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Zonotope-based quantification of the impact of renewable power generation on hybrid AC/DC distribution system

Jianing Gao[1], Bei Han[1] ✉, Chenbo Xu[2], Lijun Zhang[2], Guojie Li[1], Keyou Wang[1]
[1]*Shanghai Jiao Tong University, Shanghai, People's Republic of China*
[2]*State Grid Zhejiang Economic & Technological Research Institute, Hangzhou, People's Republic of China*
✉ *E-mail: han_bei@sjtu.edu.cn*

Abstract: Hybrid AC/DC distribution network has developed rapidly owing to its convenience for renewable energy integration, thus the authors are interested in quantifying the impact of uncertainty from renewable power on system static performance. This paper proposes a zonotope-based set-theoretic method for quantifying the impact of renewable power fluctuations on system-state variables in hybrid AC/DC distribution network, both for the assessment of the capabilities for integrating renewable resources by hybrid AC/DC system, and for the appropriate selection of operating method in hybrid power systems. The authors bound the uncertain variations of generation by a zonotope and propagate it through a linearised power flow to get another zonotope, which captures all possible values of the system state variables. The zonotope-based method is computationally tractable and takes less computation than probabilistic methods especially when considering the generation uncertainty of multiple renewable energy resources. The proposed method is applied to a hybrid AC/DC test system with different control strategies of converters. The results show that hybrid AC/DC system with proper control mode may help reduce the impact of generation uncertainty.

1 Introduction

In recent years, the penetration of renewable energy sources into power systems, motivated by demands for environmental protection, has been rapidly increasing. Since most renewable energy generators such as photovoltaic panels generate power in DC form, the hybrid AC/DC distribution system, which can help reduce unnecessary conversions between AC and DC energy and improve the efficiency of the distribution network by reducing power losses associated with the conversion, is well-suited for integrating different types of renewable energy and has become the trend of future distribution system. While these renewable resources are intermittent and variable, their uncertainty in power generation has a great impact on system operation in both static and dynamic performance, and the situation is more complex in hybrid AC/DC system considering the coupling of different types of energy and its various control strategies. Here, the authors focus on quantifying the impact of uncertainty in power generation on the static performance of hybrid AC/DC distribution system. The proposed method can be used to assess whether the system static state variables, that is, bus voltage magnitudes, will remain within acceptable ranges when the renewable power generation fluctuates.

In order to capture the effects of distributed generations on system static performance, stochastic power flow analysis needs to be used. The main approaches to analyse the power flow solutions under uncertainties include probabilistic and set-theoretic methods. In probabilistic methods [1, 2], uncertain variables are modelled as random vectors according to the statistical information of their historical values and thus the probability distribution of system state variables can be obtained by several numerical and analytical methods, such as Monte Carlo simulation [3] and point estimate method [4]. The probabilistic method can provide relatively comprehensive probability information about system operation while large amounts of historical data are needed first to determine the probability characters of the input variables. In set-theoretic methods [5–7], the uncertainties in load and generation are assumed to lie within a bounded set, resulting in the system state variables bounded within another set. For example, the interval power flow analysis [8], focuses on variable boundaries, uses intervals to describe the ranges of uncertain values, and gives the lower and upper bounds of system state variables. However, interval methods are considered to be excessively conservative, bounding the solutions with other convex hulls will be more accurate. Some typical set representations used to describe the possible values of state variables include polytopes [9], zonotopes [10], and ellipsoids [11].

Here, a set-theoretic approach using zonotope is adopted for uncertainty analysis of hybrid AC/DC system. Zonotopes are chosen for that they can efficiently represent reachable sets in high-dimensional spaces, which is suitable to consider the uncertainties of multiple generators, and they are closed under linear transformations [12]. The authors bound the uncertain variations of renewable power generation with a zonotope centred around its nominal value, and then propagate it through a linearised power flow equation to obtain another zonotope which consists of all possible values that the state variables can take. Through this zonotope, the authors can quantify the impact of renewable generation uncertainty on system static performance by predetermined fluctuation range of state variables.

The zonotope-based method has been used in previous studies to analyse the impact of generation uncertainty on both dynamic and static performance of traditional AC systems [12–14], but few researches focused on the application on hybrid AC/DC distribution system. The hybrid AC/DC distribution network connects the AC and DC system through voltage source converters (VSC), so the power flow is more complex than traditional AC systems. Furthermore, different control strategies of VSC will result in different operation states and different static response to generation fluctuations. Thus, it is significant to assess the impact of uncertainty on the power flow of hybrid AC/DC system. The main power flow algorithms for hybrid AC/DC system include unified method [15, 16] and alternating iterative method [17]. In the work here, a generalised unified power flow model for hybrid AC/DC network is established. The structure of power flow equation remains unchanged for different control modes, making it suitable for uncertainty analysis of different operation conditions. Through this model, the authors use the set-theoretic method to quantify the impact of uncertain generation and compare the results for different control modes.

The remainder of this paper is organised as follows. In Section 2, the uncertainty analysis method using zonotope for AC/DC

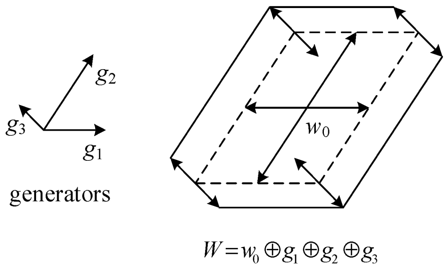

Fig. 1 *Construction of a zonotope*

network is introduced. In Section 3, the equations for VSC model and its control modes are established, and the quantification procedure is described. Section 4 applies the proposed method to a hybrid AC/DC test system for analysis. Finally, the paper is concluded in Section 5.

2 Uncertainty analysis method for AC/DC network

2.1 Zonotopes

Zonotopes are a special class of convex polytopes, which can be defined as the Minkowski sum of a finite set of line segments [18]. Formally, a zonotope is defined as

$$W = \left\{ w \in \mathbb{R}^n : w = w_0 + \sum_{i=1}^{p} \alpha_i g_i, \quad -1 \le \alpha_i \le 1 \right\} \tag{1}$$

where w_0 is the centre of the zonotope, the collection of vectors $g_1, g_2, \ldots, g_p \in \mathbb{R}^n$ is called the set of generators of the zonotope. Fig. 1 illustrates the construction of a zonotope from the Minkowski sum of three generators. It is shown that a zonotope is always centrally symmetric.

Zonotopes have the useful property of being closed under linear transformations. Let H be a linear map and Z a zonotope, then another zonotope X can be obtained by applying the linear transformation as follows

$$X = HW = \left\{ x : x = Hw_0 + H \sum_{i=0}^{n} \alpha_i g_i, \quad -1 \le \alpha_i \le 1 \right\} \tag{2}$$

2.2 Construction of AC/DC power flow

In order to obtain a state-bounding zonotope associated with the uncertainty in hybrid AC/DC system, AC/DC power flow is required first, for calculation of nominal operation point and linearisation of the system.

The AC/DC grids include three parts: VSC station, AC grid, and DC grid, thus the unified power flow equation for hybrid AC/DC system can be formulated as

$$\boldsymbol{F}\left(\boldsymbol{X}_{\rm AC}^{\rm T}, \boldsymbol{X}_{\rm DC}^{\rm T}, \boldsymbol{X}_{\rm VSC}^{\rm T}\right) = \left[\boldsymbol{F}_{\rm AC}^{\rm T}, \boldsymbol{F}_{\rm DC}^{\rm T}, \boldsymbol{F}_{\rm VSC}^{\rm T}\right]^{\rm T} \tag{3}$$

where $\boldsymbol{F}_{\rm AC}$ represents the power balance equations of AC system, including both active and reactive power equations for PQ nodes and active power equations for PV nodes; $\boldsymbol{F}_{\rm DC}$ is the power equations of DC nodes; $\boldsymbol{F}_{\rm VSC}$ indicates the power and control equations of VSC converters. The state variables $\boldsymbol{X}_{\rm AC}$, $\boldsymbol{X}_{\rm DC}$, $\boldsymbol{X}_{\rm VSC}$ are defined as

$$\begin{cases} \boldsymbol{X}_{\rm AC} = \left[\boldsymbol{\delta}^{\rm T}, \boldsymbol{U}^{\rm T}\right]^{\rm T} \\ \boldsymbol{X}_{\rm DC} = \boldsymbol{U}_{\rm dc} \\ \boldsymbol{X}_{\rm VSC} = \left[\boldsymbol{\delta}_{\rm c}^{\rm T}, \boldsymbol{U}_{\rm c}^{\rm T}\right]^{\rm T} \end{cases} \tag{4}$$

where $\boldsymbol{\delta}$ includes the phase angle of all PQ and PV nodes; $\boldsymbol{U}$ indicates the voltage magnitude of PQ nodes; $\boldsymbol{U}_{\rm dc}$ is the voltage magnitude of all DC nodes; $\boldsymbol{\delta}_{\rm c}$ and $\boldsymbol{U}_{\rm c}$ are the phase angle and voltage amplitude of converter buses.

The detailed formula of AC/DC power flow (3) considering the control modes of VSC stations will be described in Section 3. Here, for illustration of the fundamental idea of uncertainty analysis method, the authors divide the power injections into two parts and rewrite (3) as

$$w + u = f(x) \tag{5}$$

where w represents the power injections arising from renewable-based sources and u represents the generation power from conventional sources and the demands of power of each node; the non-linear function $f(\cdot)$ denotes the mapping between the system states and the power injections; x contains all the state variables of AC/DC system. Thus, the generation uncertainty can be assumed to appear only in w.

2.3 Construction of uncertainty sets for variables

Assuming the power variation of each renewable source is bounded within a range, the authors can easily construct the uncertainty set for input variables using a zonotope according to (1). For illustration, the authors take a simple system with three renewable generators for an example. Suppose the nominal output of the three renewable generators is $w_0 = \left[P_{g1}^0 \quad P_{g2}^0 \quad P_{g3}^0\right]^{\rm T} = [0.4 \quad 0.5 \quad 0.6]^{\rm T}$ p.u. and the uncertainty for each is ±50%. Thus, the authors can obtain three generator vectors $g_1 = [0.2 \quad 0 \quad 0]^{\rm T}$, $g_2 = [0 \quad 0.25 \quad 0]^{\rm T}$, and $g_3 = [0 \quad 0 \quad 0.3]^{\rm T}$ to construct a zonotope W which bounds the uncertain values of power generation. The zonotope obtained is a rectangular prism with sides having lengths of 0.4, 0.5, and 0.6, centred at $[0.4 \quad 0.5 \quad 0.6]^{\rm T}$.

Given the input uncertainty set, the possible values of system state variables can be captured through a linearised power flow model. Let x_0 represent the nominal solution to the power flow problem corresponding to $w = w_0$ and $u = u_0$, then for small variation Δw around w_0, Δx represents the corresponding variation of state variables, the authors linearise the non-linear mapping $f(\cdot)$ about its nominal solution, resulting in

$$\Delta w = J \Delta x \tag{6}$$

where $J = \partial f / \partial x|_{(x_0, w_0)}$ is the Jacobi matrix evaluated at x_0, and it is invertible since the power flow converges to the solution. Thus, the sensitivity of the state variables x to w can be expressed as

$$\Delta x = H \Delta w \tag{7}$$

where $H = J^{-1}$. Through this linear transformation matrix H, another zonotope X is obtained, which is considered as the uncertainty set of output state variables.

3 Inclusion of VSC model and control modes

This section introduces the detailed expression for the AC/DC power flow associated with the connection of VSC converters. The diverse control modes are considered in the power flow and then the quantification procedure with control mode adjustment for AC/DC system is given.

3.1 VSC station model

The equivalent model of VSC station is shown in Fig. 2, where i represents converter i; X_{ci} is the converter equivalent reactance and R_{ci} is the converter equivalent resistance for losses of the converter; $U_{si}\angle\delta_{si}$ and $U_{ci}\angle\delta_{ci}$ are, respectively, the voltage phasor at the common coupling AC bus and converter bus; U_{dci} is the voltage at the DC side of the converter.

With the steady-state model of VSC, the active and reactive power flowing from AC bus to the converter can be obtained.

$$P_{si} = U_{si}^2 G_{ci} - U_{si} U_{ci} [G_{ci} \cos(\delta_{si} - \delta_{ci}) + B_{ci} \sin(\delta_{si} - \delta_{ci})] \tag{8}$$

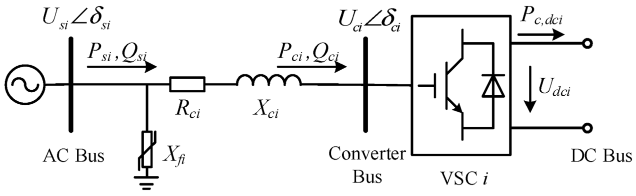

Fig. 2 *Equivalent model of VSC station*

Table 1 Control modes of VSC station

Mode	*d*-Axis control	Mode	*q*-Axis control
1	constant δ_s	1	constant U_s
2	constant P_s	2	constant Q_s
3	constant U_{dc}	—	—
4	$U_{dc} - P_{dc}$ droop	—	—

Table 2 Coefficients with diverse control modes

d-Axis control	K_{Ps}	K_{Udc}	K_{Pdc}	*q*-Axis control	K_Q	K_{Uac}
constantδ_s	1	0	0	constantU_s	0	1
constantP_s	1	0	0	constantQ_s	1	0
constantU_{dc}	0	1	0	—	—	—
U_{dc}-P_{dc} droop	0	1	non-zero	—	—	—

$$Q_{si} = -U_{si}^2 B_{ci} - U_{si}U_{ci}[G_{ci}\sin(\delta_{si} - \delta_{ci}) - B_{ci}\cos(\delta_{si} - \delta_{ci})] \quad (9)$$

The active power flowing into the converter at the converter bus can be written as

$$P_{ci} = -U_{ci}^2 G_{ci} + U_{si}U_{ci}[G_{ci}\cos(\delta_{ci} - \delta_{si}) + B_{ci}\sin(\delta_{ci} - \delta_{si})] \quad (10)$$

where G_{ci}, B_{ci} is the conductance and susceptance of the converter model.

We assume the losses of the converter are consumed on the equivalent resistance R_{ci}, for simplification, thus the power injected into the DC grid is calculated as

$$P_{c,dci} = P_{ci} \quad (11)$$

Given the power flow model of VSC station, the equations for AC and DC system can be formulated with a little modification to the traditional power flow expressions. For the AC buses which are connected with VSC stations, the power injections have to minus P_{si} and Q_{si}; For the DC buses connecting with VSCs, $P_{c,dci}$ is added into the power injections.

3.2 Control modes and control equations for VSC

By employing full-controlled electronic devices, VSC can independently control active and reactive power. As is shown in Table 1, VSC has the flexibility to control different variables on *d*-axis and *q*-axis.

In Table 1, except that mode 1 of *d*-axis control can only be used together with mode 1 of *q*-axis to supply to a passive AC network, any other mode of *d*-axis can be combined with one mode of *q*-axis control, resulting in diverse control modes for the VSC station.

According to the different control variables of different control modes, the traditional expressions for AC/DC power flow will have different forms and structures, which make it complicated for analysis when considering control mode switching. Here, the authors propose two generalised expressions of *d*-axis and *q*-axis control equations for VSCs, the dimensionality and structure of the resulting expressions stay the same for different control modes. The control equations are formulated as follows, by multiplying the control variables by their corresponding coefficients.

$$\begin{aligned}\Delta D_{\text{AXIS}i} = & K_{\text{Ps}i}\left(P_{si} - P_{si}^{\text{ref}}\right) + K_{\text{Udc}i}\left(U_{\text{dc}i} - U_{\text{dc}i}^{\text{ref}}\right) \\ & + K_{\text{Pdc}i}\left(P_{\text{dc}i} - P_{\text{dc}i}^{\text{ref}}\right) = 0\end{aligned} \quad (12)$$

$$\Delta Q_{\text{AXIS}i} = K_{\text{Q}si}\left(Q_{si} - P_{si}^{\text{ref}}\right) + K_{\text{U}si}\left(U_{si} - U_{si}^{\text{ref}}\right) = 0 \quad (13)$$

where P_{si}^{ref}, $U_{\text{dc}i}^{\text{ref}}$, $P_{\text{dc}i}^{\text{ref}}$, P_{si}^{ref}, and U_{si}^{ref} are the reference values of the corresponding variables. $K_{\text{Pdc}i}$ is the droop coefficient. The values for the coefficients when adopting different control modes are shown in Table 2.

Thus, $\Delta D_{\text{AXIS}i}$ and $\Delta Q_{\text{AXIS}i}$ make up the VSC equation $\boldsymbol{F}_{\text{VSC}}^{\text{T}}$ in (3). The Jacobi matrix can be calculated as

$$\boldsymbol{J} = \left[\begin{array}{cc|c|cc} \dfrac{\partial \Delta \boldsymbol{P}}{\partial \boldsymbol{\delta}} & \dfrac{\partial \Delta \boldsymbol{P}}{\partial \boldsymbol{U}} & 0 & \dfrac{\partial \Delta \boldsymbol{P}}{\partial \boldsymbol{\delta}_{\text{c}}} & \dfrac{\partial \Delta \boldsymbol{P}}{\partial \boldsymbol{U}_{\text{c}}} \\ \dfrac{\partial \Delta \boldsymbol{Q}}{\partial \boldsymbol{\delta}} & \dfrac{\partial \Delta \boldsymbol{Q}}{\partial \boldsymbol{U}} & 0 & \dfrac{\partial \Delta \boldsymbol{Q}}{\partial \boldsymbol{\delta}_{\text{c}}} & \dfrac{\partial \Delta \boldsymbol{Q}}{\partial \boldsymbol{U}_{\text{c}}} \\ \hline \dfrac{\partial \Delta \boldsymbol{P}_{\text{dc}}}{\partial \boldsymbol{\delta}} & \dfrac{\partial \Delta \boldsymbol{P}_{\text{dc}}}{\partial \boldsymbol{U}} & \dfrac{\partial \Delta \boldsymbol{P}_{\text{c}}}{\partial \boldsymbol{U}_{\text{dc}}} & \dfrac{\partial \Delta \boldsymbol{P}_{\text{dc}}}{\partial \boldsymbol{\delta}_{\text{c}}} & \dfrac{\partial \Delta \boldsymbol{P}_{\text{dc}}}{\partial \boldsymbol{U}_{\text{c}}} \\ \hline \dfrac{\partial \Delta \boldsymbol{D}_{\text{axis}}}{\partial \boldsymbol{\delta}} & \dfrac{\partial \Delta \boldsymbol{D}_{\text{axis}}}{\partial \boldsymbol{U}} & \dfrac{\partial \Delta \boldsymbol{D}_{\text{axis}}}{\partial \boldsymbol{U}_{\text{dc}}} & \dfrac{\partial \Delta \boldsymbol{D}_{\text{axis}}}{\partial \boldsymbol{\delta}_{\text{c}}} & \dfrac{\partial \Delta \boldsymbol{D}_{\text{axis}}}{\partial \boldsymbol{U}_{\text{c}}} \\ \dfrac{\partial \Delta \boldsymbol{Q}_{\text{axis}}}{\partial \boldsymbol{\delta}} & \dfrac{\partial \Delta \boldsymbol{Q}_{\text{axis}}}{\partial \boldsymbol{U}} & 0 & \dfrac{\partial \Delta \boldsymbol{Q}_{\text{axis}}}{\partial \boldsymbol{\delta}_{\text{c}}} & \dfrac{\partial \Delta \boldsymbol{Q}_{\text{axis}}}{\partial \boldsymbol{U}_{\text{c}}} \end{array}\right] \quad (14)$$

It is noted that the dimension of Jacobi matrix keeps constant for a given AC/DC system, and only some coefficients need to be modified when the control mode changes, thus it is suitable to be linearised for uncertainty analysis under different control modes.

3.3 Quantification procedure with control mode adjustment

Through diverse control modes of VSC stations, the hybrid AC/DC system has the advantage to adjust the system operation condition flexibly. For a given level of uncertainty in renewable generation, if the state variables cannot remain within the performance requirements, the authors are interested in whether the impact can be reduced by control mode adjustment to meet the operation requirements.

Using the set-theoretic method and the proposed unified power flow, the authors can quantify the impact of generation uncertainty

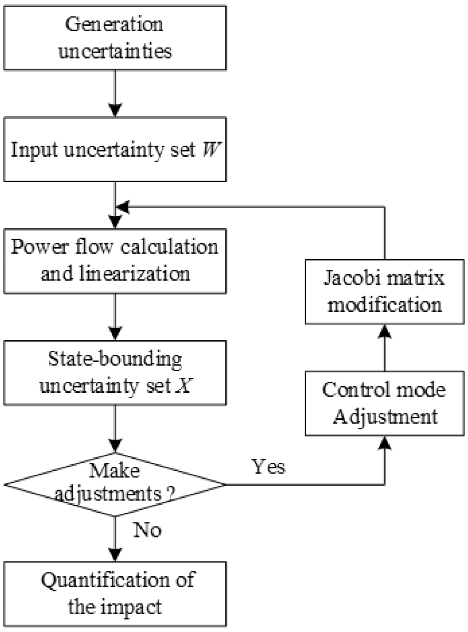

Fig. 3 *Quantification procedure for the impact of generation uncertainty*

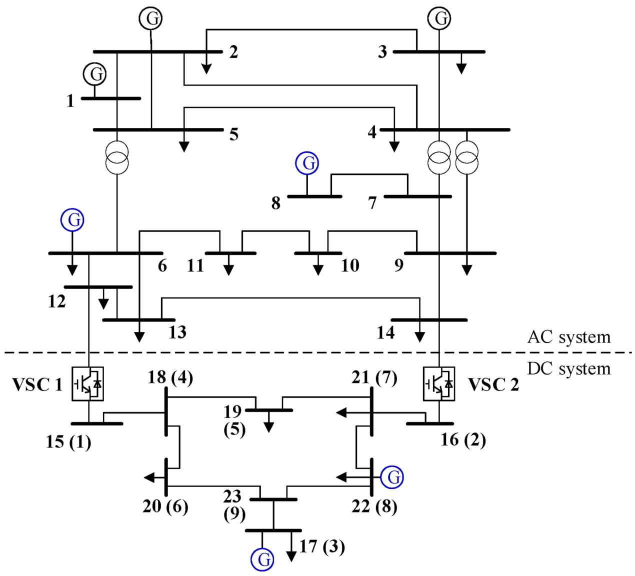

Fig. 4 *23-bus hybrid AC/DC test system*

Table 3 VSC model parameters (p.u.)

VSC no.	Resistance	Reactance	Control mode
1	0.029	0.571	constant $U_{dc} = 1$, $Q_S = 0.15$
2	0.029	0.571	constant $P_S = 0.25$, $Q_S = -0.15$

in hybrid AC/DC network under different control modes. The quantification procedure is shown in Fig. 3.

4 Case studies

Here, the authors apply the proposed method to the quantification of impact of generation uncertainty on a hybrid AC/DC system under different operation conditions. The 23-bus test system shown in Fig. 4 is formed by connecting an IEEE 14-bus system to a 9-bus DC network. The DC network structure and line resistance are the same as IEEE 9-bus system and the parameters associated with IEEE test systems are from [19]. Two VSC stations are, respectively, connected between bus 12–15 and bus 14–16. The converter parameters are shown in Table 3 and the generation and load power of DC bus is shown in Table 4. In AC system, renewable-based electricity resources are installed at bus 6 and bus 8 with nominal real power injection of 0.4 and 0.6 p.u. and an uncertainty of ±50%. In DC system, the renewable power generation at bus 17 and bus 22 are supposed to have an uncertainty of ±0.3 p.u. The authors examine the impact of uncertainty on bus voltage magnitudes using zonotope and compare the results against the exact solutions computed from non-linear power flow.

4.1 Impact of uncertainty from AC and DC systems

We first consider the impact of generation uncertainty arising from DC network at bus 17 and bus 22. The authors bound the power injection space with a zonotope W, then propagate it through the linearised power flow model to obtain a state-bounding zonotope X which capture all possible values that the state variables can take. In order to have a general view of the voltage profile and voltage

Table 4 Generation and load data of DC network

bus	15	16	17	18	19	20	21	22	23
P_{gi}	0	0	0.8	0	0	0	0	0.8	0
P_{di}	0	0	0.3	0	0.4	0.4	0.4	0.4	0

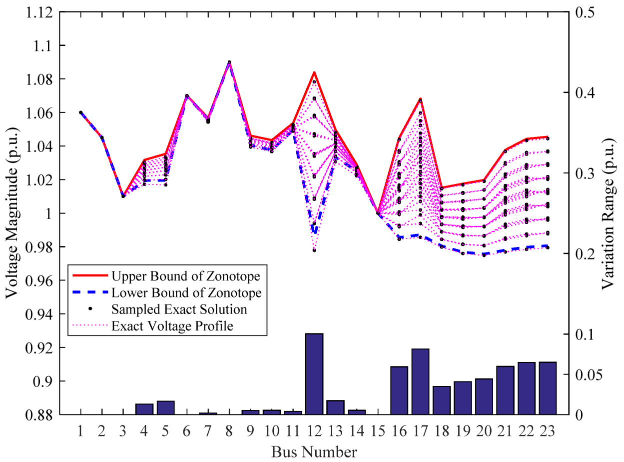

Fig. 5 *Voltage profile of the system*

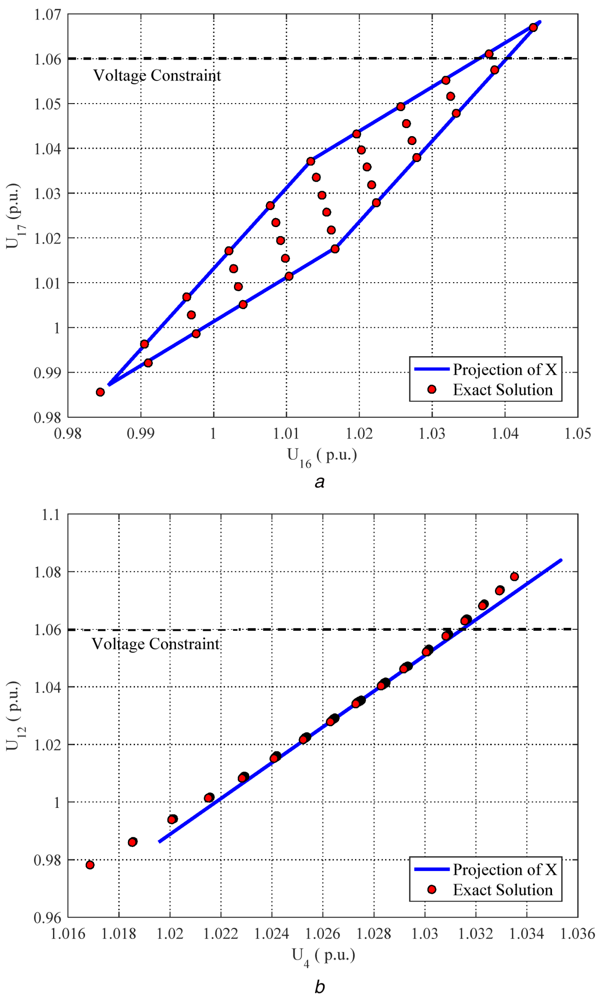

Fig. 6 *Projection of zonotope X considering uncertainty from DC network*
(a) Projection of X onto $U_{16} - U_{17}$ axes and exact solution points, ***(b)*** Projection of X onto $U_4 - U_{12}$ axes and exact solution points

variation of the system, the authors first plot the upper and lower bound value of X of each dimension (corresponding to each bus), and thus obtain the voltage variation range of each node. As is shown in Fig. 5, the authors also sample the input power injection space W, obtain the exact solutions of each case using power flow calculation, and plot them with dotted lines. It is clear that the zonotope X bounds all possible values of state variables except for one extreme condition which can be attributed to the error resulting from linearisation.

However, this figure, only containing the boundary information of each single bus, cannot reflect the correlation between different nodes and a lot of information inside the zonotope is not displayed. Thus, the authors project the output set X onto the $U_{16} - U_{17}$ and $U_4 - U_{12}$ axes as shown in Fig. 6. Additionally, the sampled exact solutions are depicted with red dots. From Fig. 6*a*, the resulting projection contains all the exact solutions but an extreme one, thus the authors can conclude that the linearisation is fairly accurate. The shape of projection indicates that the voltage magnitude of DC bus 16 and bus 17 is positively correlated with some degree. For a voltage limit of 1.06 p.u., the authors notice that the voltage at bus 17 will violate the constraint in some conditions, thus the system may not satisfy the operation requirements under this level of uncertainty. From Fig. 6*b*, the projection is a line segment, indicating the voltage magnitude of AC bus 4 and bus 12 is linearly correlated. This linear relationship can be explained, for that under this control mode, the generation variations arising from DC network are equivalent to a change in power injection at bus 12. The exact values are slightly deviated from the line, but the percentage error is calculated to be <0.1%. Through Fig. 6, the generation uncertainty arising from DC network will have a significant impact on DC system, while the impact on AC system is propagated through the connection of converters.

Now consider the generation uncertainty from AC network at bus 6 and bus 8. The projection of the obtained zonotope X onto the $U_4 - U_{12}$ axes is shown in Fig. 7 along with the exact power flow solutions. The authors can conclude that the zonotope obtained bounds the uncertain values of state variables quite well. For the several points outside of the zonotope, the maximum error resulted from linearisation is only 0.085%.

It should be noted that the voltage magnitude of DC network remains almost unchanged. This is because the slack bus 1 makes up for the power variations in AC system, the impact on the static performance of DC system is rather small.

4.2 Impact of uncertainty under different control modes

Now the authors analyse the steady-state response to generation uncertainty under different control modes. For a multi-terminal DC network, in order to maintain the power balance and voltage stability, it requires at least one converter to operate at constant U_{dc} or U_{dc} droop control mode. Thus, given the generation uncertainty

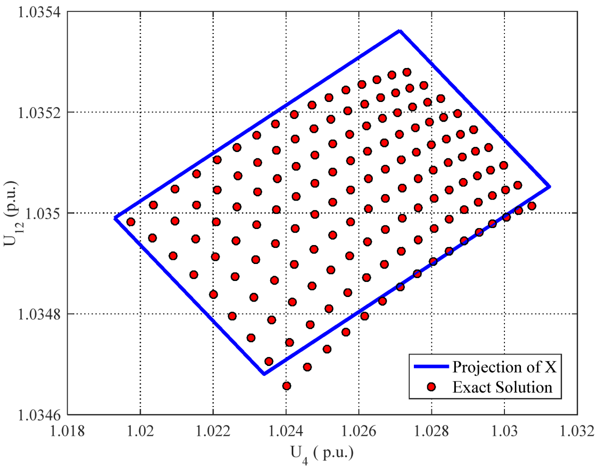

Fig. 7 *Projection of X onto $U_4 - U_{12}$ axes and exact solution points*

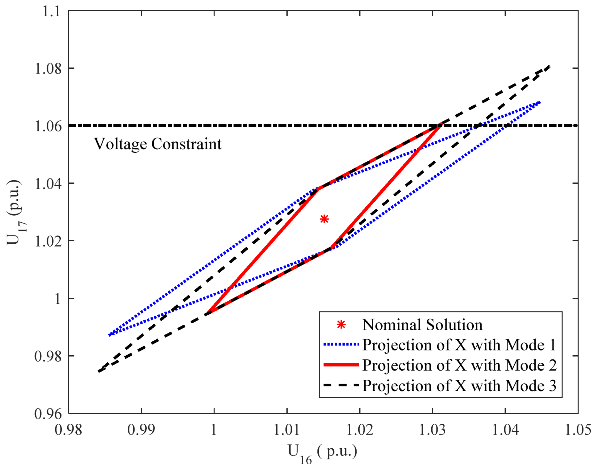

Fig. 8 *Projection of X under three control modes*

at bus 17 and bus 22 in DC system, the authors quantify the impact under three different control strategies. The three control strategies are shown in Table 5.

The control parameters are properly chosen so that the nominal solutions of the three conditions are the same. The three state-bounding zonotopes are projected onto the $U_{16} - U_{17}$ axes as shown in Fig. 8. It is clear that the three projections centred at the same point have different shapes, indicating the different variation ranges of voltage magnitude under different control modes, and the variation range under mode 2 is much smaller. The authors can also conclude that, under mode 1 and 3, a portion of input space maps to a region in the solution state space that violates the voltage constraint of 1.06 p.u. Thus, the system state variables may not remain within the operation requirements when taking mode 1 or 3, if the system were subjected to this level of uncertainty in renewable power generation. For a two-terminal DC system, the two converters, respectively, taking constant DC voltage control and voltage droop control may help reduce the voltage variation arising from generation uncertainty in renewable resources.

Now, the authors make some adjustments for control mode 2. The authors change the reference value P_{dc}^{ref} to be 0.15 p.u. (instead of 0.25 p.u.) and keep other parameters of mode 2 unchanged, the projections of the two conditions are shown in Fig. 9*a*. Then, the authors only change the droop coefficient K_{Pdc} to be 0.05 (instead of 0.1) and the results are shown in Fig. 9*b*. It can be concluded that both of the two adjustments will help the voltage magnitude of bus 17 stay within the constraint of 1.06 p.u. In Fig. 9*a*, the new projection can be considered as a movement of the original one with the change of nominal operation point, while the fluctuation amplitude of voltage magnitude is not changed. In Fig. 9*b*, the shape of the projection is changed, resulting in a smaller variation range from U_{16} to U_{17}. This indicates that, changing the reference value of droop control mode will bring a change of nominal operation point, while the change of droop coefficient will impact the correlation of bus voltages.

Table 5 Three control strategies of the hybrid system

Mode	Station 1	Station 2
1	constant $U_{dc} = 1.0$	constant $P_{dc} = 0.25$
	$Q_s = 0.15$	$Q_s = -0.15$
2	constant $U_{dc} = 1.0$	U_{dc}-P_{dc} droop, $K_{Pdc} = 0.1$
	$Q_s = 0.15$	$U_{dc}^{ref} = 1.015$, $P_{dc}^{ref} = 0.25$
		constant $Q_s = -0.15$
3	U_{dc}-P_{dc} droop, $K_{Pdc} = 0.1$	U_{dc}-P_{dc} droop, $K_{Pdc} = 0.1$
	$U_{dc}^{ref} = 1.0$, $P_{dc}^{ref} = 0.07$	$U_{dc}^{ref} = 1.015$, $P_{dc}^{ref} = 0.25$
	constant $Q_s = 0.15$	constant $Q_s = -0.15$

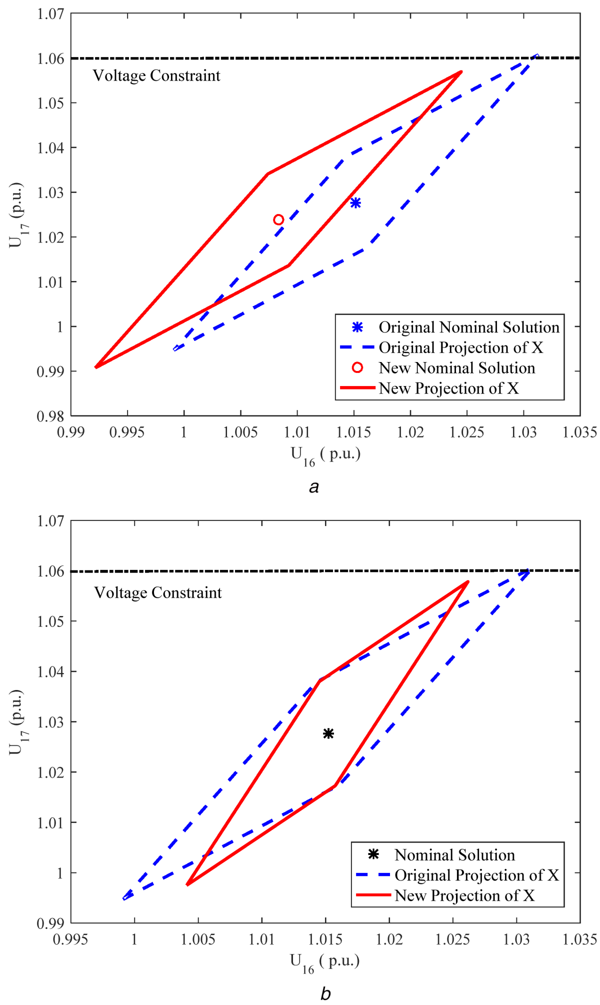

Fig. 9 *Projection of zonotope X with different control parameters*
(a) Projection of X with = 0.25 and 0.15 p.u., ***(b)*** Projection of X with = 0.1 and 0.05

5 Conclusion

This paper proposes a set-theoretic method to quantify the impact of generation uncertainty arising from renewable resources on the static performance of hybrid AC/DC system. A unified power flow model for AC/DC network considering the control modes of VSC is established. The uncertain output of renewable sources is bounded using a zonotope, and then propagated through the linearised power flow model to obtain a state-bounding zonotope which captures all possible values that the state variables can take, thus the authors can determine whether the state variables, for example, bus voltage magnitudes and angles, will remain within acceptable ranges as dictated by operational requirements.

As shown in case studies, the uncertainty set obtained using our method matches close to the set of exact solutions obtained from repeatedly sampling the input space and solving the non-linear power flow for each sample point, which verifies the validity and accuracy of our method. In addition, this method takes less computation than probabilistic methods since linear approximations are used, and it can be easily extended to consider the impact of other uncertainties of the system.

Through analysis of the performance of hybrid AC/DC system under different control modes, the authors can conclude that, given the level of uncertainty, the voltage fluctuations may be reduced or controlled to be within an acceptable range by properly selecting the control mode and its parameters. This gives an explanation for the better capability of hybrid AC/DC system than traditional AC system to integrate renewable power resources.

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