

# Second-order cone robust AC–DC DOPF considering correlation of wind power

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**Abstract:** Due to the increasing penetration, intermittent renewable energy sources (RESs), such as wind power, have added additional uncertainties to AC–DC power system operation. The strong correlation between wind power will bring larger computational errors. It makes generation dispatching incompatible with the current single-period optimal power flow practice. This study presents an AC–DC affinely adjustable robust dynamic optimal power flow (AARDOPF) model. Ellipsoidal uncertainty set is adopted to well fit the spatial-temporal correlated wind power. Affine policies are utilised in the re-dispatch process. To apply AC–DC AARDOPF, the base-point generation is calculated and determined to match the power with forecasted RES output. And once the uncertainty is revealed, generators reschedule its output through participation factors responding to the uncertain fluctuation of RES output to ensure a feasible solution for all realisations of RES output. Then original non-linear models are transformed into SOCP models. Finally, GUROBI is used to solve this model. Numerical results are obtained on modified AC–DC IEEE 30-bus and 118-bus system to minimise the expected operational cost. Comparing with stochastic dynamic optimal power flow (DOPF), this model is proved to be more practical and effective.

## Nomenclature

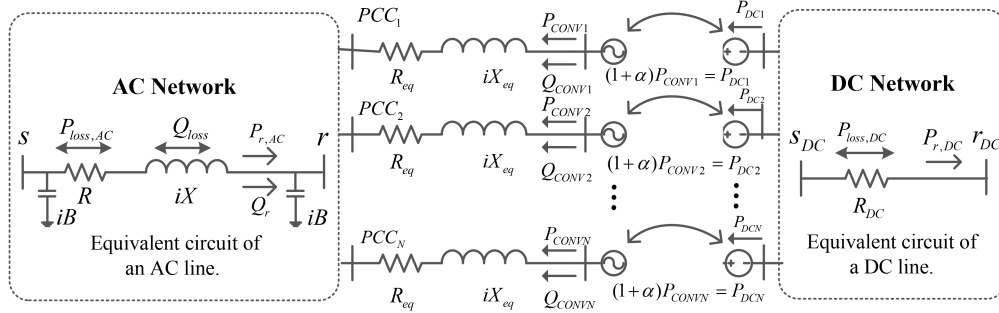
$n_{bAC}$	number of AC buses
$n_{lAC}$	number of AC lines
$n_{bDC}$	number of DC buses and converters
$n_{lDC}$	number of DC lines
$P_G$	vector of active power generation at each AC bus with size of $n_{bAC} \times 1$
$P_R$	vector of RES generation at each AC bus with size of $n_{bAC} \times 1$
$P_D$	vector containing active power loads at each AC bus with size of $n_{bAC} \times 1$
$P_{CONV}$	injected/absorbed active power vector of converters at PCCs
$P_L$	vector containing AC line flow active powers at receiving end of AC lines with size of $n_{lAC} \times 1$
$P_{DC}$	injected/absorbed active power vector at DC buses
$P_{r,DC}$	vector containing DC line flow active powers at receiving end of DC lines with size of $n_{lDC} \times 1$
$P_{loss,DC}$	vector containing DC line flow active power losses at AC lines with the size of $n_{lDC} \times 1$
$R_{DC}$	diagonal matrix where $R_{j,j}$ is resistance of DC lines with size of $n_{lDC} \times n_{lDC}$
$M_{P_{DC}}$	incidence matrix associated with DC line active powers with size of $n_{bDC} \times n_{lDC}$
$M_{l_{DC}}$	incidence matrix associated with DC line active power losses with size of $n_{bDC} \times n_{lDC}$
$P_R$	vector of nominal wind plant
$P_{0R}$	expectation of nominal wind plant
$\xi_{j,t}$	uncertain component of wind plant
$\xi_{all,t}$	uncertain component of all wind power
$P_G$	vector of controllable generation
$P_{G(base)}$	vector of base-point generation
$\beta$	vector of participation factors
$a, b, c$	cost coefficients of power generation
$\Lambda_{(j,k)}$	covariance matrix of wind plant output prediction errors

$P_D$	vector of real power demand
min/ max	minimum/maximum magnitude operator
$L$	matrix of power transfer distribution factors
$E$	identity matrix
$R_{ampGi}$	vector of generator ramp-rate limits
$\Omega_{\xi_t}/\Omega_{\xi_{all}}$	uncertainty set of $\xi_{j,t}/\xi_{all,t}$
$R_{\xi_t}/R_{\xi_{all}}$	covariance matrix of $\xi_{j,t}/\xi_{all,t}$
$C_{conf}$	budget value of the uncertainty set
$\mu_{set}$	probability of solution's feasibility
$i, j$	element $i$ or $j$ in vector or row $i$ or $j$ in matrix
$t$	time $t$

## 1 Introduction

Renewable energy sources (RESs) necessitate that the optimal power flow (OPF) solution accounts for the control of conventional power generators in response to errors of renewable power forecast, which are significantly larger than the traditional load forecast errors. It is known that RES generation can vary substantially even over short time intervals. Multiple wind power at adjacent locations may be highly correlated due to common weather factors. Thus, it requires specialised approaches for controlling dispatchable generation resources that balance the varying power [1]. OPF model considering uncertainty can be classified as probabilistic [2–4], stochastic [5–8] or robust [9–13]. It has high dimensional, non-convex, non-linear characteristics and requires an efficient optimisation technique to be solved.

Robust OPF minimises the worst-case cost or regret over the scenarios in the prescribed uncertainty set, such as a box or polyhedral uncertainty set [14]. Robust OPF is usually formulated as a two-stage problem which has a tri-level structure. But it is time consuming to solve this problem. Instead of the full recourse actions in the above tri-level structure, affine policies simplify the recourse actions to an affine form of the uncertain wind power. A robust look-ahead economic dispatch model is presented in [10] based on allowable intervals for wind power generation. Li *et al.* [11] employed affine policies for OPF and proposed a robust real-time wind power dispatch framework for coordinating wind farms. However, in these works, the forecast error is assumed to belong to



**Fig. 1** Modelling of AC–DC network for OPF formulation

interval uncertainty set, which have not considered temporal and spatial correlation of wind power thoroughly.

High-voltage direct current technology has opened opportunities to transfer the electric power between two points which in particular are remotely located from each other. For AC–DC OPF, most of the papers investigating OPF modelling of voltage source converters (VSC) has focused on VSC-based FACTS controllers embedded in AC systems [15–17]. However, given the non-convex nature of OPF problem, in all these studies, authors have tried to develop a convex optimisation problem through several methods, these methods suffer from two main drawbacks: (i) loss of accuracy in results, and (ii) complexity of algorithms. A set of AC–DC constraints developed through the line flow based equations of power flow is built in [18]. However, the issues of non-convex AC–DC OPF and the complex uncertainties of wind power have not been fully addressed by the existing methods.

This paper proposes a robust AC–DC dynamic optimal power flow (DOPF) model with adjustable conservatism. Generalised ellipsoidal uncertainty set is adopted here to well fit the spatial-temporal correlated wind power. Participation factors are employed in automatic generation control systems, and adjust generation levels as linear functions of the variations in RES generation. Then this non-convex, non-linear model is transformed to a second-order cone programming which can be solved by GUROBI efficiently. A new criterion is proposed to incorporate the statistical properties of wind power into robust optimisation. It provides an analytical relationship between the budget value and the confidence level of solution's feasibility without any assumption of the probability distribution of wind power. Comparing with classical stochastic DOPF, this model is proved to be more practical and effective.

## 2 Model for AC–DC DOPF

Based on paper [18], AC–DC network for OPF formulation is shown in Fig. 1. The converters can be modelled as a dummy generator or motor which either can absorb or inject active/reactive power to the AC network [18]. The phase reactor and coupling transformer between the points of common connection (PCCs) and converters are also represented as an AC line with equivalent inductance and resistance.

Consider a power system having  $N_B$  buses ( $N_B = n_{bAC} + n_{bDC}$ ),  $N_L$  lines ( $N_L = n_{lAC} + n_{lDC}$ ),  $N_G$  dispatchable generators, and  $N_R$  RESs. The objective of AC–DC DOPF can be formulated as a quadratic cost function [10]:

$$C_i(P_{Gi}) = aP_{Gi,t}^2 + bP_{Gi,t} + c, \quad i = 1, \dots, N_G \quad (1)$$

Let  $\Lambda_{(j,k)}$  denote the covariance matrix of the uncertainty. The quadratic cost function of unit can be rewritten as

$$\min \sum_{t=1}^{N_T} \sum_{i=1}^{N_G} (C_i(P_{Gi,t}) + c'_{2i,t} \beta_{i,t}^2) \quad (2)$$

where

$$c'_{2i,t} = \left[ \sum_{j=1}^{N_R} \sum_{k=1}^{N_R} \Lambda_{(j,k)} \right] \times c_{2i}, \quad i = 1, \dots, N_G$$

### 2.1 AC grid equations

The use of the DC-OPF is consistent with industry practice [10]. The DC-OPF can be written using balance power injection equations at each node, or it can be expressed equivalently as a generation load balance for the entire system together with line power flow constraints. Constrains for AC grid other than PCCs can be written as follows:

$$\sum_{i=1}^{N_G} P_{Gi,t} + \sum_{i=1}^{N_R} P_{Ri,t} = \sum_{i=1}^{N_B} P_{Di,t} \quad (3)$$

$$|L \times (E_1 P_{G,t} + E_2 P_{R,t} - E_3 P_{D,t})| \leq P_L^{\max} \quad (4)$$

$$P_{Gi}^{\min} \leq P_{Gi,t} \leq P_{Gi}^{\max}, \quad i = 1, \dots, N_G \quad (5)$$

$$-R_{ampGi} \leq P_{Gi,t} - P_{Gi,t-1} \leq R_{ampGi}, \quad i = 1, \dots, N_G \quad (6)$$

where  $L$  is an  $N_L \times (N_B - 1)$  matrix of power transfer distribution factors,  $E_1$ , and  $E_2$ ,  $E_3$  and are matrices that, respectively, map the vectors  $P_{G,t}$ ,  $P_{R,t}$ , and  $P_{D,t}$  into  $(N_B - 1) \times 1$  vectors such that the non-zero elements correspond to connection at one of the  $N_B$  buses, excluding the slack bus.

### 2.2 DC grid equations

As shown in Fig. 1, each DC line is specified by an optional direction from sending bus to receiving bus. Constrains for DC grid can be written as follows:

$$P_{DC,t} - (1 + \alpha)P_{CONV,t} = 0 \quad (7)$$

$$P_{DC,t} + M_{PDC} P_{r,DC,t} + M_{lDC} P_{loss,DC,t} = 0 \quad (8)$$

$$2R_{DC} P_{r,DC,t} + R_{DC} P_{loss,DC,t} - M_{PDC}^T W_{DC,t} = 0 \quad (9)$$

where

$$P_{loss,DC,j,t} = \frac{P_{r,DC,j,t}^2}{W_{DC,j,t}}, \quad j \in n_{lDC} \quad (10)$$

$$P_{r,DC,j}^{\min} \leq P_{r,DC,j,t} \leq P_{r,DC,j}^{\max}, \quad j \in n_{lDC} \quad (11)$$

$$V_{DC,i}^{\min} \leq W_{DC,i,t} \leq V_{DC,i}^{\max}, \quad i \in n_{bDC} \quad (12)$$

Equation (7) represents the relation between injected DC power at DC network and converter power at PCCs, (8) is associated with power balance constraints at DC buses and (9) represents the voltage drop constraint on DC lines.

### 2.3 Converters equations

As shown in Fig. 1, the active power balance equations at each PCC can be obtained as follows:

$$|L \times (E_1 P_{G,t} + E_2 P_{R,t} - E_3 P_{D,t} + E_4 P_{\text{CONV},t})| \leq P_L^{\max} \quad (13)$$

$$P_{\text{CONV},i}^{\min} \leq P_{\text{CONV},i,t} \leq P_{\text{CONV},i}^{\max}, \quad i \in n_{\text{bPCC}} \quad (14)$$

$E_4$  is the matrix that, respectively, map the vectors  $P_{\text{CONV},t}$  into  $(N_B - 1) \times 1$  vectors such that the non-zero elements correspond to connection at one of the  $N_B$  buses, excluding the slack bus.

## 3 Affinely adjustable robust DOPF

### 3.1 Ellipsoidal uncertainty set

$\xi_{j,t}$  and  $\xi_{\text{all},t}$  are captured by an generalised ellipsoid set  $\Omega_{\xi_t}$  and  $\Omega_{\xi_{\text{all}}}$  as follows [14]:

$$\begin{cases} \Omega_{\xi_t} = \{\xi_t: \xi_t^T R_{\xi_t}^{-1} \xi_t \leq C_{\text{conf}}\} \\ \Omega_{\xi_{\text{all}}} = \{\xi_{\text{all}}: \xi_{\text{all}}^T R_{\xi_{\text{all}}}^{-1} \xi_{\text{all}} \leq C_{\text{conf}}\} \end{cases} \quad (15)$$

where  $R_{\xi_t}$  and  $R_{\xi_{\text{all}}}$  are the covariance matrix of  $\xi_{j,t}$  and  $\xi_{\text{all},t}$ ;  $\Omega_{\xi_t}$  and  $\Omega_{\xi_{\text{all}}}$  are compact ellipsoidal uncertainty sets which consider the spatial-temporal correlation of wind power through the covariance matrix  $R_{\xi_t}$  and  $R_{\xi_{\text{all}}}$ ;  $C_{\text{conf}}$  is the budget value which measures the size of uncertainty set, and determines the economic benefits.

### 3.2 Model for AARDOPF

$P_{Rj,t}$  is the active and reactive power output of wind power  $j$  at time  $t$  and can be expressed as

$$P_{Rj,t} = P_{0Rj,t} + \xi_{j,t}, \quad j = 1, \dots, N_R \quad (16)$$

where  $P_{0Rj,t}$  and  $\xi_{j,t}$  are the expectation and random term of  $P_{Rj,t}$ , respectively.

The power generation consists of the base-point value  $P_{Gi,t(\text{base})}$  and the change in generation corresponding to fluctuations in RESs output, therefore

$$P_{Gi,t} = P_{Gi,t(\text{base})} - \beta_{i,t} \times \sum_{j=1}^{N_R} \xi_{j,t}, \quad i = 1, \dots, N_G \quad (17)$$

where  $\beta_{i,t}$  denote the participation factor of unit  $j$ , which is the rate of change of the generator output corresponding to the change in total controllable generation. The participation factor is computed in two scenarios: (i) by fixing the participation factors using the classical equation (18) [10], and (ii) by treating the participation factors as variables which are chosen by the program automatically

$$\beta_{i,t(\text{fix})} = \frac{1/a_i}{\sum_{j=1}^{N_G} 1/a_j}, \quad i = 1, \dots, N_G \quad (18)$$

Let  $x_t = P_{G,t(\text{base})}$  denote the vector of non-adjustable variables. Let  $y_t = W\xi_t$  denote the vector of adjustable variables, which is affinely related to the uncertainty vector ( $W$  is an  $N_G \times N_R$  matrix of ones). Define

$$\begin{aligned} A_1 &= \begin{bmatrix} E_1 \\ -E_1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -\text{diag}\{\beta\} \\ \text{diag}\{\beta\} \end{bmatrix} \\ d_1 &= \begin{bmatrix} P_G^{\max} \\ -P_G^{\min} \end{bmatrix}, \quad d_3 = \begin{bmatrix} R_{\text{amp}G} \\ R_{\text{amp}G} \end{bmatrix} \end{aligned} \quad (19)$$

$$\begin{aligned} A_2 &= \begin{bmatrix} LE_1 \\ -LE_1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -LE_1 \text{diag}\{\beta\} \\ LE_1 \text{diag}\{\beta\} \end{bmatrix} \\ D &= \begin{bmatrix} -LE_2 \\ LE_2 \end{bmatrix}, \quad d_2 = \begin{bmatrix} P_L^{\max} + LE_3 P_{D,t} - LE_2 P_{R,t} - LE_4 P_{\text{CONV},t} \\ P_L^{\max} - LE_3 P_{D,t} + LE_2 P_{R,t} + LE_4 P_{\text{CONV},t} \end{bmatrix} \end{aligned} \quad (20)$$

Therefore, constrains (3)–(6) and (13) for given  $\xi_{j,t}$  and  $\xi_{\text{all},t}$  can be written as

$$\begin{aligned} \sum_{i=1}^{N_G} x_{i,t} &= \sum_{i=1}^{N_B} P_{Di,t} - \sum_{i=1}^{N_R} P_{Ri,t} \\ A_1 x_t + B_1 y_t &\leq d_1 \end{aligned} \quad (21)$$

$$A_2 x_t + B_2 y_t - D \xi_t \leq d_2$$

$$A_1(x_t - x_{t-1}) + B_1(y_t - y_{t-1}) \leq R_{\text{amp}G}$$

## 4 Solution approach

Let  $\xi_{\text{all}} = [\xi_{\text{all},1}, \dots, \xi_{\text{all},T}]^T$  denote all wind power uncertainty at time  $t$ . As an example, the second constraints in (21) can be rewritten as

$$A_1 x_t + B_1 W \xi_t \leq d_1 \quad (22)$$

The solution of AARDOPF is to ensure the feasibility for infinite scenarios of uncertainties within the uncertainty set. More exactly, the following robust counterpart of (22), namely the worst case, must hold:

$$A_1 x_t + \max_{\xi \in \Omega_{\xi_t}} B_1 W \xi_t \leq d_1 \quad (23)$$

By using Cholesky decomposition  $R_{\xi_t} = L_{\xi_t} L_{\xi_t}^T$ ,  $\xi_t$  can be replaced by auxiliary  $\eta$ , and  $\Omega_{\xi_t}$  can be reformulated as follows:

$$\Omega_{\xi_t} = \{\xi_t: \xi_t = \sqrt{C_{\text{conf}}} L_{\xi_t} \eta, \|\eta\|_2 \leq 1\} \quad (24)$$

Then (23) can be rewritten as

$$A_1 x_t + \max_{\|\eta\|_2 \leq 1} \{\sqrt{C_{\text{conf}}} L_{\xi_t} B_1 W \eta\} \leq d_1 \quad (25)$$

Which is equivalent to

$$A_1 x_t + \sqrt{C_{\text{conf}}} \|L_{\xi_t} B_1 W\|_2 \leq d_1 \quad (26)$$

Same as (26), the last two equations in (21) can be rewritten as

$$A_2 x_t + \sqrt{C_{\text{conf}}} \|L_{\xi_t} (B_2 W - D)\|_2 \leq d_2 \quad (27)$$

$$A_1 M x_t + \sqrt{C_{\text{conf}}} \|L_{\xi_t} B_1 W M\|_2 \leq d_3 \quad (28)$$

where

$$M = \begin{bmatrix} -1 & 1 & \dots & 0 & 0 \\ 0 & -1 & \dots & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & -1 & 1 \end{bmatrix}_{N_R \times N_R}$$

Let  $Z_{r,DC_j} = P_{r,DC_j}^2 / W_{DC_j}$  (10) can be transformed into

$$P_{\text{loss},DC_j} = Z_{r,DC_j} R_{DC_j,j} \quad (29)$$

$$\| \frac{2P_{r,DC_j}}{Z_{r,DC_j} - W_{DC_j}} \| \leq (Z_{r,DC_j} + W_{DC_j}) \quad (30)$$

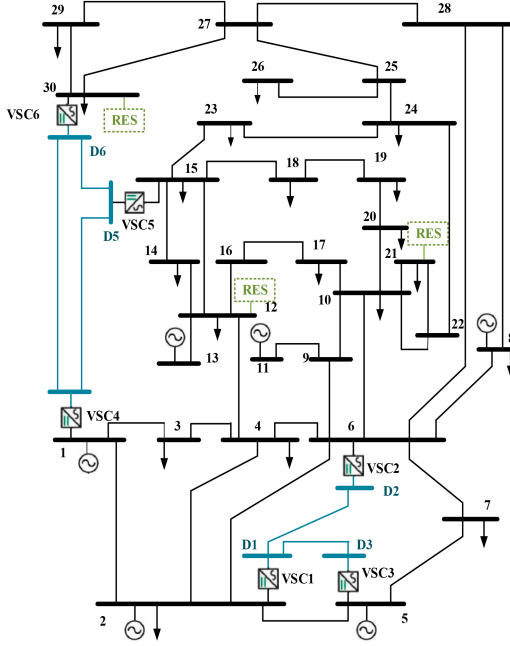


Fig. 2 Modified AC-DC IEEE-30 system with RES

Table 1 Percentage increase in expected cost

$\mu_{\text{set}}$	$\beta$ by (4)		$\beta$ as variables	
	Cost, \$	$r\%$	Cost, \$	$r\%$
0	1,324,555.78	0	1,324,555.78	0
0.1	1,327,509.53	0.223	1,325,284.28	0.055
0.2	1,328,979.79	0.334	1,325,708.14	0.087
0.3	1,329,827.51	0.398	1,326,264.45	0.129
0.4	1,331,032.85	0.489	1,327,085.68	0.191
0.5	1,333,271.35	0.658	1,327,959.88	0.257
0.6	1,336,013.18	0.865	1,329,178.48	0.349
0.7	1,337,761.60	0.997	1,331,933.55	0.557
0.8	1,341,231.93	1.259	1,332,383.90	0.591
0.9	1,348,371.29	1.798	1,336,264.85	0.884
1	—	—	1,341,245.18	1.26

Table 2 DC lines of modified IEEE 118-bus system

$\mu_{\text{set}}$	AARDOPF		Stochastic DOPF	
	$r\%$	Success rate %	$r\%$	Success rate %
0.1	0.201	0.15	0.177	0.10
0.2	0.304	0.22	0.244	0.17
0.3	0.396	0.36	0.571	0.25
0.4	0.497	0.59	0.853	0.44
0.5	0.677	0.77	0.991	0.49
0.6	0.826	0.83	1.216	0.57
0.7	1.032	0.91	—	—
0.8	1.359	1	—	—
0.9	1.867	1	—	—
1	—	—	—	—

Therefore, all the non-linear constraints are replaced by linear or second-order cone constraints which can be efficiently solved by GUROBI [19].

## 5 Case study

### 5.1 AARDOPF with different participation factors strategies

The proposed method is first tested on modified AC-DC IEEE 30-bus. The original data sets are given with the distribution files of MATPOWER [20]. Three wind farms replace the generators at nodes 12, 21, 30 in 30-bus system. AC network and DC network

are connected with these DC lines by VSCs. The data of wind farms in the Wind Integration Datasets of NREL are from [21]. The forecasted values translate into an RES penetration (relative to the total load) of 20% for the 30-bus network. The AC-DC AAROPF problem is solved by GUROBI on PC with Intel i3-4160@3.60 GHz 8 GB RAM. The optimality gap 'MIPGap' is set as 0.005 in GUROBI (Fig. 2).

In this case study, two strategies of participation factors are used to compute the corresponding expected cost: (i) by classical equation (18) which fixes the participation factors, and (ii) by treating participation factors as variables. Table 1 shows the percentage increase  $r\%$  in cost with respect to the nominal value.  $\mu_{\text{set}} = 1 - 1/(1 + C_{\text{conf}})$  is the setting probability of solution's feasibility.  $C_{\text{conf}}$  is the budget value which measures the size of uncertainty set, and determines the economic benefits.

As shown in Table 1, affinely adjustable robust solutions allowing the choice of the participation factors by the program yields adjustable robust solutions with 100%, whereas with predefined participation factors only can be obtained for up to 90%. It is evident that the treatment of  $\beta$  as a variable gives lower cost. And the increase in cost for 100% probability of solution's feasibility is only 1.26%. The overall results reveal that AAROPF by treating participation factors as variables is superior to predefined participation factors because it gives a 100% success rate even at high uncertainty levels, and its associated base (and expected) costs are lower.

### 5.2 Comparison with stochastic DOPF

In this section, the proposed method is tested on modified AC-DC 118-bus network. The original data sets are given with the distribution files of MATPOWER [20]. In the modified 118-bus system, eight wind farms replace the generators at nodes 2, 11, 13, 23, 53, 68, 93, 115. The forecasted values translate into an RES penetration (relative to the total load) of 20% for the 118-bus network. The DC lines of 118-bus system are shown in Table 2. AC network and DC network are connected with these DC lines by VSCs.

In order to substantiate the superiority of the robust solutions, this case uses stochastic DOPF as a comparison. Participation factors are considered by classical equation (18). The classical chance-constrained optimisation [22] is used to solve stochastic DOPF

$$\begin{aligned}
 \min_{x,y} \quad & \sum_{t=1}^{N_T} \sum_{i=1}^{N_G} (C_i(x_{i,t}) + c'_{2i,t} \beta_{i,t}^2) \\
 \text{s.t.} \quad & \begin{cases} \sum_{i=1}^{N_G} x_{i,t} = \sum_{i=1}^{N_B} P_{Di,t} - \sum_{i=1}^{N_R} P_{Ri,t} \\ A_1 x_t \leq d_1 - K_\alpha \sqrt{B_1 W \Lambda_t (B_1 W)^T} \\ A_2 x_t \leq d_2 - K_\alpha \sqrt{(B_2 W - D) \Lambda_t (B_2 W - D)^T} \\ A_1 M x_t \leq d_3 - K_\alpha \sqrt{B_1 W M \Lambda_{\text{all}} (B_1 W M)^T} \end{cases} \quad (31)
 \end{aligned}$$

where  $K_\alpha$  is standard Gaussian value corresponding to probability level,  $\Phi(K_\alpha) = \mu_{\text{set}}$  is cumulative distribution function of Gauss distribution;  $\Lambda_t$  and  $\Lambda_{\text{all}}$  are the covariance matrix of wind output prediction error  $\xi_{j,t}$  and  $\xi_{\text{all},t}$ .

Table 3 shows 100 000 Monte Carlo trials which is defined as successful if none of the power flow or generation limits are violated after the generation is rescheduled. As shown in Table 3, when setting probability of solution's feasibility is below 20%, AARDOPF gives a higher percentage increase in cost as the uncertainty level increases. However, stochastic DOPF gives lower success rates and higher percentage increase in cost up to 30%. And when the setting probability of solution's feasibility is up to 80%, AARDOPF produces a 100% success rate. It reveals that AARDOPF is superior to stochastic DOPF.

**Table 3** Percentage increase in expected cost and success rate

No.	IEEE 118-bus	
	From bus	To bus
1	10	18
2	15	18
3	26	27
4	26	28
5	30	33
6	33	34
7	34	30
8	38	43
9	43	42
10	42	38

## 6 Conclusion

This paper proposes an affinely adjustable robust AC–DC DOPF which considers spatial-temporal correlation of wind power. Generalised ellipsoidal uncertainty set is used to well fit uncertainty set. Affine policy is applied in recourse decision to get tractable formulation. The non-convex, non-linear model is transformed to a second order cone programming which can be solved by GUROBI efficiently.

AARDOPF deduces the relationship between the budget value of uncertainty set and probabilistic guarantees of feasibility for all possible scenarios. The results show a modest increase in expected cost for reasonable levels of uncertainty. Numerical results are obtained on modified AC–DC IEEE 30-bus and 118-bus systems. The proposed method gives a more robust solution with higher successful rates with a modest increase in expected cost for reasonable levels of uncertainty representing forecast error confidence intervals. And load uncertainty using a Gaussian distribution and AC OPF will be discussed in the future study.

## 7 Acknowledgments

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