
500	37.876	33.758	21.480
1000	20.409	18.269	11.904
2000	10.429	9.470	6.648
3000	6.459	5.982	4.598
4000	4.658	4.418	3.727
5000	4.263	4.149	3.825
5500	5.673	5.611	5.439
5700	8.643	8.609	8.523
5900	37.842	37.937	38.260
6000	42.178	42.059	41.745
6100	13.986	13.951	13.861
6300	6.829	6.771	6.613
6500	5.268	5.184	4.947
7000	4.778	4.622	4.169
8000	12.727	11.966	9.687
9000	7.940	7.500	6.199
10000	4.034	3.907	3.536

Table 6. Maneuver cost as a function of time of flight for three different along track travels form (a) (1, 10, 1) km to (0, 1, 0) km, (b) (1, 10, 1) km to (0, 2, 0) km, and (c) (1, 10, 1) km to (0, 5, 0) km for M1 model.

Time of Flight	Cost Function in case (a)	Cost Function in case (b)	Cost Function in case (c)
100	182.534	162.770	104.104
200	91.717	81.782	52.294
500	37.584	33.492	21.354
1000	19.153	17.029	10.761
2000	8.477	7.572	5.051
3000	87.171	86.650	85.141
4000	3.433	3.341	3.207
5000	3.664	3.679	3.764
5500	6.606	6.623	6.686
5700	11.425	11.440	11.486
5900	53.882	54.014	54.393
6000	60.056	59.883	59.348
6100	19.375	19.363	19.324
6300	8.428	8.440	8.484
6500	5.587	5.609	5.690
7000	3.471	3.497	3.637
8000	3.717	3.432	3.660
9000	28.169	28.107	27.969
10000	3.184	3.204	3.310

3. Model M1 and M2 are closer in the form of position profile compared to M3 model for a given time of flight and transfer distance.
4. For only along-track separation, maneuver cost is minimum while it is maximum for (along-track, radial, across-track) separations.
5. For along-track motion, maneuver cost shows a minimal around one orbital period of target, but with the inclusion of radial separation, its shows one maximal at same point.
6. For three-dimensional motion, local maximal are found around half period and one period of target.

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