The Journal of Engineering

The 14th IET International Conference on AC and DC Power Transmission (ACDC 2018)

Square-root-extended complex Kalman filter for estimation of symmetrical components in power system

eISSN 2051-3305
Received on 24th August 2018
Accepted on 19th September 2018
E-First on 14th January 2019
doi: 10.1049/joe.2018.8642
www.ietdl.org

Bowen Cui[1] ✉
[1]*School of Marine Engineering, Jimei University, Xiamen, People's Republic of China*
✉ *E-mail: tsuibowen@gmail.com*

Abstract: The paper presents a square-root-extended complex Kalman filter (SRECKF) by decomposing covariance matrix with its square-root forms to improve stability of the filter for estimating complex number. *αβ* transformation is used to map three-phase instantaneous voltages in the *abc* phases into instantaneous voltages on the *αβ* axes, and a non-linear state equation and observation equation of the three-phase voltages are built by introducing a complex vector and defining state variables. Positive symmetrical component, negative symmetrical components, and frequency of the three-phase voltages are estimated using traditional extended complex Kalman filter (ECKF), the estimation results show that the method proposed here are superior to traditional extended complex Kalman filter on estimation accuracy and convergence rate.

1 Introduction

Under normal operating conditions, three-phase industrial power system voltage and current waveforms are to a certain extent unbalanced and contain harmonics for a number of reasons. Unbalanced power system is usually caused by system faults and other reasons. Information on the symmetrical components symmetrical components are widely used in the analysis of unsymmetrical faults in a power system, digital protection of power system components, open-phase and inter-turn short-circuit faults detection of electrical machines [1–8]. An accurate and fast algorithm is needed to measure the symmetrical components especially from a harmonic polluted unbalanced three-phase system.

Several techniques have been proposed to estimate the symmetrical components of a three-phase voltage and current. In [9], methods for online calculation of the phasors of symmetrical components from the complex space-phasor are presented. In [1], the author presented a symmetrical components estimation through adaptive transformation matrix of phase shifters to get instantaneous symmetrical components independently on the frequency variation with modest computation requirements. In [2], the author presented a new method for decomposing a set of three-phase signals into its constituting instantaneous positive-sequence, negative-sequence, and zero-sequence components. In [6, 10], the fast Fourier transform (FFT) was used to estimate symmetrical components. In [11], state observer based method was applied to estimation of the current and voltage symmetrical components in a three-phase electrical network. In [12], stochastic estimation theory-based dynamic technique was presented to estimate the symmetrical components of three-phase voltage or current waveforms in electrical power systems. In [13], the authors proposed the use of a complex Kalman filter for the estimation of positive and negative sequences from three-phase voltages. A complex voltage is obtained by applying the *αβ*-transform followed by the dq-transform using a rotational operator. In [14], the authors introduced a new state-space model to estimate the symmetrical components of distorted and time-changing power systems using extended Kalman Filter.

However, in real filtering algorithms, the major source of numerical instability comes from round-off errors of finite-precision arithmetics implemented in modern computational devices. One of the most serious consequences of rounding is that it may destroy the theoretical semi-positiveness of the covariance matrices[15]. To overcome this problem, the square-root approach, based on finding square-root factors of the covariance matrix, has been developed.

Here, the paper presents a new square-root extended complex Kalman filter (SRECKF) by decomposing covariance matrix with its square root forms to improve stability of the filter for estimating symmetrical components in complex number domain.

2 Extended complex Kalman filter

Kalman filtering is used to estimate time varying parameters of a system. The estimates produced are optimal in the least squares. To apply Kalman filtering, the system model should be in a state-variable form given by

$$\boldsymbol{X}(k+1) = \varphi[\boldsymbol{X}(k), k] + \boldsymbol{u}(k) \tag{1}$$

$$Z(k) = h[\boldsymbol{X}(k), k] + v(k) \tag{2}$$

where, $\boldsymbol{X}(k) \in C^{n\times 1}$ and $Z(k) \in C^{m\times 1}$ are complex state and measurement vector, respectively. k is a discrete time index, $\boldsymbol{X}(k)$ that is, means $\boldsymbol{X}(t_k)$. $\varphi[X(k), k] \in C^{n\times 1}$ and $h[X(k), k] \in C^{m\times 1}$ are non-linear state and measurement complex function vector, respectively. The process noise $u(k)$ and the measurement noise $v(k)$ are Gaussian white-noise processes with covariance matrices $Q \geq 0$ and $R \geq 0$, respectively.

The well-known extended Kalman filter is a two-step prediction-correction process, it can be summarised as follows:

(1) State Prediction: Calculate the one-step prediction of the system state along with the associated covariance matrix of the prediction state estimation error.

$$\hat{\boldsymbol{X}}(k+1\big|k) = \varphi\Big[\hat{\boldsymbol{X}}(k), k\Big] \tag{3}$$

$$\boldsymbol{P}(k+1\big|k) = \boldsymbol{\Phi}(k+1, k)\boldsymbol{P}(k\big|k)\boldsymbol{\Phi}^{*T}(k+1, k) + Q(k) \tag{4}$$

where $\boldsymbol{\Phi}(k+1, k) = \partial\phi/\partial X|_{X(k)=\hat{X}(k|k)}$ is the Jacobian matrix, $\boldsymbol{P}(k|k) = E\Big[\big(x(k|k) - \hat{x}(k|k-1)\big)\big(x(k|k) - \hat{x}(k|k-1)\big)^{*T}\Big]$ is the state prediction error covariance matrix and can be derived using a first-order Taylor series expansion of $\varphi[X(k), k]$ about $\hat{x}(k|k-1)$.

(2) State Correction: Calculate the Kalman filter gain and update the state estimate and the estimation error covariance matrix using

$$\boldsymbol{K}(k+1) = \boldsymbol{P}(k+1|k)\boldsymbol{H}^{*T}(k+1)\left[\boldsymbol{H}(k+1)\boldsymbol{P}(k+1|k)\boldsymbol{H}^{*T}(k+1) + R(k)\right]^{-1} \tag{5}$$

$$\hat{\mathbf{X}}(k+1|k+1) = \hat{\mathbf{X}}(k+1|k) + \mathbf{K}(k+1)\left\{\left[Z(k+1) - h\left[\hat{\mathbf{X}}(k+1|k), k+1\right]\right]\right\} \tag{6}$$

$$\boldsymbol{P}(k+1|k+1) = [\boldsymbol{I} - \boldsymbol{K}(k+1)]\boldsymbol{P}(k+1|k) \tag{7}$$

where, *represents complex conjugate, ^{T} is transpose of matrix, $\boldsymbol{H}(k+1) = \partial h/\partial X|_{\hat{X}(k+1|k)}$.

3 Square-root-extended complex Kalman filter

The Cholesky decomposition is applied to factorise the error covariance matrix

$$\boldsymbol{P}(k-1|k-1) = \boldsymbol{S}(k-1|k-1)\boldsymbol{S}^T(k-1|k-1) \tag{8}$$

where $\boldsymbol{S}(k-1|k-1)$ stands for the lower-triangular matrix in (8). Then, we can rewrite (6) as the following,

$$\boldsymbol{P}(k|k-1) = \tilde{\boldsymbol{S}}(k|k-1)\tilde{\boldsymbol{S}}^{*T}(k|k-1) \tag{9}$$

where $\tilde{\boldsymbol{S}}(k|k-1) = \boldsymbol{\Phi}(k, k-1)\boldsymbol{S}(k-1|k-1)$.Then, substituting the results obtained in (9) into (6) yields

$$\begin{aligned}
\boldsymbol{P}(k|k) &= [\boldsymbol{I} - \boldsymbol{K}(k)\boldsymbol{H}(k)]\boldsymbol{P}(k|k-1) \\
&= \boldsymbol{P}(k|k-1) - \boldsymbol{K}(k)\boldsymbol{H}(k)\boldsymbol{P}(k|k-1) \\
&= \boldsymbol{P}(k|k-1) - \boldsymbol{P}(k|k-1)\boldsymbol{H}^{*T}(k) \\
&\quad \times \left[\boldsymbol{H}(k)\boldsymbol{P}(k|k-1)\boldsymbol{H}^{H}(k) + R(k)\right]^{-1}\boldsymbol{H}(k)\boldsymbol{P}(k|k-1) \\
&= \tilde{\boldsymbol{S}}(k|k-1)\tilde{\boldsymbol{S}}^{*T}(k|k-1) - \tilde{\boldsymbol{S}}(k|k-1) \\
&\quad \times \tilde{\boldsymbol{S}}^{*T}(k|k-1)\boldsymbol{H}^{*T}(k) \\
&\quad \times \left[\boldsymbol{H}(k)\tilde{\boldsymbol{S}}(k|k-1)\tilde{\boldsymbol{S}}^{*T}(k|k-1)\boldsymbol{H}^{*T}(k) + \boldsymbol{R}(k)\right]^{-1} \\
&\quad \times \boldsymbol{H}(k)\tilde{\boldsymbol{S}}(k|k-1)\tilde{\boldsymbol{S}}^{*T}(k|k-1)
\end{aligned} \tag{10}$$

so we have,

$$\begin{aligned}
\boldsymbol{P}(k|k) &= \tilde{\boldsymbol{S}}(k|k-1)\left\{\boldsymbol{I} - \boldsymbol{F}_k\left[\boldsymbol{F}_k^{*T}\boldsymbol{F}_k + R(k)\right]^{-1}\right\}\tilde{\boldsymbol{S}}^{*T}(k|k-1) \\
&= \tilde{\boldsymbol{S}}(k|k-1)\left[\boldsymbol{I} - \alpha_k\boldsymbol{F}_k\boldsymbol{F}_k^{*T}\right]\tilde{\boldsymbol{S}}^{*T}(k|k-1)
\end{aligned} \tag{11}$$

where $\boldsymbol{F}_K = \tilde{\boldsymbol{S}}(k|k-1)\boldsymbol{H}^{*T}(k)$, $\alpha_k = \left[\boldsymbol{F}_k^{*T}\boldsymbol{F}_k + R(k)\right]^{-1}$.

In order to simplify (11), we suppose that,

$$\begin{aligned}
\boldsymbol{I} - \alpha_k\boldsymbol{F}_k\boldsymbol{F}_k^{*T} &= \left[\boldsymbol{I} - \alpha_k\gamma_k\boldsymbol{F}_k\boldsymbol{F}_k^{*T}\right]\left[\boldsymbol{I} - \alpha_k\gamma_k\boldsymbol{F}_k\boldsymbol{F}_k^{*T}\right]^{*T} \\
&= \boldsymbol{I} - 2\alpha_k\gamma_k\boldsymbol{F}_k\boldsymbol{F}_k^{*T} + \alpha_k^2\gamma_k^2\boldsymbol{F}_k\boldsymbol{F}_k^{*T}\boldsymbol{\Phi}_k\boldsymbol{F}_k^{*T} \\
&= \boldsymbol{I} - \alpha_k\boldsymbol{F}_k\left[2\gamma_k\boldsymbol{F}_k^{*T} - \alpha_k\gamma_k^2\boldsymbol{F}_k^{*T}\boldsymbol{F}_k\right]\boldsymbol{F}_k^{*T}
\end{aligned} \tag{12}$$

To satisfy the equation, comparing value of two sides of (12), we have the following equation

$$2\gamma_k\boldsymbol{F}_k^{*T} - \alpha_k\gamma_k^2\boldsymbol{F}_k^{*T}\boldsymbol{F}_k = 1 \tag{13}$$

Solving the equation, we have,

$$\gamma_k = \frac{1 \pm \sqrt{\alpha_k R(k)}}{1 - \alpha_k R(k)} \tag{14}$$

and then,

$$\boldsymbol{P}(k|k) = \tilde{S}(k|k)\tilde{S}^{*T}(k|k) \tag{15}$$

where $\tilde{S}(k|k) = \tilde{S}(k|k-1)\left[I - \alpha_k\gamma_k F_k F_k^{*T}\right]$.

So, SRECKF can be summarised as follows,

$$\begin{aligned}
&\boldsymbol{X}(k|k) = \boldsymbol{X}(k-1|k-1) + \boldsymbol{K}(k) \\
&[z(k) - \boldsymbol{H}(k)\boldsymbol{\Phi}(k, k-1)\boldsymbol{X}(k|k-1)]
\end{aligned} \tag{16}$$

$$K(k) = \alpha_k\tilde{S}(k|k-1)F_k \tag{17}$$

$$F_k = \tilde{S}^{*T}(k|k-1)H^{*T}(k) \tag{18}$$

$$\alpha_k = \left[F_k^{*T}F_k + R(k)\right]^{-1} \tag{19}$$

$$\tilde{\boldsymbol{S}}(k|k-1) = \boldsymbol{\Phi}(k, k-1)\tilde{\boldsymbol{S}}(k|k-1) \tag{20}$$

$$\tilde{\boldsymbol{S}}(k|k-1) = \tilde{\boldsymbol{S}}(k|k-1)\left[\boldsymbol{I} - \alpha_k\gamma_k F_k F_k^{*T}\right] \tag{21}$$

$$\gamma_k = \frac{1 \pm \sqrt{\alpha_k R(k)}}{1 - \alpha_k R(k)} \tag{22}$$

4 Symmetrical components and system modelling

Considering a set of three-phase voltage signal $\boldsymbol{u}(k) = [u_a(k), u_b(k), u_c(k)]^T$ associated with a three-phase set of measurements. We make no assumption on these signals as they can be unbalanced and/or carry other kinds of distortions such as harmonic pollution and noise. We particularly assume that $u(k)$ has a fundamental component

$$\begin{cases}
u_a(k) = \sqrt{2}V_a\sin(\omega kT + \varphi_a) \\
u_b(k) = \sqrt{2}V_b\sin(\omega kT + \varphi_b) \\
u_c(k) = \sqrt{2}V_c\sin(\omega kT + \varphi_c)
\end{cases} \tag{23}$$

which comprises three symmetrical components as $\boldsymbol{u}(k) = \boldsymbol{u}_p(k) + \boldsymbol{u}_n(k) + \boldsymbol{u}_0(k)$, in which k is sampling time, T is sampling period, $\boldsymbol{u}_p(k)$, $\boldsymbol{u}_n(k)$, $\boldsymbol{u}_0(k)$ are positive sequence, negative sequence, and zero sequence, respectively, and in which

$$\boldsymbol{u}_p(k) = \begin{pmatrix} V_p\sin(\omega kT + \phi_+) \\ V_p\sin(\omega kT + \phi_+ - \frac{2}{3}\pi) \\ V_p\sin(\omega kT + \phi_+ + \frac{2}{3}\pi) \end{pmatrix} \tag{24}$$

$$\boldsymbol{u}_n(k) = \begin{pmatrix} V_n\sin(\omega kT + \phi_-) \\ V_n\sin(\omega kT + \phi_- + \frac{2}{3}\pi) \\ V_n\sin(\omega kT + \phi_- - \frac{2}{3}\pi) \end{pmatrix} \tag{25}$$

$$\boldsymbol{u}_0(k) = \begin{pmatrix} V_0\sin(\omega kT + \phi_0) \\ V_0\sin(\omega kT + \phi_0) \\ V_0\sin(\omega kT + \phi_0) \end{pmatrix} \tag{26}$$

The $\alpha\beta0$ transformation is used to separate zero sequence components from the abc-phase components. The α and β axes make no contribution to zero sequence components [16]. Using $\alpha\beta0$ transformation to transform positive sequence and negative sequence components in the $\alpha\beta$ frame, respectively

$$\begin{bmatrix} u_{P\alpha} \\ u_{P\beta} \end{bmatrix} = \sqrt{\frac{2}{3}}\begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} V_p\sin(\omega kT + \phi_+) \\ V_p\sin(\omega kT + \phi_+ - \frac{2}{3}\pi) \\ V_p\sin(\omega kT + \phi_+ - \frac{2}{3}\pi) \end{bmatrix} \tag{27}$$

$$\begin{bmatrix} u_{n\alpha} \\ u_{n\beta} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} V_n \sin(\omega kT + \phi_+) \\ V_n \sin(\omega kT + \phi_+ + \frac{2}{3}\pi) \\ V_n \sin(\omega kT + \phi_+ - \frac{2}{3}\pi) \end{bmatrix} \tag{28}$$

where $u_{p\alpha}$ and $u_{p\beta}$ are positive components in the $\alpha\beta$ reference frame, $u_{n\alpha}$ and $u_{n\beta}$ are negative components in the $\alpha\beta$ reference frame. Instantaneous voltage vector is defined from the instantaneous α- and β-voltage components, that is,

$$\boldsymbol{u}_1(k) = u_{p\alpha} + ju_{p\beta} = A_p \,\mathrm{e}^{j\omega kT} \tag{29}$$

$$\boldsymbol{u}_2(k) = u_{n\alpha} + ju_{n\beta} = A_n \,\mathrm{e}^{-j\omega kT} \tag{30}$$

Define new vector, that is,

$$\boldsymbol{u} = \boldsymbol{u}_1(k) + \boldsymbol{u}_2(k) = A_p \,\mathrm{e}^{j\omega kT} + A_n \,\mathrm{e}^{-j\omega kT} \tag{31}$$

we chose $x_1(k) = \mathrm{e}^{j\omega T} = \cos(\omega T) + j\sin(\omega T)$, $x_2(k) = A_p \,\mathrm{e}^{j\omega kT}$, $x_3(k) = A_n \,\mathrm{e}^{-j\omega kT}$ as state variable of three-phase voltage system, and the state equation of the system can be written as

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & x_1(k) & 0 \\ 0 & 0 & \dfrac{1}{x_1(k)} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} \tag{32}$$

the measurement equation is

$$y == [0 \quad 1 \quad 1]\,[0 \quad x_2(k) \quad x_3(k)]^T \tag{33}$$

5 Symmetrical components estimation

The system model described by (32) and (33) can be expressed with the following non-linear equation

$$\begin{aligned} x(k+1) &= \varphi(x(k)) \\ y &= h[x(k), k] \end{aligned} \tag{34}$$

where

$$\boldsymbol{X}(k) = [x_1(k) \quad x_2(k) \quad x_3(k)]^T \tag{35}$$

$$\varphi(x(k)) = [x_1(k) \quad x_1(k)x_2(k) \quad x_3(k)/x_1(k)]^T \tag{36}$$

$$y = x_2(k) + x_3(k) \tag{37}$$

Equation (36) is linearised, and we have,

$$\boldsymbol{\Phi}(k+1, k) = \frac{\partial \phi}{\partial \boldsymbol{X}}\bigg|_{X(k) = \hat{X}(k|k)} = \begin{bmatrix} 1 & 0 & 0 \\ x_2(k) & x_1(k) & 0 \\ -\dfrac{x_3(k)}{x_1^2(k)} & 0 & \dfrac{1}{x_1(k)} \end{bmatrix} \tag{38}$$

$$\boldsymbol{H}(k+1) = \frac{\partial h}{\partial \boldsymbol{X}}\Big|_{\hat{X}(k+1|k)} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \tag{39}$$

Then, the state variable x can be estimated directly using the SRECKF expressed by (18)~(24). In order to get the symmetrical components, $\alpha\beta$ inverse transformation is used. Defining u_{pa}, u_{pb}, u_{pc} as phase a, b, and c positive sequence components, respectively, u_{na}, u_{nb}, u_{nc} as phase a, b, and c negative sequence components, respectively, so we have the following results

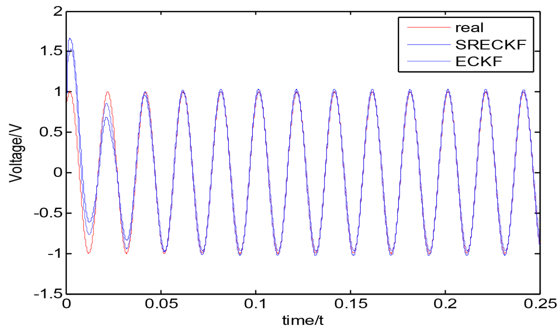

Fig. 1 *Positive symmetrical components of Phase a*

$$\begin{aligned} \begin{bmatrix} u_{\mathrm{pa}}(k) \\ u_{\mathrm{pb}}(k) \\ u_{\mathrm{pc}}(k) \end{bmatrix} &= \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} u_{p\alpha}(k) \\ u_{p\beta}(k) \end{bmatrix} \\ &= \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \mathrm{Re}(x_2(k)) \\ \mathrm{Im}(x_2(k)) \end{bmatrix} \end{aligned} \tag{40}$$

$$\begin{aligned} \begin{bmatrix} u_{\mathrm{na}}(k) \\ u_{\mathrm{nb}}(k) \\ u_{\mathrm{nc}}(k) \end{bmatrix} &= \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} u_{n\alpha}(k) \\ u_{n\beta}(k) \end{bmatrix} \\ &= \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \mathrm{Re}(x_3(k)) \\ \mathrm{Im}(x_3(k)) \end{bmatrix} \end{aligned} \tag{41}$$

If the frequency of the system is needed to estimate, the variable x_1 is converted appropriately, and the estimated frequency can be expressed as follows

$$f = \sin^{-1}\left(\frac{\mathrm{Im}(x_1(k))}{2\pi T}\right) = \cos^{-1}\left(\frac{\mathrm{Re}(x_1(k))}{2\pi T}\right) \tag{42}$$

6 Simulation results

In order to verify the effectiveness of the method presented here, three-phase voltages with positive sequence, negative sequence, and zero sequence is used to describe the method presented here, where the value of the positive sequence, negative sequence, and zero sequence voltage are 1.0, 0.4, and 0.2, respectively, and their phases are $\varphi_+ = \pi/3$ $\varphi_- = \pi/6$, $\varphi_0 = 0$, respectively, the sampling period $T = 0.0001$ s. The three-phase voltage can be given as follows,

$$\begin{aligned} u_a(k) &= V_p \sin(\omega kT + \varphi_+) + V_n \sin(\omega kT + \varphi_-) + V_0 \sin(\omega kT + \varphi_0) \\ u_b(k) &= V_p \sin\left(\omega kT + \varphi_+ - \frac{2}{3}\pi\right) + V_n \sin\left(\omega kT + \varphi_- + \frac{2}{3}\pi\right) \\ &\quad + V_0 \sin(\omega kT + \varphi_0) \\ u_c(k) &= V_p \sin\left(\omega kT + \varphi_+ + \frac{2}{3}\pi\right) + V_n \sin\left(\omega kT + \varphi_- - \frac{2}{3}\pi\right) \\ &\quad + V_0 \sin(\omega kT + \varphi_0) \end{aligned}$$

where, $V_p = 1$, $V_n = 0.4$, $V_0 = 0.2$.

The traditional extended complex Kalman filter (ECKF) and SRECKF are used to estimate positive sequence, negative sequence, and frequency of the three-phase voltage system. The results of positive and negative sequence estimation of phase a using ECKF and SRECKF are shown in Figs. 1 and 2, respectively. The results of positive and negative sequence estimation of phase b using ECKF and SRECKF are shown in Figs. 3 and 4, respectively.

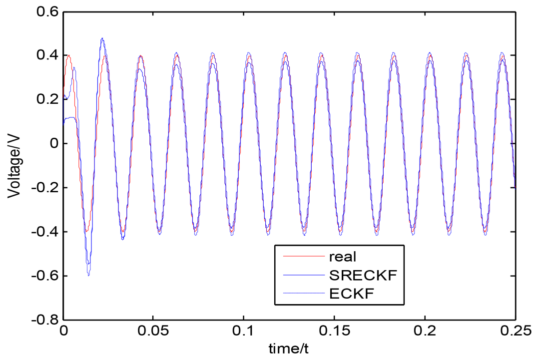

Fig. 2 *Negative symmetrical component of Phase a*

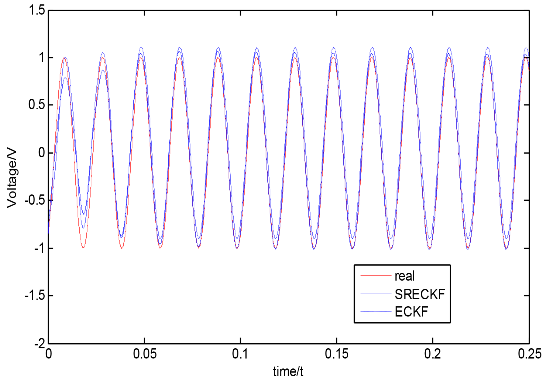

Fig. 3 *Positive symmetrical components of phase b*

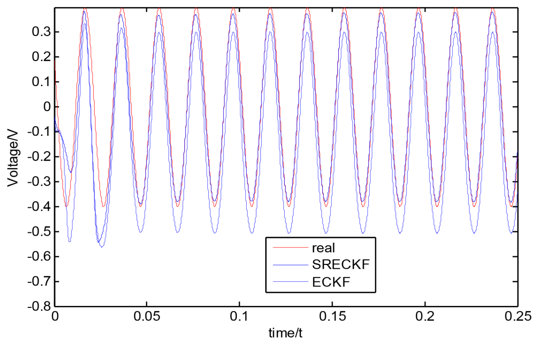

Fig. 4 *Negative symmetrical component of phase b*

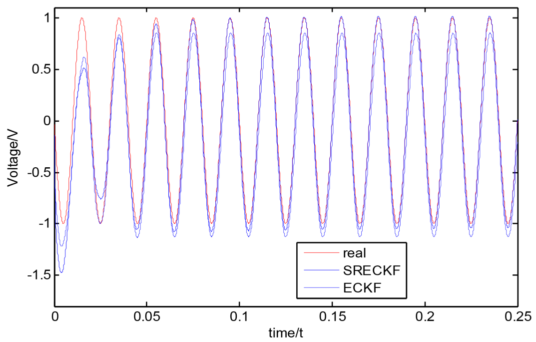

Fig. 5 *Positive symmetrical components of phase c*

The results of positive and negative sequence estimation of phase c using ECKF and SRECKF are shown in Figs. 5 and 6, respectively. The results of frequency estimation using ECKF and SRECKF are shown in Fig. 7. The results of positive and negative sequence estimation of phase a using ECKF and SRECKF are shown in Figs. 1 and 2, respectively. Comparing the estimation results of phase a shown in Figs. 1–6 using ECKF and SRECKF, we can find that there are no significant difference between the results of positive and negative estimation of phase *a* using ECKF and SEECKF and the real value. However, there are obvious difference

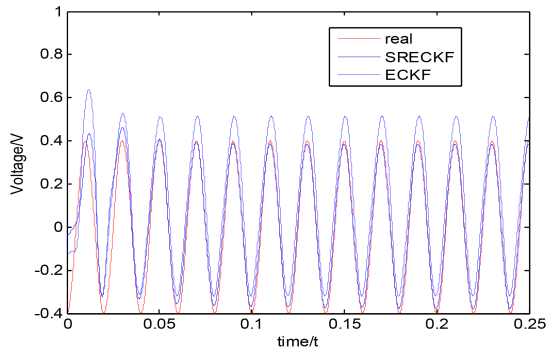

Fig. 6 *Negative symmetrical component of phase c*

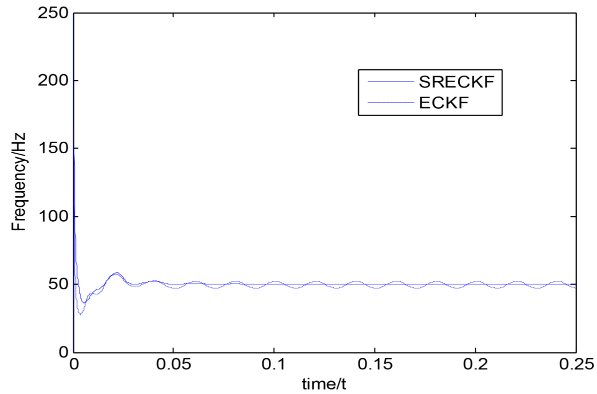

Fig. 7 *Frequency estimation*

between the estimation results using ECKF and their real value for phases *b* and *c*. The estimation results using SRECKF are more close to real value. For frequency estimation results using ECKF and SRECKF shown in Fig. 7, the frequency estimated by the traditional ECKF is fluctuating near the truth value, but it reaches to the real value quickly using SRECKF.

The results of positive and negative sequence amplitude estimation and their minimum mean square error (MMSE) for phases *a*, *b* and *c* using ECKF are shown in Table 1. The results of positive and negative sequence amplitude estimation and their MMSE for phases *a*, *b* and *c* using SRECKF are shown in Table 2. Table 3 summarises the comparisons of the frequency estimation results and MMSE using the ECKF, SRECKF, MMSE is 0.0025 using SRECKF to estimate system frequency, and it is 0.0227 using ECKF to estimate system frequency, so the SRECKF presented here is superior to ECKF on estimation accuracy.

7 Conclusion

Here, we propose a square-root-extended complex Kalman filter to estimate symmetrical components of three-phase voltage system. In order to improving numerical stability of the traditional ECKF due to propagation of round-off errors, the Cholesky decomposition is applied to factorise the error covariance matrix, and a new iterated SRECKF algorithm is proposed. In order to separate zero sequence components from the *abc*-phase components, $\alpha\beta$ transformation is used to map three-phase instantaneous voltages in the *abc* phases into instantaneous voltages on the $\alpha\beta$ axes, and a non-linear state equation and observation equation of the three-phase voltages are built by introducing a complex vector and defining state variables. Positive symmetrical component, negative symmetrical components and frequency of the three-phase voltages are estimated using traditional extended complex Kalman filter and the method proposed here, the estimation results show that the method proposed here is superior to traditional extended complex Kalman filter on estimation accuracy and convergence rate.

Table 1 Amplitude estimations by ECKF

Parameters	*Vpa*	*Vna*	*Vpb*	*Vnb*	*Vpc*	*Vnc*
true value	1.0	0.4	1.0	0.4	1.0	0.4
ECKF	1.0014	0.4146	1.1055	0.3015	0.8524	0.5145
MMSE	5.8×10^{-4}	0.0023	0.0202	0.0194	0.0288	0.0221

Table 2 Amplitude estimations by SRECKF

Parameters	*Vpa*	*Vna*	*Vpb*	*Vnb*	*Vpc*	*Vnc*
true value	1.0	0.4	1.0	0.4	1.0	0.4
SRECKF	1.0198	0.3936	1.0295	0.3936	1.0063	0.3956
MMSE	0.0037	0.0032	0.0062	0.0027	0.0032	0.0014

Table 3 Frequency estimations and its performance

	ECKF	SRECKF
mean value, Hz	49.6925	50.1092
MMSE	0.0277	0.0025

8 Acknowledgments

The work was supported by the National Natural Science Foundation of China [grant no. 51779102], and Natural Science Foundation of Fujian Province [grant no. 2015J01640]

9 References

[1] Yazdani, D., Mojiri, M., Bakhshai, A., *et al.*: 'A fast and accurate synchronization technique for extraction of symmetrical components', *IEEE Trans. Power Electron.*, 2009, **24**, (3), pp. 674–684

[2] Karimi-Ghartemani, M., Karimi, H.: 'Processing of symmetrical components in time-domain', *IEEE Trans. Power Syst.*, 2007, **22**, (2), pp. 572–579

[3] Thiringer, T.: 'Frequency scanning for power system property determination – applied to a wind power grid', *IEEE Trans. Power Syst.*, 2006, **21**, (2), pp. 702–708

[4] Aghatehrani, R., Kavasseri, R.: 'Sensitivity-analysis-based sliding mode control for voltage regulation in microgrids', *IEEE Trans. Sustain. Energy*, 2013, **4**, (1), pp. 50–57

[5] Tummuru, N.R., Mishra, M.K., Srinivas, S.: 'Multifunctional VSC controlled microgrid using instantaneous symmetrical components theory', *IEEE Trans. Sustain. Energy*, 2014, **5**, (1), pp. 313–322

[6] Lobos, T.: 'Fast estimation of symmetrical components in real time', *IEE Proc. C*, 1992, **139**, (1), pp. 27–30

[7] Arafat, A., Choi, S., Baek, J.: 'Open-phase fault detection of a five-phase permanent magnet assisted synchronous reluctance motor based on symmetrical components theory', *IEEE Trans. Ind. Electron.*, 2017, **64**, (8), pp. 6465–6474

[8] Ballal, M.S., Suryawanshi, H.M., Mishra, M.K.: 'Stator winding inter-turn insulation fault detection in induction motors by symmetrical components method', *Electr. Power Compon. Syst.*, 2008, **36**, pp. 741–753

[9] Kuljevic, M.D.: 'Symmetrical components estimation through maximum likelihood algorithm and adaptive filtering', *IEEE Trans. Instrum. Meas.*, 2007, **56**, (6), pp. 2386–2394

[10] Andria, G., Salvatore, L.: 'Inverter drive signal processing via DFT and EKF', *IEE Proc. B*, 1990, **137**, (2), pp. 111–119

[11] Rosolowski, E., Michalik, M.: 'Fast estimation of symmetrical components by use of a state observer', *IEE Proc. Gener. Transm. Distrib.*, 1994, **141**, (6), pp. 617–622

[12] El-Naggar, K. M.: 'A fast method for identification of symmetrical components for power system protection', *Electr. Power Energy Syst.*, 2001, **23**, pp. 813–817

[13] Flores, R.A., Gu, I.Y.H., Bollent, M.H.J.: 'Positive and negative sequence estimation for unbalanced voltage dips'. IEEE Power Engineering Society General Meeting, Toronto, Ontario, Canada, 2003, pp. 2498–2502

[14] Phan, A.T., Hermann, G., Wira, P.: 'Kalman filtering with a new state-space model for three-phase systems: application to the identification of symmetrical components'. IEEE Int. Conf. on Evolving and Adaptive Intelligent Systems (EAIS), Douai, France, December 2015, pp. 1–6

[15] Kulikova, M.V., Pacheco, A.: 'Kalman filter sensitivity evaluation with orthogonal and J-orthogonal transformations', *IEEE Trans. Autom. Control*, 2013, **58**, (7), pp. 1798–1804

[16] Akagi, H., Watanabe, E.H., Aredes, M.: '*Instantaneous power theory and applications to power conditioning*' (Wiley, Hoboken, 2007)