# Distributed demand-side management optimisation for multi-residential users with energy production and storage strategies

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**Abstract:** This study considers load control in a multi-residential setup where energy scheduler (ES) devices installed in smart meters are employed for demand-side management (DSM). Several residential end-users share the same energy source and each residential user has non-adjustable loads and adjustable loads. In addition, residential users may have storage devices and renewable energy sources such as wind turbines or solar as well as dispatchable generators. The ES devices exchange information automatically by executing an iterative distributed algorithm to locate the optimal energy schedule for each end-user. This will reduce the total energy cost and the peak-to-average ratio (PAR) in energy demand in the electric power distribution. Users possessing storage devices and dispatchable generators strategically utilise their resources to minimise the total energy cost together with the PAR. Simulation results are provided to evaluate the performance of the proposed game theoretic-based distributed DSM technique.

## 1 Introduction

The smart grid presents several opportunities for end-users to save energy and for the utility company to operate the grid in a more efficient, effective and reliable way. In recent years, much attention has been paid to optimisation of energy consumption in smart grid. Demand-side management (DSM) is one of the notable functions in a smart grid that enables end-users to modify their demand for energy through various methods, such as financial incentives and education [1–6]. The deployment of DSM will motivate end-users to utilise less energy during peak hours, or to shift the time of energy use to off-peak times [1, 7–9], which will help the utility company to reduce the peak load demand and reshape the load profile. Consequently, end-users will save money on electricity and the society will conserve electricity [5, 7, 10, 11].

Basically, the outcomes of DSM programmes depend on a portion of the total load that can be controllable [1, 8, 12]. End-users consisting of adjustable loads, such as plug-in hybrid electric vehicles (PHEVs) and dishwashers, offer significant benefit to this end [5, 7, 11]. Moreover, end-users with storage devices and dispatchable energy generators offer an exceptional opportunity to increase the percentage of controllable load. The control of end-users demand and supply of energy can be done through various methods such as financial incentives, new tariff schemes and education. The end-users agree to involvement, if they may be charged less for consuming electricity during off-peak hours and paid for supplying electricity during peak hours. Suppose the utility company pays more for user-generated energy during peak hours of energy demand, and pay less for off-peak power, and then end-users will be motivated to generate more energy and consume minimum energy during peak hours, which in turn achieves the main goal of DSM [3, 4, 8, 11].

Practically, to meet all energy demands from the end-users, the grid capacity should be designed such that it satisfies the peak power demand instead of just the average power demand [6, 13–15]. However, the utility company supplying energy to the grid prefers to use the least expensive sources of energy to generate electricity (which might not be enough to meet the required grid capacity) and use expensive energy sources only when the demand increases [3, 4, 16]. When costly energy sources are employed by the utility company, end-users will also pay high prices for the energy. Thus, by strategically engaging end-users in energy production, storage and shifting the energy consumptions of their adjustable load appliances, the utility company will alleviate the use of expensive base load generators and both end-users and the utility company will benefit from the strategy [1, 8, 10, 12].

In addition, renewable energy sources (RESs) such as solar and wind turbines play an important role in reducing the total load. Typically solar and wind turbines (without some added component for storage) are non-dispatchable because the sunlight or wind is periodic and cannot be predicted and controlled [10, 12, 17]. With a rapid advancement of battery technology, it is likely that storage devices will become an integral part of the means by which energy generated from renewable sources can be stored and utilised when needed. Storage devices can be used to store some of the energy generated by renewable sources and discharge them during peak hours.

Home automation systems play an important role in determining the success of the proposed energy strategies. Through the use of automation, the decision about the energy schedule, amount of power consumption, charging and discharging as well as the running of dispatchable generators can be facilitated. An automatic scheme that requires minimum effort from the end-users is desired since most end-users do not have knowledge and/or interest to respond to the energy costs [8].

In this study, we present an energy scheduling strategy for the future smart grid network. A distributed game-theoretic cost minimisation demand-side optimisation and energy scheduling scheme that takes into account load uncertainty are presented. We consider a scenario where the main source of energy is shared by several end-users; some of the users are equipped with the RESs, some with dispatchable distribution generators (DGs) and/or storage devices. Each end-user is utilising an energy scheduler (ES) deployed inside the smart meters for the adjustable loads, charging and discharging of their storage devices as well as generating energy from the dispatchable generators.

The smart meters are linked to the electric power line and communication network [5, 8, 11, 18]. The smart meters with ES scheme communicate automatically by executing a distributed algorithm to obtain the optimised energy schedule for each end-user [5, 7, 8, 11]. The DSM optimisation and scheduling problem is formulated as a non-cooperative cost minimisation game among the end-users and an iterative algorithm that optimises user energy costs is proposed.

Unlike the methods in [5, 8, 11], where ES is considered for a system with adjustable energy appliances without energy storage and DGs, the proposed scheme considers a setup with both storage devices, RESs as well as dispatchable generators. The scheme in [12] includes the storage devices and dispatchable energy sources as well as non-dispatchable energy sources; however, it does not include energy shifting of adjustable appliances, which are crucial in reducing peak-to-average ratio (PAR) energy costs of the end-users as well as the cost incurred by the utility company. The proposed algorithm includes optimisation of energy consumption of adjustable appliances and that of the energy generators and storers.

The proposed iterative algorithm demands each end-user to give some information related to his/her daily total load. Residential users can exchange limited information related to their daily total load (without giving detailed information regarding the energy consumption of their appliances or their storage and production strategies [8, 11]). A simple pricing mechanism can give motivation for the end-users to cooperate (i.e. self-enforcing mechanism) [5, 11]. Thus, it is in the end-user's personal interest to disclose local information accurately to improve the overall system performance and minimise his/her daily energy costs.

Simulation results demonstrate that, with our proposed distributed scheme, the ES can substantially minimise the PAR as well as the daily energy cost of each end-user. Furthermore, by utilising energy producers and storers, end-users without energy strategies (dispatchable generators and storage devices) can also benefit and pay less for the same daily total energy load.

## 2 System model and problem statement

Fig. 1 depicts the smart power system model considered, where each residential user has non-adjustable load appliances and adjustable load appliances. Non-adjustable load appliances include electric bulbs, TVs, refrigerators etc., and these are loads whose instantaneous power or starting time cannot be adjusted. Adjustable load refers to the loads whose instantaneous power, starting time or both can be adjusted. Adjustable load appliances include PHEVs, dish washers, washing machines etc. In addition, some residential users possess storage devices, and/or RESs, DGs or both (see Fig. 1).

We consider $N$ multiple end-users connected to the electrical grid from the same energy source. Each end-user has a smart meter that communicates with several appliances per end-user as well as the utility company via the advanced metering infrastructure. It is generally assumed that the cost of supplied energy from the utility company is dictated in advance within a specified period of time $1, \ldots, T$. Each time slot of the scheduling horizon can stand for, for example, 1 h, with $T = 24$ representing one day. For an end-user $n \in N$, let $\mathcal{K}_n$ denote a set of adjustable load appliances. For each device $k \in \mathcal{K}_n$, we define energy consumption scheduling vector $\boldsymbol{x}_{n,k} \triangleq [x_{n,k}^1, \ \ldots, \ x_{n,k}^T]$. Here, $x_{n,k}^t$ corresponds to the 1 h energy consumption scheduled for device $k$ of user $n$, whereas the energy consumption for non-adjustable load at slot $t$ is denoted as $y_{n,0}^t$.

### 2.1 Energy storage

The residential end-user $n$ may have a storage device (such as a battery). Let $p_n^t \geq 0$, $t = 1, \ \ldots, \ T$, be the available energy in a battery at the end of slot $t$; and $p_n^{\max}$ is the capacity of the battery. The energy available at the beginning of the horizon can be represented as $p_n^0$. The battery can be either charged or discharged throughout the time slot $t$. Let $b_n^t$ be the energy discharged from or charged to the battery at slot $t$. Here $b_n^t < 0$ represents that the battery is discharging while $b_n^t > 0$ represents that the battery is charging. The charge/discharge variables and the accumulated

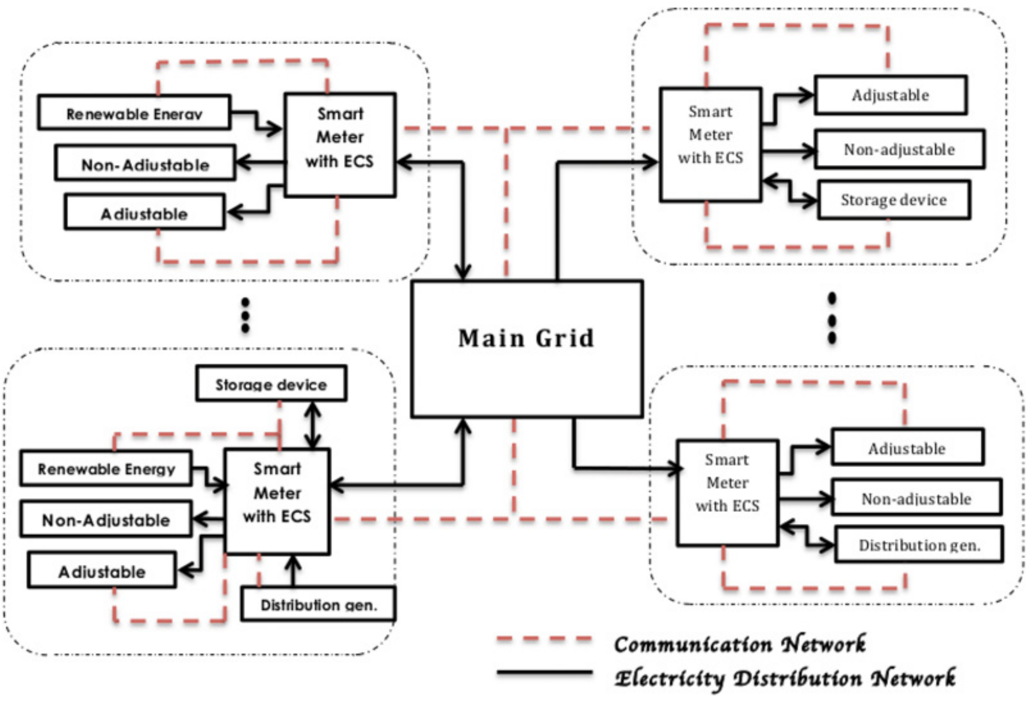


**Fig. 1** *Links between connecting end-users and the smart grid*

energy in the battery at time slot $t$ are given by the equation [9]

$$p_n^t = p_n^{t-1} + b_n^t, \quad t = 1, \ldots, T \tag{1}$$

The variable $b_n^t$ is constrained by the maximum charge and discharge as $b_n^{\text{dis}} < b_n^t < b_n^{\text{ch}}$, also we assume that the batteries have limited capacity, thus the energy supplied by the battery is no more than the current consumed energy, that is, $b_n^t + \sum_k x_{n,k}^t + p_{n,0}^t \geq 0$. Every single battery has an efficiency $\eta_n \in (0, 1)$, implying that if $p_n^{t-1}$ is accumulated at the end of the time slot $t-1$, then the discharge at time slot $t$ is constrained by $b_n^t \geq -\eta_n p_n^{t-1}$. With ES, residential users can utilise energy storage devices to store energy during off-peak hours and discharge them during peak hours [9].

### 2.2 Energy production

Apart from storage devices, some end-users own RES and/or distributed generators (DGs). Users deploy DG and/or RES to produce energy rather than just consuming energy supplied by the utility company. By integrating DG and/or RES, end-users reduce their energy cost since they can produce energy to power their own appliances, to sell it to the utility company or to charge their batteries during peak hours. These energy sources are classified as dispatchable or non-dispatchable energy producers [12]. Typically, RESs such as solar and wind turbines (without some added component for storage) are non-dispatchable, since the supply of sunlight or wind is periodic and cannot be predicted and controlled. Possessing only fixed (initial plus maintenance) costs, they produce electricity at their maximum available power, which indicates no optimal scheme regarding the production of energy [12]. Conversely, dispatchable energy sources are those sources that can be turned on or off or can adjust their energy productions on demand; these include fuel generators, gas turbines or internal combustion engines [12]. DGs are categorised as dispatchable energy sources, thus end-users possessing DGs are concerned with the optimisation of their energy production strategies.

We denote the non-dispatchable energy generated by user $n$ per-time slot as $g_{n,r}^t$. The dispatchable energy generated by user $n$ per-time slot is denoted as $g_{n,d}^t$. We introduce the production cost function $\mathcal{W}(g_{n,d}^t)$, which provides the variable costs for producing a certain quantity of energy $g_{n,d}^t$ at a time slot $t$, where $\mathcal{W}(0) = 0$.

Let $g_n^{\max}$ denote the maximum energy production capability for end-user $n$ during a time slot $t$. Then, energy production profile per-time slot is bounded as

$$0 \leq g_{n,d} \leq g_n^{\max} \tag{2}$$

Here, $g_n^{\max}$ represents the quantity of energy produced when user $n$'s energy source is operated throughout the time slot $t$. Besides, the cumulative energy production has to satisfy the constraint

$$\sum_{t=1}^{T} g_{n,d}^t \leq \lambda_n^{\max} \tag{3}$$

where $0 \leq \lambda_n^{\max} \leq T \times g_{n,d}^{\max}$. We define the non-dispatchable energy production vector and the dispatchable energy production scheduling vector as $\boldsymbol{g}_{n,r} = (g_{n,r}^t)_t^T$ and $\boldsymbol{g}_{n,d} = (g_{n,d})_t^T$, respectively.

The total hourly energy profile for user $n \in N$ can be defined as

$$l_n^t \triangleq \sum_{k \in \mathcal{K}_n} x_{n,k}^t + y_{n,0}^t + b_n^t - g_{n,r}^t - g_{n,d}^t \tag{4}$$

and the daily total load for user $n$ is defined as $\mathcal{I}_n = [l_n^1, l_n^2, \ldots, l_n^T]$. Here, $l_n^t \geq 0$ if the energy flows from the utility company to the end-user $n$, else $l_n^t \leq 0$.

### 2.3 Energy scheduler

The aim of utilising ESs is to minimise the operational costs and PAR as well as the energy costs of the end-users. The involvement of the end-users is a response to aspects such as incentive pricing, tariff schemes etc. End-users participation may involve either active behavioural changes or passive responses, using automation techniques. For example, if someone chooses not to charge his/her PHEV battery with regard to a period of high demand, he/she is only re-scheduling (deferring) that use and will still charge his/her PHEV battery later, thus the energy is consumed at a different time instead of being minimised [9].

The total hourly energy consumption for $n \in N$ is given by

$$e_n^t = \sum_{k \in \mathcal{K}_n} x_{n,k}^t + y_{n,0}^t + b_n^t \tag{5}$$

Hourly energy consumption of each user includes the energy consumed by the devices as well as that consumed to charge the battery. Since the net energy of the battery charge is zero at the time of observation, the total energy used to charge the battery is equivalent to the discharged energy.

Thus we have

$$l_n^t = e_n^t - g_{n,d}^t - g_{n,r}^t \tag{6}$$

For users without generators, either $g_{n,r}^t$ or $g_{n,d}^t$ can be set to zero based on what they possess. Although RESs are intermittent and uncertain, the use of storage devices can absorb this variability. Thus, for end-users possessing both RES and storage device, RES can improve the active participation of users with storage devices.

Operation of the adjustable loads can be shifted to a different time, so each residence selects the time interval $[\alpha_{n,k}, \beta_{n,k}]$ (i.e. beginning time $\alpha_{n,k}$ and end time $\beta_{n,k}$) that the energy consumption for device $k$ can practically be scheduled. We define the total energy consumption for device $k$ from user $n$ as [8, 11]

$$E_{n,k} = \sum_{t=\alpha_{n,k}}^{\beta_{n,k}} x_{n,k}^t \tag{7}$$

and $x_{n,k}^t = 0$, $\forall t \notin \{\alpha_{n,k}, \ldots, \beta_{n,k}\}$.

Similar to [5, 8, 11], the proposed scheme does not intend to change the amount of power consumed by appliances, but systematically control and adjust it to minimise the energy cost of the end-users as well as to reduce the PAR that likely happen during the peak hours. Thus, ES is about optimising energy consumption over a number of factors, rather than just a simple ‘do not consume at these hours’ command.

Many devices may have some maximum power levels $\gamma_{n,k}^{\max}$ as well as the minimum power level $\gamma_{n,k}^{\min}$; this sets the upper bound and lower bound constraints on the ES vector $\boldsymbol{x}_{n,k}$ for each device [1], that is

$$\gamma_{n,k}^{\min} \leq x_{n,k}^t \leq \gamma_{n,k}^{\max}, \quad \forall t \in \{\alpha_{n,k}, \ldots, \beta_{n,k}\} \tag{8}$$

The total load for $N$ residences at each hour of the day is given by

$$L_t = \sum_{n \in N} l_n^t \tag{9}$$

Let $C^t(L_t)$ denote the energy cost over a time slot $t$. This is the cost that the utility company incurs to provide energy to the end-users or the cost that the utility company pays to buy electricity from the end-users. We also consider that the price of the same load may differ at different times of the day [5, 7, 8, 11]. The cost paid by end-user $n$ to buy electricity $l_n^t$ from the utility company (if $l_n^t > 0$) or the cost paid to the end-user for selling the same energy load to the utility company (if $l_n^t < 0$) is given by

$$C_t(L_t)\frac{l_n^t}{L_t} \tag{10}$$

The dispatchable energy production cost function is given by $\mathcal{W}(y) = \nu y$, where $\nu$ is a constant. The cumulative expenses incurred by user $n$ over the period of analysis $T$ is given by

$$\mathcal{F}(\boldsymbol{x}_n, \boldsymbol{g}_{n,d}, \boldsymbol{b}_n) = \sum_{t=1}^{T}\left(C^t(L_t)\frac{l_n^t}{L_t} + \mathcal{W}(g_{n,d}^t)\right) \tag{11}$$

We define the total cumulative expenses of all users as

$$\boldsymbol{\Gamma}(\boldsymbol{\mathcal{I}}) = \sum_{n\in N}\mathcal{F}(\boldsymbol{x}_n, \boldsymbol{g}_{n,d}, \boldsymbol{b}_n) \tag{12}$$

where $\boldsymbol{\mathcal{I}}$ is an $N \times T$ matrix representing the daily total load of all users, that is, $\boldsymbol{\mathcal{I}} = \mathcal{I}_1, \ldots, \mathcal{I}_N{}^T$.

The multi-residential load control task amounts to minimising the cumulative expense of electricity, that is

$$\begin{aligned}
\min_{\substack{\boldsymbol{x}_1,\ldots,\boldsymbol{x}_N \\ \boldsymbol{g}_{1,d},\ldots,\boldsymbol{g}_{N,d} \\ \boldsymbol{b}_1,\ldots,\boldsymbol{b}_N}} \quad & \boldsymbol{\Gamma}(\mathcal{I}_1, \ldots, \mathcal{I}_N) \\
\text{subject to} \quad & \sum_{t=\alpha_{n,k}}^{\beta_{n,k}} x_{n,k}^t = E_{n,k} \\
& \gamma_{n,k}^{\min} \le x_{n,k}^t \le \gamma_{n,k}^{\max}, \quad \forall t \in \{\alpha_{n,k}, \ldots, \beta_{n,k}\} \\
& x_{n,k}^t = 0, \quad \forall t \notin \{\alpha_{n,k}, \ldots, \beta_{n,k}\} \\
& b_n^{\text{dis}} < b_n^t < b_n^{\text{ch}}, \quad t = 1, \ldots, T \\
& p_n^t = p_n^{t-1} + b_n^t, \quad t = 1, \ldots, T \\
& b_n^t \ge -\eta_n p_n^{t-1} \\
& 0 \le g_{n,d} \le g_n^{\max} \\
& \sum_{t=1}^{T} g_{n,d}^t \le \lambda_n^{\max} \\
& 0 \le \lambda_n^{\max} \le T \times g_{n,d}^{\max}
\end{aligned} \tag{13}$$

The constraints of the optimisation problem in (13) are linear, thus if the objective function is convex then the problem can be solved using convex optimisation techniques. The problem above is in a centralised fashion, thus some modifications are required to solve the problem distributively. A distributed approach is desirable in order to address possible concerns regarding data privacy and integrity [7].

## 3 Energy consumption game

We assume that the price that each user pays or receives is proportional to his/her daily energy load. For each end-user $n \in N$, let $d_n$ represent the daily price in dollars to be charged to the end-user $n$ by the utility company or the amount of money that user $n \in N$ receives from the utility company for generating energy. Thus

$$d_n \propto \sum_{t}^{T} l_n^t, \quad \forall n \in N \tag{14}$$

Using the proportionality constant, we can equate users' energy consumption and their bill as

$$\frac{d_n}{d_m} = \frac{\sum_{t=1}^{T} l_n^t}{\sum_{t=1}^{T} l_m^t}, \quad \forall n, m \in N \tag{15}$$

From (15), we have

$$d_m = d_n \frac{\sum_{t=1}^{T} l_m^t}{\sum_{t=1}^{T} l_n^t} \tag{16}$$

The total monetary expenses for all end-users can be expressed as

$$\sum_{m\in N} d_m = \sum_{m\in N}\left(d_n \frac{\sum_{t=1}^{T} l_m^t}{\sum_{t=1}^{T} l_n^t}\right) = d_n \frac{\sum_{m\in N}\sum_{t=1}^{T} l_m^t}{\sum_{t=1}^{T} l_n^t} \tag{17}$$

From (17), we can express $d_n$ as

$$\begin{aligned}
d_n &= \frac{\sum_{t=1}^{T} l_n^t}{\sum_{m\in N}\sum_{t=1}^{T} l_m^t}\left(\sum_{m\in N} d_m\right) \\
&= \Phi_n \sum_{m\in N} d_m
\end{aligned} \tag{18}$$

where

$$\Phi_n = \frac{\sum_{t=1}^{T} l_n^t}{\sum_{m\in N}\sum_{t=1}^{T} l_m^t} = \frac{\sum_{t=1}^{T} l_n^t}{\sum_{t=1}^{T} l_n^t + \sum_{m\in N\backslash n}\sum_{t=1}^{T} l_m^t} \tag{19}$$

$\Phi_n$ is not constant for daily energy because of the uncertainty of the RESs such as solar or wind turbine in producing energy. At some hours in a day, users with such generators might produce more power than their own energy demand and thereby feed their excess energy to the grid, which may lead to zero or negative aggregate load $l_n^t$ for such a user $n$.

From (18), it can be seen that the charge on each user depends on his/her energy strategy and the strategies of other users. This leads to the game theory among the users. In this game, users are players and their strategies are their daily energy schedule. Next, we investigate different approaches of end-users in responding to the price values.

The cumulative cost of user $n$ $\mathcal{F}(\boldsymbol{x}_n, \boldsymbol{g}_{n,d}, \boldsymbol{b}_n)$ is proportional to his/her daily load, that is

$$\mathcal{F}(\boldsymbol{x}_n, \boldsymbol{g}_{n,d}, \boldsymbol{b}_n) \propto \sum_{t=1}^{T} l_n^t \tag{20}$$

For the utility company to generate profit, it is expected that the cost of electricity for the end-users $d_n$ to be equal or slightly higher than the cumulative cost [8, 11], that is

$$d_n \ge \mathcal{F}(\boldsymbol{x}_n, \boldsymbol{g}_{n,d}, \boldsymbol{b}_n) \tag{21}$$

where the left-hand side represents the total daily charge to the end-users, whereas the right-hand side indicates the daily cumulative cost. Following the inequality in (21), we can define

$$\mu = \frac{d_n}{\mathcal{F}(\boldsymbol{x}_n, \boldsymbol{g}_{n,d}, \boldsymbol{b}_n)} \ge 1 \tag{22}$$

For $\mu = 1$, the billing system is budget balanced and the energy supplier pays/charges the end-users equivalent amount corresponding to their cumulative costs. From (18) and (22), it can be shown that regardless of the value of $\mu$

$$\mathcal{F}(\boldsymbol{x}_n, \boldsymbol{g}_{n,d}, \boldsymbol{b}_n) = \Phi_n \sum_{m\in N}\mathcal{F}(\boldsymbol{x}_m, \boldsymbol{g}_{m,d}, \boldsymbol{b}_m) \tag{23}$$

### 3.1 Equilibrium among users

Given the daily total load for user $n$ as $\mathcal{I}_n$, we define the daily total load of other users as $\mathcal{I}_{-n}$ such that $\mathcal{I}_{-n} = \boldsymbol{\mathcal{I}}\backslash\mathcal{I}_n$. The problem can be formulated as a non-cooperative energy cost minimisation game. In game theory, a non-cooperative game is one in which players make decisions independently [19]. The game consists of

- *Player*: a set of end-users $n = 1, \ldots, N$.
- *Strategies*: energy scheduling vectors $\boldsymbol{x}_n, \boldsymbol{g}_{n,d}, \boldsymbol{b}_n$ for all end-users with adjustable load devices, dispatchable generators, battery or both.
- Payoff functions $P_n(\mathcal{I}_n, \mathcal{I}_{-n}) = -\mathcal{F}(\boldsymbol{x}_n, \boldsymbol{g}_{n,d}, \boldsymbol{b}_n)$, define the user payoffs for the joint strategies.

To maximise payoff, the goal of the users is to minimise the expected overall cost of energy. From (23), the payoff can be expressed as

$$P_n(\mathcal{I}_n, \mathcal{I}_{-n}) = -\Phi_n \sum_{m \in N} \mathcal{F}(\boldsymbol{x}_m, \boldsymbol{g}_{m,d}, \boldsymbol{b}_m) \tag{24}$$

Using (12), the payoff of user $n$ can be expressed as

$$P_n(\mathcal{I}_n, \mathcal{I}_{-n}) = -\Phi_n \boldsymbol{\Gamma}(\mathcal{I}) \tag{25}$$

End-users attempt to determine their energy strategies to minimise the cost paid to the utility company or maximise their profit. Using Nash equilibrium, we can characterise how players play a game [8, 11]. The optimal performance with regard to energy cost minimisation achieves at Nash equilibrium of power consumption game. The Nash equilibrium of this game always exists. The energy consumption variable $(\mathcal{I}_n^*, \forall n \in N)$ is in a Nash equilibrium of the game if for every user $n \in N$

$$P_n(\mathcal{I}_n^*, \mathcal{I}_{-n}^*) \geq P_n(\mathcal{I}_n; \mathcal{I}_{-n}^*) \tag{26}$$

Once the energy scheduling game is at unique Nash equilibrium, none of the end-users would attempt to diverge from the schedule $(\mathcal{I}_n^*, \forall n \in N)$. Moreover, the user cannot influence the value of $\boldsymbol{\Phi}_n$ with the choice of their strategies.

### 3.2 Distributed algorithm

Suppose all other end-users fix their corresponding energy schedule $\mathcal{I}_{-n}$, then the end-user $n$ can maximise his/her own payoff by solving the local optimisation problem

$$\begin{aligned}
\max_{\boldsymbol{x}_n, \boldsymbol{g}_{n,d}, \boldsymbol{b}_n} \quad & P_n(\mathcal{I}_n; \mathcal{I}_{-n}) \\
\text{subject to} \quad & \sum_{t=\alpha_{n,k}}^{\beta_{n,k}} x_{n,k}^t = E_{n,k} \\
& \gamma_{n,k}^{\min} \leq x_{n,k}^t \leq \gamma_{n,k}^{\max}, \quad \forall t \in \{\alpha_{n,k}, \ldots, \beta_{n,k}\} \\
& x_{n,k}^t = 0, \quad \forall t \notin \{\alpha_{n,k}, \ldots, \beta_{n,k}\} \\
& b_n^{\text{dis}} < b_n^t < b_n^{\text{ch}}, \quad t = 1, \ldots, T \\
& p_n^t = p_n^{t-1} + b_n^t, \quad t = 1, \ldots, T \\
& b_n^t \geq -\eta_n p_n^{t-1} \\
& 0 \leq g_{n,d}^t \leq g_n^{\max} \\
& \sum_{t=1}^{T} g_{n,d}^t \leq \lambda_n^{\max} \\
& 0 \leq \lambda_n^{\max} \leq T \times g_{n,d}^{\max}
\end{aligned} \tag{27}$$

This is equivalent to the minimisation of the cost function

$$\begin{aligned}
\min_{\boldsymbol{x}_n, \boldsymbol{g}_{n,d}, \boldsymbol{b}_n} \quad & \Phi_n \left( \mathcal{F}(\boldsymbol{x}_n, \boldsymbol{g}_{n,d}, \boldsymbol{b}_n) + \sum_{m \in N \setminus n} \mathcal{F}(\boldsymbol{x}_m, \boldsymbol{g}_{m,d}, \boldsymbol{b}_m) \right) \\
\text{subject to} \quad & \sum_{t=\alpha_{n,k}}^{\beta_{n,k}} x_{n,k}^t = E_{n,k} \\
& \gamma_{n,k}^{\min} \leq x_{n,k}^t \leq \gamma_{n,k}^{\max}, \quad \forall t \in \{\alpha_{n,k}, \ldots, \beta_{n,k}\} \\
& x_{n,k}^t = 0, \quad \forall t \notin \{\alpha_{n,k}, \ldots, \beta_{n,k}\} \\
& b_n^{\text{dis}} < b_n^t < b_n^{\text{ch}}, \quad t = 1, \ldots, T \\
& p_n^t = p_n^{t-1} + b_n^t, \quad t = 1, \ldots, T \\
& b_n^t \geq -\eta_n p_n^{t-1} \\
& 0 \leq g_{n,d}^t \leq g_n^{\max} \\
& \sum_{t=1}^{T} g_{n,d}^t \leq \lambda_n^{\max} \\
& 0 \leq \lambda_n^{\max} \leq T \times g_{n,d}^{\max}
\end{aligned} \tag{28}$$

We assume that each end-user has a predetermined amount of energy consumption and active end-users have some limit in the capacity of generating power for each particular day, thus even with the uncertainty of the RESs in generating power, user influence on the value of $\boldsymbol{\Phi}_n$ is minimum. Consequently, the value of $\boldsymbol{\Phi}_n$ can be assumed to be constant for daily consumption. Under this assumption, (28), can be written as

$$\begin{aligned}
\min_{\boldsymbol{x}_n, \boldsymbol{g}_{n,d}, \boldsymbol{b}_n} \quad & \mathcal{F}(\boldsymbol{x}_n, \boldsymbol{g}_{n,d}, \boldsymbol{b}_n) + \sum_{m \in N \setminus n} \mathcal{F}(\boldsymbol{x}_m, \boldsymbol{g}_{m,d}, \boldsymbol{b}_m) = \boldsymbol{\Gamma}(\mathcal{I}) \\
\text{subject to} \quad & \sum_{t=\alpha_{n,k}}^{\beta_{n,k}} x_{n,k}^t = E_{n,k} \\
& \gamma_{n,k}^{\min} \leq x_{n,k}^t \leq \gamma_{n,k}^{\max}, \quad \forall t \in \{\alpha_{n,k}, \ldots, \beta_{n,k}\} \\
& x_{n,k}^t = 0, \quad \forall t \notin \{\alpha_{n,k}, \ldots, \beta_{n,k}\} \\
& b_n^{\text{dis}} < b_n^t < b_n^{\text{ch}}, \quad t = 1, \ldots, T \\
& p_n^t = p_n^{t-1} + b_n^t, \quad t = 1, \ldots, T \\
& b_n^t \geq -\eta_n p_n^{t-1} \\
& 0 \leq g_{n,d}^t \leq g_n^{\max} \\
& \sum_{t=1}^{T} g_{n,d}^t \leq \lambda_n^{\max} \\
& 0 \leq \lambda_n^{\max} \leq T \times g_{n,d}^{\max}
\end{aligned} \tag{29}$$

The optimisation problem in (29) is equivalent to the optimisation problem in (13). The problem in (29) can be solved distributively. The following is the proposed algorithm to solve the optimisation problem in (29) distributively.

The proposed algorithm requires only some limited information exchange between end-users when each of them attempts to maximise his/her own benefit. From Algorithm 1 (see Fig. 2), end-users minimise the optimisation problem in (29) based on the random order sequence $\mathcal{S}$ by optimising their load scheduling for adjustable loads $\boldsymbol{x}_n$, storage devices, dispatchable generators or both, such that the objective function $\boldsymbol{\Gamma}(\mathcal{I})$ is strictly decreasing, that is, $\boldsymbol{\Gamma}(\mathcal{I}^c) < \boldsymbol{\Gamma}(\mathcal{I}^{c-1})$.

Each user minimises the cost function with respect to $l_n^t, \forall t \in T$, whereas the load of the other users (i.e. $\sum_{m \in N \setminus n} l_m^t, \forall t \in T$) is fixed. User $n$ broadcasts its new load $l_n^t, \forall t \in T$ (without giving detailed information about his/her storage strategies, generators strategies or energy consumption of his/her appliances) provided that the objective function is decreasing. The energy consumption schedule for users $\mathcal{I}$ is updated and the next user in the generated sequence minimises the objective function in (29) with respect to its local load. This process is repeated until none of the users can improve their payoff by scheduling his/her load. The parameter $\epsilon_0$ is a small fraction value for adjusting the cost function at the beginning of the algorithm to make the first two cost functions $\boldsymbol{\Gamma}(\mathcal{I}^0)$ and $\boldsymbol{\Gamma}(\mathcal{I}^1)$ different.

**Algorithm 1:**

Initialise $\mathcal{I}^0$ randomly, and calculate the corresponding cost function $\Gamma(\mathcal{I}^0)$ ;
Initialise counter $c = 1$ and set $\Gamma(\mathcal{I}^c) = \Gamma(\mathcal{I}^0) - \epsilon_0$ ;
**while** $\Gamma(\mathcal{I}^c) < \Gamma(\mathcal{I}^{c-1})$ **do**
  Generate a random sequence $\mathcal{S}$ for $N$ users ;
  **for** $n \leftarrow 1$ **to** $N$ **do**
    $\mathcal{I}_n = \mathcal{I}(\mathcal{S}(n), :)$ and $\mathcal{I}_{-n} = \mathcal{I} \backslash \mathcal{I}_n$ ;
    Optimise $\Gamma(\mathcal{I}_n, \mathcal{I}_{-n})$ and update $\mathcal{I}_n$;
    Convey a control message to make $\mathcal{I}_n$ known to all ES units and update $\mathcal{I}$;
  **end**
  Increment the counter, $c \leftarrow c + 1$ ;
  Update the cost function $\Gamma(\mathcal{I}^c) = \Gamma(\mathcal{I}_n, \mathcal{I}_{-n})$
**end**

**Fig. 2** *Performed by every user* $n \in \mathcal{N}$

The fact that each user broadcasts its load schedule implies that each user reveals his/her strategy to all other users. The Nash equilibrium exists if no users change their strategy, despite knowing the actions of the other users [19]. From the algorithm it is clear that the termination will be reached when there is no change in cost function. Users are in Nash equilibrium since each user is making the best decision, taking into account the decisions of the others.

## 4 Simulation results

Simulation results are presented to access the performance of the proposed ES algorithm. We evaluate our distributed demand-side optimisation in a scenario consisting of $N = 1000$ end-users each having random devices from 15 to 25 non-adjustable loads and 15 to 25 adjustable loads. Non-adjustable loads have a fixed schedule and consume energy continuously; examples of these devices are electric stove, electric bulbs, refrigerators and TV. The adjustable loads include electrical appliances with flexible schedule such as PHEVs, dish washers, washing machines, clothes driers etc.

Out of $N$ users, 10% of them possess either storage devices, RESs, DGs or both. These users are termed as active users because they can utilise ES together with the energy storage devices and/or their energy generators to optimise their interests.

The daily usage of adjustable and non-adjustable devices is set to be similar to those in [8, 9]. The average daily consumption of each user ranges from 12 to 16 kWh, and for users with storage devices the capacity of each user is between 2 and 5 kWh. We also considered that the RESs can produce a maximum of 6 kWh in a day. Users with dispatchable generators can produce up to 8 kWh in a day.

We consider that the higher energy demand (peak) occurs during day time, from 8:00 to 00:00, and the low energy peak occurs at night-time, from 00:00 to 8:00. This implies that the cost for day time covers the first 16 h of simulation and night-time cost covers the last 8 h of simulation. The selected cost function is quadratic given by $C^t(L_t) = \Phi_{\text{Day}} \times L_t^2$ for day time and $C^t(L_t) = \Phi_{\text{night}} \times L_t^2$ for night-time. We select $\Phi_{\text{night}} = \frac{1}{2}\Phi_{\text{Day}}$.

First, we examine a scenario where RESs together with ES are deployed compared with one without ES and RESs. Fig. 3 depicts hourly energy consumption and cost for users with RESs (i.e. 10% of the users possess RESs) compared with those without RESs. When ES is not deployed, the cost of electricity as well as the energy drawn from the utility company is reduced for users possessing RESs compared with the scenario where all users rely only on the utility company. However, RESs alone, without any optimisation scheme to effectively manage the hourly energy consumption of the users appliances, do not necessarily reduce the PAR. Using smart meters running the ES, users can optimally minimise the amount they pay to the utility company as well as PAR [2]. As it can be seen, the users without ES pay much more during the peak hour and they pay much less during the off-peak because they cannot shift their loads to the off-peak hours. Fig. 3 shows that the utilisation of the ES can help to shave off the peak load. The grid cost per-time slot depicted in Fig. 3*b* suggests that the ES helps in reducing the cost by shifting the adjustable loads to the valley of the energy cost. The results also show that the use of ES and RESs minimises the PAR as well as the daily energy cost. This is because, for each time slot, the deployed ES controls shiftable or adjustable load devices to operate at a certain power within the specified operational time of the appliances while taking the advantages of the energy generated by the RESs.

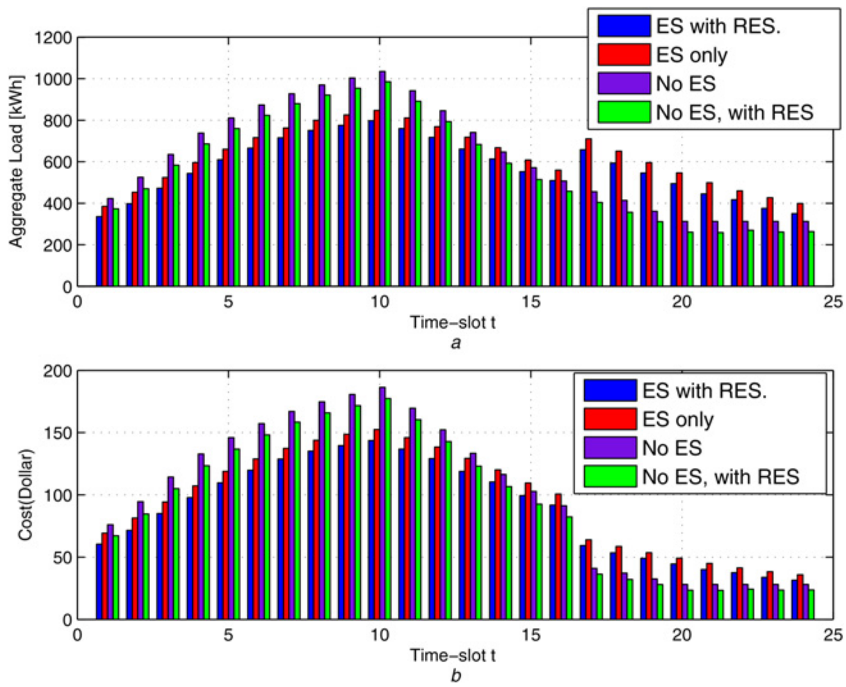


**Fig. 3** *Scheduled energy load when the RESs and ES units are deployed as well as when they are not deployed*
*a* Comparison of the aggregate load of $N$ users for different energy strategies
*b* Hourly cumulative energy cost of the aggregate load

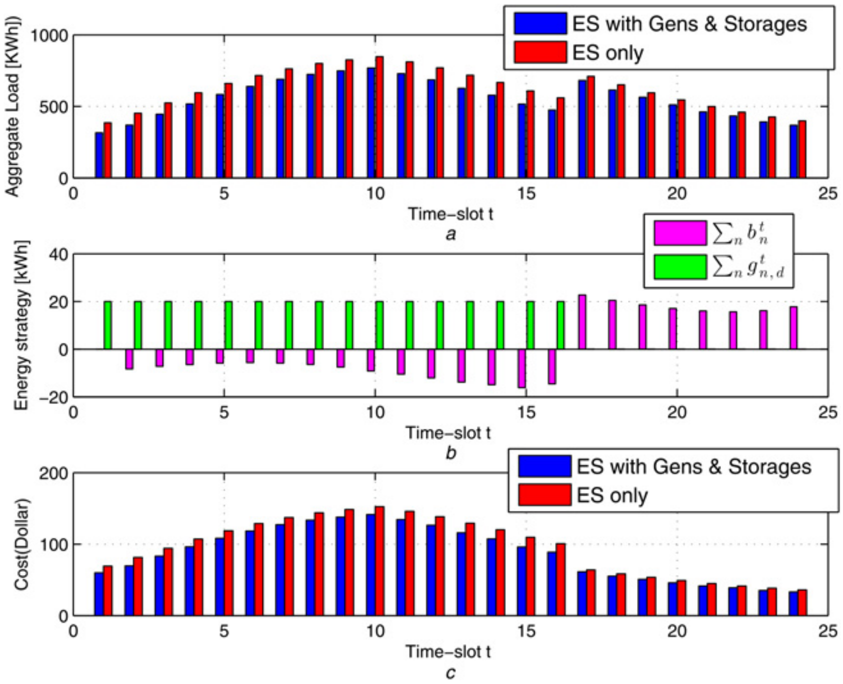


**Fig. 4** *Effects of deploying ES for users with/without energy generators and storage strategies*
*a* Hourly aggregate load of *N* users with ES and storage and production devices compared with those with ES only
*b* Energy storage and production strategies
*c* Cumulative cost of the aggregate load

Next, we compare a scenario where some users possess RESs and/or storage devices and/or dispatchable generators or both. Again for $N = 1000$, only 10% of them possess storage and/or generators. For a fair comparison, we considered that at the end of the day, each storage device remains with its initial charging state. Fig. 4*a* depicts the aggregate energy consumption per-time

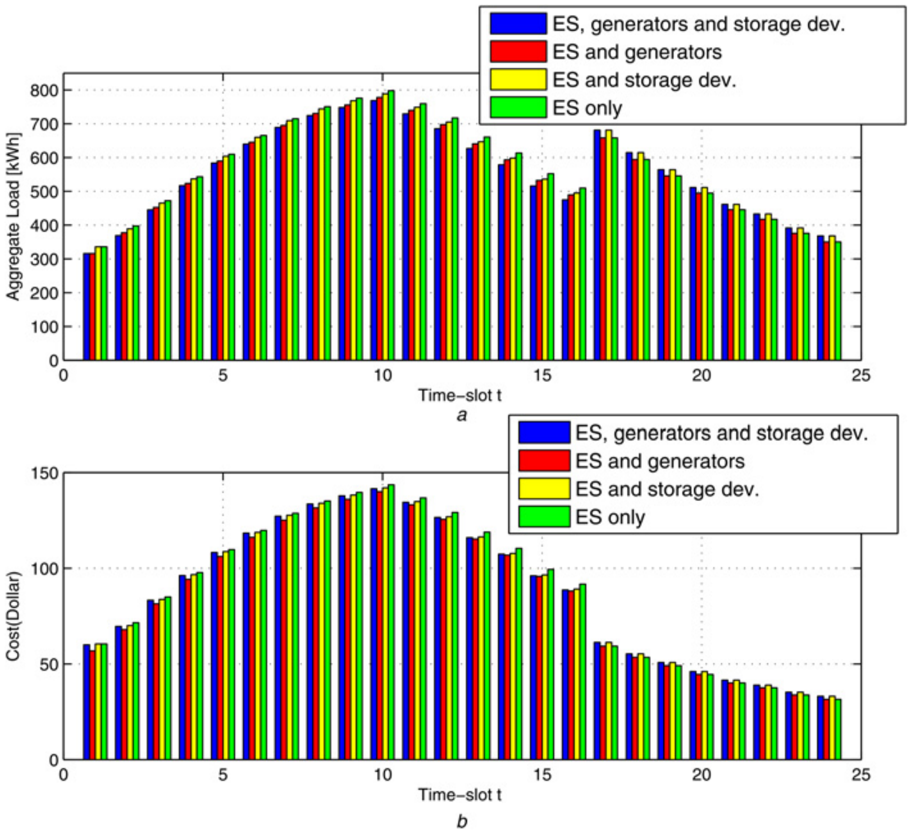


**Fig. 5** *Comparison of ES together with influence of various energy strategies*
*a* Comparison of the aggregate load of *N* users for different energy strategies
*b* Hourly cumulative energy cost of aggregate load

slot $t$ when ES is deployed. From the results, it is clear that there is no peak load, which implies that the utilisation of the ES can help to shave off the peak load. Moreover, from the results, it is obvious that deploying ES and energy generators together with energy storage devices significantly shaves the peak power and results into a stable grid. Both utility company and individual consumers benefit from the strategy as they can all maximise their payoffs.

Users with RES cannot directly optimise their energy production as they produce energy based on their capacity and the weather. On the contrary, users with dispatchable generators and/or storage devices strategically produce electricity or store electricity to optimise their payoffs. Fig. 4*b* shows that users with dispatchable generators produce power only when the cost of buying energy from the utility company is higher than that of generating energy with their dispatchable energy sources. These users can produce energy to power their own appliances or to sell it to the utility company. Similarly, users with storage devices (see Fig. 4*b*) strategically charges their devices during the low rate time and discharge their storage devices during the peak hours where the cost from the utility company is higher. In addition, the use of energy storage devices has a potential in increasing revenue earned by RESs by storing the excess produced energy and discharge it during the peak hours. It should be noted that charging and discharging are mutually exclusive (cannot occur at the same time) operations within the same time slot.

Most of the utility companies charge their customer based on the total hourly load they supply to the customers. Thus, users without energy generators or storage devices can benefit from the strategies as long as there are some active users who strategically generate and store energy. The grid cost per-time slot is depicted in Fig. 4*c*. From the results, it is clear that the ES helps in reducing the cost by shifting the adjustable loads to the valley of the energy cost. The use of energy storers and generators can further reduce the total cost of energy.

To realise the effect of deploying various energy strategies, we compare hourly load and the hourly energy cost. Fig. 5 shows the aggregate load for different strategies. From this figure, it is clear that ES minimises the PAR as well as the daily energy cost. By employing ES together with other energy strategies such as energy storage and/or dispatchable energy generators, the PAR and the daily cost of energy can further be minimised. Users utilising both ES and storage devices schedule their storage devices to be charged during low-price off-peak hours and discharge stored energy during peak hours to further minimise their consumption cost. The result also illustrates the influence of each energy strategy in optimising the utilisation of energy. Increasing the capacity of the deployed strategies may lead to better performance; however, there is a tradeoff between the initial costs as well as operational costs and the performance.

## 5 Conclusion

This paper demonstrated a game theoretic-based distributed DSM scheme, where end-users utilise ES and various energy strategies to minimise daily energy costs of the end-users as well as the utility costs. We verified that difference in pricing mechanisms employed by utility companies gives incentive for users to trade energy. Furthermore, increasing the hourly load results in increasing unit costs since more expensive energy sources are brought online. Thus, end-users deploying ES with energy strategies such as dispatchable energy generators and/or energy storers may substantially minimise the daily energy price of the electricity. Simulation results show that there is a significant reduction in the cost by just shifting the adjustable loads and strategic utilisation of energy storers and/or energy generators.

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