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REMARKS ON D-INTEGRAL COMPLETE MULTIPARTITE GRAPHS

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Abstract. A graph is called distance integral (or D-integral) if all eigenvalues of its distance matrix are integers. In their study of D-integral complete multipartite graphs, Yang and Wang (2015) posed two questions on the existence of such graphs. We resolve these questions and present some further results on D-integral complete multipartite graphs. We give the first known distance integral complete multipartite graphs K_{p_1,p_2,p_3} with $p_1 < p_2 < p_3$, and K_{p_1,p_2,p_3,p_4} with $p_1 < p_2 < p_3 < p_4$, as well as the infinite classes of distance integral complete multipartite graphs $K_{a_1p_1,a_2p_2,\ldots,a_sp_s}$ with $s = 5, 6$.

Keywords: distance spectrum; integral graph; distance integral graph; complete multipartite graph

MSC 2010: 05C50

1. Introduction and preliminaries

The study of graphs with integral adjacency spectrum was initiated by Harary and Schwenk in 1974 (see [7]). A survey of papers up to 2002 appears in [3], but more than a hundred new studies on integral graphs have been published in the last ten years.

Let $G = (V, E)$ be a simple, connected graph with $n = |V|$ vertices. A distance matrix of G is the $n \times n$ matrix D, indexed by V, such that $D_{u,v}$ is the distance between the vertices u and v. Among the earliest users of a distance matrix in chemistry were Clark and Kettle in 1975 (see [4]). Topological indices based on the distance matrix, in particular its largest eigenvalue and its energy, play a significant role in research (see, for example, [5], [6], [8], [9], [13], [16]). A survey on the distance spectra of graphs appears in [2].

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The distance characteristic polynomial (or D-polynomial) of G is $D_G(x) = |xI_n - D(G)|$. A graph G is called D-integral if all the eigenvalues of its D-polynomial are integers. Distance integral graphs are studied only in [8], [11] in the case of some special, highly symmetric graphs, and in [10], [14], [15].

Complete multipartite graphs, in the case of integer distance spectrum, are studied in [14], [15]. In [15], Yang and Wang show that the D-characteristic polynomial of a complete multipartite graph $K_{p_1,p_2,\ldots,p_r}$ with $p_1+p_2+\ldots+p_r = n$ vertices is equal to

$$(1.1)\qquad P(K_{p_1,p_2,\ldots,p_r};x) = \prod_{i=1}^{r}(x+2)^{(p_i-1)}\prod_{i=1}^{r}(x-p_i+2)\left(1-\sum_{i=1}^{r}\frac{p_i}{x-p_i+2}\right).$$

If $p_1', p_2', \ldots, p_s'$ denote all the distinct integers among $p_1, p_2, \ldots, p_r$ and a_i, $i = 1, 2, \ldots, s$, denotes the multiplicity of p_i' in the family $p_1, p_2, \ldots, p_r$, then $K_{p_1,p_2,\ldots,p_r}$ will also be denoted by $K_{a_1p_1',a_2p_2',\ldots,a_sp_s'}$.

In [15], the following sufficient and necessary conditions for complete r-partite graphs to be distance integral are given.

Theorem 1.1 ([15], Theorem 2.6). *If a complete r-partite graph $K_{p_1,p_2,\ldots,p_r} = K_{a_1p_1,a_2p_2,\ldots,a_sp_s}$ on n vertices is distance integral, then there exist integers μ_i, $i = 1, 2, \ldots, s$, such that $-2 < p_1 - 2 < \mu_1 < p_2 - 2 < \mu_2 < \ldots < p_{s-1} - 2 < \mu_{s-1} < p_s - 2 < \mu_s < \infty$, and the numbers $a_1, a_2, \ldots, a_s$ defined by*

$$(1.2)\qquad a_k = \frac{\prod_{i=1}^{s}(\mu_i - p_k + 2)}{p_k\prod_{i=1,\,i\neq k}^{s}(p_i - p_k)},\quad k = 1, 2, \ldots, s$$

are positive integers.

Conversely, suppose that there exist integers μ_i, $i = 1, 2, \ldots, s$, such that $-2 < p_1 - 2 < \mu_1 < p_2 - 2 < \mu_2 < \ldots < p_{s-1} - 2 < \mu_{s-1} < p_s - 2 < \mu_s < \infty$ and that the numbers a_k, in (1.2) *are positive integers. Then the complete r-partite graph $K_{p_1,p_2,\ldots,p_r} = K_{a_1p_1,a_2p_2,\ldots,a_sp_s}$ is distance integral.*

Corollary 1.1 ([15], Corollary 2.9). *For any positive integer q, the complete r-partite graph $K_{p_1q,p_2q,\ldots,p_rq} = K_{a_1p_1q,a_2p_2q,\ldots,a_sp_sq}$ is distance integral if and only if the complete r-partite graph $K_{p_1,p_2,\ldots,p_r} = K_{a_1p_1,a_2p_2,\ldots,a_sp_s}$ is distance integral.*

Theorem 1.2 ([15], Theorem 3.2). *Let a complete r-partite graph $K_{p_1,p_2,\ldots,p_r} = K_{a_1p_1,a_2p_2,\ldots,a_sp_s}$ be distance integral with eigenvalues μ_i. Let $\mu_i \geqslant 0$ and $p_i > 0$, $i = 1, 2, \ldots, s$, be integers such that $-2 < p_1 - 2 < \mu_1 < p_2 - 2 < \mu_2 < \ldots < p_{s-1} - 2 < \mu_{s-1} < p_s - 2 < \mu_s < \infty$ and let*

$$(1.3)\qquad a_k = \frac{\prod_{i=1}^{s}(\mu_i - p_k + 2)}{p_k\prod_{i=1,\,i\neq k}^{s}(p_i - p_k)},\quad k = 1, 2, \ldots, s$$

be positive integers. Then for

$$(1.4)\quad b_k = \frac{\prod_{i=1}^{s-1}(\mu_i - p_k + 2)(\mu_s - p_k + 2 + rt)}{p_k \prod_{i=1,\, i\neq k}^{s}(p_i - p_k)}, \quad k = 1, 2, \ldots, s,$$

$$(1.5)\quad r = \mathrm{LCM}(r_1, r_2, \ldots, r_s), \quad r_k = \frac{p_k \prod_{i=1,\, i\neq k}^{s}(p_i - p_k)}{d_k}, \quad k = 1, 2, \ldots, s,$$

$$(1.6)\quad d_k = \mathrm{GCD}\Bigl(\prod_{i=1}^{s-1}(\mu_i - p_k + 2), p_k \prod_{i=1,\, i\neq k}^{s}(p_i - p_k)\Bigr), \quad k = 1, 2, \ldots, s,$$

the complete m-partite graph $K_{p_1,p_2,\ldots,p_m} = K_{b_1p_1,b_2p_2,\ldots,b_sp_s}$ is distance integral for every nonnegative integer t with eigenvalues $\mu_1, \mu_2, \ldots, \mu_{s-1}$, $\mu_s' = \mu_s + rt$.

In [15], Yang and Wang concluded their study with the following questions. The first of them is answered affirmatively in [14], the other we answer affirmatively here.

Question 1.1 ([15], Question 4.1). Are there any distance integral complete r-partite graphs $K_{p_1,p_2,\ldots,p_r} = K_{a_1p_1,a_2p_2,\ldots,a_sp_s}$ for $s \geqslant 5$?

Question 1.2 ([15], Question 4.2). Are there any distance integral complete r-partite graphs $K_{p_1,p_2,\ldots,p_r} = K_{a_1p_1,a_2p_2,\ldots,a_sp_s}$ with $a_1 = a_2 = \ldots = a_s = 1$ for $s \geqslant 3$?

The rest of the present paper is organized as follows. In Section 2, we study complete multipartite graphs $K_{a_1p_1,a_2p_2}$ and give sufficient and necessary conditions for their distance integrality. Our conditions are more easily applicable than the conditions published in Theorem 3.1 of [15]. In Section 3, we give the first known distance integral complete multipartite graphs K_{p_1,p_2,p_3} with $p_1 < p_2 < p_3$, and K_{p_1,p_2,p_3,p_4} with $p_1 < p_2 < p_3 < p_4$. In Section 4, we give infinite classes of distance integral complete multipartite graphs $K_{a_1p_1,a_2p_2,\ldots,a_sp_s}$ with $s = 5, 6$, which are different from those of Yang and Wang in [14].

2. Distance integral complete multipartite graphs $K_{a_1p_1,a_2p_2}$

Let us start with the definition of the join of graphs G_1 and G_2 and the notation of the spectrum of the adjacency matrix $A(G)$ of G and the spectrum of the distance matrix $D(G)$ of G.

Definition 2.1. The join $G_1 \nabla G_2$ of graphs G_1 and G_2 is the graph obtained from the union of G_1 and G_2 by adding the edges joining every vertex of G_1 to every vertex of G_2.

Definition 2.2. Let $\lambda_1 < \lambda_2 < \ldots < \lambda_t$ be t distinct eigenvalues of the adjacency matrix $A(G)$ of G with the corresponding multiplicities $k_1, k_2, \ldots, k_t$. The spectrum of $A(G)$ is also called the spectrum of G and denoted by $\mathrm{Spec}(G) = \{\lambda_1^{(k_1)}, \lambda_2^{(k_2)}, \ldots, \lambda_t^{(k_t)}\}$.

Definition 2.3. Let $\mu_1 < \mu_2 < \ldots < \mu_t$ be t distinct eigenvalues of the distance matrix $D(G)$ of G with the corresponding multiplicities $k_1, k_2, \ldots, k_t$. The spectrum of $D(G)$ is also called the distance spectrum of G and denoted by $\mathrm{Spec}_D(G) = \{\mu_1^{(k_1)}, \mu_2^{(k_2)}, \ldots, \mu_t^{(k_t)}\}$.

The following theorem is useful for getting conditions for D-integrality of $K_{a_1p_1,a_2p_2}$.

Theorem 2.1 ([12]). *For $i = 1, 2$, let G_i be an r_i-regular graph with n_i vertices and the eigenvalues $\lambda_{i,1} = r_i \geqslant \ldots \geqslant \lambda_{i,n_i}$ of the adjacency matrix of G_i. The distance spectrum of $G_1 \nabla G_2$ consists of the eigenvalues $-\lambda_{i,j} - 2$ for $i = 1, 2$ and $j = 2, 3, \ldots, n_i$, and two further simple eigenvalues $n_1 + n_2 - 2 - (r_1 + r_2)/2 \pm \sqrt{(n_1 - n_2 - (r_1 - r_2)/2)^2 + n_1 n_2}$.*

It is clear that $K_{a_1p_1,a_2p_2} = K_{a_1p_1} \nabla K_{a_2p_2}$. Using the above theorem for $K_{a_1p_1}$, $K_{a_2p_2}$, we have the following theorem.

Theorem 2.2. *The graph $K_{a_1p_1,a_2p_2}$ is D-integral if and only if*

$$\frac{(a_1+1)p_1 + (a_2+1)p_2 - 4}{2} \pm \sqrt{\frac{((a_1+1)p_1 - (a_2+1)p_2)^2}{4} + a_1a_2p_1p_2}$$

are integers and its distance spectrum is

$$\left\{\frac{(a_1+1)p_1 + (a_2+1)p_2 - 4}{2} \pm \sqrt{\frac{((a_1+1)p_1 - (a_2+1)p_2)^2}{4} + a_1a_2p_1p_2},\right.$$
$$\left.(p_1-2)^{(a_1-1)}, (p_2-2)^{(a_2-1)}, (-2)^{(a_1p_1-a_1+a_2p_2-a_2)}\right\}.$$

P r o o f. The A-spectrum of $K_{a_1p_1}$ is $\{p_1(a_1-1), 0^{(p_1a_1-a_1)}, (-p_1)^{(a_1-1)}\}$ and the A-spectrum of $K_{a_2p_2}$ is $\{p_2(a_2-1), 0^{(p_2a_2-a_2)}, (-p_2)^{(a_2-1)}\}$. Now it is sufficient to use Theorem 2.1. $\square$

Using $(a_1, a_2) = (1,1), (2,1), (2,2), (3,1)$ in Theorem 2.2, we have the following corollary.

Corollary 2.1.

1. *The graph K_{p_1,p_2} is D-integral if and only if $p_1^2 - p_1p_2 + p_2^2$ is a perfect square. Moreover, its distance spectrum is* $\{(-2)^{(p_1+p_2-2)}, p_1 + p_2 - 2 \pm \sqrt{p_1^2 - p_1p_2 + p_2^2}\}$.
2. *The only distance integral graph among stars is* K_2.
3. *The graph K_{2p_1,p_2} is distance integral if and only if $9p_1^2 - 4p_1p_2 + 4p_2^2$ is a perfect square. Moreover, its distance spectrum is* $\{(-2)^{(2p_1+p_2-3)}, p_1 - 2, (3p_1 + 2p_2 - 4 \pm \sqrt{9p_1^2 - 4p_1p_2 + 4p_2^2})/2\}$.
4. *The graph $K_{2p_1,2p_2}$ is distance integral if and only if $9p_1^2 - 2p_1p_2 + 9p_2^2$ is a perfect square. Moreover, its distance spectrum is* $\{(-2)^{(2p_1+2p_2-4)}, p_1 - 2, p_2 - 2, (3p_1 + 3p_2 - 4 \pm \sqrt{9p_1^2 - 2p_1p_2 + 9p_2^2})/2\}$.
5. *The graph K_{3p_1,p_2} is distance integral if and only if $4p_1^2 - p_1p_2 + p_2^2$ is a perfect square. Moreover, its distance spectrum is* $\{(-2)^{(3p_1+p_2-4)}, (p_1-2)^2, 2p_1 + p_2 - 2 \pm \sqrt{4p_1^2 - p_1p_2 + p_2^2}\}$.

The following corollary gives sufficient and necessary conditions for complete bipartite graphs to be D-integral.

Corollary 2.2. *K_{p_1,p_2} is D-integral if and only if there exist integers k, u and v such that $p_1 = k(v^2 + 2uv)$, $p_2 = k(v^2 - u^2)$, or $p_1 = k(v^2 - u^2)$, $p_2 = k(v^2 + 2uv)$, where $u, v \in \mathbb{Z}$ and $k \in \mathbb{Q}$ are such that $3k \in \mathbb{Z}$.*

P r o o f. Part 1 of Corollary 2.1 yields that the necessary and sufficient condition for K_{p_1,p_2} to be D-integral is that for some integer r, $p_1^2 - p_1p_2 + p_2^2 = r^2$. According to [1], page 90, all integral solutions to $p_1^2 - p_1p_2 + p_2^2 = r^2$ are given by $p_1 = k(v^2 + 2uv)$, $p_2 = k(v^2 - u^2)$, or $p_1 = k(v^2 - u^2)$, $p_2 = k(v^2 + 2uv)$, where $u, v \in \mathbb{Z}$ and $k \in \mathbb{Q}$ is such that $3k \in \mathbb{Z}$. □

3. Distance integral complete multipartite graphs K_{p_1,p_2,p_3} and K_{p_1,p_2,p_3,p_4}

Using computers, we have found 292 D-integral complete 3-partite graphs K_{p_1,p_2,p_3} for $p_1 < p_2 < p_3 \leqslant 1{,}000$. The primitive graphs (those, where $\mathrm{GCD}(p_1, p_2, p_3) = 1$) with less than 180 vertices are given in Table 1, rows 2–7.

Using Theorem 1.2, we can construct infinite classes of D-integral complete multipartite graphs for each graph from Table 1.

Corollary 3.1. *Let K_{p_1,p_2,p_3} be a D-integral complete 3-partite graph from Table 1, rows 2–4. Then $K_{b_1p_1,b_2p_2,b_3p_3}$ is a D-integral complete multipartite graph for every $t \in \mathbb{N}$, where b_1, b_2, b_3 are those of Table 1, rows 9–11.*

No.	1	2	3	4	5	6	7	8
p_1	12	7	28	25	20	23	39	35
p_2	21	33	33	30	39	39	48	54
p_3	28	81	60	81	84	81	56	75
μ_1	12	7	28	25	22	25	40	38
μ_2	22	42	42	43	50	50	50	61
μ_3	82	187	166	198	208	205	190	223
r	504	9,828	3,780	5,950	11,970	11,592	2,448	8,550
b_1	$1+7t$	$1+54t$	$1+27t$	$1+34t$	$1+63t$	$1+63t$	$1+16t$	$1+45t$
b_2	$1+8t$	$1+63t$	$1+28t$	$1+35t$	$1+70t$	$1+69t$	$1+17t$	$1+50t$
b_3	$1+9t$	$1+91t$	$1+35t$	$1+50t$	$1+95t$	$1+92t$	$1+18t$	$1+57t$

Table 1. D-integral complete multipartite graphs K_{p_1,p_2,p_3}.

P r o o f. It is sufficient to use the formulas (1.3)–(1.6) from Theorem 1.2. $\square$

Similarly, using computers, we have found the D-integral complete 4-partite graph $K_{143,192,228,468}$. Using Theorem 1.2, we have the following corollary.

Corollary 3.2. *The graph $K_{(1+1{,}368t)\cdot 143,(1+1{,}425t)\cdot 192,(1+1{,}470t)\cdot 228,(1+1{,}862t)\cdot 468}$ is a D-integral complete multipartite graph for every $t\in\mathbb{N}$.*

P r o o f. It is sufficient to use (1.3)–(1.6) from Theorem 1.2 for $\mu_1 = 154$, $\mu_2 = 206$, $\mu_3 = 328$, $\mu_4 = 1,366$, $r = 1,675,800$. $\square$

4. Distance integral complete multipartite graphs $K_{a_1p_1,a_2p_2,\ldots,a_sp_s}$ with $s = 5,6$

Using a computer search based on Theorem 1.1, we have found examples of D-integral complete multipartite graphs $K_{a_1p_1,a_2p_2,a_3p_3,a_4p_4,a_5p_5}$; they are given in Table 2, rows 2–11. Using Theorem 1.2, we can construct infinite classes of D-integral complete multipartite graphs for each graph from Table 2.

Corollary 4.1. *Let $K_{a_1p_1,a_2p_2,a_3p_3,a_4p_4,a_5p_5}$ be a D-integral complete multipartite graph from Table 2, rows 2–11. Then $K_{b_1p_1,b_2p_2,b_3p_3,b_4p_4,b_5p_5}$ is a D-integral complete multipartite graph for every $t\in\mathbb{N}$, where b_1,b_2,b_3,b_4,b_5 are those of Table 2, rows 18–22.*

P r o o f. It is sufficient to use (1.3)–(1.6) from Theorem 1.2. $\square$

Similarly, using a computer search based on Theorem 1.1, we have found an example of D-integral complete multipartite graph $K_{a_1p_1,a_2p_2,a_3p_3,a_4p_4,a_5p_5,a_6p_6}$.

No.	1	2	3	4	5	6	7
a_1	11	31	44	56	23	39	44
p_1	3	11	4	10	10	7	8
a_2	1	9	52	2	39	37	52
p_2	12	35	8	22	14	10	16
a_3	2	2	12	13	6	23	12
p_3	18	45	23	37	22	23	46
a_4	3	3	11	9	6	31	11
p_4	28	49	25	46	35	28	50
a_5	1	1	6	3	21	7	6
p_5	39	56	29	57	55	50	58
μ_1	4	19	3	17	9	6	8
μ_2	11	40	13	22	18	12	28
μ_3	19	45	22	40	26	23	46
μ_4	34	53	26	53	38	44	54
μ_5	226	978	1,332	1,700	2,308	2,413	2,666
r	37,800	10,445,820	22,621,305	100,792,440	8,208,200	1,721,720	45,242,610
b_1	$11+1,848t$	$31+334,180t$	$44+748,374t$	$56+3,335,920t$	$23+82,082t$	$39+27,885t$	$44+748,374t$
b_2	$1+175t$	$9+99,484t$	$52+887,110t$	$2+119,991t$	$39+139,425t$	$37+26,488t$	$52+887,110t$
b_3	$2+360t$	$2+22,344t$	$12+207,060t$	$13+786,968t$	$6+21,525t$	$23+16,555t$	$12+207,060t$
b_4	$3+567t$	$3+33,660t$	$11+190,095t$	$9+547,785t$	$6+21,648t$	$31+22,360t$	$11+190,095t$
b_5	$1+200t$	$1+11,305t$	$6+104,006t$	$3+183,816t$	$21+76,440t$	$7+5,096t$	$6+104,006t$

Table 2. D-integral complete multipartite graphs $K_{a_1p_1,a_2p_2,a_3p_3,a_4p_4,a_5p_5}$.

Corollary 4.2. 1. *The graph* $K_{722,608\cdot 4,706,668\cdot 8,364,041\cdot 14,73,308\cdot 23,73,420\cdot 25,214,524\cdot 32}$ *is a D-integral complete multipartite graph and* $\mu_1 = 3$, $\mu_2 = 9$, $\mu_3 = 18$, $\mu_4 = 22$, $\mu_5 = 26$, $\mu_6 = 24,026,718$.

2. *Let* $b_1 = 722,608+825,792t$, $b_2 = 706,668+807,576t$, $b_3 = 364,041+416,024t$, $b_4 = 73,308+83,776t$, $b_5 = 73,420+83,904t$, $b_6 = 214,524+245,157t$. *The graph* $K_{b_1\cdot 4,b_2\cdot 8,b_3\cdot 14,b_4\cdot 23,b_5\cdot 25,b_6\cdot 32}$ *is a D-integral complete multipartite graph for every* $t \in \mathbb{N}$.

P r o o f. For case 1 it is sufficient to use Theorem 1.1. For Case 2 it is sufficient to use (1.3)–(1.6) from Theorem 1.2 ($r = 27,457,584$). □

5. Conclusion

In the paper, we give new results for D-integrality of complete multipartite graphs $K_{a_1p_1,a_2p_2,\ldots,a_sp_s}$, where $s = 1,2,3,4,5,6$, and answer affirmatively questions 4.1 and 4.2 of Yang and Wang (see [15]). However, when $s > 6$, we have not found such D-integral graphs. Thus, we raise the following questions.

Question 5.1. Are there any distance integral complete multipartite graphs $K_{a_1p_1,a_2p_2,\ldots,a_sp_s}$ for $s \geqslant 7$?

Question 5.2. Are there any distance integral complete multipartite graphs $K_{a_1p_1,a_2p_2,\ldots,a_sp_s}$ with $a_1 = a_2 = \ldots = a_s = 1$ for $s \geqslant 5$?

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