

**Math Gains in Early Elementary Grades Using Traditional  
and Montessori Math Manipulatives**

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## MATH GAINS IN EARLY ELEMENTARY GRADES

### **Abstract**

I have moved from teaching in a traditional public school to a Montessori public school. Both environments share similarities, in terms of grade level learning targets and the use of manipulative objects to help students develop mathematical understandings. For this study, I used the district high-priority learning targets for grades 1, 2, and 3, and aligned them with three key mathematical strands: Numbers and Operations, Algebra, and Geometry and Measurement. (See Appendices A, B, and C.) I conducted this study with 14 students: 3 first-graders, 4 second-graders, and 7 third-graders, using both traditional and Montessori-specific manipulative objects to help students acquire key mathematical concepts. I wanted to discover whether Montessori-specific manipulative materials are more effective than manipulatives used in traditional public schools, in helping students build their understanding of key mathematical concepts.

Keywords: concrete, abstract, manipulatives, mathematics, symbols,

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### **Literature Review**

Multiple studies show that using manipulatives consistently over a period of time allows students to work from the concrete to the abstract to build deeper understandings of key mathematical concepts. Using manipulatives frequently results in increased test scores for these concepts.

Laski, Jor'dan, Daoust, and Murray (2015) defined manipulatives as “concrete materials (e.g., blocks, tiles) used to demonstrate a mathematics concept or to support the execution of a mathematical procedure.” Uribe-Flórez, and Wilkins (2010), described math manipulatives as “objects that can be touched, moved about, and rearranged, or stacked.”

Earlier, Burns (1996) explained the importance of using pieces with shapes that are familiar to children: “Early advocates of manipulatives posited that concrete objects that resemble everyday objects (e.g., teddy bear counters) help children draw on their practical knowledge for understanding concepts.” Students use the concrete objects in hands-on experiences that help them to build connections between the manipulatives and the concept. According to Johnson (2015), students enjoy using manipulatives to help them understand mathematical concepts. Using manipulatives helps them gain more in-depth knowledge about key math concepts. According to McNeil and Uttal (2009), “children are not capable of thinking about the world in terms of abstract concepts or symbolic representations; rather, they need to develop these capabilities through their

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experiences with concrete materials. This position suggests that concrete materials should be used to teach young children because young children cannot yet interpret the world in any other way.” It may be a challenge for children to look at a math problem and understand the relationship between the math problem and manipulatives, but they can find the connection if they learn how to manipulate and build the connections.

When students use manipulatives in a classroom, it helps them understand the mathematical concepts *and* the relationship between the manipulatives and the concepts. Laski, et al (2015) cited research from cognitive science to support their findings of the effective use of manipulatives in building the connection between the manipulatives and math concepts. Children learn most effectively when they move from the concrete to the abstract, especially in the early developmental stages. Laski and Siegler explained, “ The cognitive alignment approach provides a theoretical framework for considering how and when physical materials are most likely to produce effective learning. Its basic principle is this: The more precisely that physical materials and learning activities are aligned with the desired mental representation, the more likely students are to acquire that representation.”

One source of support for this framework comes from research on the development of the use of symbols and analogical reasoning. This research suggests that physical materials which are closely aligned to the desired mental representation increase analogical transfer (2014). Therefore, multiple practice sessions with manipulatives that represent key mathematical concepts will help students gain understanding of those math

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concepts.

Laski and Siegler described a scenario in which students played physical board games (e.g., *Chutes and Ladders*) that required them to count on in sequence as on a number line, from 0 - 10. Preschoolers who played such a game improved on two tasks that measured numerical magnitude knowledge—number line estimation and numerical magnitude comparison—as well as counting, numeral identification, and ability to learn the answers to arithmetic problems (Laski and Sieger, 2014).

DeLoache (2004) had reported that “Children do not easily interpret the meaning of symbols to use them for problem solving. Children under the age of five are unable to make the connection between a scale model of a room and a regular-sized room, to locate a hidden toy, without receiving explicit guidance from an experimenter” (DeLoache, Peralta de Mendoza, & Anderson, 1999). Children become better able to interpret the relationship between a symbol and its referent with age, but even older children need cumulative experience with a symbol to use it for sophisticated reasoning (Liben & Myers, 2007). Gick & Holyoak (1983) and later, Son, Smith, & Goldstone (2011), concluded that “Children are better able to identify the relationship between a concept and manipulative when they have multiple opportunities to compare them.”

Researchers assert it is important for students to use manipulatives consistently, over a period of time, to help them effectively move from concrete to abstract thinking. According to Laski, et al (2015), using manipulatives over a long period of time with

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clear explanations of the relationship between the manipulatives and the math concepts, consistently strengthens students' abilities in problem solving, critical thinking, and overall knowledge in mathematics – and results in increased test scores. Teachers must begin with clear, concrete demonstrations that explain the relationship between the manipulatives and the mathematical concept, and then, move on to more abstract demonstrations.

Post's Dynamic Principle (1981) suggested three temporally-ordered stages for optimum results in using manipulatives for moving from the concrete to the abstract. First is the preliminary, or play stage: students interact with the concept in an unstructured manner. Post (1981) states, "When children are exposed to a new type of manipulative material, they naturally 'play' with their new 'toy.' " The second stage involves structured, interactive activities that connect to the concepts. In the third or final stage, students re-apply the concept again, playing with the materials as at the start, building and strengthening the connection to the concept.

Laski, et al (2015) also suggested that using the same or similar manipulatives over a long period of time will help students develop a deeper understanding of the relationship between the physical material and the abstract concept, because it allows students to see the connection abstractly. Uttal, O'Doherty, Newland, Hand and DeLoache agreed with Laski et al, as they stated, " In comparing American and Japanese teachers, Stevenson and Stigler (1992) noted that American teachers "seek variety. . . . They may use Popsicle sticks in one lesson, and marbles, Cheerios, M&Ms, checkers,

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poker chips, or plastic animals in another” (p. 187).

In contrast, Japanese teachers tended to use the same, standard kit of manipulatives. “The Asian view is that using a variety of representational materials may confuse children, and thereby, make it more difficult for them to use the objects for the representation and solution of mathematics problems. Over time, as children continue to practice, they will be able to build the connections between the mathematical problems and manipulatives.”

Laski, et al (2015) stated that students who attended Montessori programs scored higher on standardized math tests, when, as previously noted, manipulatives were used consistently and over a long period of time. In the article, *Using Manipulatives to Teach Elementary Mathematics*, Boggan, Harper & Whitmire (2010) describe experiments testing both the importance and the benefits of math manipulatives in developing key concepts. Research results showed that students whose teachers used manipulatives in their math lessons scored higher in math post-tests than those whose teachers did not use manipulatives (2010). According to Johnson’s (2015) research, *Manipulatives in the Math Classroom*, concrete learners who used manipulatives scored higher on their MCA math tests (i.e, (Minnesota Comprehensive Assessments) than those students who did not use manipulatives.

Does the type of manipulative make a difference in how readily and deeply students acquire key mathematical concepts?

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Maria Montessori was one of the first among many inventors of manipulatives to create several hands-on materials to align with key mathematical concepts and skills.

The article, *How Important Are Montessori Materials?* explains how each type of object helps students acquire different skills. For example, the *Pink Tower* is designed to educate students in both sensory and fine motor systems, as well as to introduce them to the decimal system (Lillard 2008).

Laski, et al (2015) describe an example of how Montessori golden beads can support place-value learning. “When children are first introduced to the golden bead materials, the teacher explicitly points out to the child the value of the beads. The teacher places a single unit bead in front of the child and says, ‘This is a unit.’ Later, when the golden bead materials are used to teach children about numbers and counting, the teacher points as she counts each bead, helping them to make the connection between the quantity and the number words.”

Another example illustrates how students can move from the concrete to abstract understanding of a key mathematical concept, using Montessori materials/manipulatives (i.e., golden beads or base ten blocks): dynamic subtraction,  $54 - 26$ . Without manipulatives, a student might subtract  $6 - 4 = 2$ , and  $5 - 4 = 1$ , resulting in  $54 - 26 = 12$ . But, if students use the objects, they can see the concept visually, and understand that there are 4 beads (i.e., unit cubes) in the ones place – not enough to subtract 6 - so they’ll need to regroup. With consistent practice with manipulatives over time, students will be



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able to do dynamic subtraction, abstractly. “The sequence in which the Montessori materials are introduced is structured to move children to increasingly abstract representations over time (Laski et al, 2015) Uribe-Florez et al (2010) stated that “Mathematical manipulatives offer students a way of understanding abstract mathematical concepts by enabling them to connect the concepts to more informal concrete ideas.”

In conclusion, Boggan et al (2010) stated that when teachers use manipulatives in the classroom, it allows students to: (1) use their prior knowledge, then (2) connect it with their thinking, next (3), understand what they are working with, and finally (4) make thorough connections with the mathematical concepts being taught. Using manipulatives helps students understand key mathematics concepts, gain mathematical knowledge, abstractly, and increase test scores.

Based on research findings, teachers should use manipulatives – whether standard or Montessori – on a daily basis in their classrooms, to support student learning of mathematical concepts. Manipulatives help students move from concrete, hands-on experiences, to understanding abstract mathematical concepts.

### **Purpose**

The purpose of this study is to assess the extent to which learners (grades 1-3) experienced gains in math test scores after using traditional or Montessori-specific math manipulatives. I wanted to explore these questions:

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1. What are the gains in student math scores of a certain specific math concept after being exposed to a traditional math manipulative, designed to support and reinforce the concept?
2. What are the gains in student math scores of a certain specific math concept after being exposed to a *Montessori* math manipulative, designed to support and reinforce the concept?
3. What are the gains in student math scores of a certain specific math concept after being exposed to a traditional *and* a Montessori math manipulative, designed to support and reinforce the concept?

### **Participants**

Participants in this study are 14 students in first, second, and third grades. Students are ages 6-8, enrolled in an urban public Montessori elementary school, in the Midwest. The classroom is mixed-gender, comprised of White, Asian, Hispanic, and African- American students.

### **Method**

The study used a mixed methods approach, involving qualitative and quantitative data. Qualitative data were taken from observing students using manipulatives, between pre- and post-tests. Students were observed weekly, and anecdotal notes were kept to answer the following questions:

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1. Are the students actively using the manipulatives to help them solve the math problems?
2. How are students solving problems (e.g., individually, with the help of another student, or with the help of a teacher or teacher aide?)
3. What problems do students encounter in using the manipulatives?

Quantitative data used in the study came from pre- and post-tests of district standard learning targets.

### **Procedure**

There are six standards for each grade level, prioritized and reported at the end of the year. I focused on these three standards for the purposes of this study:

First grade standard **1.1.2.1** - Use words, pictures, objects, length-based models (connecting cubes), numerals and number lines to model and solve addition and subtraction problems in part-part-total, adding to, taking away from, and comparing situations.

Second grade standard **2.1.2.2**

Demonstrate fluency with basic addition facts and related subtraction facts.

Third grade standard **3.1.2.4**

Solve real-world and mathematical problems involving multiplication and division, including both "how many in each group" and "how many groups" division problems.

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For each standard, there are pre- and post-tests to assess student learning. Students took a pretest before using any manipulatives and taking a daily math fact assessment. After each math pre-test, students used manipulatives to practice the math skills and concepts, and then, re-tested with the same assessment after one week to measure growth and effective use of the manipulative. Each math fact assessment was timed at two minutes per section.

### **Materials**

Traditional math manipulatives used in the study included counters, such as teddy bears, glass beads, colored cubes, Unifix<sup>®</sup> cubes, and base ten blocks. Here is a brief description of each:

- Teddy bear counters are colorful, small, plastic teddy bears. There are six colors: orange, red, yellow, blue, green, and purple. Each teddy bear counter is one inch tall.
- Glass beads are round on top and have a flat bottom surface. Each glass bead is the same size as a penny. They come in two colors: clear white and clear blue.
- Colored cubes also known as “counting cubes,” come in six colors: red, orange, yellow, blue, green, and black. They are small, one-inch wooden cubes.
- Unifix<sup>®</sup> cubes are colorful, interlocking cubes that come in ten colors: green, blue, light blue, yellow, pink, red, white, orange, black, and brown.
- Base ten blocks are yellow and made of plastic. They help students represent math

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concepts such as place value, addition, subtraction, number sense, and counting.

The smallest cube represents one unit. There is a tens block made of ten units.

There is a hundreds bar made of 10 tens blocks and a thousands cube made of ten hundreds blocks.

Montessori materials included in this study are the strip board, colored bead bar, golden beads and stamp game. (See Appendix E.) Here is a brief description of the materials:

- The strip board is a board with two sets of color-coded numbers. The red set has numbers from one through ten, and the blue set has numbers eleven through eighteen.
- The colored bead bars contain beads that represent a number. Red represents one, because there is one bead. Green represents two, with two green beads. Pink is three, yellow is four, light blue is five, purple is six, white is seven, brown is eight, dark blue is nine, and gold is ten.
- Golden beads are yellow. There is a golden bead that represents ones or a unit, a ten bar that represents tens, a hundreds square with ten tens that represent one hundred, and a thousands cube with ten hundreds, representing one thousand.
- The Stamp Game is a wooden box with green, blue and red square chips. The wooden box is divided into four sections. In each section is a square with one hundred chips. Beginning from the right to left, the first section is one hundred green square chips with the number 1 each chip. In the second section are one hundred blue square chips with the number 10 on each chip. In the third section are one hundred red square chips with the number 100 on each chip. The last

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section has one hundred square chips with the number 1000 on each chip.

### **Observations**

After each pre-test and lesson during the student work cycle, I observed students daily for 6 weeks. I took anecdotal notes of students working with the manipulatives as I helped them to answer questions on their assignments. After my observations, I used the following questions to help analyze my notes:

1. Are the students using the materials to help them solve the math problems?
2. How are students solving problems (e.g., by themselves, with the help of another student, with the help of a teacher or teacher's aide or Montessori control of error materials)?
3. What are some problems that students seemed to encounter in using the materials?

### **Observations**

Through my observations, I noticed that most students preferred the Montessori manipulatives, such as the golden beads and stamp game, over the traditional choices. About 75% of the time, the first and second graders took out the golden beads and colored bead bars to help with their addition and subtraction assignments. About 85% of the time, the third graders took out the stamp game to help them with their multiplication and division assignments. I noticed that most students worked individually. About 30% of first graders worked individually, while the others worked with a partner. About 90% of the second and third graders worked individually.

During my observations, I noticed that four third graders were having a few challenges with division. The students were using the stamp game and dividing,

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beginning with the units place instead of starting from the biggest number. I had to re-teach division to those four students.

Overall, students took out Montessori manipulatives more often than traditional materials to help them solve their math assignments. My observations showed students using both types of manipulatives effectively, the majority of the time. Both types of manipulatives helped students achieve similar results.

### **Learning Target: Score Chart According to SPPS Assessment Data**

Level	Points
Prerequisite skills	0-1
Developing	2

### **First Grade Addition Facts Pretest and Posttest Results**

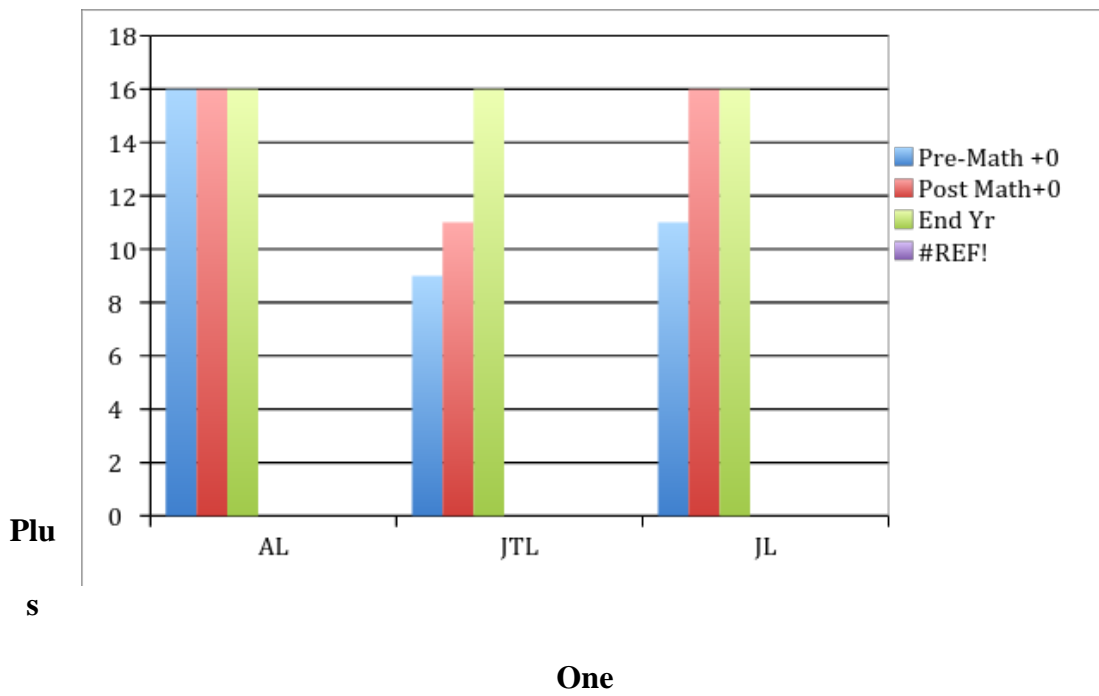
During the first week, one student used the golden beads and two students used the base ten blocks; all three students scored 100% on the Plus Zero post-test. In the second week, the students switched materials: one student used base ten blocks and the

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other two students used the golden beads. All three students scored 100% on the Plus Zero post-test. During the third and fourth weeks, two students used the strip board and one student used the ten frames. Again, all of the students scored 100% on the Making Ten assessments.

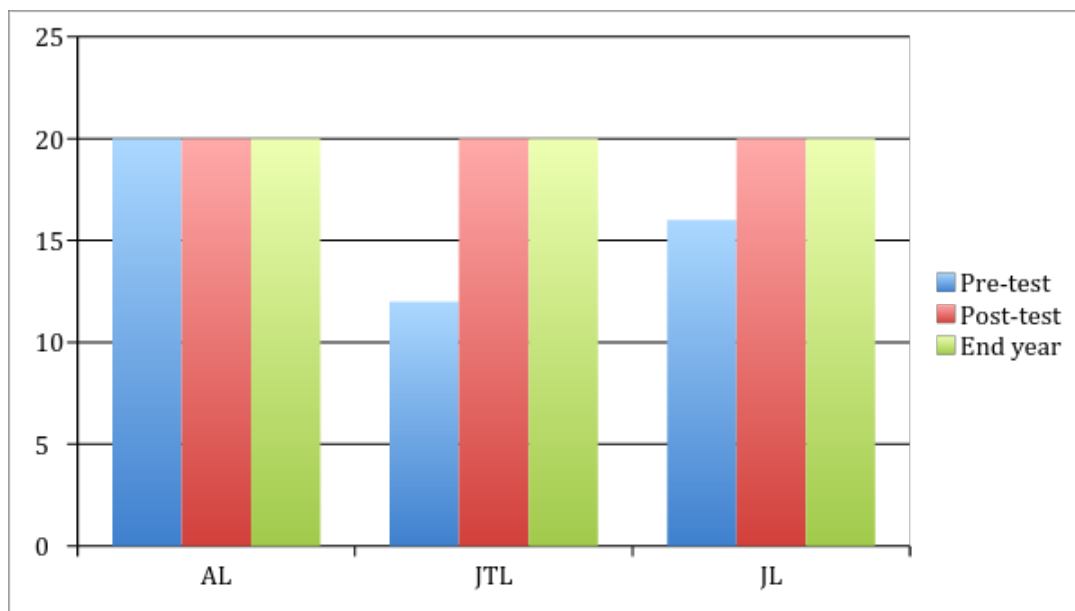
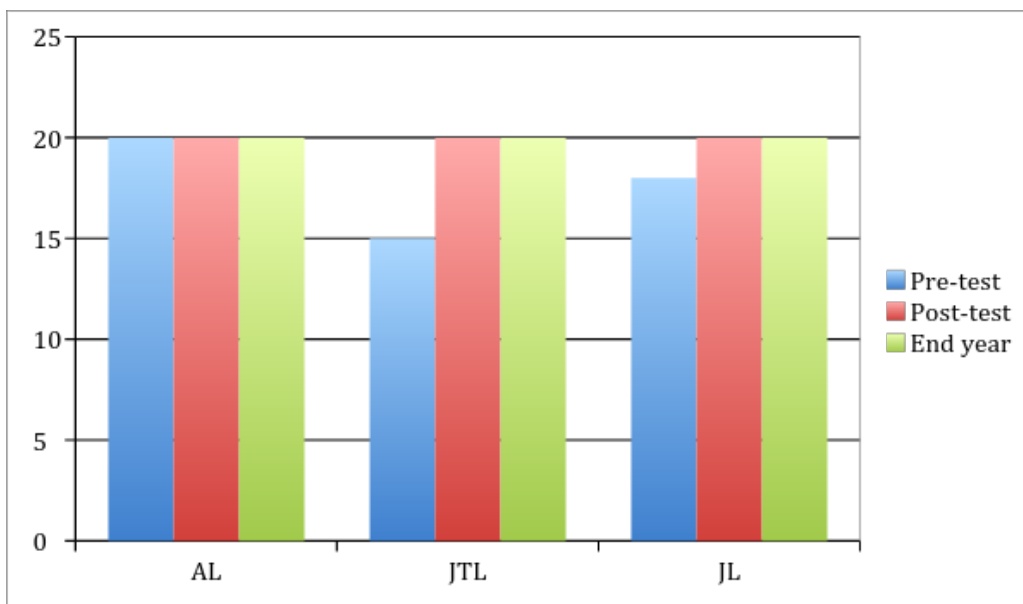
In the fifth week, one student used the strip board, another student used the glass bead counters, and one student used the golden beads. Two students made a 15% gain, and one student made a 10% gain on the Using Ten in Addition post-test. Overall, 67% of the students scored at Proficient, and the remaining 33% are at Developing and made gains.

### Plus Zero

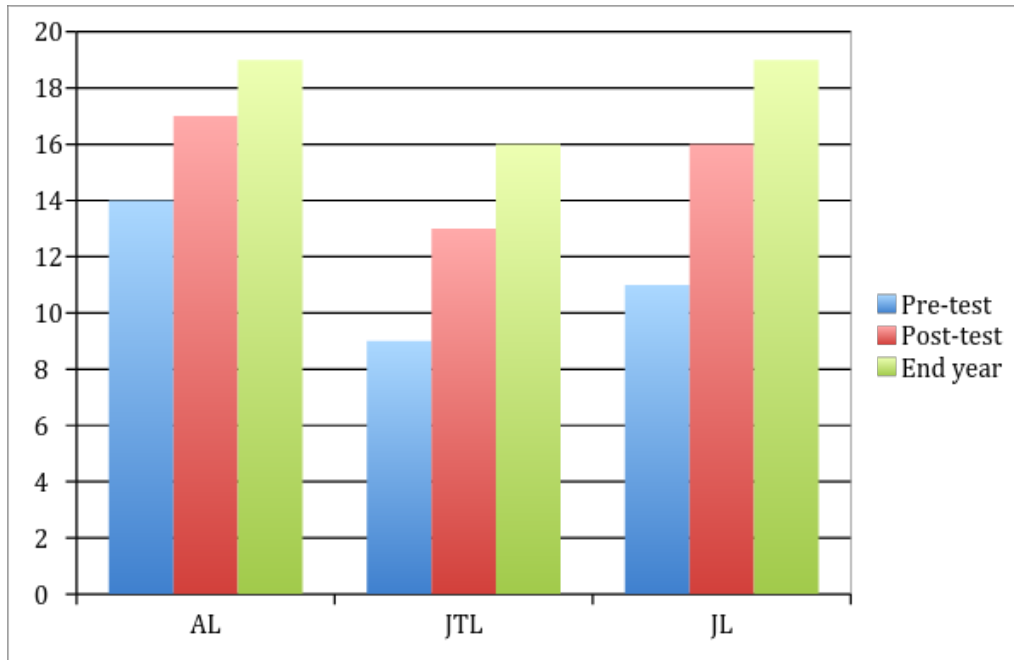




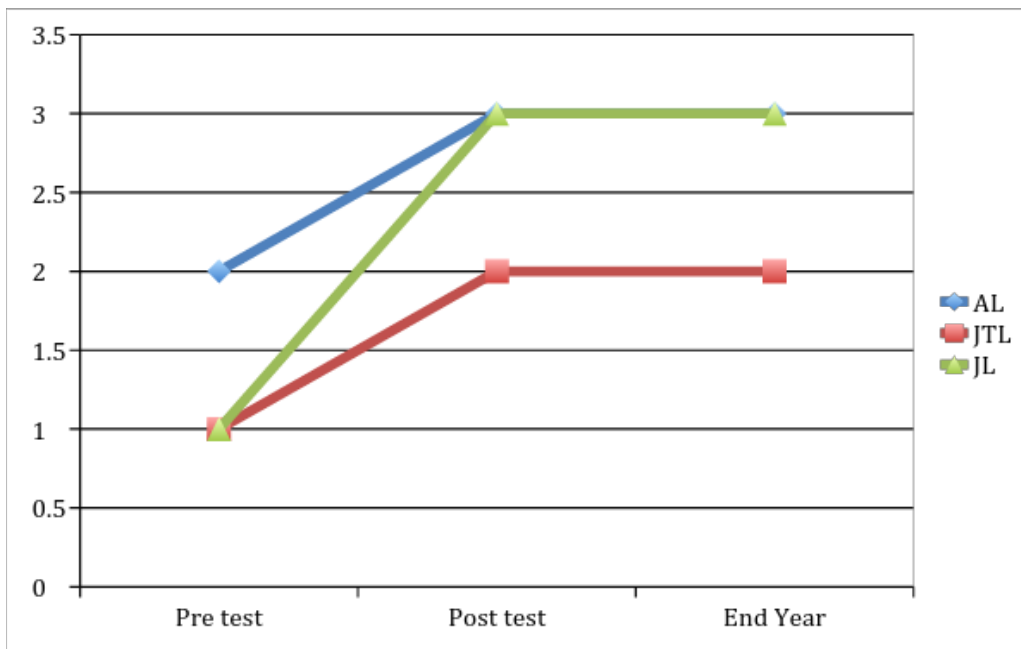
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**Making Ten +****Using Ten +**

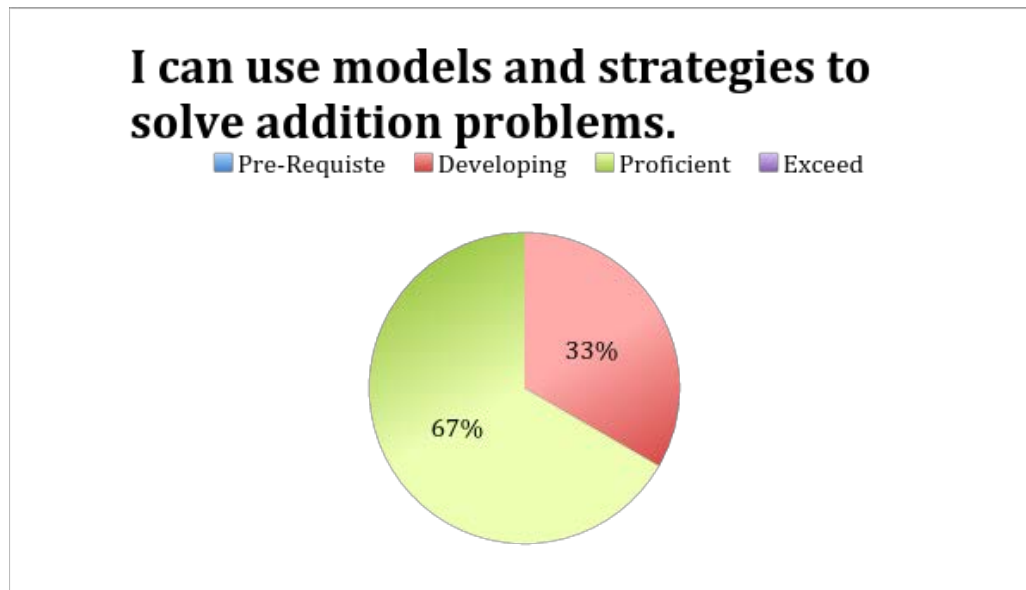
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**Learning Target 1.1.2.1 I can use models and strategies to solve additions problems.**



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**First Grade Subtraction Pretest and Posttest Results**

During the first week, two first-graders used the golden beads and scored 87% on the Subtract Zero post-test. One student used base ten blocks and scored 68% on the Subtract Zero Fact Check assessment.

In the second week, the students switched materials: two students used base ten blocks and one student used the golden beads. Each student scored 100% on the Subtract One and Two assessments.

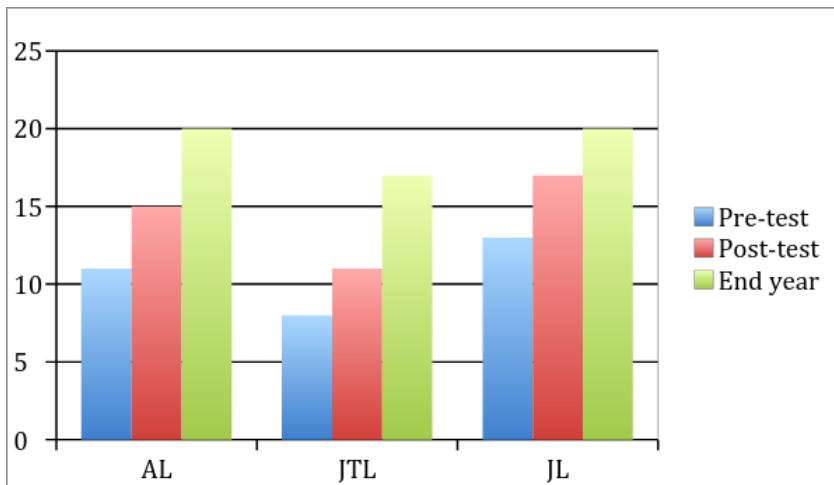
During the third week, two students used the strip board and one student used the golden beads. Two students scored 100% on the Subtract Ten post-test, while one of the students scored 85%.

In the fourth week, one student used the golden beads and scored 95% on the Using Ten Subtraction post-test. Another student used the color cubes counters and scored 75% on the Using Ten Subtraction post-test. One student used the strip board and

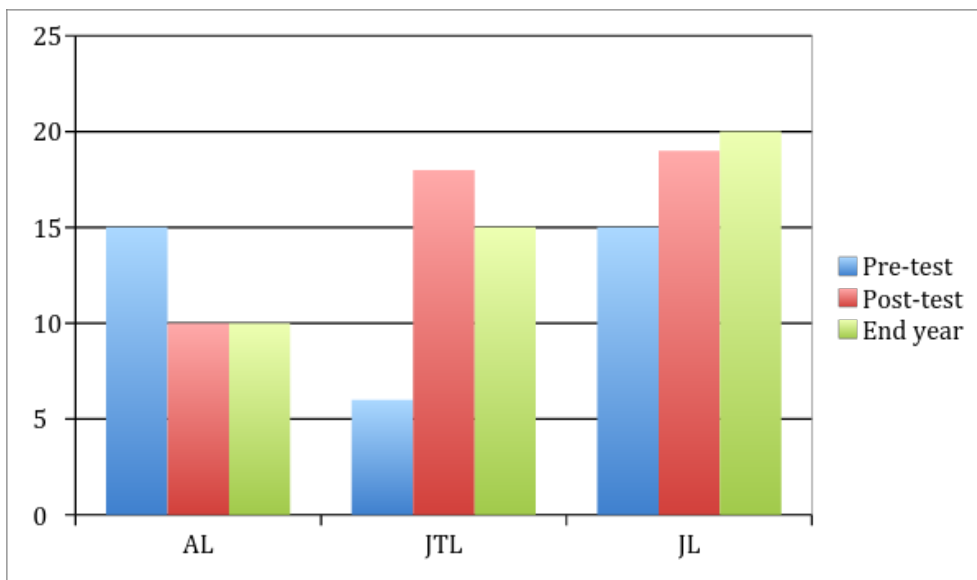
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scored 50% on the Using Ten Subtraction post-test. While the overall results showed that 67% of the students are At Developing, they still showed growth.

### Minus One

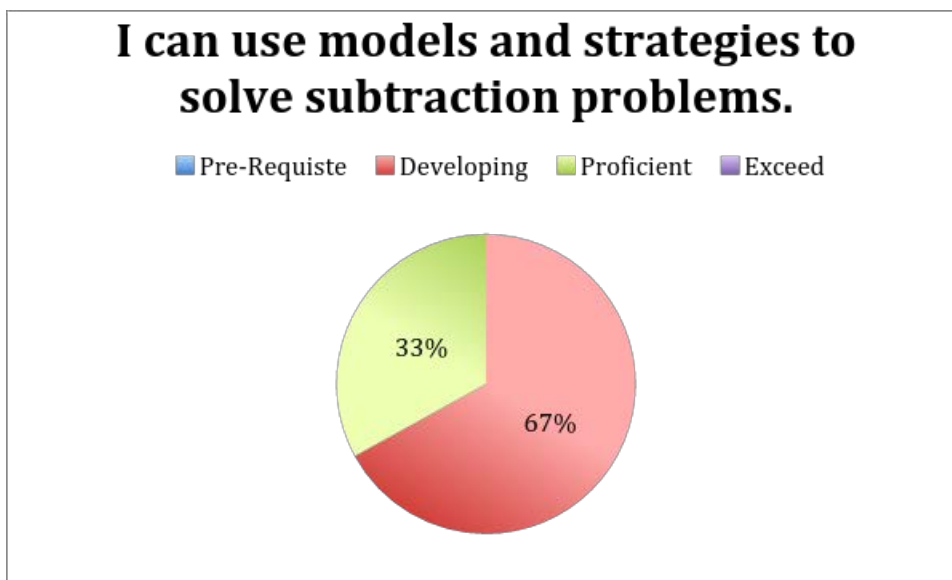
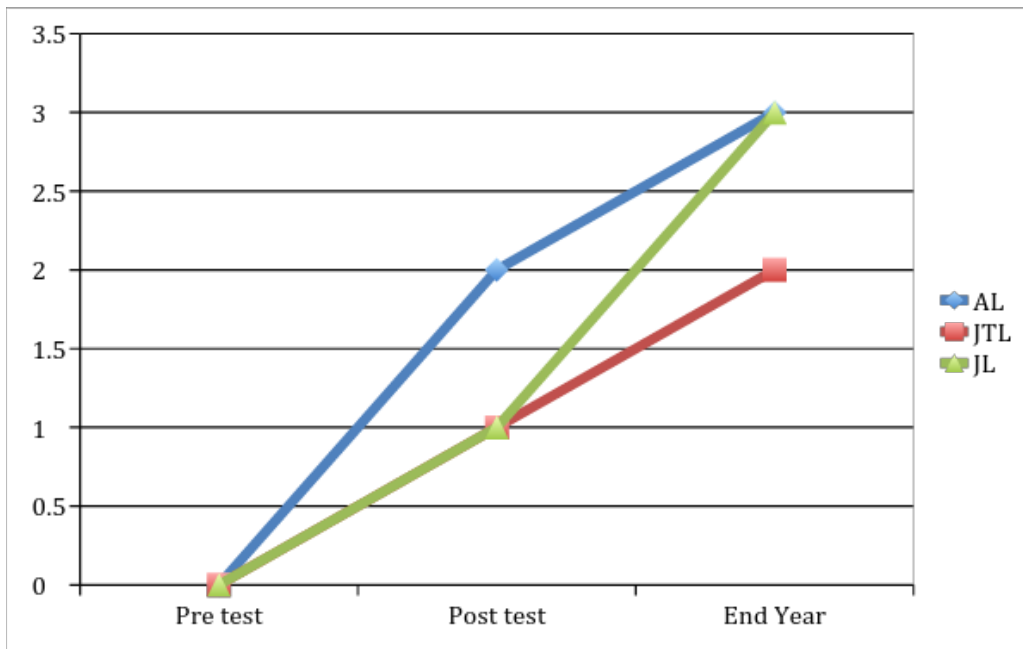


### Using Ten Minus



**Learning target 1.1.2.1 - I can use models and strategies to solve subtraction problems.**

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### Second Grade Addition Pretest and Posttest Results

#### Learning target: 2.1.2.2 - I can demonstrate additions quickly.

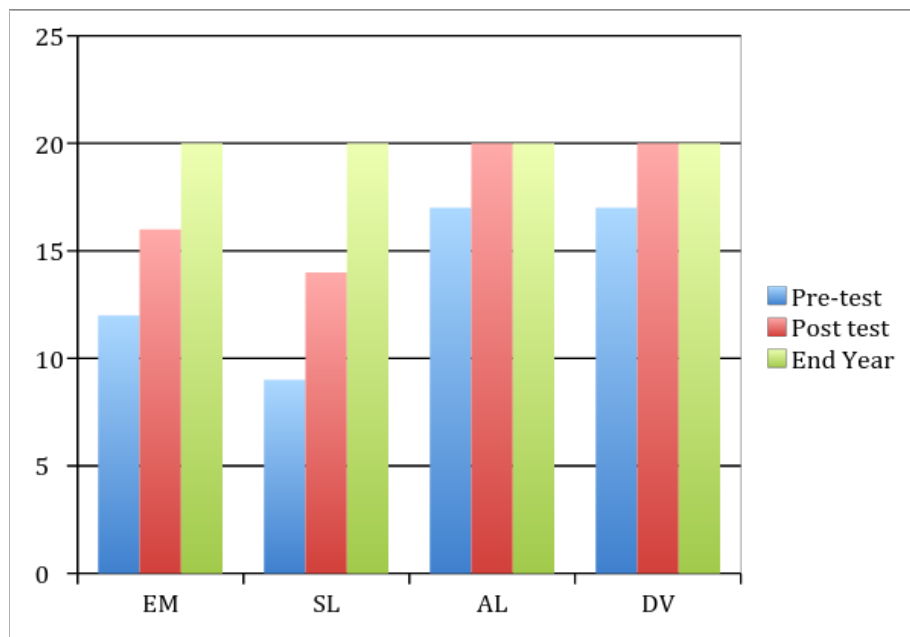
The second graders took a pretest on 'addition using ten.' 50% of the students scored 85%, 20% of the students scored 60%, and 30% of the students scored 45%.

During the first week, two second-graders used the golden beads and the other

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two second-graders used base ten blocks. In the second week, the students switched materials. During the third week, two students scored 100% on the Addition Using Ten post-test. One student scored 80% and one student scored 70%. Overall, the results showed that students used both types of manipulatives effectively and they made gains in their assessment scores.

### Using Ten +



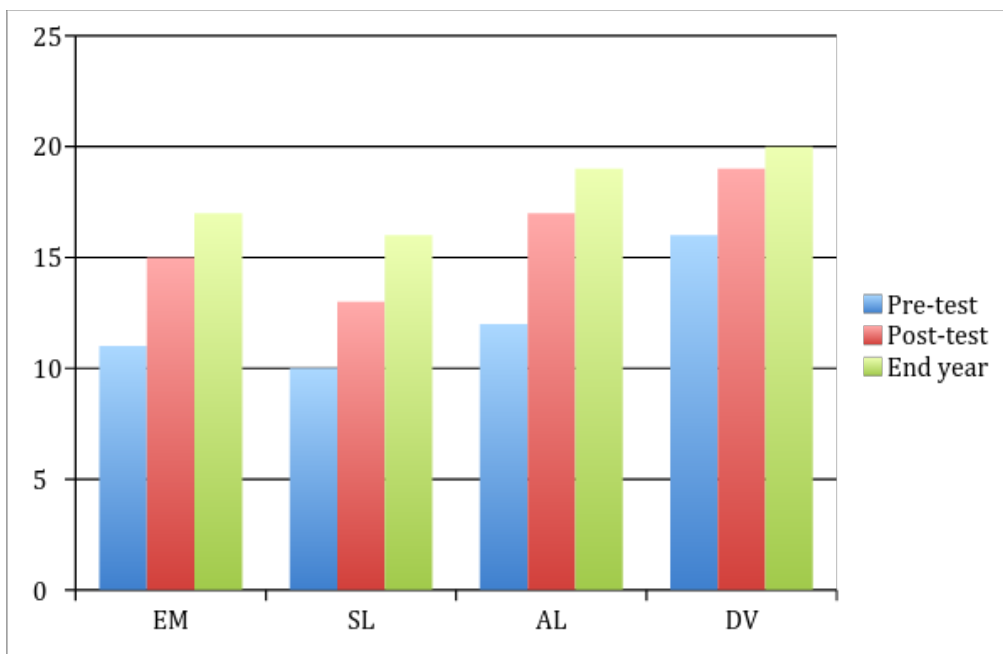
### Second Grade Subtraction Pretest and Posttest Results

For the pre-test on Subtraction Using Ten, 25% of the students scored 80%, and 75% of students scored lower than 60%.

During the first week, one student used the golden beads and scored 75% on the post-test. Two students used the glass counters; one scored 65%, and the other scored

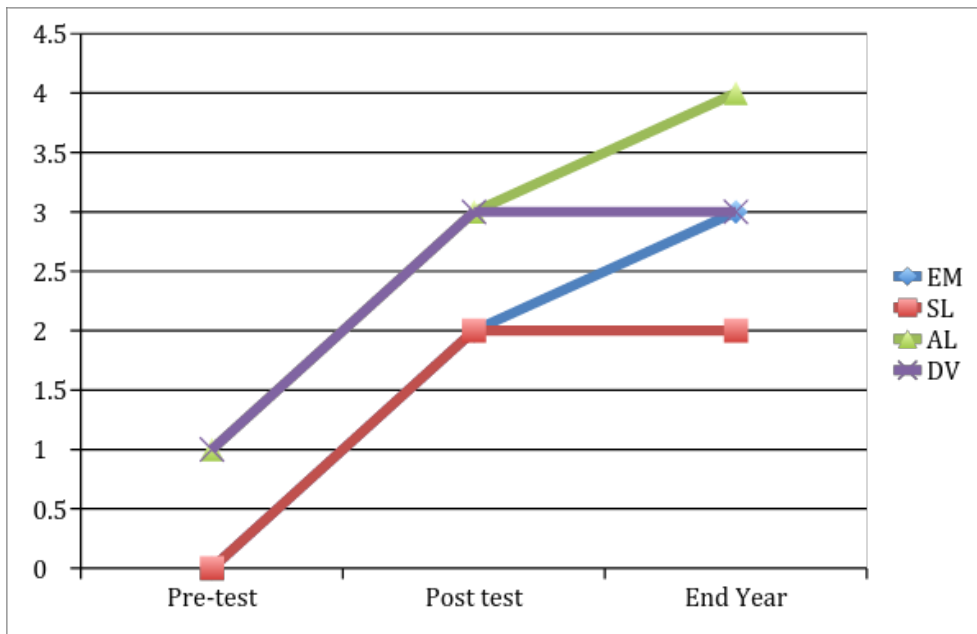
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85%. One student used the strip board and scored 95%. The overall results showed 50% at Proficient and 50% at Developing. The students used the materials effectively and showed gains in subtraction skills on post-tests.

**Using Ten -**

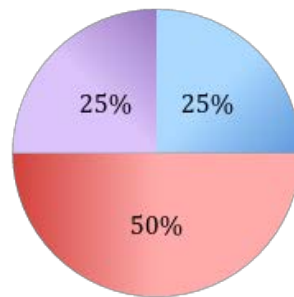
**Learning target: I can use fact families to understand addition and subtraction.**

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**I can use fact families to understand addition and subtraction.**

Pre-Requisite Developing Proficient Exceed

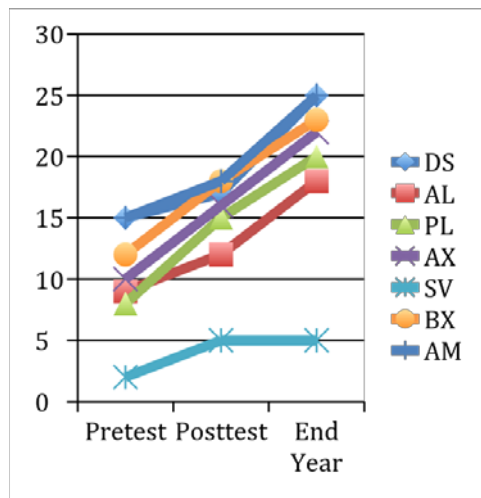


## Third Grade Multiplication Fact Assessment Results

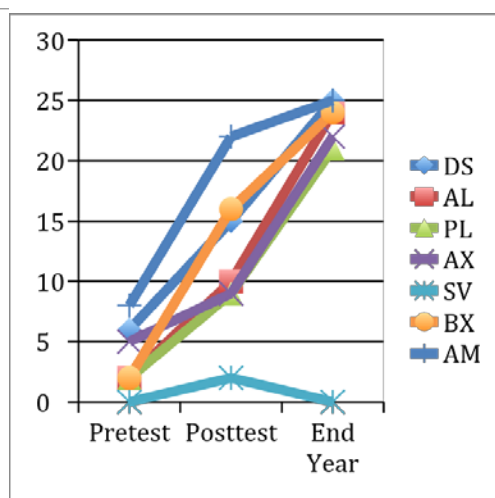


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### Multiplies of 6



### Division



Each section of the Math Facts assessment has twenty-five math problems. The assessments are timed at 2 minutes for each section. According to the district Math Facts program, each student must score 24 points or higher to pass. The goal is for students to recall a math fact quickly and understand the strategies being taught.

The results in the Multiplies of 6 assessments indicated that 29% of students scored 24 points or higher, and 71% of students scored below 24 points. For the division section, 57% of students scored 24 points or higher, and 43% scored below 24 points.

For the Multiplies of 9 assessments, the results showed that 71% of students scored 24 points or higher, and 29% students scored below 24 points. For the division sections, the results were the same: about 71% scored 24 points or higher, and 29% students scored below 24 points.

On the Multiplies of 8 assessments, the results showed that 57% of students scored 24 points or higher, and 43% of students scored below 24 points. For the division section, 43% of students scored 24 points or higher, and 57% of students scored below 24

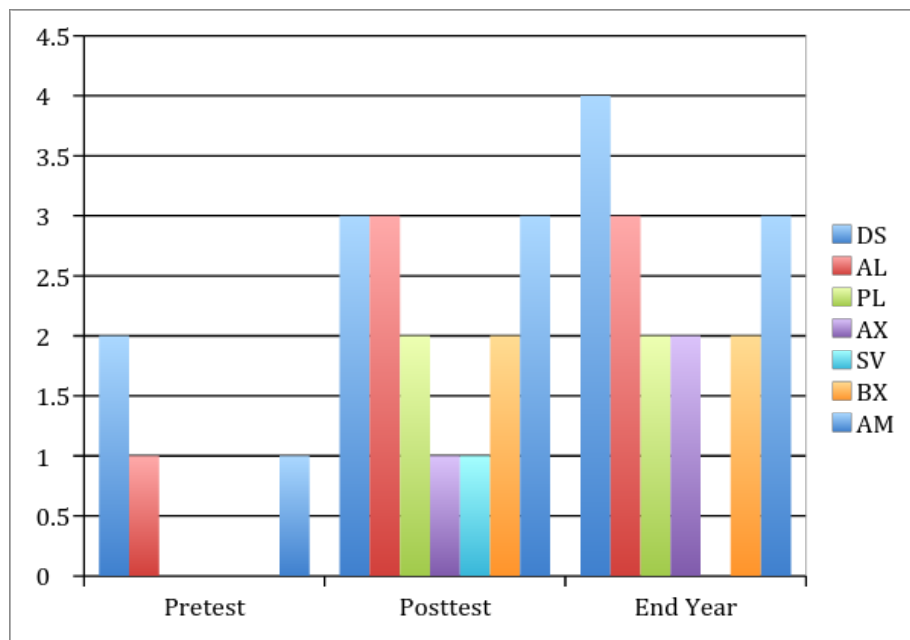
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points.

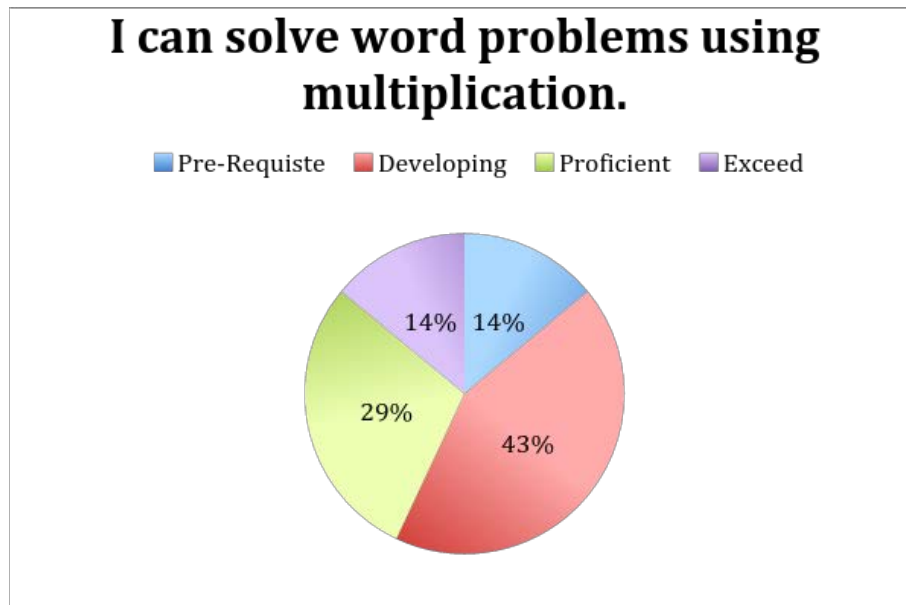
On the Multiples of 7 assessment, the results showed that 29% of students scored 24 points or higher, and 71% scored below 24 points. For the division section, 43% of students scored 24 points or higher, and 57% scored below 24 points.

### Third Grade Learning target 3.1.2.4 a - I can solve word problems using multiplication.

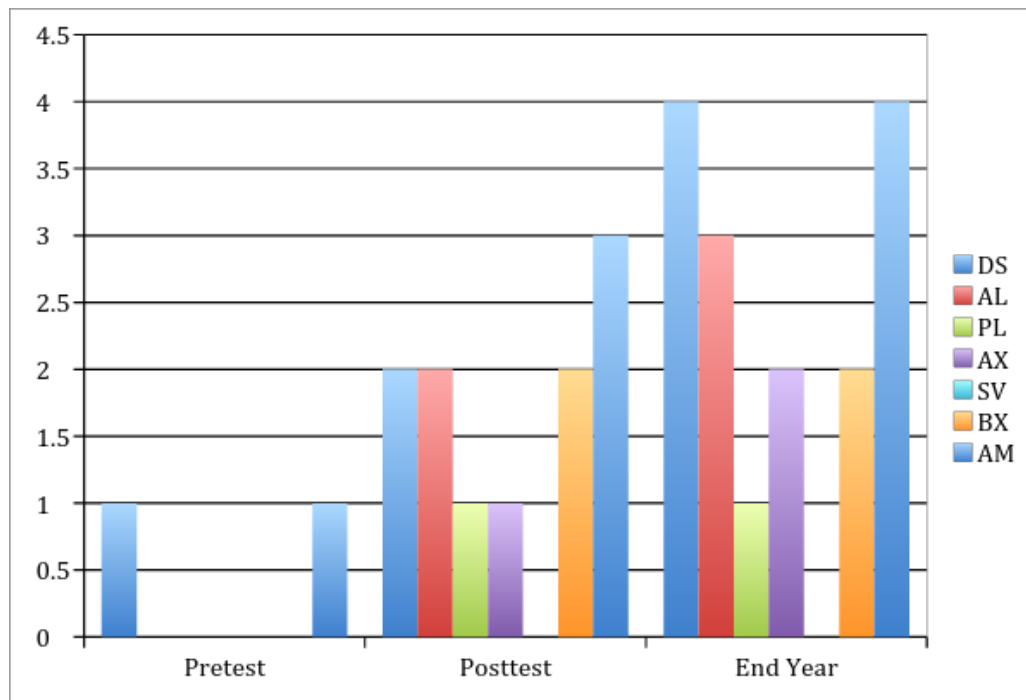
Test results for the third-grade students showed 14% at Exceed (Proficient), 29% at Proficient, 43% Developing, and 14% at Prerequisite.



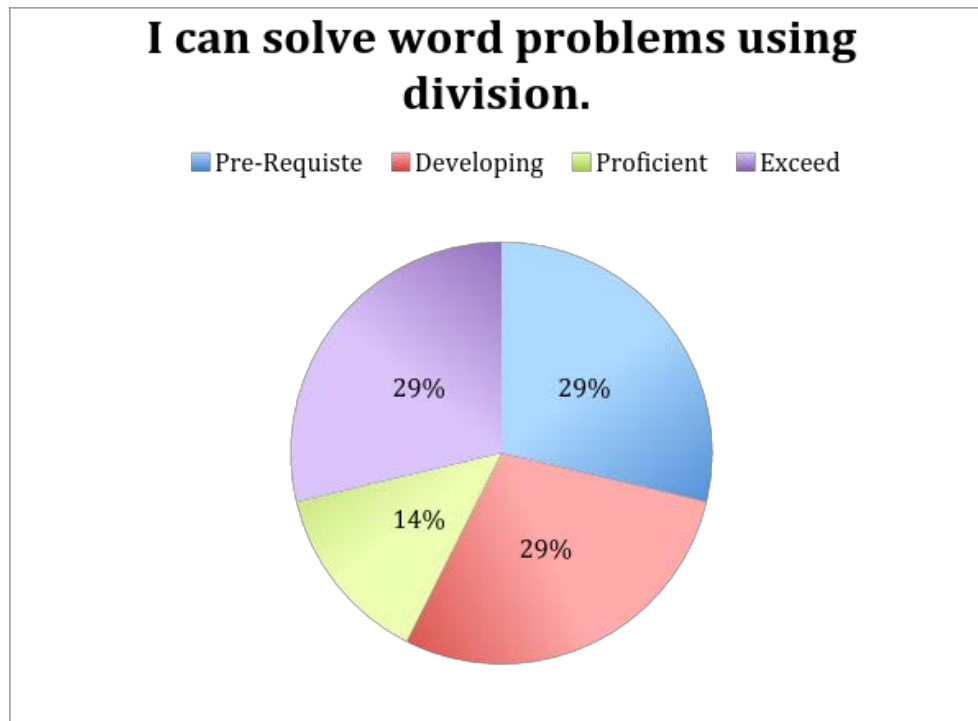
# MATH GAINS IN EARLY ELEMENTARY GRADES



## Learning Target 3.1.2.4 b - I can solve word problems using division.



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**The impact of the results****Analysis**

Some changes in instruction need to occur in order to help all students learn their math skills proficiently. Students could use alternative strategies, such as drawing pictures to represent math concepts. For example, students could solve the math problems with manipulatives, then draw the pictures. This would allow them to draw representations of the manipulatives to help them solve math problems during testing. Another change is to provide more teacher guidance during students' individual work time, and to offer immediate feedback on their problem-solving attempts. Checking in with students on their assignments more often before the post-test might help ensure more steady progress through the continuum of lessons. Students would also benefit

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from reviewing concept-related math vocabulary, and participating in lessons that address reading skills needed for understanding word problems.

### **Results**

Overall, the quantitative results showed positive improvement in pre- to posttest scores when students used either traditional or Montessori manipulatives, or a combination of both. One of the highest score increases was from 1 point (Pre-requisite) to 4 points (Exceeds). Some students who scored Proficient did not achieve further increases, but remained at the same level at the end of the year.

The qualitative results also demonstrated positive gains. The majority of students used a variety of manipulatives to help them solve math problems and answer assessment questions correctly, achieving gains. Throughout the course of the study, students were engaged and focused as they used manipulatives during work time.

### **Conclusion**

Both the quantitative and qualitative results from this study showed that using manipulatives – whether traditional, Montessori, or a combination of both – produced positive gains in student test scores measuring understanding of key mathematical concepts. Even those students with the lowest pre-test scores showed gains in the post-tests.

However, a significant limitation of the study involves students switching

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between manipulative materials every week, resulting in difficulty in precisely comparing the use of both types of materials. Therefore, I cannot answer my original question; whether Montessori-specific manipulative materials are more effective than manipulatives used in traditional public schools, in helping students to build their understanding of key mathematical concepts.

In a future study, I would allow students more time - about 2 weeks - to practice with manipulatives before post-testing. Also, in a future study, I would use Montessori-specific manipulatives and traditional public school manipulatives separately for a period of time, and then compare which manipulative materials are more effective; then, I would conduct a study comparing the use of manipulatives with another classroom that uses no or fewer math manipulatives.

### **Future Actions**

I will continue to use manipulatives in my classroom to help support student understanding of key mathematical concepts and to help increase post-test scores and performance on high-stakes standardized math testing, such as end-of-the-year Minnesota Comprehensive Assessments (MCAs). As Uribe-Flórez & Wilkins (2010) stated, “Teachers who tend to believe that it is important to have students participate in appropriate hands-on activities for effective mathematics instruction tend to use manipulatives more frequently in their mathematics lessons.” I believe the use of manipulatives in my classroom will help support my students’ learning and academic gains. I also want to implement technology tools and iPad apps to help increase student understanding of mathematical concepts and to help increase test scores.

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# MATH GAINS IN EARLY ELEMENTARY GRADES

## Appendix A

### 1st Grade Math High Priority Benchmarks & Learning Targets (progress monitor on electronic data wall and progress report)

MCA-III Strand	SPPS High Priority Benchmark	Learning Target
Number & Operation	<b>1.1.1.2</b> Read, write and represent whole numbers up to 120. Representations may include numerals, addition and subtraction, pictures, tally marks, number lines and manipulatives, such as bundles of sticks and base 10 blocks.	a.) I can read numbers up to 120. b.) I can write numbers up to 120. c.) I can represent numbers up to 120 in different ways.
	<b>1.1.2.1</b> Use words, pictures, objects, length-based models (connecting cubes), numerals and number lines to model and solve addition and subtraction problems in part-part-total, adding to, taking away from and comparing situations.	a.) I can use models and strategies to solve addition problems. b.) I can use models and strategies to solve subtraction problems. c.) I can solve real-world addition problems. d.) I can solve real-world subtraction problems.
	<b>1.1.2.3</b> Recognize the relationship between counting and addition and subtraction. Skip count by 2s, 5s, and 10s.	a.) I can skip count by 2. b.) I can skip count by 5. c.) I can skip count by 10. d.) I can subtract by counting backwards. e.) I can add by counting forward.
Algebra	<b>1.2.1.1</b> Create simple patterns using objects, pictures, numbers and rules. Identify possible rules to complete or extend patterns. Patterns may be repeating, growing or shrinking. Calculators can be used to create and explore patterns.	a.) I can create simple patterns using objects, pictures, numbers and rules. b.) I can describe simple patterns using objects, pictures, numbers and rules. c.) I can recognize simple patterns, using objects, pictures, numbers and rules. d.) I can identify rules to complete or extend patterns that are repeating, growing and shrinking. e.) I can extend and complete patterns that are repeating. f.) I can extend and complete patterns that are growing. g.) I can extend and complete patterns that are shrinking. h.) I can use a calculator to create and explore patterns.

### 1st Grade Math High Priority Benchmarks & Learning Targets (progress monitor on electronic data wall and progress report)

MCA-III Strand	SPPS High Priority Benchmark	Learning Target
Geometry & Measurement	<b>1.3.1.1</b> Describe characteristics of two- and three-dimensional objects, such as triangles, squares, rectangles, circles, rectangular prisms, cylinders, cones and spheres.	a.) I can describe a triangle. b.) I can describe a square. c.) I can describe a rectangle. d.) I can describe a circle. e.) I can describe a rectangular prism. f.) I can describe a cylinder. g.) I can describe a cone. h.) I can describe a sphere.
	<b>1.3.2.3</b> Identify pennies, nickels and dimes; find the value of a group of these coins, up to one dollar.	a.) I can identify pennies. b.) I can identify nickels. c.) I can identify dimes. d.) I can count groups of pennies. e.) I can count groups of nickels. f.) I can count groups of dimes. g.) I can count mixed groups of coins (pennies, nickels, dimes) up to one dollar.

# MATH GAINS IN EARLY ELEMENTARY GRADES

## Appendix B

### 2nd Grade Math High Priority Benchmarks & Learning Targets (progress monitor on electronic data wall and progress report)

MCA-III Strand	SPPS High Priority Benchmark	Learning Target
Number & Operation	2.1.1.1 Read, write and represent whole numbers up to 1000. Representations may include numerals, addition, subtraction, multiplication, words, pictures, tally marks, number lines and manipulatives, such as bundles of sticks and base 10 blocks. 2.1.1.2 Use place value to describe whole numbers between 10 and 1000 in terms of hundreds, tens and ones. Know that 100 is 10 tens, and 1000 is 10 hundreds.	a.) I can read a number from 0-1000. b.) I can write a number from 0-1000. c.) I can show a number from 0-1000 in a variety of ways. d.) I can describe whole numbers 10-1000 in terms of ones, tens and hundreds.
	2.1.2.2 Demonstrate fluency with basic addition facts and related subtraction facts.	a.) I can use fact families to understand addition and subtraction. b.) I can demonstrate addition facts quickly. c.) I can demonstrate subtraction facts quickly.
	2.1.2.5 Solve real-world and mathematical addition and subtraction problems involving whole numbers with up to 2 digits.	a.) I can solve one- and two- digit addition problems. b.) I can solve one- and two- digit subtraction problems. c.) I can solve real-world addition problems with one- and two- digit numbers. d.) I can solve real-world subtraction problems with one- and two-digit numbers.
Algebra	2.2.1.1 Identify, create and describe simple number patterns involving repeated addition or subtraction, skip counting and arrays of objects such as counters or tiles. Use patterns to solve problems in various contexts.	a.) I can identify a number pattern with addition and subtraction. b.) I can create a number pattern with addition and subtraction. c.) I can describe a number pattern with addition and subtraction. d.) I can use patterns and rules to solve mathematical problems. e.) I can use patterns and rules to solve real world problems.

### 2nd Grade Math High Priority Benchmarks & Learning Targets (progress monitor on electronic data wall and progress report)

MCA-III Strand	SPPS High Priority Benchmark	Learning Target
Geometry & Measurement	2.3.1.1 Describe, compare, and classify two- and three-dimensional figures according to number and shape of faces, and the number of sides, edges and vertices (corners).	a.) I can describe two-dimensional figures by the number of sides and vertices. b.) I can compare two-dimensional figures by the number of sides and vertices. c.) I can classify two-dimensional figures by the number and shape of sides and vertices. d.) I can describe three-dimensional figures by the number and shape of faces, edges and vertices. e.) I can compare three-dimensional figures by the number and shape of faces, edges and vertices. f.) I can classify three-dimensional figures by the number and shape of faces, edges and vertices.
	2.3.2.2 Demonstrate an understanding of the relationship between length and the numbers on a ruler by using a ruler to measure lengths to the nearest centimeter or inch.	a.) I can show that the numbers on a ruler name a length. b.) I can use a ruler to measure to the nearest centimeter. c.) I can use a ruler to measure to the nearest inch.
	2.3.3.2 Identify pennies, nickels, dimes and quarters. Find the value of a group of coins and determine combinations of coins that equal a given amount.	a.) I can identify pennies, nickels, dimes and quarters. b.) I can find the value of a group of coins. c.) I can combine coins to make a given amount.

# MATH GAINS IN EARLY ELEMENTARY GRADES

## Appendix C

### 3rd Grade Math High Priority Benchmarks & Learning Targets (progress monitor on electronic data wall and progress report)

MCA-III Strand	SPPS High Priority Benchmark	Learning Target
Number & Operation	<b>3.1.1.1</b> Read, write and represent whole numbers up to 100,000. Representations may include numerals, expressions with operations, words, pictures, number lines, and manipulatives such as bundles of sticks and base 10 blocks.	a.) I can read whole numbers up to 100,000. b.) I can write whole numbers up to 100,000. c.) I can show whole numbers up to 100,000.
	<b>3.1.2.2</b> Use addition and subtraction to solve real-world and mathematical problems involving whole numbers. Use various strategies, including the relationship between addition and subtraction, the use of technology, and the context of the problem to assess the reasonableness of results.	a.) I can add multi-digit whole numbers. b.) I can subtract multi-digit whole numbers. c.) I can solve real-world math problems using addition. d.) I can solve real-world math problems using subtraction. e.) I can use different strategies to check my answers.
	<b>3.1.2.4</b> Solve real-world and mathematical problems involving multiplication and division, including both "how many in each group" and "how many groups" division problems.	a.) I can solve word problems using multiplication. b.) I can solve word problems using division. c.) I can figure out how many are in each group when I divide. d.) I can figure out how many groups there are when I divide.
	<b>3.1.3.1</b> Read and write fractions with words and symbols. Recognize that fractions can be used to represent parts of a whole, parts of a set, points on a number line, or distances on a number line.	a.) I can read fractions with words. b.) I can read fractions with symbols. c.) I can write fractions with words. d.) I can write fractions with symbols. e.) I can show parts of a whole. f.) I can show parts of a set. g.) I can find points on a number line. h.) I can find distances on a number line.
Algebra	<b>3.2.1.1</b> Create, describe, and apply single-operation input-output rules involving addition, subtraction and multiplication to solve problems in various contexts.	a.) I can describe patterns using input and output rules. b.) I can create input and output rules with addition. c.) I can create input and output rules with subtraction. d.) I can create input and output rules with multiplication. e.) I can use input and output rules to solve problems. f.) I can apply input and output rules in real-world problems.

### 3rd Grade Math High Priority Benchmarks & Learning Targets (progress monitor on electronic data wall and progress report)

MCA-III Strand	SPPS High Priority Benchmark	Learning Target
Geometry & Measurement	<b>3.2.2.2</b> Use multiplication and division basic facts to represent a given problem situation using a number sentence. Use number sense and multiplication and division basic facts to find values for the unknowns that make the number sentences true.	a.) I can solve true number sentences/equations using multiplication. b.) I can solve true number sentences/equations using division. c.) I can create true number sentences/equations using multiplication. d.) I can create true number sentences/equations using division. e.) I can find values of unknowns in equations/number sentences. f.) I can use number sentences/equations to represent a situation.
	<b>3.3.2.2</b> Find the perimeter of a polygon by adding the lengths of the sides.	a.) I can find the perimeter of a polygon by adding the length of the sides.
	<b>3.3.3.3</b> Make change up to one dollar in several different ways, including with as few coins as possible.	a.) I can make change up to one dollar with the fewest coins possible. b.) I can solve real-world money problems for making change.
Analysis & Probability	<b>3.4.1.1</b> Collect, display and interpret data using frequency tables, bar graphs, picture graphs and number line plots having a variety of scales. Use appropriate titles, labels, and units in my displays.	a.) I can collect data using a frequency table. b.) I can display data in a frequency table. c.) I can display data in a bar graph. d.) I can display data in a pictograph. e.) I can display data in a number line plot. f.) I can use titles, labels, and units in my displays. g.) I can use a variety of scales in my displays. h.) I can interpret data in frequency tables.



## MATH GAINS IN EARLY ELEMENTARY GRADES

## Appendix D

Teddy Bears



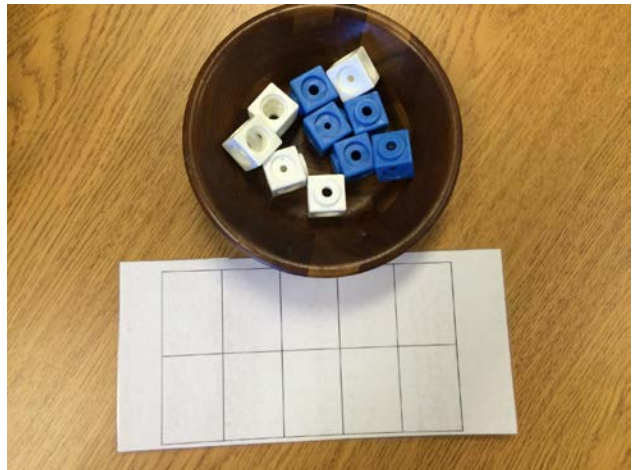
Colored Cubes



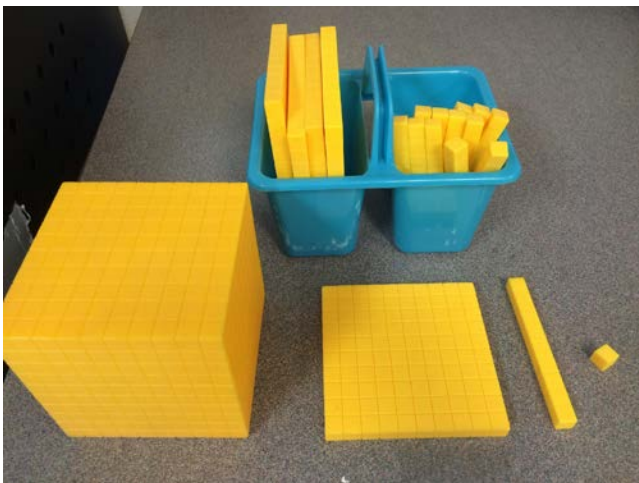
Glass Beads



Ten Frame



Base Ten Blocks



Unifix® Cubes



# MATH GAINS IN EARLY ELEMENTARY GRADES

## Appendix E

### Colored Bead Bar

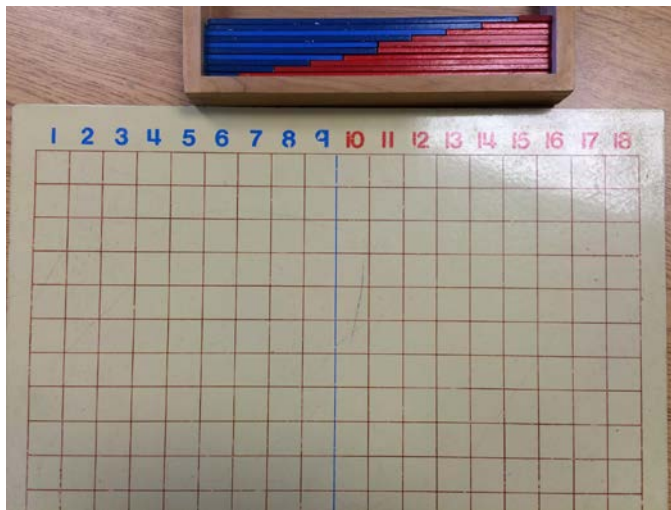


### Stamp Game



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### Strip Board



### Golden Beads

