Approximate symbol error rate of cooperative communication over generalised $\kappa{-}\mu$ and $\eta{-}\mu$ fading channels

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Abstract: Closed form expressions for approximate symbol error rate are obtained using moment generating function for a two branch co-operative communication system over generalised $\kappa{-}\mu$ and $\eta{-}\mu$ i.i.d. fading channels for BPSK and QAM modulation schemes. Selective decode and forward protocol is used at the relay transmitter. At the destination maximal-ratio combining is used. Monte Carlo simulations are performed to verify the analytical results.

1 Introduction

Cooperative communication [1] is a solution for reliable reception in single antenna systems. In cooperative communication, each user not only sends their information but also forward other user's information to the destination. Owing to single antenna system there is no inter-antenna interference and also there is diversity gain advantage in cooperative communication. Generally amplify and forward (AF) and decode and forward (DF) protocols are used at the relay node. In AF protocol, the received noisy signal at the relay will be amplified and forwarded to the destination, whereas in DF protocol noisy signal at the relay will be first decoded and forwarded to destination if the signal decodes correctly.

The performance analysis of cooperative communication over classical fading channels has been studied in the past. Performance analysis of cooperative communication over Rayleigh, Weibull and Nakagami-*m* fading channels is reported in [2–4]. In incremental relaying cooperative communication, the channel resources are used efficiently by not forwarding the information from relay to destination if the destination itself decodes it correctly [5].

The physical model of $\kappa{-}\mu$ and $\eta{-}\mu$ distributions are described in [6] and moment generating function (MGF) of $\kappa{-}\mu$ and $\eta{-}\mu$ distributions are given in [7]. The $\kappa{-}\mu$ and $\eta{-}\mu$ distributions are best suited to model practical small scale fading channels in line of sight (LOS) and non-LOS (NLOS) environments, respectively [6]. $\kappa{-}\mu$ and $\eta{-}\mu$ are generalised models which accommodate the best known fading models such as Rayleigh, Rician, Nakagami-*m*, Nakagami-*n* and Hoyt distributions etc. These models assume non-homogenous physical environment which is closer to the practical scenario.

In this paper, we derived, for the first time, closed-form expression for approximate SER for a two branch cooperative communication system over generalised $\kappa{-}\mu$ and $\eta{-}\mu$ fading channels for quadrature amplitude modulation (QAM) and binary phase shift keying (BPSK) modulation schemes. In our analysis, we considered DF protocol at the relay. We also show SER results obtained from Monte Carlo simulations to validate our analytical results.

Section 2 describes the system model and about $\kappa{-}\mu$ and $\eta{-}\mu$ fading distributions. In Section 3 approximate SER expression is derived for BPSK and QAM modulation schemes. In Section 4 simulation results were shown. The paper is concluded in Section 5.

2 System model

Let us consider a cooperative communication system with one source node (S) communicating with one destination node (D)

and one relay node (R) as shown in Fig. 1 [8]. The transmission of information from the source to destination is performed in two orthogonal phases. In the first phase, the source broadcasts data to the relay and the destination. The received signals $y_{s,d}$ and $y_{s,r}$ at the destination and the relay, respectively, can be written as

$$y_{s,d} = \sqrt{p_1} f x + n_1 \tag{1}$$

$$y_{s,r} = \sqrt{p_1} g x + n_2 \tag{2}$$

in which p_1 is the transmitted power at the source, x is the transmitted information symbol, n_1 and n_2 are zero-mean complex Gaussian random variables with variance N_0 and channel gains f and g are $\kappa{-}\mu$ or $\eta{-}\mu$ distribution fading coefficients between source and destination, and between source and relay, respectively.

In second phase, for a DF cooperation protocol, if the relay is able to decode the transmitted symbol correctly, then it forwards the decoded symbol with power p_2 to the destination, otherwise the relay does not send or remains idle. The received signal $y_{r,d}$ at the destination can be written as

$$y_{r,d} = \sqrt{\tilde{p}_2} h x + n_3 \tag{3}$$

where $\tilde{p}_2 = p_2$ if the relay decodes the transmitted symbol correctly, otherwise $\tilde{p}_2 = 0$. n_3 is zero-mean complex Gaussian random variables with variance N_0 and channel gain h is $\kappa{-}\mu$ or $\eta{-}\mu$ distribution fading coefficients between relay and destination. The fading coefficients f, g and h are assumed to be known at the receiver, but not at the transmitter. The destination jointly combines the
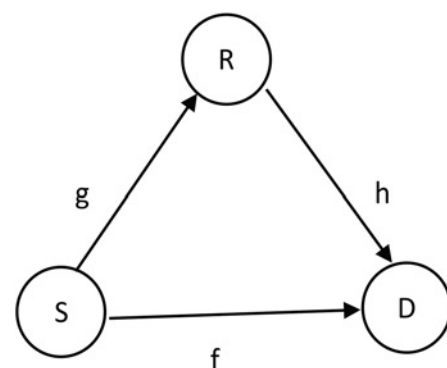

Fig. 1 *Cooperative communication system with single relay*

received signal from the source in phase 1 and that from the relay in phase 2, and detects the transmitted symbols by using maximum-ratio combining (MRC) [9]. The total transmitted power has to satisfy $p_1 + p_2 = p$.

2.1 $\kappa - \mu$ distribution

The probability density function of $\kappa - \mu$ distributed random variable (RV) is given as [6]

$$p_{x_{\kappa-\mu}}(x) = \frac{2\mu x^{\mu}}{\kappa^{(\mu-1/2)}e^{\mu\kappa}} \left(\frac{1+\kappa}{\Omega}\right)^{(\mu+1/2)} e^{-(\mu(1+\kappa)/\Omega)x^2}$$

$$\times I_{\mu-1}\left(2\mu\sqrt{\frac{\kappa(1+\kappa)}{\Omega}}x\right) \qquad (4)$$

where $\Omega = E[x^2]$, $E[\cdot]$ is the expectation operator, $\kappa > 0$ and $\mu > 0$ are the parameters of the distribution and $I_{\nu}(\cdot)$ is the modified Bessel function of the first kind and νth order. The parameter κ is the ratio of the total power because of dominant components to the total power because of scattered components and μ is the number of multipath clusters. This model includes Rice ($\mu = 1$ and $\kappa = K$), Nakagami-m ($\kappa \to 0$ and $\mu = m$), Rayleigh ($\mu = 1$ and $\kappa \to 0$) and one sided Gaussian distribution ($\mu = 0.5$ and $\kappa \to 0$) fading models as special cases. This distribution is better suited for LOS propagation.

2.2 $\eta - \mu$ distribution

The probability density function (PDF) of $\eta - \mu$ distributed RV is given as [6]

$$p_{x_{\eta-\mu}}(x) = \frac{4\sqrt{\pi}\mu^{\mu+(1/2)}h^{\mu}x^{2\mu}}{\Gamma(\mu)H^{\mu-(1/2)}\Omega^{\mu+(1/2)}} e^{-(2\mu x/\Omega)x^2}$$

$$\times I_{\mu-(1/2)}\left(\frac{2\mu H}{\Omega}x^2\right) \qquad (5)$$

where $\Gamma(\cdot)$ is the gamma function, and h and H are functions of the parameter η defined for two formats in next subsections. μ denotes the number of multipath clusters. This fading model includes Hoyt ($\eta = q^2$, $\mu = 0.5$), Nakagami-m ($\eta = 1$, $\mu = m/2$), Rayleigh and one sided Gaussian distribution as special cases.

(1) The $\eta - \mu$ distribution: format 1: In this format $0 < \eta < \infty$ is the power ratio of the in-phase and quadrature components of the scattered-waves of each multipath. It is assumed that the in-phase and quadrature phase components of fading signal within each cluster are independent of each other and have different average powers. In this case, $h = [(2 + \eta^{-1} + \eta)/4]$ and $H = [(\eta^{-1} - \eta)/4]$. Within $0 < \eta \leq 1$, we have $H \geq 0$, on the other hand, within $0 < \eta^{-1} \leq 1$, we have $H \leq 0$. Since $I_{\nu}(-z) = (-1)^{\nu}I_{\nu}(z)$, the distribution is symmetrical around $\eta = 1$. Therefore as far as the envelope (or power) distribution is concerned, it is sufficient to consider η only within one of the ranges. In format 1, $H/h = (1 - \eta)/(1 + \eta)$.

(2) The $\eta - \mu$ distribution: format 2: In this format $-1 < \eta < 1$ is the correlation coefficient between the in-phase and quadrature components of the scattered-waves of each cluster. It is assumed that the in-phase and quadrature phase components of fading signal within each cluster are correlated and have identical powers. In this case, $h = (1/1 - \eta^2)$ and $H = (\eta/1 - \eta^2)$. Within $0 \leq \eta < 1$, we have $H \geq 0$, on the other hand, within $-1 < \eta \leq 0$, we have $H \leq 0$. Since $I_{\nu}(-z) = (-1)^{\nu}I_{\nu}(z)$, the distribution is symmetrical around $\eta = 0$. Therefore as far as the envelope (or power) distribution is concerned, it is sufficient to consider η only within one of the ranges. In format 2, $H/h = \eta$.

3 Analysis of probability of error

The combined signal at the MRC detector can be written as in [8]

$$y = \frac{\sqrt{p_1}f^*}{N_0}y_{s,d} + \frac{\sqrt{\tilde{p}_2}h^*}{N_0}y_{r,d} \qquad (6)$$

where f^* and h^* are the complex conjugates of f and h, respectively.

The signal-to-noise ratio (SNR) of the MRC output is

$$\gamma = \frac{p_1|f|^2 + \tilde{p}_2|h|^2}{N_0} \qquad (7)$$

3.1 BPSK modulation

The conditional SER can be calculated as in [8]

$$P_{bpsk}^{cond}(e) = \psi_{bpsk}(\gamma)\big|_{\tilde{p}_2} = 0 \times \psi_{bpsk}\left(\frac{p_1|g|^2}{N_0}\right)$$

$$+ \psi_{bpsk}(\gamma)\big|_{\tilde{p}_2} = p_2 \times \left[1 - \psi_{bpsk}\left(\frac{p_1|g|^2}{N_0}\right)\right] \qquad (8)$$

where γ is SNR, $\psi_{bpsk}(\gamma)$ is conditional SER and is given by [10]

$$\psi_{bpsk}(\gamma) = Q(\sqrt{2\gamma}) \qquad (9)$$

and $Q(.)$ is Q function and is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} \exp\left(-\frac{y^2}{2}\right)dx \qquad (10)$$

by using approximation on $Q(.)$ function [11]

$$Q(x) \simeq \frac{1}{12}\exp\left(-\frac{x^2}{2}\right) + \frac{1}{4}\exp\left(-\frac{2x^2}{3}\right) \qquad (11)$$

The final approximated average SER can be given by

$$P_{bpsk}(e) \simeq \varepsilon(p_1) \times \xi_1(p_1) + (1 - \varepsilon(p_1)) \times \xi_2(p_1 + p_2) \qquad (12)$$

where $\varepsilon(p_1)$ is the probability of error at the relay after the first phase and is given by

$$\varepsilon(p_1) \simeq \frac{1}{12}\text{MGF}_{p_1}(1) + \frac{1}{4}\text{MGF}_{p_1}\left(\frac{4}{3}\right) \qquad (13)$$

$\xi_1(p_1)$ is the probability of error at the destination after the first phase and is given by

$$\xi_1(p_1) \simeq \frac{1}{12}\text{MGF}_{p_1}(1) + \frac{1}{4}\text{MGF}_{p_1}\left(\frac{4}{3}\right) \qquad (14)$$

and $\xi_2(p_1 + p_2)$ is the probability of error at the destination after the second phase and is given by

$$\xi_2(p_1 + p_2) \simeq \frac{1}{12}\text{MGF}_{p_1}(1) \times \text{MGF}_{p_2}(1) + \frac{1}{4}\text{MGF}_{p_1}\left(\frac{4}{3}\right)$$

$$\times \text{MGF}_{p_2}\left(\frac{4}{3}\right) \qquad (15)$$

and MGF(.) is given by [7]

$$\text{MGF}_{\gamma_{\kappa-\mu}}(s) = \left[\frac{\mu(1+\kappa)}{\mu(1+\kappa) + s\tilde{\gamma}}\right]^{\mu} \exp\left[\frac{\mu^2\kappa(1+\kappa)}{\mu(1+\kappa) + s\tilde{\gamma}} - \mu\kappa\right] \qquad (16)$$

$$\text{MGF}_{\gamma_{\eta-\mu}}(s) = \left(\frac{4\mu^2 h}{(2(h-H)\mu + s\tilde{\gamma})(2(h+H)\mu + s\tilde{\gamma})}\right)^{\mu} \qquad (17)$$

3.2 4-QAM modulation

The conditional SER can be calculated as [8]

$$P_{qam}^{cond}(e) = \psi_{qam}(\gamma)\big|_{\tilde{p}_2=0} \times \psi_{qam}\left(\frac{p_1|g|^2}{N_0}\right)$$

$$+ \psi_{qam}(\gamma)\big|_{\tilde{p}_2=p_2} \times \left[1 - \psi_{qam}\left(\frac{p_1|g|^2}{N_0}\right)\right] \quad (18)$$

where $\psi_{qam}(\gamma)$ is conditional SER and is given by [10]

$$\psi_{qam}(\gamma) = 2Q\left(\sqrt{\frac{3}{4}\gamma}\right) - Q^2\left(\sqrt{\frac{3}{4}\gamma}\right) \quad (19)$$

by using $Q^2(.)$ and is given by

$$Q^2(x) \simeq \frac{1}{144}\exp(-x^2) + \frac{1}{16}\exp\left(-\frac{4x^2}{3}\right) + \frac{1}{24}\exp\left(-\frac{7x^2}{6}\right) \quad (20)$$

The final approximated average SER can be given by

$$P_{qam}(e) \simeq \varepsilon(p_1) \times \xi_1(p_1) + (1 - \varepsilon(p_1)) \times \xi_2(p_1 + p_2) \quad (21)$$

In the above equation, $\varepsilon(p_1)$, $\xi_1(p_1)$ and $\xi_2(p_1 + p_2)$ are defined as follows

$$\varepsilon(p_1) \simeq 2\left\{\frac{1}{12}\text{MGF}_{p_1}\left(\frac{1}{2}\right) + \frac{1}{4}\text{MGF}_{p_1}\left(\frac{2}{3}\right)\right\}$$
$$- \left\{\frac{1}{144}\text{MGF}_{p_1}(1) + \frac{1}{16}\text{MGF}_{p_1}\left(\frac{4}{3}\right) + \frac{1}{24}\text{MGF}_{p_1}\left(\frac{7}{6}\right)\right\} \quad (22)$$

$$\varepsilon(p_1) \simeq 2\left\{\frac{1}{12}\text{MGF}_{p_1}\left(\frac{1}{2}\right) + \frac{1}{4}\text{MGF}_{p_1}\left(\frac{2}{3}\right)\right\}$$
$$- \left\{\frac{1}{144}\text{MGF}_{p_1}(1) + \frac{1}{16}\text{MGF}_{p_1}\left(\frac{4}{3}\right) + \frac{1}{24}\text{MGF}_{p_1}\left(\frac{7}{6}\right)\right\} \quad (23)$$

$$\xi_2(p_1 + p_2) \simeq 2\left\{\frac{1}{12}\text{MGF}_{p_1}\left(\frac{1}{2}\right) \times \text{MGF}_{p_2}\left(\frac{1}{2}\right) + \frac{1}{4}\text{MGF}_{p_1}\left(\frac{2}{3}\right)\right.$$
$$\left. \times \text{MGF}_{p_2}\left(\frac{2}{3}\right)\right\}$$
$$- \left\{\frac{1}{144}\text{MGF}_{p_1}(1) \times \text{MGF}_{p_2}(1) + \frac{1}{16}\text{MGF}_{p_1}\left(\frac{4}{3}\right)\right.$$
$$\left. \times \text{MGF}_{p_2}\left(\frac{4}{3}\right) + \frac{1}{24}\text{MGF}_{p_1}\left(\frac{7}{6}\right) \times \text{MGF}_{p_2}\left(\frac{7}{6}\right)\right\} \quad (24)$$

4 Simulation results

The derived approximate expressions of SER for different modulation schemes have been evaluated numerically and plotted with respect to average SNR, and are compared with the simulation results for different values of $\kappa-\mu$ and $\eta-\mu$ distributions.

Fig. 2 curves represent approximate average SER against average SNR for BPSK modulation scheme showing the results for Rayleigh ($\kappa = 0$, $\mu = 1$), Rice ($\kappa = 1$, $\mu = 1$) and Nakagami-m ($\kappa = 0$, $\mu = 2, 4$) as special cases of $\kappa-\mu$ distribution.

Fig. 3 curves represent approximate average SER against average SNR for 4-QAM modulation scheme showing the results for Rayleigh ($\kappa = 0$, $\mu = 1$), Rice ($\kappa = 2$, $\mu = 1$) and Nakagami-m ($\kappa = 0$, $\mu = 2, 3$) as special cases of $\kappa-\mu$ distribution. Fig. 4 curves represent approximate average SER against average SNR for BPSK
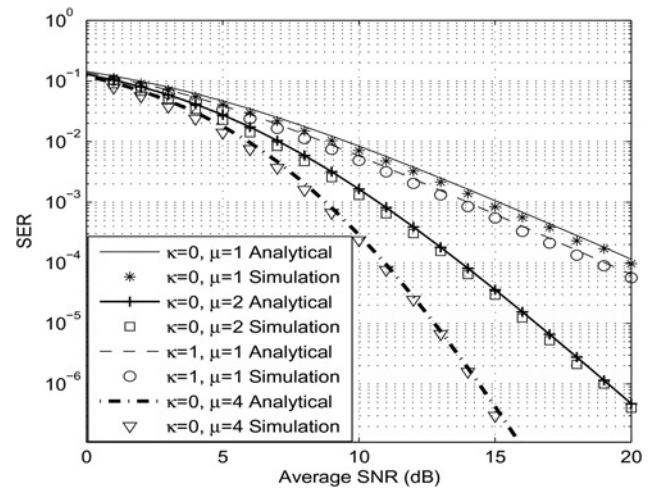

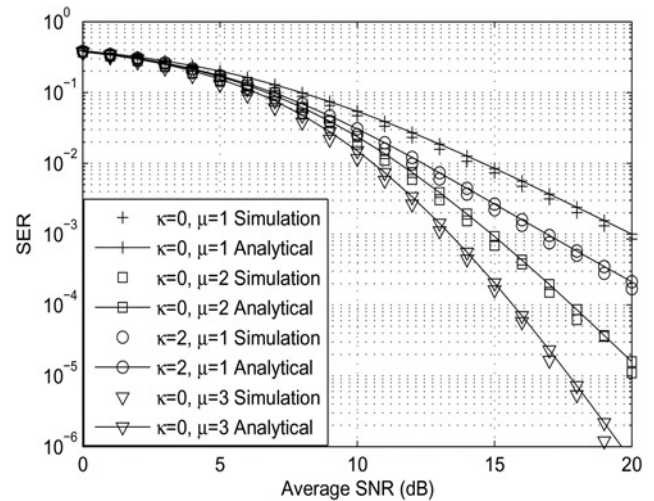**Fig. 2** *SNR against SER of BPSK for different values of κ and μ with $p_1 = p_2 = p/2$*

Fig. 3 *SNR against SER of QAM for different values of κ and μ with $p_1 = p_2 = p/2$*
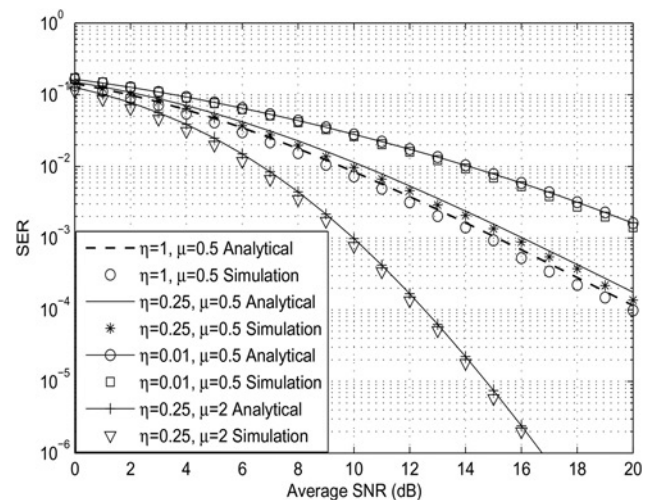

Fig. 4 *SNR against SER of BPSK for different values of η and μ with $p_1 = p_2 = p/2$*
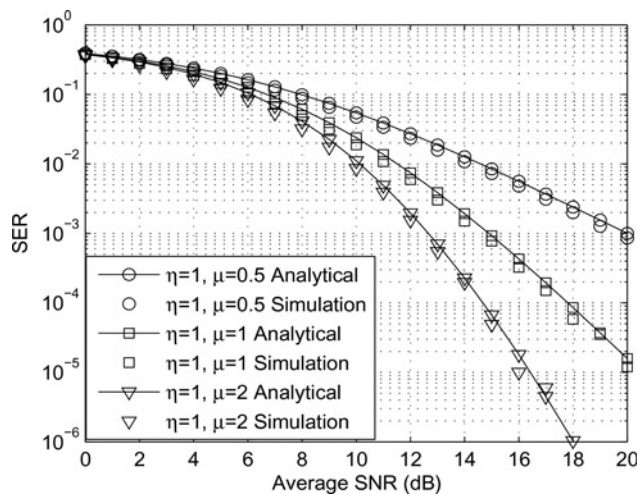
Fig. 5 *SNR against SER of QAM for different values of η and μ with $p_1 = p_2 = p/2$*

modulation scheme showing the results for Rayleigh ($\eta = 1, \mu = 0.5$) and Nakagami-*q*/Hoyt ($\eta = 0.01, 0.25, \mu = 0.5$) as special cases of $\eta - \mu$ distribution. Fig. 5 curves represent approximate average SER against average SNR for 4-QAM modulation scheme showing the results for Rayleigh ($\eta = 1, \mu = 0.5$), and Nakagami-*m* ($\eta = 1, \mu = 1, 2$) as special cases of $\eta - \mu$ distribution.

5 Conclusion

In this paper, approximate SER expression is derived for a cooperative communication system over the generalised $\kappa - \mu$ and $\eta - \mu$ distributed fading channels for BPSK and QAM modulation schemes using DF protocol. Representing Q function as sum of exponentials

makes the analysis simpler at the cost of a small deviation between the analytical and simulation results. The advantage of $\kappa - \mu$ and $\eta - \mu$ distribution is that we can model Rayleigh, Rice, Nakagami-*m*, Hoyt and one sided Gaussian channel models as special cases.

6 References

[1] Nosratinia A., Hunter T., Hedayat A.: 'Cooperative communication in wireless networks', *IEEE Commun. Mag.*, 2004, **42**, (10), pp. 74–80

[2] Anghel P., Kaveh M.: 'Exact symbol error probability of a cooperative network in a rayleigh-fading environment', *IEEE Trans. Wirel. Commun.*, 2004, **3**, (5), pp. 1416–1421

[3] Ikki S., Ahmed M.: 'Performance analysis of cooperative diversity wireless networks over nakagami-m fading channel', *IEEE Commun. Lett.*, 2007, **11**, (4), pp. 334–336

[4] Lei Y., Cheng W., Zeng Z.: 'Performance analysis of selection combining for amplify-and-forward cooperative diversity networks over Weibull fading channels'. Proc. IEEE Int. Conf. Communications Technology and Applications, 2009, pp. 648–651

[5] Ikki S., Ahmed M.: 'Performance analysis of incremental-relaying cooperative-diversity networks over rayleigh fading channels', *IET Commun.*, 2011, **5**, (3), pp. 337–349

[6] Yacoub M.: 'The $\kappa - \mu$ distribution and the $\eta - \mu$ distribution', *IEEE Antennas Propag. Mag.*, 2007, **49**, (1), pp. 68–81

[7] Ermolova N.: 'Moment generating functions of the generalized $\eta - \mu$ and $\kappa - \mu$ distributions and their applications to performance evaluations of communication systems', *IEEE Commun. Lett.*, 2008, **12**, (7), pp. 502–504

[8] Liu K.J.R., Kwasinski A., Su W., Sadek A.K.: 'Cooperative communications and networking' (Cambridge University Press, New York, 2009, 1st edn.)

[9] Brennan D.G.: 'Linear diversity combining techniques', *IEEE Proc.*, 2003, **91**, (2), pp. 331–356

[10] Simon M.K., Alouini M.-S.: 'Digital communication over fading channels' (Wiley Interscience, 2005, 2nd edn.)

[11] Chiani M., Dardari D., Simon M.K.: 'New exponential bounds and approximations for the computation of error probability in fading channels', *IEEE Trans. Wirel. Commun.*, 2003, **2**, (4), pp. 840–845