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Computational soliton solutions to (3 + 1)-dimensional generalised Kadomtsev–Petviashvili and (2 + 1)-dimensional Gardner–Kadomtsev–Petviashvili models and their applications

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Abstract. In this paper, the auxiliary equation method is successfully applied to compute analytical solutions for (3+1)-dimensional generalised Kadomtsev–Petviashvili and (2+1)-dimensional Gardner–Kadomtsev–Petviashvili equations, by introducing simple transformations. These results hold numerous travelling wave solutions that are of key importance which provide a powerful mathematical tool for solving nonlinear wave equations in recent era of applied science and engineering. The method can also be extended to other nonlinear evolution models arising in contemporary physics.

Keywords. Soliton solutions; Gardner–Kadomtsev–Petviashvili equation; generalised Kadomtsev–Petviashvili equation; auxiliary equation method.

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1. Introduction

Nonlinear evolution equations (NEEs) have been studied in the last few decades. A veriety of NEEs are integrated with the help of various interesting computational techniques. To understand the physical structure, described by nonlinear partial differential equations (PDEs), exact solutions to the nonlinear PDEs play a crucial role in the study of the nonlinear models appearing in diverse disciplines, for instance electromagnetic theory, geochemistry, astrophysics, fluid dynamics, elastic media, nuclear physics, optical fibres, high-energy physics, gravitation, statistical and condensed matter physics, biology, solid-state physics, chemical kinematics, chemical physics, electrochemistry, fluid dynamics, acoustics, cosmology, plasma physics, etc. [1–52].

Many well-known models have been established to describe the dynamics of nonlinear waves arising in recent era of modern science and engineering, such as the Korteweg–de Vries (KdV) equation [15], Korteweg–de Vries Burgers equation [16,20], modified Korteweg–de Vries (mKdV) equation [18], modified Korteweg–de Vries Kadomtsev–Petviashvili (mKdV-KP) equation [39], Boussinesq equation [21], Zakharov–Kuznetsov–Burgers equation [31], modified Korteweg–de Vries Zakharov–Kuznetsov equation [14, 25], Perergrine equation [45], Kawahara equation [36], Benjamin–Bona–Mahoney equation [51], Kadomtsev–Petviashvili–Benjamin–Bona–Mahony (KP–BBM) equation [53], coupled Korteweg–de Vries equation [13], coupled Boussinesq equation [52], Gardner equation [38], a combination of KdV and mKdV equations. Numerous approaches, such as the travelling wave solution [29,32], Cole–Hopf transformation, Painlevé method, Bäcklund transformation, amplitude ansatz method [46], sine–cosine method, Darboux transformation, Hirota method, function transformation method [50], Lie group analysis, extended simple equation method [33], homogeneous balance method [23], similarity reduced method, tanh method, fractional direct algebraic function method [48], inverse scattering method [19], Hirota's bilinear method [17], homogeneous balance method [21], variational method [26,28], algebraic method [30], sine–cosine method [35], Jacobi elliptic function method [22,27], F-expansion method [24], extended Fan sub-equation method [41], (G'/G) expansion method [42–44], tanh and extended tanh method [37,47], extended direct algebraic method [34] etc.

In this paper, the auxiliary equation method is applied to compute the soliton solution of (3 + 1)-dimensional generalised KP and (2 + 1)-dimensional Gardner–KP equations. The completely integrable KP equation is one of the models that describes the evolution of nonlinear waves, which is the extension of the well-known KdV equation, where the effect of the surface tension and the viscosity is negligible [54–56].

The (3 + 1)-dimensional generalised KP equation reads as

$$u_{xxxy} + u_{tx} + u_{ty} - u_{zz} + 3(u_x u_y)_x = 0. \tag{1}$$

Another model that is also considered and studied at times is the combined KdV–mKdV equations, which is recognised as the Gardener equation [15]. This describes the internal solitary waves in shallow seas. The (2 + 1)-dimensional Gardner–KP equation [40,49] is given by

$$\left(v_t + 6vv_x \pm 6v^2 v_x + v_{xxx}\right)_x + v_{yy} = 0. \tag{2}$$

This paper is organised as follows: in §2, the auxiliary equation method is introduced, while in §3, the solutions of the nonlinear PDEs have been presented and the physical interpretation of the solutions is discussed in §4. In §5, the conclusions have been drawn.

2. The description of the auxiliary equation method

We shall briefly present the auxiliary equation method (AEM) in the following steps:

Step 1: Let us have a general form of the nonlinear PDE:

$$F(u, u_t, u_x, u_y, u_z, u_{xx}, u_{yy}, u_{zz}, \dots) = 0, \tag{3}$$

where F is a polynomial function with respect to the indicated variables.

Step 2: The following wave variable is presented to solve (3):

$$u(x, y, z, t) = F(\xi). \tag{4}$$

Transformation (4) converts the PDE (3) to an ODE:

$$O_i(F, F_\xi, F_{\xi\xi}, F_{\xi\xi\xi}, \dots), \tag{5}$$

where $F = F(\xi)$ is an unknown function.

Step 3: The main idea of the auxiliary equation method based on expanding the travelling wave solution $F(\xi)$ of eq. (5) as a finite series is

$$F(\xi) = \sum_{j=0}^{n} a_j \;\; \psi^j(\xi). \tag{6}$$

ψ satisfies

$$\frac{\mathrm{d}\psi}{\mathrm{d}\xi} = C_0 + C_1\psi(\xi) + C_2\psi^2(\xi) + C_3\psi^3(\xi) + C_4\psi^4(\xi), \tag{7}$$

$$\xi = k_1 x + k_2 y + k_3 z - \omega t, \tag{8}$$

where C_i $(i = 0, 1, 2, 3, 4)$ and k_k $(k = 1, 2, 3)$ are constants.

Step 4: Applying the homogeneous balance to (3), the parameters n in (6) can be obtained.

Step 5: Substitute (6)–(8) into (3) and collect the coefficients of $\psi^j\psi^{(k)}$, then solve the system for ω and C_i.

Step 6: Substitute ω, C_i and $\psi(\xi)$ obtained in step 5 into (6) to obtain solutions for (1) and (2).

3. Soliton extraction

3.1 *Three-dimensional generalised KP equation*

Consider the transformation

$$u(x, y, z, t) = u(\xi), \quad \xi = k_1 x + k_2 y + k_3 z - \omega t. \tag{9}$$

Using (9) into (1),

$$k_1^3 k_2 u'''' - \left(k_1\omega + k_2\omega + k_3^2\right)u'' + 6k_1^2 k_2 u' u'' = 0. \tag{10}$$

Integrating and neglecting the constants of integration

$$k_1^3 k_2 u''' - \left(k_1\omega + k_2\omega + k_3^2\right)u' + 6k_1^2 k_2 (u')^2 = 0. \tag{11}$$

Considering the homogeneous balance between $(u')^2$ and u''', gives $n = 3$. Suppose the solution of (11) is of the form

$$u = a_0 + a_1\psi(\xi) + a_2\psi(\xi)^2 + a_3\psi(\xi)^3. \tag{12}$$

Substitute (6), (7) and (12) into (11) and collect the coefficients of $\psi^j\psi^{(k)}$.

Case I: $C_4 = 0$.

(a)

$$\psi_1(\xi) = -\frac{1}{8a_2k_2k_1^2\mathrm{e}^{\theta_1\xi}+8}\left(a_1k_1^2k_2\left(4\mathrm{e}^{\theta_1\xi}+\theta_2\right)\right.$$
$$+\frac{1}{a_2}\left(64\mathrm{e}^{\theta_1\xi}\sqrt{a_2^4k_1^3k_2((k_1+k_2)\omega+k_3^2)}\right.$$
$$\times\left(a_2k_2k_1^2\mathrm{e}^{\theta_1\xi}+1\right)+a_1^2\left(a_2\theta_2k_1^2k_2-4\right)$$
$$\left.\left.\times\left(a_2k_2k_1^2\left(8\mathrm{e}^{\theta_1\xi}+\theta_2\right)+4\right)\right)^{1/2}\right), \qquad (13)$$

where

$$\theta_1 = \frac{a_2^2\left(k_1\omega+k_2\omega+k_3^2\right)}{\sqrt{a_2^4k_1^3k_2\left(k_1\omega+k_2\omega+k_3^2\right)}},$$
$$\theta_2 = \frac{4k_3^2}{a_2k_1^2k_2\left(k_1\omega+k_2\omega+k_3^2\right)}+\frac{4\omega}{a_2k_1^2\left(k_1\omega+k_2\omega+k_3^2\right)}$$
$$+\frac{4\omega}{a_2k_1k_2\left(k_1\omega+k_2\omega+k_3^2\right)}.$$

The parameters C_i and a_j become

$$C_0 = \frac{1}{16a_2^2}\left(\frac{4a_1\sqrt{a_2^4k_1^3k_2\left(k_1\omega+k_2\omega+k_3^2\right)}}{a_2k_1^3k_2}-\frac{a_1^3}{k_1}\right),$$
$$C_1 = \frac{1}{8a_2^2k_1^3k_2}\left(4\sqrt{a_2^4k_1^3k_2^2\omega+a_2^4k_1^4k_2\omega+a_2^4k_1^3k_2k_3^2}\right.$$
$$\left.-3a_1^2a_2k_1^2k_2\right),$$
$$C_2 = -\frac{3a_1}{4k_1}, \quad C_3 = -\frac{a_2}{2k_1},$$
$$a_0 = 0, \quad a_1 = \lambda_1, \quad a_2 = \lambda_2, \quad a_3 = 0,$$

where λ_1 and λ_2 are arbitrary constants. Hence, the solution of (1) will be

$$u_1 = -\frac{\theta_1^2\mathrm{e}^{\eta\xi}\sqrt{k_1^3k_2\lambda_2^4\left((k_1+k_2)\omega+k_3^2\right)}\left(k_1^2k_2\lambda_2\mathrm{e}^{\eta\xi}-1\right)}{\lambda_2^2\left(k_2\lambda_2k_1^2\mathrm{e}^{\eta\xi}+1\right)^5}$$
$$\times\left(k_2^2\lambda_2^3k_1^4(-\mathrm{e}^{2\eta\xi})\left(\eta^2(-k_2)k_1^3+(k_1+k_2)\omega+k_3^2\right)\right.$$
$$-2k_2\lambda_2^2k_1^2\mathrm{e}^{\eta\xi}\left(5\theta_1^2k_2k_1^3+(k_1+k_2)\omega+k_3^2\right)$$
$$+6\theta_1k_2k_1^2\mathrm{e}^{\theta_1\xi}\sqrt{k_1^3k_2\lambda_2^4\left((k_1+k_2)\omega+k_3^2\right)}$$
$$\left.-\lambda_2\left(\eta^2(-k_2)k_1^3+(k_1+k_2)\omega+k_3^2\right)\right), \qquad (14)$$

where

$$\eta = \frac{\sqrt{k_1^3k_2\lambda_2^4((k_1+k_2)\omega+k_3^2)}}{k_1^3k_2\lambda_2^2}.$$

(b)

$$\psi_2(\xi) = \frac{\sqrt{\frac{\sqrt{a_2^4k_1^3k_2((k_1+k_2)\omega+k_3^2)}(\mathrm{e}^{\theta_1\xi}-a_2k_1^2k_2)}{a_2^2}}}{a_2k_1^2k_2-\mathrm{e}^{\theta_1\xi}}$$
$$-\frac{a_1}{2a_2}, \qquad (15)$$

where

$$\theta_1 = \frac{a_2^2(k_1\omega+k_2\omega+k_3^2)}{\sqrt{a_2^4k_1^3k_2(k_1\omega+k_2\omega+k_3^2)}}.$$

The parameters C_i and a_j become

$$C_0 = \frac{1}{16a_2^2}\left(-\frac{4a_1\sqrt{a_2^4k_1^3k_2(k_1\omega+k_2\omega+k_3^2)}}{a_2k_1^3k_2}-\frac{a_1^3}{k_1}\right),$$
$$C_1 = \frac{1}{8a_2^2k_1^3k_2}\left(-4\sqrt{a_2^4k_1^3k_2^2\omega+a_2^4k_1^4k_2\omega+a_2^4k_1^3k_2k_3^2}\right.$$
$$\left.-3a_1^2a_2k_2k_1^2\right),$$
$$C_2 = -\frac{3a_1}{4k_1}, \quad C_3 = -\frac{a_2}{2k_1},$$
$$a_0 = 0, \quad a_1 = \lambda_1, \quad a_2 = \lambda_2, \quad a_3 = 0,$$

where λ_1 and λ_2 are arbitrary constants, and hence the solution of (1) will be

$$u_2 = \frac{\eta^2\mathrm{e}^{\eta\xi}\sqrt{k_1^3k_2\lambda_2^4\left((k_1+k_2)\omega+k_3^2\right)}(\mathrm{e}^{\eta\xi}+k_2\lambda_2k_1^2)}{\lambda_2^2\left(k_1^2k_2\lambda_2-\mathrm{e}^{\eta\xi}\right)^5}$$
$$\times\left(-2k_2\lambda_2^2k_1^2\mathrm{e}^{\eta\xi}\left(k_2(5\eta^2k_1^3+\omega\right)+k_1\omega+k_3^2\right)$$
$$+\lambda_2\mathrm{e}^{2\eta\xi}\left(k_2\left(\omega-\eta^2k_1^3\right)+k_1\omega+k_3^2\right)$$
$$+k_2^2\lambda_2^3k_1^4\left(k_2\left(\omega-\eta^2k_1^3\right)+k_1\omega+k_3^2\right)$$
$$+6\eta k_2k_1^2\mathrm{e}^{\eta\xi}\sqrt{k_1^3k_2\lambda_2^4\left((k_1+k_2)\omega+k_3^2\right)}),$$
$$(16)$$

where

$$\eta = \frac{\sqrt{k_1^3k_2\lambda_2^4((k_1+k_2)\omega+k_3^2)}}{k_1^3k_2\lambda_2^2}.$$

Case II: $C_3 = 0, C_4 = 0$.
(a)

$$\psi_3(\xi) = \frac{k_1\lambda_1}{2a_1} - \frac{\sqrt{-k_1\omega-k_2\omega-k_3^2}\tan\left(\frac{\xi\sqrt{-k_1\omega-k_2\omega-k_3^2}}{2k_1^{3/2}\sqrt{k_2}}\right)}{2a_1\sqrt{k_1}\sqrt{k_2}}. \tag{17}$$

The parameters C_i and a_j become

$$C_0 = \frac{-c_1^2k_2k_1^3 + k_1\omega + k_2\omega + k_3^2}{4a_1k_1^2k_2}, \quad C_1 = \lambda_1,$$
$$C_2 = -\frac{a_1}{k_1},$$
$$a_0 = 0, \quad a_1 = \lambda_2, \quad a_2 = 0, \quad a_3 = 0,$$

where λ_1 and λ_2 are arbitrary constants, and hence the solution of (1) will be

$$u_3 = \eta_1^2\eta_2\tan(\eta_1\xi)(-\sec^2(\eta_1\xi))\left(3\eta_1k_2k_1^2(4\eta_1k_1 - \eta_2)\sec^2(\eta_1\xi) - 4\eta_1^2k_2k_1^3 - k_1\omega + k_2\omega - k_3^2\right), \tag{18}$$

where

$$\eta_1 = \frac{\sqrt{-(k_1+k_2)\omega - k_3^2}}{2k_1^{3/2}\sqrt{k_2}},$$
$$\eta_2 = \frac{\sqrt{-(k_1+k_2)\omega - k_3^2}}{\sqrt{k_1}\sqrt{k_2}}.$$

(b)

$$\psi_4(\xi) = -\frac{\sqrt{-k_1\omega-k_2\omega-k_3^2}\tan\left(\frac{\xi\sqrt{-k_1\omega-k_2\omega-k_3^2}}{2k_1^{3/2}\sqrt{k_2}}\right)}{2a_1\sqrt{k_1}\sqrt{k_2}}. \tag{19}$$

The parameters C_i and a_j become

$$C_0 = \frac{k_1\omega + k_2\omega + k_3^2}{4a_1k_1^2k_2}, \quad C_1 = 0, \quad C_2 = -\frac{a_1}{k_1},$$
$$a_0 = 0, \quad a_1 = \lambda_1, \quad a_2 = 0, \quad a_3 = 0,$$

where λ_1 is an arbitrary constant, and hence the solution of (1) will be

$$u_4 = 2\eta_1^2\eta_2\tanh(\eta_1\xi)\mathrm{sech}^2(\eta_1\xi) \times \left(6\eta_1k_2k_1^2(2\eta_1k_1 - \eta_2)\mathrm{sech}^2(\eta_1\xi) - 4\eta_1^2k_2k_1^3 + k_1\omega - k_2\omega + k_3^2\right), \tag{20}$$

where

$$\eta_1 = \frac{\sqrt{-(k_1+k_2)\omega - k_3^2}}{2k_1^{3/2}\sqrt{k_2}},$$
$$\eta_2 = -\frac{\sqrt{-(k_1+k_2)\omega - k_3^2}}{2\sqrt{k_1}\sqrt{k_2}}.$$

Case III: $C_0 = 0, C_4 = 0$.
(a)

$$\psi_5(\xi) = \frac{\sqrt{a_2\theta_2\sqrt{k_1}\sqrt{k_2} + \theta_2\mathrm{e}^{\theta_1\xi}}}{\sqrt{\mathrm{e}^{2\theta_1\xi} - a_2^2k_1k_2}}, \tag{21}$$

where

$$\theta_1 = \frac{\sqrt{k_1\omega + k_2\omega + k_3^2}}{k_1^{3/2}\sqrt{k_2}}, \tag{22}$$

$$\theta_2 = \sqrt{k_1\omega + k_2\omega + k_3^2}. \tag{23}$$

The parameters C_i and a_j become

$$C_1 = -\frac{\sqrt{k_1\omega + k_2\omega + k_3^2}}{2k_1^{3/2}\sqrt{k_2}}, \quad C_2 = 0, \quad C_3 = -\frac{a_2}{2k_1},$$
$$a_0 = 0, \quad a_1 = 0, \quad a_2 = \lambda_1, \quad a_3 = 0,$$

where λ_1 is an arbitrary constant, and hence the solution of (1) will be

$$\begin{aligned} u_5 = & \frac{\theta_1^2\lambda_1\mathrm{e}^{\theta_1\xi}\sqrt{(k_1+k_2)\omega + k_3^2}(\mathrm{e}^{\theta_1\xi} + \sqrt{k_1}\sqrt{k_2}\lambda_1)}{(\sqrt{k_1}\sqrt{k_2}\lambda_1 - \mathrm{e}^{\theta_1\xi})^5} \\ & \times \Big(6\theta_1k_2\lambda_1k_1^2\mathrm{e}^{\theta_1\xi}\sqrt{(k_1+k_2)\omega + k_3^2} \\ & - 2\sqrt{k_1}\sqrt{k_2}\lambda_1\mathrm{e}^{\theta_1\xi}\left(5\theta_1^2k_2k_1^3 + (k_1+k_2)\omega + k_3^2\right) \\ & + k_2\lambda_1^2k_1\left(\theta_1^2(-k_2)k_1^3 + (k_1+k_2)\omega + k_3^2\right) \\ & + \mathrm{e}^{2\theta_1\xi}\left(\theta_1^2(-k_2)k_1^3 + (k_1+k_2)\omega + k_3^2\right)\Big). \end{aligned} \tag{24}$$

(b)

$$\psi_6(\xi) = -\frac{\sqrt{\theta_2(-\mathrm{e}^{\theta_1\xi}) - a_2\theta_2\sqrt{k_1}\sqrt{k_2}\mathrm{e}^{2\theta_1\xi}}}{\sqrt{1 - a_2^2k_1k_2\mathrm{e}^{2\theta_1\xi}}}, \tag{25}$$

where

$$\theta_1 = \frac{\sqrt{k_1\omega + k_2\omega + k_3^2}}{k_1^{3/2}\sqrt{k_2}}, \tag{26}$$

$$\theta_2 = \sqrt{k_1\omega + k_2\omega + k_3^2}. \tag{27}$$

The parameters C_i and a_j become

$$C_1 = \frac{\sqrt{k_1\omega + k_2\omega + k_3^2}}{2k_1^{3/2}\sqrt{k_2}}, \quad C_2 = 0, \quad C_3 = -\frac{a_2}{2k_1},$$
$$a_0 = 0, \quad a_1 = 0, \quad a_2 = \lambda_1, \quad a_3 = 0,$$

where λ_1 is an arbitrary constant, and hence the solution of (1) will be

$$u_6 = \frac{\eta^2\lambda_1 e^{\eta\xi}\sqrt{(k_1+k_2)\omega + k_3^2}(e^{\eta\xi} + \sqrt{k_1}\sqrt{k_2}\lambda_1)}{(\sqrt{k_1}\sqrt{k_2}\lambda_1 - e^{\eta\xi})^5} \times \Big(6\eta k_2\lambda_1 k_1^2 e^{\eta\xi}\sqrt{(k_1+k_2)\omega + k_3^2} + 2\sqrt{k_1}\sqrt{k_2}\lambda_1 e^{\eta\xi}\left(5\eta^2 k_2 k_1^3 + (k_1+k_2)\omega + k_3^2\right) - k_2\lambda_1^2 k_1\left(\eta^2(-k_2)k_1^3 + (k_1+k_2)\omega + k_3^2\right) - e^{2\eta_1\xi}\left(\eta^2(-k_2)k_1^3 + (k_1+k_2)\omega + k_3^2\right)\Big), \tag{28}$$

where

$$\eta = -\frac{\sqrt{(k_1+k_2)\omega + k_3^2}}{k_1^{3/2}\sqrt{k_2}}.$$

3.2 *Two-dimensional Gardner–KP equation*

Consider the transformation

$$v(x, y, t) = v(\xi), \quad \xi = k_1x + k_2y - \omega t. \tag{29}$$

Using (29) into (2),

$$k_1^4 v^{(4)} + (k_2^2 - k_1\omega)v'' + 6k_1^2(v^2(-v'') + vv'' - 2v(v')^2 + (v')^2) = 0. \tag{30}$$

Integrating (30) and neglecting the constants of integration

$$k_1^4 v''' + (k_2^2 - k_1\omega)v' + k_1^2(vv' - v^2v') = 0. \tag{31}$$

The homogeneous balance between v^2v' and v''' gives $n = 3$. Suppose the solution of (31) is of the form

$$v = a_0 + a_1\psi(\xi) + a_2\psi(\xi)^2 + a_3\psi(\xi)^3. \tag{32}$$

Substitute (6), (7) and (32) into (31) and collect the coefficients of $\psi^j\psi^{(k)}$.

Case I: $C_4 = 0$.
(a)

$$\psi_1(\xi) = \frac{\sqrt{6}k_1\lambda_1}{2a_1} - \frac{\sqrt{3}\sqrt{4k_1\omega - k_1^2 - 4k_2^2}\tan(\theta\xi)}{2a_1k_1}, \tag{33}$$

where

$$\theta = \frac{\sqrt{4k_1\omega - k_1^2 - 4k_2^2}}{2\sqrt{2}k_1^2}.$$

The parameters C_i and a_j become

$$C_0 = \frac{-2\sqrt{6}c_1^2k_1^4 - 4\sqrt{6}k_1\omega + \sqrt{6}k_1^2 + 4\sqrt{6}k_2^2}{8a_1k_1^3},$$
$$C_1 = \lambda_1, \quad C_2 = -\frac{a_1}{\sqrt{6}k_1}, \quad C_3 = 0,$$
$$a_0 = \frac{1}{2}(1 - \sqrt{6}c_1k_1), \quad a_1 = \lambda_2, \quad a_2 = 0, \quad a_3 = 0,$$

where λ_1 and λ_2 are arbitrary constants, and hence the solution of (2) will be

$$v_1 = \frac{1}{2k_1}\Big(\sqrt{3}\theta^2\sqrt{4k_1\omega - k_1^2 - 4k_2^2}\tan(\theta\xi)\sec^4(\theta\xi) \times (k_1\omega(55 - 17\cos(2\theta\xi)) + 4\theta^2k_1^4(\cos(2\theta\xi) - 5) + 3k_1^2(\cos(2\theta\xi) - 5) + k_2^2(17\cos(2\theta\xi) - 55))\Big). \tag{34}$$

(b)

$$\psi_2(\xi) = \frac{\sqrt{3}\sqrt{\theta_2\left(-e^{\theta_1\xi/\sqrt{2}}\right) - \sqrt{3}a_2\theta_2k_1e^{\sqrt{2}\theta_1\xi}}}{\sqrt{3a_2^2k_1^2e^{\sqrt{2}\theta_1\xi} - 1}}, \tag{35}$$

where

$$\theta_1 = \frac{\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}}{k_1^2},$$
$$\theta_2 = \sqrt{-4k_1\omega + k_1^2 + 4k_2^2}.$$

The parameters C_i and a_j become

$$C_0 = 0, \quad C_1 = \frac{\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}}{2\sqrt{2}k_1^2}, \quad C_2 = 0,$$
$$C_3 = \frac{a_2}{2\sqrt{6}k_1},$$
$$a_0 = \frac{1}{2}\left(\frac{\sqrt{3}\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}}{k_1} + 1\right), \quad a_1 = 0,$$
$$a_2 = \lambda_1, \quad a_3 = 0,$$

where λ_1 is an arbitrary constant, and hence the solution of (2) will be

$$v_2 = \frac{3\eta^2\lambda_1 e^{\eta\xi}\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}}{(e^{\eta\xi} - \sqrt{3}k_1\lambda_1)^5} \times \big(3k_1^2\lambda_1^2 e^{\eta\xi}\big(11\eta^2 k_1^4 + 127k_1\omega - 33k_1^2 - 127k_2^2\big) + \sqrt{3}k_1\lambda_1 e^{2\eta\xi}\big(11\eta^2k_1^4 + 127k_1\omega - 33k_1^2 - 127k_2^2\big) + 3\sqrt{3}k_1^3\lambda_1^3(\eta^2k_1^4 + 17k_1\omega - 3k_1^2 - 17k_2^2) + e^{3\eta\xi}\big(\eta^2k_1^4 + 17k_1\omega - 3k_1^2 - 17k_2^2\big)\big), \tag{36}$$

where

$$\eta = -\frac{\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}}{\sqrt{2}k_1^2}.$$

(c)

$$\psi_3(\xi) = -\frac{3\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}\, e^{\frac{\xi\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}}{\sqrt{2}k_1^2}}}{\sqrt{3}a_1k_1 e^{\frac{\xi\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}}{\sqrt{2}k_1^2}} - 1}. \tag{37}$$

The parameters C_i and a_j become

$$C_0 = 0, \quad C_1 = \frac{\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}}{\sqrt{2}k_1^2},$$

$$C_2 = \frac{a_1}{\sqrt{6}k_1}, \quad C_3 = 0,$$

$$a_0 = \frac{1}{2}\left(\frac{\sqrt{3}\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}}{k_1} + 1\right),$$

$$a_1 = \lambda_1, \quad a_2 = 0, \quad a_3 = 0,$$

where λ_1 is an arbitrary constant, and hence the solution of (2) will be

$$v_3 = -\frac{3\eta^2\lambda_1 e^{\eta\xi}\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}}{(\sqrt{3}k_1\lambda_1 e^{\eta\xi} - 1)^5} \times \Big(k_1\Big(3\sqrt{3}\lambda_1^3k_1^2e^{3\eta\xi}\big(\eta^2k_1^4 + 17k_1\omega - 3k_1^2 - 17k_2^2\big) + 3\lambda_1^2k_1e^{2\eta\xi}\big(11\eta^2k_1^4 + 127k_1\omega - 33k_1^2 - 127k_2^2\big) + \sqrt{3}\lambda_1e^{\eta\xi}\big(11\eta^2k_1^4 + 127k_1\omega - 33k_1^2 - 127k_2^2\big) + \eta^2k_1^3 - 3k_1 + 17\omega\Big) - 17k_2^2\Big), \tag{38}$$

where

$$\eta = \frac{\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}}{\sqrt{2}k_1^2}.$$

Case II: $C_3 = 0, C_4 = 0$.

(a)

$$\psi_4(\xi) = \frac{\sqrt{6}k_1\lambda_1}{2a_1} - \frac{\sqrt{3}\sqrt{4k_1\omega - k_1^2 - 4k_2^2}\tan(\theta\xi)}{2a_1k_1}, \tag{39}$$

where

$$\theta = \frac{\sqrt{4k_1\omega - k_1^2 - 4k_2^2}}{2\sqrt{2}k_1^2}.$$

The parameters C_i and a_j become

$$C_0 = \frac{-2\sqrt{6}c_1^2k_1^4 - 4\sqrt{6}k_1\omega + \sqrt{6}k_1^2 + 4\sqrt{6}k_2^2}{8a_1k_1^3},$$

$$C_1 = \lambda_1, \quad C_2 = -\frac{a_1}{\sqrt{6}k_1},$$

$$a_0 = \frac{1}{2}(1 - \sqrt{6}c_1k_1), \quad a_1 = \lambda_2, \quad a_2 = 0, \quad a_3 = 0,$$

where λ_1 and λ_2 are the arbitrary constants, and hence the solution of (2) will be

$$v_4 = \frac{\sqrt{3}\theta^2\sqrt{4k_1\omega - k_1^2 - 4k_2^2}\tan(\theta\xi)\sec^4(\theta\xi)}{2k_1} \times \big(k_1\omega(55 - 17\cos(2\theta\xi)) + 4\theta^2k_1^4(\cos(2\theta\xi) - 5) + 3k_1^2(\cos(2\theta\xi) - 5) + k_2^2(17\cos(2\theta\xi) - 55)\big). \tag{40}$$

(b)

$$\psi_5(\xi) = -\frac{3\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}\, e^{\frac{\xi\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}}{\sqrt{2}k_1^2}}}{\sqrt{3}a_1k_1 e^{\frac{\xi\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}}{\sqrt{2}k_1^2}} - 1}. \tag{41}$$

The parameters C_i and a_j become

$$C_0 = 0, \quad C_1 = \frac{\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}}{\sqrt{2}k_1^2},$$

$$C_2 = \frac{a_1}{\sqrt{6}k_1},$$

$$a_0 = \frac{1}{2}\left(\frac{\sqrt{3}\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}}{k_1} + 1\right), \quad a_1 = \lambda_1,$$

$$a_2 = 0, \quad a_3 = 0,$$

where λ_1 is an arbitrary constant, and hence the solution of (2) will be

$$v_5 = -\frac{3\eta^2\lambda_1 e^{\eta\xi}\sqrt{-4k_1\omega+k_1^2+4k_2^2}}{(\sqrt{3}k_1\lambda_1 e^{\eta\xi}-1)^5} \times \Big(k_1\Big(3\sqrt{3}\lambda_1^3k_1^2e^{3\eta\xi}\big(\eta^2k_1^4+17k_1\omega -3k_1^2-17k_2^2\big)+3\lambda_1^2k_1e^{2\eta\xi}\big(11\eta^2k_1^4 +127k_1\omega-33k_1^2-127k_2^2\big) +\sqrt{3}\lambda_1e^{\eta\xi}\big(11\eta^2k_1^4+127k_1\omega-33k_1^2-127k_2^2\big) +\eta^2k_1^3-3k_1+17\omega\Big)-17k_2^2\Big), \tag{42}$$

where

$$\eta = \frac{\sqrt{-4k_1\omega+k_1^2+4k_2^2}}{\sqrt{2}k_1^2}.$$

(c)

$$\psi_6(\xi) = \frac{\sqrt{3}\sqrt{4k_1\omega-k_1^2-4k_2^2}\tan(\theta\xi)}{2a_1k_1}, \tag{43}$$

where

$$\theta = \frac{\sqrt{4k_1\omega-k_1^2-4k_2^2}}{2\sqrt{2}k_1^2}.$$

The parameters C_i and a_j become

$$C_0 = \frac{4\sqrt{6}k_1\omega-\sqrt{6}k_1^2-4\sqrt{6}k_2^2}{8a_1k_1^3},$$

$$C_1=0,\quad C_2=\frac{a_1}{\sqrt{6}k_1},$$

$$a_0=\frac{1}{2},\quad a_1=\lambda_1,\quad a_2=0,\quad a_3=0,$$

where λ_1 is an arbitrary constant, and hence the solution of (2) will be

$$v_6 = -\frac{\sqrt{3}\theta^2\sqrt{4k_1\omega-k_1^2-4k_2^2}\tan(\theta\xi)\sec^4(\theta\xi)}{2k_1} \times\big(k_1\omega(17\cos(2\theta\xi)-55)-4\theta^2k_1^4(\cos(2\theta\xi)-5) -3k_1^2(\cos(2\theta\xi)-5)+k_2^2(55-17\cos(2\theta\xi))\big). \tag{44}$$

Case III: $C_0=0, C_4=0$.
(a)

$$\psi_7(\xi) = \frac{\sqrt{3}\sqrt{\theta_2\big(-e^{\theta_1\xi/\sqrt{2}}\big)-\sqrt{3}a_2\theta_2k_1e^{\sqrt{2}\theta_1\xi}}}{\sqrt{3a_2^2k_1^2e^{\sqrt{2}\theta_1\xi}-1}}, \tag{45}$$

where

$$\theta_1 = \frac{\sqrt{-4k_1\omega+k_1^2+4k_2^2}}{k_1^2},$$

$$\theta_2 = \sqrt{-4k_1\omega+k_1^2+4k_2^2}.$$

The parameters C_i and a_j become

$$C_1 = \frac{\sqrt{-4k_1\omega+k_1^2+4k_2^2}}{2\sqrt{2}k_1^2},\quad C_2=0,\quad C_3=\frac{a_2}{2\sqrt{6}k_1},$$

$$a_0=\frac{1}{2}\left(\frac{\sqrt{3}\sqrt{-4k_1\omega+k_1^2+4k_2^2}}{k_1}+1\right),$$

$$a_1=0,\quad a_2=\lambda_1,\quad a_3=0,$$

where λ_1 is an arbitrary constant, and hence the solution of (2) will be

$$v_7 = -\frac{3\eta^2\lambda_1 e^{\eta\xi}\sqrt{-4k_1\omega+k_1^2+4k_2^2}}{(\sqrt{3}k_1\lambda_1 e^{\eta\xi}-1)^5} \times \Big(k_1\Big(3\sqrt{3}\lambda_1^3k_1^2e^{3\eta\xi}\big(\eta^2k_1^4+17k_1\omega -3k_1^2-17k_2^2\big)+3\lambda_1^2k_1e^{2\eta\xi}\big(11\eta^2k_1^4 +127k_1\omega-33k_1^2-127k_2^2\big) +\sqrt{3}\lambda_1e^{\eta\xi}\big(11\eta^2k_1^4+127k_1\omega-33k_1^2-127k_2^2\big) +\eta^2k_1^3-3k_1+17\omega\Big)-17k_2^2\Big), \tag{46}$$

where

$$\eta = \frac{\sqrt{-4k_1\omega+k_1^2+4k_2^2}}{\sqrt{2}k_1^2}.$$

(b)

$$\psi_8(\xi) = -\frac{3\sqrt{-4k_1\omega+k_1^2+4k_2^2}\,e^{\frac{\xi\sqrt{-4k_1\omega+k_1^2+4k_2^2}}{\sqrt{2}k_1^2}}}{\sqrt{3}a_1k_1e^{\frac{\xi\sqrt{-4k_1\omega+k_1^2+4k_2^2}}{\sqrt{2}k_1^2}}-1}. \tag{47}$$

The parameters C_i and a_j become

$$C_1 = \frac{\sqrt{-4k_1\omega+k_1^2+4k_2^2}}{\sqrt{2}k_1^2},\quad C_2=\frac{a_1}{\sqrt{6}k_1},\quad C_3=0,$$

$$a_0=\frac{1}{2}\left(\frac{\sqrt{3}\sqrt{-4k_1\omega+k_1^2+4k_2^2}}{k_1}+1\right),\quad a_1=\lambda_1,$$

$$a_2=0,\quad a_3=0,$$

where λ_1 is an arbitrary constant, and hence the solution of (2) will be

$$v_8 = -\frac{3\eta^2\lambda_1 e^{\eta\xi}\sqrt{-4k_1\omega+k_1^2+4k_2^2}}{(\sqrt{3}k_1\lambda_1 e^{\eta\xi}-1)^5} \times \Big(k_1\Big(3\sqrt{3}\lambda_1^3k_1^2e^{3\eta\xi}\left(\eta^2k_1^4+17k_1\omega-3k_1^2-17k_2^2\right) + 3\lambda_1^2k_1e^{2\eta\xi}\left(11\eta^2k_1^4+127k_1\omega-33k_1^2-127k_2^2\right) + \sqrt{3}\lambda_1e^{\eta\xi}\left(11\eta^2k_1^4+127k_1\omega-33k_1^2-127k_2^2\right) + \eta^2k_1^3-3k_1+17\omega\Big)-17k_2^2\Big), \tag{48}$$

where

$$\eta = \frac{\sqrt{-4k_1\omega+k_1^2+4k_2^2}}{\sqrt{2}k_1^2}.$$

(c)

$$\psi_9(\xi) = \frac{3\sqrt{-4k_1\omega+k_1^2+4k_2^2}}{\sqrt{3}a_2k_1 - e^{\frac{\xi\sqrt{-4k_1\omega+k_1^2+4k_2^2}}{2\sqrt{2}k_1^2}}}. \tag{49}$$

The parameters C_i and a_j become

$$C_1 = -\frac{\sqrt{-4k_1\omega+k_1^2+4k_2^2}}{2\sqrt{2}k_1^2},$$

$$C_2 = \frac{a_2}{2\sqrt{6}k_1}, \quad C_3 = 0,$$

$$a_0 = \frac{1}{2}\left(1-\frac{\sqrt{3}\sqrt{-4k_1\omega+k_1^2+4k_2^2}}{k_1}\right),$$

$$a_1 = 0, \quad a_2 = \lambda_1, \quad a_3 = 0,$$

where λ_1 is an arbitrary constant, and hence the solution of (2) will be

$$v_9 = \frac{9\eta_1^2\eta_2\lambda_1 e^{\eta_1\xi/\sqrt{2}}}{2(e^{\eta_1\xi/\sqrt{2}}-\sqrt{3}k_1\lambda_1)^8} \times \Big(-6k_1^2\lambda_1^2e^{\sqrt{2}\eta_1\xi}\Big(378\eta_2^{3/2}+\sqrt{3}k_1\lambda_1\left(\eta_1^2k_1^4 - 2k_1\omega+3k_1^2+2k_2^2\right) - 9\sqrt{3}\eta_2k_1\lambda_1\Big) + 18k_1^2\lambda_1^2e^{\eta_1\xi/\sqrt{2}}\Big(-324\eta_2^2 + k_1^2\lambda_1^2\left(8\eta_1^2k_1^4+2k_1\omega-3k_1^2-2k_2^2\right) + 36\sqrt{3}\eta_2^{3/2}k_1\lambda_1+9\eta_2k_1^2\lambda_1^2\Big) + \sqrt{3}k_1\lambda_1e^{2\sqrt{2}\eta_1\xi} \times \left(63\eta_2+17\eta_1^2k_1^4+14k_1\omega-21k_1^2-14k_2^2\right) + 24k_1\lambda_1e^{3\eta_1\xi/\sqrt{2}} \times \Big(18\sqrt{3}\eta_2^{3/2} + k_1\lambda_1\left(-5\eta_1^2k_1^4-2k_1\omega+3k_1^2+2k_2^2\right) - 9\eta_2k_1\lambda_1\Big) + 9k_1^3\lambda_1^3\Big(-108\sqrt{3}\eta_2^2+\sqrt{3}k_1^2\lambda_1^2\left(-9\eta_2+\eta_1^2k_1^4 - 2k_1\omega+3k_1^2+2k_2^2\right)+108\eta_2^{3/2}k_1\lambda_1\Big) + 2e^{5\eta_1\xi/\sqrt{2}}\left(-9\eta_2+4\eta_1^2k_1^4 -2k_1\omega+3k_1^2+2k_2^2\right)\Big), \tag{50}$$

where

$$\eta_1 = \frac{\sqrt{-4k_1\omega+k_1^2+4k_2^2}}{2k_1^2},$$

$$\eta_2 = -4k_1\omega+k_1^2+4k_2^2.$$

Case IV: $C_0=0, C_2=0, C_4=0$.
(a)

$$\psi_{10}(\xi) = \frac{\sqrt{3}\sqrt{\theta_2(-e^{\theta_1\xi/\sqrt{2}})-\sqrt{3}a_2\theta_2k_1}}{\sqrt{e^{\sqrt{2}\theta_1\xi}-3a_2^2k_1^2}}, \tag{51}$$

where

$$\theta_1 = \frac{\sqrt{-4k_1\omega+k_1^2+4k_2^2}}{k_1^2},$$

$$\theta_2 = \sqrt{-4k_1\omega+k_1^2+4k_2^2}.$$

The parameters C_i and a_j become

$$C_1 = -\frac{\sqrt{-4k_1\omega+k_1^2+4k_2^2}}{2\sqrt{2}k_1^2}, \quad C_3 = \frac{a_2}{2\sqrt{6}k_1},$$

$$a_0 = \frac{1}{2}\left(1-\frac{\sqrt{3}\sqrt{-4k_1\omega+k_1^2+4k_2^2}}{k_1}\right),$$

$$a_1 = 0, \quad a_2 = \lambda_1, \quad a_3 = 0,$$

where λ_1 is an arbitrary constant, and hence the solution of (2) will be

$$v_{10} = \frac{3\theta_1^2\theta_2\lambda_1e^{\theta_1\xi/\sqrt{2}}}{4(e^{\theta_1\xi/\sqrt{2}}-\sqrt{3}k_1\lambda_1)^5} \times \Big(-108\theta_2k_1^2\lambda_1^2e^{\theta_1\xi/\sqrt{2}}\sqrt{-4k_1\omega+k_1^2+4k_2^2} - 3k_1^2\lambda_1^2e^{\theta_1\xi/\sqrt{2}}\left(-108\theta_2^2+11\theta_1^2k_1^4 - 34k_1\omega+6k_1^2+34k_2^2\right)$$

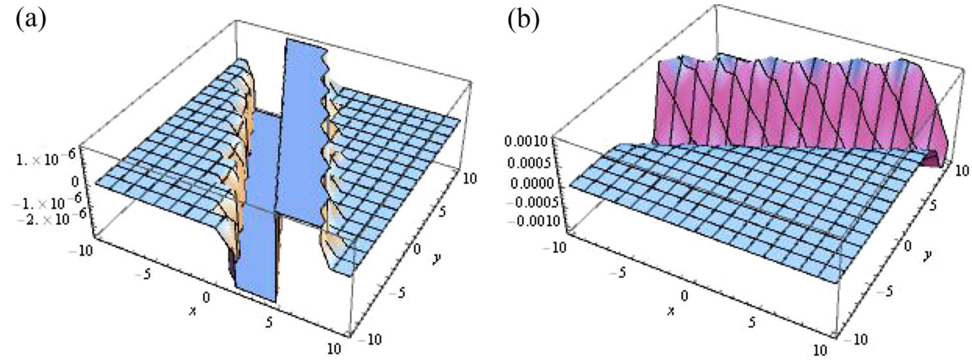

Figure 1. The solution of $(3+1)$-dimensional generalised KP equation (Case I) plotted as: (**a**) $u_1(x, y, z, t)$: $\lambda_2 = 4, k_1 = 2, k_2 = 1, k_3 = 3, \omega = 4, t = 1, z = 1$; (**b**) $u_2(x, y, z, t)$: $\lambda_2 = 3, k_1 = 2, k_2 = -4, k_3 = 1, \omega = 1, t = 1, z = 1$.

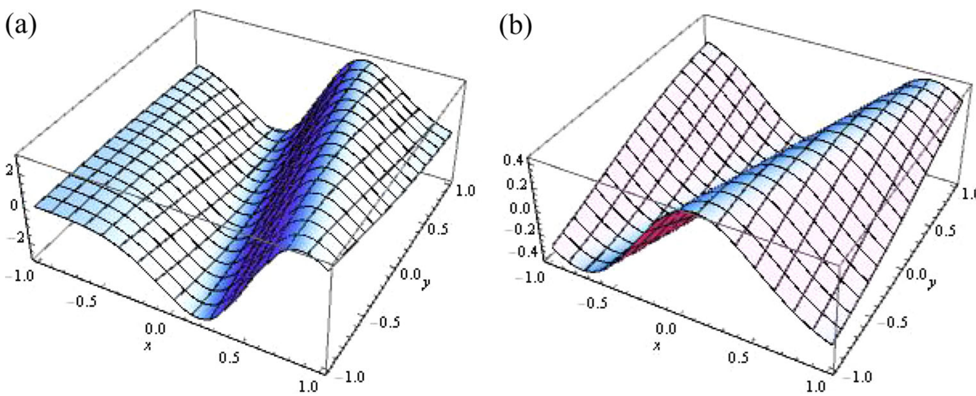

Figure 2. The solution of $(3+1)$-dimensional generalised KP equation (Case II) plotted as: (**a**) $u_3(x, y, z, t)$: $k_1 = 4, k_2 = 1, k_3 = 3, \omega = 4, t = 1, z = 1$; (**b**) $u_4(x, y, z, t)$: $k_1 = 3, k_2 = -1, k_3 = 2, \omega = 2, t = 1, z = 1$.

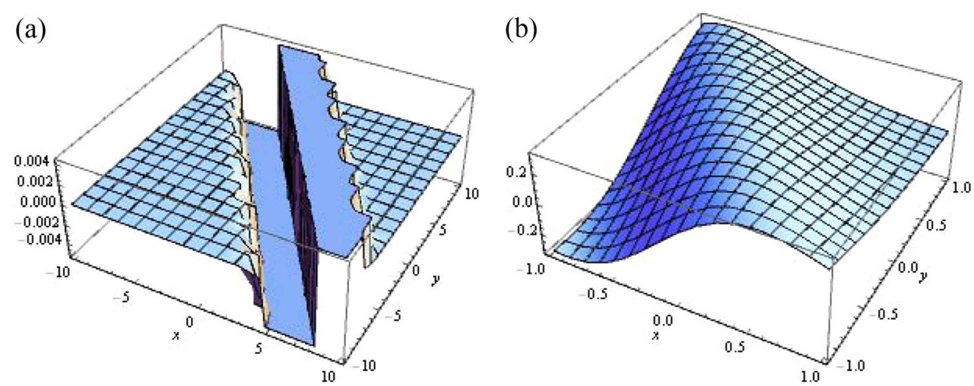

Figure 3. The solution of $(3+1)$-dimensional generalised KP equation (Case III) plotted as: (**a**) $u_5(x, y, z, t)$: $\lambda_1 = -2, k_1 = 3, k_2 = 2, k_3 = 1, \omega = 2, t = 1, z = 1$; (**b**) $u_6(x, y, z, t)$: $\lambda_1 = -2, k_1 = 3, k_2 = 2, k_3 = 1, \omega = 2, t = 1, z = 1$.

$$
\begin{aligned}
&- \sqrt{3}k_1\lambda_1 \mathrm{e}^{\sqrt{2}\theta_1\xi}\left(11\theta_1^2k_1^4 - 34k_1\omega\right.\\
&\left.+ 6k_1^2 + 34k_2^2\right)\\
&+ 72\sqrt{3}\theta_2k_1\lambda_1\mathrm{e}^{\sqrt{2}\theta_1\xi}\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}\\
&- 108\sqrt{3}\theta_2k_1^3\lambda_1^3\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}\\
&- 3\sqrt{3}k_1^3\lambda_1^3\left(-36\theta_2^2 + \theta_1^2k_1^4\right.\\
&\left.+ 34k_1\omega - 6k_1^2 - 34k_2^2\right) + \mathrm{e}^{3\theta_1\xi/\sqrt{2}}\\
&\left.\times\left(-\theta_1^2k_1^4 - 34k_1\omega + 6k_1^2 + 34k_2^2\right)\right).
\end{aligned}
\tag{52}
$$

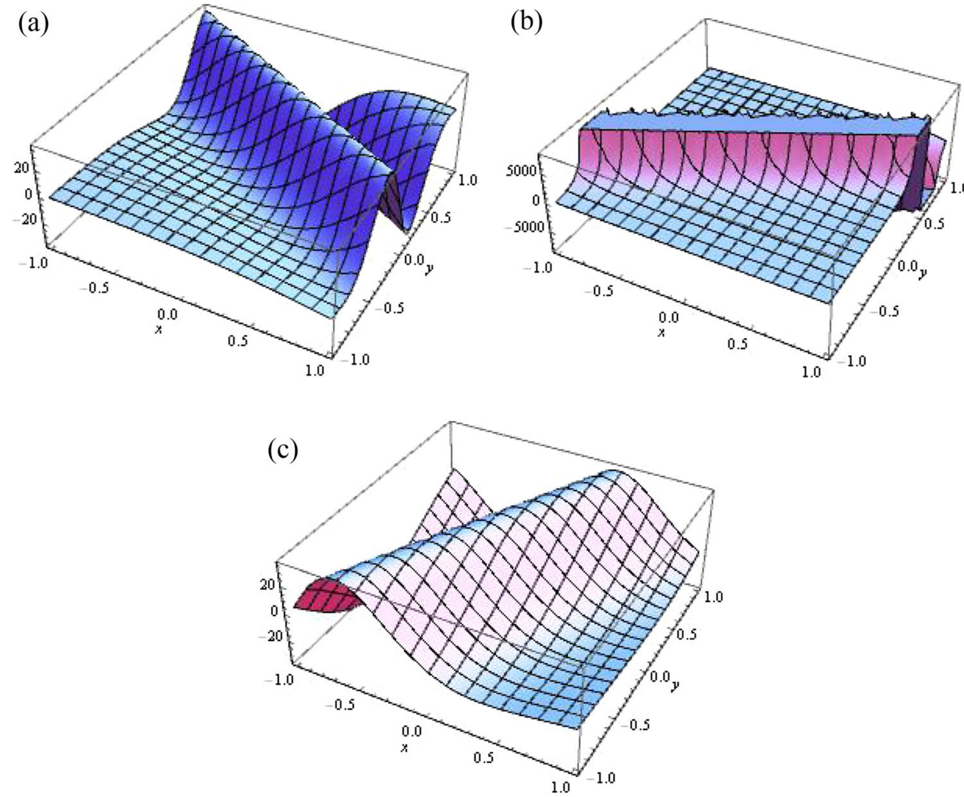

Figure 4. The solution of (2 + 1)-dimensional Gardner–KP equation (Case I) plotted as: (**a**) $v_1(x, y, t)$: $\lambda_1 = 1$, $k_1 = 3$, $k_2 = 5$, $\omega = 3$, $t = 1$; (**b**) $v_2(x, y, t)$: $\lambda_1 = 2$, $k_1 = 2$, $k_2 = -4$, $\omega = 1$, $t = 1$; (**c**) $v_3(x, y, t)$: $\lambda_1 = 2$, $k_1 = -2$, $k_2 = 1$, $\omega = 3$, $t = 1$.

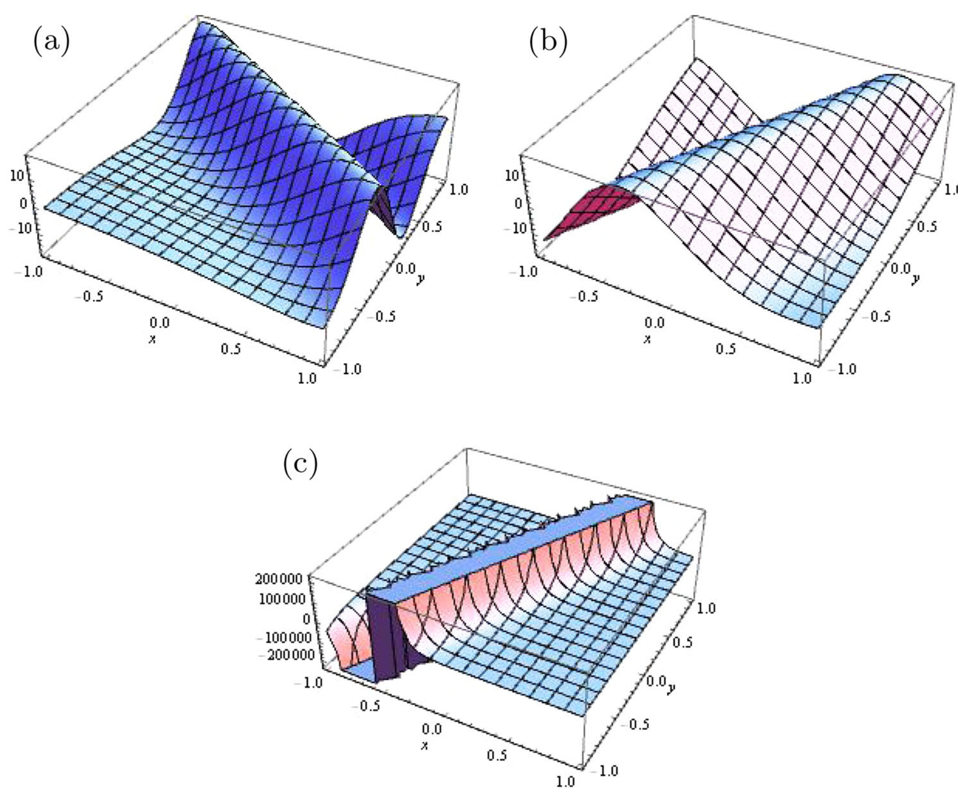

Figure 5. The solution of (2 + 1)-dimensional Gardner–KP equation (Case II) plotted as: (**a**) $v_4(x, y, t)$: $k_1 = 2$, $k_2 = 3$, $\omega = 2$, $t = 1$; (**b**) $v_5(x, y, t)$: $\lambda_1 = 1$, $k_1 = -2$, $k_2 = 1$, $\omega = 2$, $t = 1$; (**c**) $v_6(x, y, t)$: $k_1 = 2$, $k_2 = -1$, $\omega = 2$, $t = 3$.

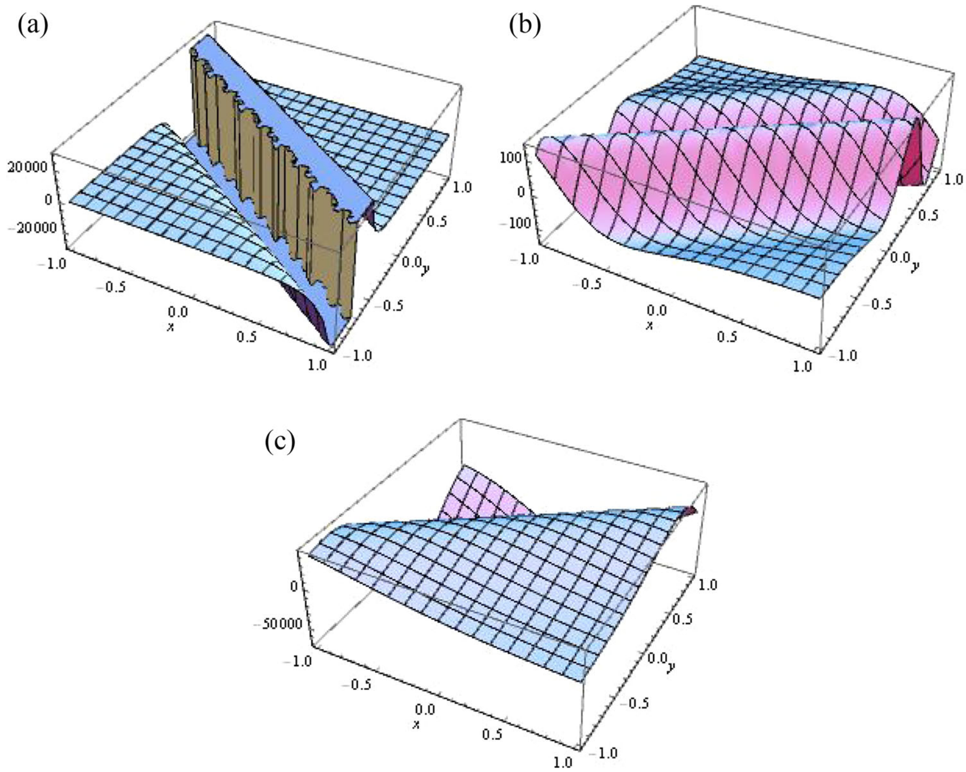

Figure 6. The solution of (2 + 1)-dimensional Gardner–KP equation (Case III) plotted as: (**a**) $v_7(x, y, t)$: $\lambda_1 = 2$, $k_1 = 2$, $k_2 = 3$, $\omega = 2$, $t = 1$; (**b**) $v_8(x, y, t)$: $\lambda_1 = 5$, $k_1 = -2$, $k_2 = 3$, $\omega = 2$, $t = 1$; (**c**) $v_9(x, y, t)$: $\lambda_1 = 1$, $k_1 = -2$, $k_2 = 3$, $\omega = 2$, $t = 1$.

(b)

$$\psi_{11}(\xi) = \frac{\sqrt{3}\sqrt{\theta_2(-e^{\theta_1\xi/\sqrt{2}}) - \sqrt{3}a_2\theta_2 k_1 e^{\sqrt{2}\theta_1\xi}}}{\sqrt{3a_2^2k_1^2e^{\sqrt{2}\theta_1\xi} - 1}}, \tag{53}$$

where

$$\theta_1 = \frac{\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}}{k_1^2},$$

$$\theta_2 = \sqrt{-4k_1\omega + k_1^2 + 4k_2^2}.$$

The parameters C_i and a_j become

$$C_1 = \frac{\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}}{2\sqrt{2}k_1^2}, \quad C_3 = \frac{a_2}{2\sqrt{6}k_1},$$

$$a_0 = \frac{1}{2}\left(\frac{\sqrt{3}\sqrt{-4k_1\omega + k_1^2 + 4k_2^2}}{k_1} + 1\right),$$

$$a_1 = 0, \quad a_2 = \lambda_1, \quad a_3 = 0,$$

where λ_1 is an arbitrary constant, and hence the solution of (2) will be

$$\begin{aligned} v_{11} = {} & \frac{3\theta_2^3\lambda_1 e^{\theta_2\xi/\sqrt{2}k_1^2}}{4k_1^4(\sqrt{3}k_1\lambda_1 e^{\theta_2\xi/\sqrt{2}k_1^2} - 1)^5} \\ & \times \big(8\theta_2^2 + k_1\big(- 3\sqrt{3}k_1^2\lambda_1^3 e^{3\theta_2\xi/\sqrt{2}k_1^2}\big(-8\theta_2^2 - 2k_1\omega \\ & + 3k_1^2 + 2k_2^2\big) + 3k_1\lambda_1^2 e^{\sqrt{2}\theta_2\xi/k_1^2}\big(52\theta_2^2 - 2k_1\omega \\ & + 3k_1^2 + 2k_2^2\big) + \sqrt{3}\lambda_1 e^{\theta_2\xi/\sqrt{2}k_1^2}\big(52\theta_2^2 - 2k_1\omega \\ & + 3k_1^2 + 2k_2^2\big) - 3k_1 + 2\omega\big) - 2k_2^2\big). \end{aligned} \tag{54}$$

4. Discussion and results

The graphical representation of solitons has been illustrated in the following figures for various values of the parameters. Mathematica 10.4 is used to carry out simulations and to visualise the behaviour of nonlinear waves.

In Case I, the solution for eq. (1) is shown in figure 1, obtained from eq. (14) with $\lambda_2 = 4$, $k_1 = 2$, $k_2 = 1$, $k_3 = 3$, $\omega = 4$, $t = 1$, $z = 1$ and eq. (16) with $\lambda_2 = 3$, $k_1 = 2$, $k_2 = -4$, $k_3 = 1$, $\omega = 1$, $t = 1$, $z = 1$, while

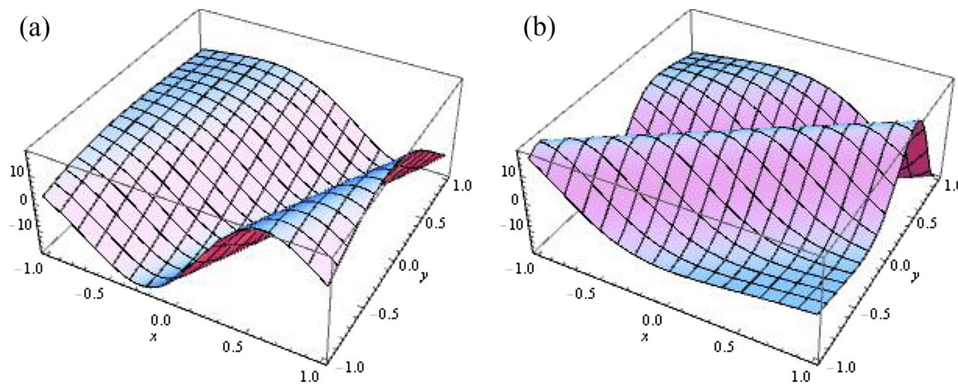

Figure 7. The solution of $(2+1)$-dimensional Gardner–KP equation (Case IV) plotted as: (**a**) $v_{10}(x, y, t)$: $\lambda_1 = -5$, $k_1 = 2$, $k_2 = -1$, $\omega = -2$, $t = 1$; (**b**) $v_{11}(x, y, t)$: $\lambda_1 = -2$, $k_1 = 2$, $k_2 = -3$, $\omega = 2$, $t = 1$.

in Case II, the solution for eq. (1) is shown in figure 2, obtained from eq. (18) with $k_1 = 4$, $k_2 = 1$, $k_3 = 3$, $\omega = 4$, $t = 1$, $z = 1$ and eq. (20) with $k_1 = 4$, $k_2 = 1$, $k_3 = 3$, $\omega = 4$, $t = 1$, $z = 1$. In Case III, the solution for eq. (1) is shown in figure 3, obtained from eq. (24) with $\lambda_1 = -2$, $k_1 = 3$, $k_2 = 2$, $k_3 = 1$, $\omega = 2$, $t = 1$, $z = 1$ and (28) with $\lambda_1 = -2$, $k_1 = 3$, $k_2 = 2$, $k_3 = 1$, $\omega = 2$, $t = 1$, $z = 1$.

Similarly, the solution for eq. (2) for Case I is shown in figure 4, obtained from eq. (34) with $\lambda_1 = 1$, $k_1 = 3$, $k_2 = 5$, $\omega = 3$, $t = 1$; (36) with $\lambda_1 = 2$, $k_1 = 2$, $k_2 = -4$, $\omega = 1$, $t = 1$ and eq. (38) with $\lambda_1 = 2$, $k_1 = -2$, $k_2 = 1$, $\omega = 3$, $t = 1$, while in Case II, the solution for eq. (2) is shown in figure 5, obtained from eq. (40) with $k_1 = 2$, $k_2 = 3$, $\omega = 2$, $t = 1$; (42) with $\lambda_1 = 1$, $k_1 = -2$, $k_2 = 1$, $\omega = 2$, $t = 1$ and eq. (44) with $k_1 = 2$, $k_2 = -1$, $\omega = 2$, $t = 3$. Moreover, in Case III, the solution for eq. (2) is shown in figure 6, obtained from eq. (46) with $\lambda_1 = 2$, $k_1 = 2$, $k_2 = 3$, $\omega = 2$, $t = 1$; (48) with $\lambda_1 = 5$, $k_1 = -2$, $k_2 = 3$, $\omega = 2$, $t = 1$ and eq. (50) with $\lambda_1 = 1$, $k_1 = -2$, $k_2 = 3$, $\omega = 2$, $t = 1$, while in Case IV, the solution for eq. (2) is shown in figure 7, obtained from eq. (52) with $\lambda_1 = -5$, $k_1 = 2$, $k_2 = -1$, $\omega = -2$, $t = 1$ and eq. (54) with $\lambda_1 = -2$, $k_1 = 2$, $k_2 = -3$, $\omega = 2$, $t = 1$.

5. Conclusion

The aim of the study is to find some new travelling wave solutions for $(3+1)$-dimensional generalised KP and $(2+1)$-dimensional Gardner–KP model equations. The auxiliary equation method is observed as one of the most powerful tools to find a variety of analytical solutions for more complex problems. Depending on the real parameters, a collection of new exact solutions are obtained. These results are very favourable for further investigation and have a strong basis for the solution of NPDEs.

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