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Nonlinear propagation of ion plasma waves in dust-ion plasma including quantum-relativistic effect

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Abstract. In this paper we have theoretically investigated the quantum and relativistic effects on ion plasma wave in an unmagnetised dust-ion plasma. By using the method of normal mode analysis, we have obtained a linear dispersion relation. It has been analysed numerically for quantum and relativistic effects on the propagation of ion plasma wave. By using the standard reductive perturbation technique, we have derived a Korteweg–de Vries (KdV) equation which describes the nonlinear propagation of the wave. Numerically, it is shown that only compressive type of soliton can exist in the plasma under consideration. It is found that the solitary wave profile depends significantly on the quantum and relativistic parameters. The dust size, dust charge and the dust number density are also shown to have significant influences on these solitary waves. The results of this present investigation have some relevance to the nonlinear propagation of ion plasma wave in some astrophysical, space and laboratory plasma environments.

Keywords. Relativistic effect; quantum plasma; ion plasma wave; ion streaming.

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1. Introduction

In recent years, dusty plasma has become an important area of plasma research. Dusty plasma is ubiquitous in astrophysical, space and in some laboratory plasma environments. Relativistic and quantum effects in plasma are a relatively new and rapidly growing field of research in plasma physics. The relativistic and quantum effects in plasmas can be important in many practical situations such as in metal nanostructures [1], space plasma [2,3], Van Allen radiation belts [4], laser–solid interaction experiments [5], cool vibes [6], semiconductor devices [7,8], dense astrophysical environments [3,9], etc. Many researchers have reported the linear and nonlinear behaviour of waves in dusty plasma by considering classical non-relativistic plasma [10–12]. Rao *et al* [13] first predicted theoretically the propagation of dust-acoustic wave in a dusty plasma. Here the dust grains provide the inertia and pressure of electrons and the ions provide the necessary restoring force. In some dusty plasmas, it may so happen that almost all the electrons in the plasma system may get attached to the surface of the dust grains. In this situation, we have a dust-ion plasma. Such a plasma has practical existence in Saturn's F-ring [14], Halley's Comet [15] and some laboratory plasma experiments [16].

In dust-ion plasma, heavier dust particles may be assumed to be at rest and provide a uniform neutralising background. If ions are displaced from their equilibrium position, an electrostatic field is generated due to charge separation and it tries to bring the ions back to their original position. Thus, we can have an ion oscillation similar to the electron oscillation in a two-component electron–ion plasma. These oscillations lead to the propagation of a wave mode which is the ion plasma wave mode.

For lower number density and higher temperature, the plasma can be treated classically and the quantum effect is negligible. However, ion number density in some dusty plasma may become very high and plasma temperature may be relatively low. In such dusty plasma, the quantum effect cannot be ignored. It may significantly influence the linear and nonlinear propagation of the wave. On the other hand, the relativistic effect may become important in a variety of environments and laboratories where the ion velocity is very high. The relativistic effect is also likely to influence the linear and nonlinear propagation of the wave in the dust-ion plasma. The importance of relativistic effect for the formation of ion-acoustic solitary waves in the presence of particle streaming was first pointed out by Das and Paul [17]. Recently, some researchers [18–23] have considered the relativistic effect on different types of waves and obtained a few interesting results relevant to some space and laboratory plasmas. Gill *et al* [24] have studied the relativistic effect on ion-acoustic solitons in electron–positron–ion (epi) plasma. Many works on relativistic effect on the waves in dusty plasma have been carried out by considering the classical plasma. But, for high-density and low-temperature plasmas, the thermal de-Broglie wavelength of plasma particles may become comparable to the interparticle distances. In such situations, wave functions of the neighbouring particles may overlap and the quantum effect becomes important. Quantum effects in plasma cannot be neglected in many situations such as dense astrophysical and cosmological plasmas and laser–solid interaction experiments [25,26]. Ali and Shukla [27] have studied the nonlinear properties of dust-acoustic wave in unmagnetised quantum dusty plasma with inertial charged dust particles. Only a few works on the relativistic effect in quantum plasma have been reported so far. Recently, by considering relativistic electrons and non-relativistic ions, Sahu [28] has studied the relativistic effect on ion-acoustic solitary waves in quantum plasma in the weak relativistic limit. Gill *et al* [29] have investigated the quantum-relativistic effect on ion-acoustic shock waves in epi plasma. The relativistic effect on the characteristics of the electron plasma waves in quantum plasma has been investigated by Ghosh *et al* [30]. Electrons, because of their lighter mass, attain relativistic speed faster than ions. However, there are some practical environments where ions can also attain relativistic speed. For example, in laser–plasma interaction experiment, ions, because of their collective effect in plasma, can gain relativistic speed at relatively less laser intensity compared with the direct laser acceleration of ions [31]. Compact astrophysical objects such as neutron stars, white dwarfs and block holes are sources of energetic plasma flow. As the surrounding plasma cloud is attracted towards the compact object, ions are accelerated to relativistic speeds [32–34]. In solar flare and pulsar wind, where the ion temperature is very high, the relativistic ion motion becomes important [35]. Different space plasma observations indicate that the presence of large-amplitude plasma wave can drive ions to relativistic speed. An example of such a situation is the pulsar relativistic wind in which the strong electromagnetic radiation drives ion to relativistic speed [36].

Stenflo and Tsintsadze [37] have shown that even a small-amplitude wave, in the presence of an external magnetic field, can make the ion motion relativistic. Stenflo and Shukla [38] have shown that the inclusion of relativistic ion motion in dust-ion plasma leads to the generation of low-frequency circularly polarised electromagnetic wave. Thus, the inclusion of relativistic ion motion in dust-ion plasma becomes important.

A survey of the past literature shows that no work has been reported on the ion plasma waves in dust-ion plasma including the combined effects of quantum nature and relativistic motion of plasma particles. The purpose of the present work is to study quantum-relativistic effect on the propagation of ion wave in dust-ion plasma.

This paper is organised as follows: in §2 the basic equations governing the dynamics of the waves have been presented. In §3 we have derived a general type of linear dispersion relation. In §4 we have made nonlinear analysis for the wave propagation. In §5 we have presented and discussed the results. Finally, in §6, we give some concluding remarks.

2. Basic equations

Let us consider a two-component homogeneous quantum and relativistic plasma containing negatively charged dust grains and positively charged ions. In this two-component dust-ion plasma, we assume that almost all the electrons from the surrounding plasma are attached to the dust grains and we have a two-component dust-ion plasma which has relevance to some space and astrophysical environments. We assume the dust particles to be cold and ions streaming with some constant velocity. The basic equations that govern the dynamics of one-dimensional ion plasma wave in this two-component dust-ion plasma are the following:

$$\frac{\partial(\gamma_i n_i)}{\partial t} + \frac{\partial(\gamma_i n_i u_i)}{\partial x} = 0, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + u_i\frac{\partial}{\partial x}\right)\gamma_i u_i = -\frac{e}{m_i}\cdot\frac{\partial\phi}{\partial x} - \frac{1}{m_i n_i}\cdot\frac{\partial p_i}{\partial x} + \frac{\hbar^2}{2m_i^2}\cdot\frac{\partial}{\partial x}\left[\frac{\partial^2\sqrt{n_i}}{\partial x^2}\Big/\sqrt{n_i}\right], \tag{2}$$

$$\frac{\partial n_d}{\partial t} + \frac{\partial(n_d u_d)}{\partial x} = 0, \tag{3}$$

$$\left(\frac{\partial}{\partial t} + u_d\frac{\partial}{\partial x}\right)u_d = \frac{z_d e}{m_d}\cdot\frac{\partial\phi}{\partial x}, \tag{4}$$

$$\frac{\partial^2\phi}{\partial x^2} = \frac{e}{\varepsilon_0}\left(z_d n_d - \gamma_i n_i\right), \tag{5}$$

$$z_d n_{d0} = \gamma_i n_{i0}, \tag{6}$$

$$\gamma_i = \left(1 - \frac{u_i^2}{c^2}\right)^{-(1/2)} \approx 1 + \frac{u_i^2}{2 \cdot c^2}, \tag{7}$$

where n_i, m_i and u_i are the number density, mass and velocity of the ions, respectively, and n_d, m_d and u_d are the corresponding quantities of the dust; z_d, ϕ, ε_0 and $\hbar$ denote the number of electrons attached to the dust grain, electric potential, free space permittivity and the Planck constant divided by 2π, respectively; p_i is the pressure of the ions, n_{i0} is the equilibrium number density of the ions and eq. (6) represents the quasineutrality condition. In eq. (7) we have assumed a weakly relativistic limit. The plasma particles are assumed to behave as one-dimensional Fermi gas at zero temperature. So they will obey the following pressure law [39,40]:

$$p_i = \frac{m_i v_{Fi}^2}{3n_{i0}^2} \cdot n_i^3 = \frac{2k_B T_{Fi}}{3n_{i0}^2} \cdot n_i^3, \tag{8}$$

where k_B is the Boltzmann constant, T_{Fi} is the ion Fermi temperature and v_{Fi} is the ion Fermi velocity. Note that eq. (2) is the hydrodynamic equation of motion for the quantum-relativistic ion fluid. The factor γ_i takes care of the relativistic variation of ion mass. The second term on the right-hand side represents the force arising out of quantum-statistical effect and the third term on the right-hand side is a quantum force related to the quantum Bohm potential [41]. Now, after normalisation eqs (1)–(5) can be rewritten in the following form:

$$\frac{\partial(\gamma_i n_i)}{\partial t} + \frac{\partial(\gamma_i n_i u_i)}{\partial x} = 0, \tag{9}$$

$$\left(\frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x}\right)\gamma_i u_i = -\frac{\partial \phi}{\partial x} - n_i \cdot \frac{\partial n_i}{\partial x} + \frac{H^2}{2} \cdot \frac{\partial}{\partial x}\left[\frac{\partial^2 \sqrt{n_i}}{\partial x^2} \Big/ \sqrt{n_i}\right], \tag{10}$$

$$\frac{\partial n_d}{\partial t} + \frac{\partial(n_d u_d)}{\partial x} = 0, \tag{11}$$

$$\left(\frac{\partial}{\partial t} + u_d \frac{\partial}{\partial x}\right) Q u_d = z_d \cdot \frac{\partial \phi}{\partial x}, \tag{12}$$

$$\frac{\partial^2 \phi}{\partial x^2} = (z_d n_d - \gamma_i n_i), \tag{13}$$

where all the velocities are given by the quantum ion-acoustic speed $c_{is} = \sqrt{2k_B T_{Fi}/m_i}$, time ($t$) by the inverse of ion plasma frequency, $\omega_{pi} = \left(e^2 n_{i0}/m_i \varepsilon_0\right)^{1/2}$, potential ϕ by $2k_B T_{Fi}/e$ and all number densities by n_{i0}. In the above equation $Q = m_d/m_i$ is the normalised dust mass and $H = \hbar\omega_{pi}/2k_B T_{Fi}$ is a non-dimensional quantum parameter which is a measure of quantum diffraction effect.

3. Linear wave

In order to examine the linear characteristics of ion plasma wave, the following perturbation expansion is made for the field quantities $F(n_i, n_d, u_i, u_d, \phi)$:

$$F = F_0 + F_1 + \cdots, \tag{14}$$

where $F_0(1, n_{d0}, u_{i0}, 0, 0,)$ represents the equilibrium values and $F_1(n_{i1}, n_{d1}, u_{i1}, u_{d1}, \phi_1)$ represents the first-order perturbed values. Substituting perturbation expansion (14) into eqs (9)–(13) and linearising, we obtain

$$\frac{\partial n_{i1}}{\partial t} + \frac{u_{i0}}{c^2}\frac{\partial u_{i1}}{\partial t} + \frac{u_{i0}^2}{2 \cdot c^2}\frac{\partial n_{i1}}{\partial t} + \frac{\partial u_{i1}}{\partial x} + u_{i0} \cdot \frac{\partial n_{i1}}{\partial x} + \frac{3}{2} \cdot \frac{u_{i0}^2}{c^2}\frac{\partial u_{i1}}{\partial x} + \frac{u_{i0}^3}{2 \cdot c^2}\frac{\partial n_{i1}}{\partial x} = 0, \tag{15}$$

$$\frac{\partial u_{i1}}{\partial t} + \frac{3}{2} \cdot \frac{u_{i0}^2}{c^2}\frac{\partial u_{i1}}{\partial t} + u_{i0} \cdot \frac{\partial n_{i1}}{\partial x} + u_{i0} \cdot \frac{3}{2} \cdot \frac{u_{i0}^2}{c^2}\frac{\partial u_{i1}}{\partial x} = -\frac{\partial \phi}{\partial x} - \frac{\partial n_{i1}}{\partial x} + \frac{H^2}{4} \cdot \frac{\partial^3 n_{i1}}{\partial x^3}, \tag{16}$$

$$\frac{\partial n_{d1}}{\partial t} + u_{d1} \cdot \frac{\partial n_{d1}}{\partial x} + (n_{d0} + n_{d1})\frac{\partial u_{d1}}{\partial x} = 0, \tag{17}$$

$$\frac{\partial u_{d1}}{\partial t} + u_{d1} \cdot \frac{\partial u_{d1}}{\partial x} + = \frac{z_d}{Q}\frac{\partial \phi_1}{\partial x}, \tag{18}$$

$$\frac{\partial^2 \phi_1}{\partial x^2} = z_d n_{d1} - \gamma_{i0} n_{i1} - \frac{u_{i0} \cdot u_{i1}}{c^2}, \tag{19}$$

where we have used the weakly relativistic limit given by eq. (7). In order to consider plain-wave propagation, the field parameters are assumed to vary harmonically with space and time according to $\exp[i(\vec{k} \cdot \vec{r} - \omega t)]$, where ω is the normalised wave frequency and k is the wave number. Using the standard normal mode analysis, we obtain the following linear dispersion relation for the propagation of ion plasma wave:

$$\left[k\beta\left(\frac{\omega\sigma^2}{u_{i0}} - k\alpha\right) + \alpha\gamma_{i0}(u_{i0}k - \omega)^2\right] \cdot \left(Q\omega^2 - n_{d0}z_d^2\right) = \omega^2 Q \gamma_{i0}^2, \tag{20}$$

where

$$\beta = 1 + \frac{k^2 H^2}{4}, \quad \alpha = 1 + \frac{3}{2} \cdot \frac{u_{i0}^2}{c^2} \quad \text{and} \quad \sigma = \frac{u_{i0}}{c}.$$

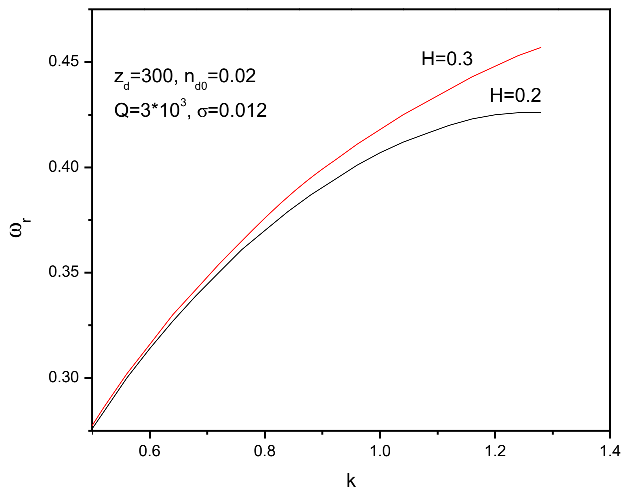

Figure 1. Quantum effect on the linear dispersion of ion plasma wave.

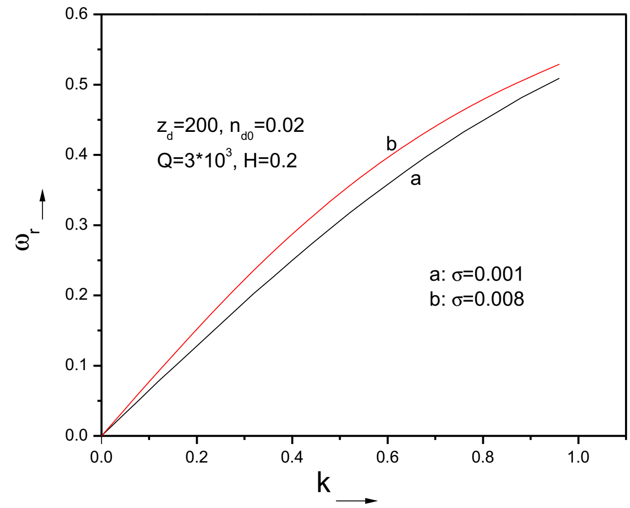

Figure 2. Relativistic effect on the linear dispersion of ion plasma wave.

The dispersion law (20) can also be written as

$$A\omega^4 + B\omega^3 + C\omega^2 + D\omega + E = 0, \tag{21}$$

where $A = Q = m_{\mathrm{d}}/m_{\mathrm{i}}$, $B = (Qk/u_{\mathrm{i0}})(\beta\sigma^2 - 2\alpha\gamma_{\mathrm{i0}}u_{\mathrm{i0}}^2)$, $C = Qk^2\alpha(\gamma_{\mathrm{i0}}u_{\mathrm{i0}}^2 - \beta) - \gamma_{\mathrm{i0}}(z_{\mathrm{d}}^2 n_{\mathrm{d0}}\alpha + Q\gamma_{\mathrm{i0}})$, $D = ((z_{\mathrm{d}}^2 n_{\mathrm{d0}}k)/u_{\mathrm{i0}})(2\alpha\gamma_{\mathrm{i0}}u_{\mathrm{i0}}^2 - \beta\sigma^2)$ and $E = z_{\mathrm{d}}^2 n_{\mathrm{d0}}k^2\alpha(\beta - \gamma_{\mathrm{i0}}u_{\mathrm{i0}}^2)$.

An examination of eq. (21) reveals that the dispersion character of ion plasma wave depends on both the quantum and relativistic effects in an intricate way. To study the dependence of the dispersive character of the wave on quantum and relativistic parameters, we have numerically analysed eq. (21) by using typical plasma parameters appropriate to Saturn's F-ring and Halley's Comet. Equation (21) is a fourth-degree equation in ω and has four roots. In the parametric region under consideration, we find two negative roots which are unphysical; one positive root greater than ω_{pi} and another positive root smaller than ω_{pi}. As we are interested in the low-frequency ion plasma wave mode, for our study, we consider that the root is smaller than ω_{pi}.

In figure 1 we show the quantum effect on the linear dispersion characteristic of ion plasma wave. It shows that as the quantum diffraction parameter H increases, the wave frequency also increases. In figure 2 we show the relativistic effect on the linear dispersion character of the ion plasma wave. It shows that as the relativistic parameter σ increases, the wave frequency also increases.

4. Nonlinear wave

In order to study the nonlinear propagation of the wave, the following stretched coordinates are introduced:

$$\eta = \varepsilon^{1/2}(x - vt) \quad \text{and} \quad \tau = \varepsilon^{3/2}t, \tag{22}$$

where ε is a smallness parameter. Following the standard procedure, eqs (9)–(13) are first written in terms of the stretched coordinates η and τ. Then the following perturbation expansion is introduced for the field quantities $X(n_{\mathrm{i}}, n_{\mathrm{d}}, u_{\mathrm{i}}, u_{\mathrm{d}}, \phi)$:

$$X = X_0 + \varepsilon X_1 + \varepsilon^2 X_2 + \varepsilon^3 X_3 + \cdots, \tag{23}$$

where $X_0(1, n_{\mathrm{d0}}, u_{\mathrm{i0}}, 0, 0,)$ stands for the equilibrium values of the field quantities: $X_1(n_{\mathrm{i1}}, n_{\mathrm{d1}}, u_{\mathrm{i1}}, u_{\mathrm{d1}}, \phi_1)$, $X_2(n_{\mathrm{i2}}, n_{\mathrm{d2}}, u_{\mathrm{i2}}, u_{\mathrm{d2}}, \phi_2)$ and $X_3(n_{\mathrm{i3}}, n_{\mathrm{d3}}, u_{\mathrm{i3}}, u_{\mathrm{d3}}, \phi_3)$ stand for the first-order, second-order and third-order perturbed quantities, respectively. Now, by solving the lowest-order equations in ε with the boundary condition $\phi_1 \to 0$ as $|\eta| \to \infty$, we obtain the following solutions:

$$n_{\mathrm{i1}} = -\frac{\left((vu_{\mathrm{i0}}/c^2) - \alpha\right)\phi_1}{(u_{\mathrm{i0}} - v)^2\,\alpha\gamma_{\mathrm{i0}} + \left((vu_{\mathrm{i0}}/c^2) - \alpha\right)}, \tag{24}$$

$$u_{\mathrm{i1}} = -\frac{(u_{\mathrm{i0}} - v)\,\gamma_{\mathrm{i0}}\phi_1}{(u_{\mathrm{i0}} - v)^2\,\alpha\gamma_{\mathrm{i0}} + \left((vu_{\mathrm{i0}}/c^2) - \alpha\right)}, \tag{25}$$

$$n_{\mathrm{d1}} = -\frac{Z_{\mathrm{d}}n_{\mathrm{d0}}\phi_1}{Qv^2}, \tag{26}$$

$$u_{\mathrm{d1}} = -\frac{Z_{\mathrm{d}}\phi_1}{Qv}. \tag{27}$$

Considering the next higher-order equations and following standard technique, we finally obtain the following desired KdV equation for the ion plasma wave:

$$\frac{\partial\phi_1}{\partial\tau} + a\phi_1\frac{\partial\phi_1}{\partial\eta} + b\frac{\partial^3\phi_1}{\partial\eta^3} = 0, \tag{28}$$

where

$$a = \frac{\frac{3z_{\mathrm{d}}}{Qv^2} + \frac{\xi}{\Gamma}\left(\frac{6\xi}{\Gamma} - 5\right) + \frac{(u_{\mathrm{i0}}-v)^2\gamma_{\mathrm{i0}}}{\Gamma^2}\left\{\begin{array}{l} 3\alpha\xi - \gamma_{\mathrm{i0}}\alpha + \frac{3\gamma_{\mathrm{i0}}u_{\mathrm{i0}}(u_{\mathrm{i0}}-v)^2}{c^2} + \frac{v}{c^2} - \frac{3u_{\mathrm{i0}}}{c^2} \\ + \frac{Qv^2}{z_{\mathrm{d}}^2 n_{\mathrm{d0}}}\left(\frac{3u_{\mathrm{i0}}}{c^2} - \frac{v}{c^2} - \frac{2(u_{\mathrm{i0}}-v)}{c^2}\right) \end{array}\right\}}{\frac{1}{\Gamma}\left(\frac{\xi}{(u_{\mathrm{i0}}-v)} + \frac{u_{\mathrm{i0}}}{c^2} - \gamma_{\mathrm{i0}}\alpha\,(u_{\mathrm{i0}} - v)\right) - \frac{2}{v} - \frac{1}{(u_{\mathrm{i0}}-v)}},$$

$$b = \frac{\frac{Qv^2}{z_{\mathrm{d}}^2 n_{\mathrm{d0}}} - \frac{\xi H^2}{\Gamma^4}}{\frac{1}{\Gamma}\left(\frac{\xi}{(u_{\mathrm{i0}}-v)} + \frac{u_{\mathrm{i0}}}{c^2} - \gamma_{\mathrm{i0}}\alpha(u_{\mathrm{i0}} - v)\right) - \frac{2}{v} - \frac{1}{(u_{\mathrm{i0}}-v)}}$$

in which $\xi = (v\sigma^2/u_{\mathrm{i0}}) - \alpha$ and $\Gamma = \gamma_{\mathrm{i0}}\alpha\,(u_{\mathrm{i0}} - v)^2 + \xi$. An examination of the KdV equation (28) shows that the nonlinear coefficient a depends on the relativistic parameter σ, whereas the dispersive coefficient b depends on both the relativistic parameter σ and the quantum diffraction parameter H.

With a view to find a steady-state solution of eq. (28), we introduce a new variable $\mu = \eta - M\tau$, where M is normalised by quantum ion-acoustic speed c_{is}. Applying the following boundary conditions:

$$\mu \to \pm\infty, \phi, \frac{\partial \phi}{\partial \mu}, \frac{\partial^2 \phi}{\partial \mu^2} \to 0,$$

we obtain the following stationary wave solution of the KdV equation (28):

$$\phi_1 = \phi_m \operatorname{sech}^2(\mu/\Delta)\,, \tag{29}$$

where ϕ_m is the amplitude and Δ is the width of the soliton. These are given by

$$\phi_m = 3M/a \quad \text{and} \quad \Delta = \sqrt{4b/M}. \tag{30}$$

Note that the amplitude of the solitary wave depends only on the relativistic parameter σ, whereas the width of the solitary wave depends on both quantum diffraction parameter H and the relativistic parameter σ.

5. Results and discussion

Here we have investigated quantum and relativistic effects on the ion plasma wave in a two-component dust-ion plasma model which is pertinent to many astrophysical and laboratory plasma environments such as Saturn's F-ring, surroundings of Halley's Comet, relativistic cosmic structures and laser–solid interaction experiments. To begin with, we have examined quantum and relativistic effects on the linear dispersion characteristic of the ion plasma wave. Numerically, it is shown that both the quantum and the relativistic effects increase the wave frequency. Next, to describe the nonlinear propagation of the wave, we have derived a KdV equation including the quantum and relativistic effects. From its solution, it is shown that the ion plasma wave can propagate as a solitary wave whose amplitude and width significantly depend on quantum and relativistic parameters. The plasma model under consideration is found to support only the compressive type of a soliton.

To study the relativistic effect on the solitary wave propagation, we have plotted in figure 3 the solitary wave profile for different values of relativistic streaming factor σ. It shows that the amplitude of the soliton increases and the width of the soliton decreases with increase in relativistic streaming factor σ. Physically, it indicates the increase of nonlinearity in the system due to the relativistic effect.

In order to study the effect of quantum diffraction on the solitary wave propagation, we have plotted in figure 4 the solitary wave profile for different values of quantum diffraction parameter H. It shows that the amplitude of the soliton remains unaffected by quantum effect but the width of the soliton increases significantly with an increase in quantum diffraction parameter H. Physically, it indicates that the quantum effect increases dispersion in the system.

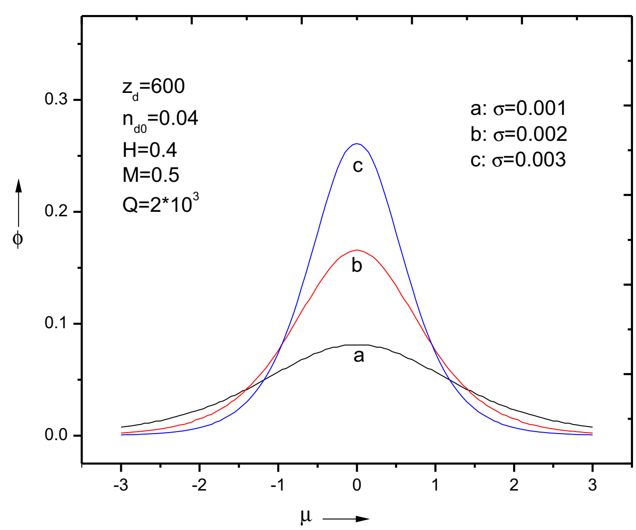

Figure 3. Relativistic effect on the solitary wave profile.

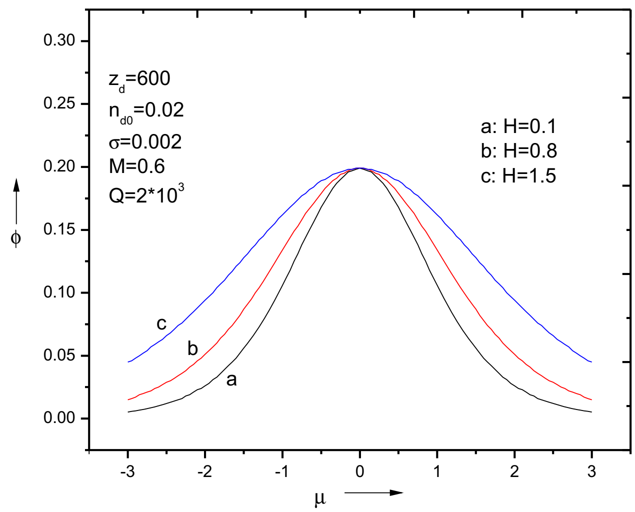

Figure 4. Quantum effect on the solitary wave profile.

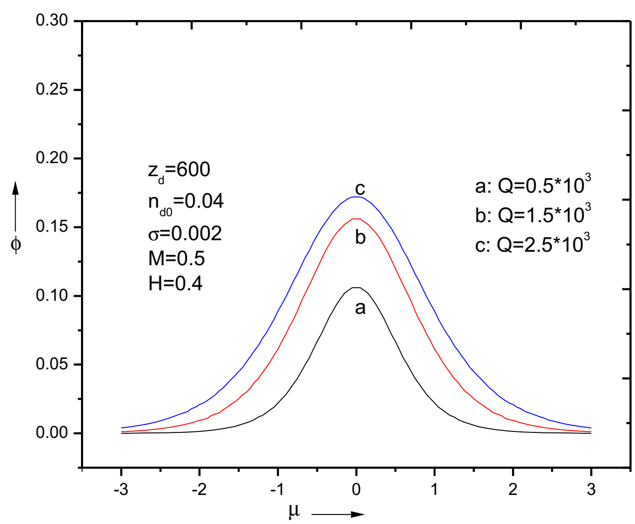

Figure 5. Solitary wave profile for different values of the dust mass Q.

The solitary wave profile is also found to depend on other plasma parameters such as dust size Q, dust number density n_{d0} and dust charge z_d. Figure 5 shows the solitary wave profile for different values of dust size. Obviously, both the amplitude and width of the solitary wave increase with the increase in dust size Q.

6. Concluding remarks

To conclude, we have investigated linear and nonlinear properties of ion plasma waves in a dust-ion plasma model including the quantum and relativistic effects. The quantum effect is found to increase dispersion in the system, whereas the relativistic effect tends to increase nonlinearity in the system. The result of this investigation has relevance to Saturn's F-ring, surroundings of Halley's Comet, laser–solid interaction experiments, cosmic relativistic structures such as active galactic nuclei, γ-ray bursts, supernova remnants and presumably the early period of evolution of the Universe.

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References

[1] G Manfredi, *Fields Inst. Commun.* **46**, 263 (2005)
[2] C Grabbe, *J. Geophys. Res.* **94**, 17299 (1989)
[3] M Opher, L O Silva, D E Dauger, V K Decyk and J M Dawson, *Phys. Plasmas* **8**, 2454 (2001)
[4] H Ikezi, *Phys. Fluids* **16**, 1668 (1973)
[5] M Marklund and P K Shukla, *Rev. Mod. Phys.* **78**, 591 (2006)
[6] S H Glenzer and R Redmer, *Rev. Mod. Phys.* **81**, 1625 (2009)
[7] P A Markowich, C A Ringhofer and C Schmeiser, *Semiconductor equations* (Springer, Vienna, New York, 1990)
[8] P K Shukla and B Eliasson, *Phys. Rev. Lett.* **96**, 245001 (2006)
[9] P K Shukla and B Eliasson, *Phys. Rev. Lett.* **99**, 096401 (2007)
[10] H Sahoo, B Ghosh and K K Mondal, *The African Rev. Phys.* **11**, 0035 (2016)
[11] B Ghosh and K P Das, *J. Plasma Phys.* **40(3)**, 545 (1988)
[12] Shalini and N S Saini, *J. Plasma Phys.* **81**, 905810316 (2015)
[13] N N Rao, P K Shukla and M Y Yu, *Planet. Space Sci.* **38**, 543 (1990)
[14] R Z Sagdeev, *Reviews of plasma physics* edited by M A Leontovich (Consultants Bureau, New York, 1966) Vol. 4, pp. 23–91
[15] N Y Kotsarenko *et al*, *Planet Space Sci.* **46**, 429 (1998)
[16] A Barkan, R L Merlino and N D Angelo, *Phys. Plasmas* **2**, 3563 (1995)
[17] G C Das and S N Paul, *Phys. Fluids* **28**, 823 (1985)
[18] Y Nejoh, *J. Plasma Phys.* **38**, 439 (1987)
[19] H H Kuehl and G Y Zhang, *Phys. Fluids B* **3**, 26 (1991)
[20] A Roychowdhury, G Pakira and S N Paul, *IEEE Trans. Plasma Sci.* **26**, 987 (1998)
[21] T S Gill, A S Bains and N S Saini, *Can. J. Phys.* **87**, 861 (2009)
[22] B C Kalita, R Das and H K Sarmah, *Phys. Plasmas* **18**, 012304 (2011)
[23] H Pakzad, *Astrophys. Space Sci.* **332**, 269 (2011).

[24] T S Gill, A Singh, H Kaur, N S Saini and P Bala, *Phys. Lett.* A **361**, 364 (2007)
[25] M Opher, L O Silva, D E Dauger, V K Decyk and J M Dawson, *Phys. Plasmas* **8**, 2544 (2001)
[26] Y D Jung, *Phys. Plasmas* **8**, 3842 (2001)
[27] S Ali and P K Shukla, *Phys. Plasmas* **13**, 022313 (2006)
[28] B Sahu, *Pramana – J. Phys.* **76**, 933 (2011)
[29] T S Gill, A S Bains and C Bedi, *J. Phys. Conf. Ser.* **208**, 012040 (2010)
[30] B Ghosh, S Chandra and S N Paul, *Pramana – J. Phys.* **78(5)**, 779 (2012)
[31] T Esirkepov, M Borghesi, S V Bulanov, G Mourou and T Tajima, *Phys. Rev. Lett.* **92**, 175003-1 (2004)
[32] F Melia and H Falcke, *Ann. Rev. Astron. Astrophys.* **39**, 309 (2001)
[33] M E Dieckmann, *Nonlinear Process. Geophys.* **15**, 831 (2008)
[34] S L Shapiro and S.A Teukolsky, *Black holes, white dwarfs and neutron stars: The physics of compact objects* (Wiley-VCH, Verlag, Weinhelm, 2004)
[35] N S Saini and K Singh, *Phys. Plasmas* **23**, 103701 (2016)
[36] J E Gunn and J P Ostriker, *Astrophys. J.* **165**, 523 (1971)
[37] L Stenflo and N L Tsintsadze, *Astrophys. Space Sci.* **64**, 513 (1979)
[38] L Stenflo and P K Shukla, *IEEE Trans. Plasma Sci.* **29**, 208 (2001)
[39] F Haas, L G Garcia, J Goedert and G Manfredi, *Phys. Plasmas* **10**, 3858 (2003)
[40] G Manfredi and F Haas, *Phys. Rev. B* **64**, 075316 (2001)
[41] C L Gardner and C Ringhofer, *Phys. Rev. E* **53**, 157 (1996)