Edge adaptive directional total variation

Hua Zhang, Yuanquan Wang

School of Computer Science, Tianjin University of Technology, Tianjin 300191, People's Republic of China
E-mail: yqwang@bit.edu.cn

Published in *The Journal of Engineering*; Received on 15th October 2013; Accepted on 21st October 2013

Abstract: The directional total variation (DTV) model has been proposed very recently for image denoising. However, the DTV model works well when there is just one dominant direction in the image. In this Letter, the authors propose to make the DTV model adaptive to image edge direction so that the proposed model can handle images with several dominant directions. Experiment and comparison show the effectiveness of the proposed method.

1 Introduction

During the past two decades, variational and partial differential equation (PDE)-based methods have gained popularity for image restoration [1]. The anisotropic diffusion proposed by Perona and Malik [2] is usually taken as the seminal work on PDEs for image restoration. The variational method can date back to the seminal work of Rudin *et al.* [3] known as the total variation (TV) model. The TV model measures the amount of oscillation and favours discontinuities found in an image. Since its debut in 1992, there has been a flurry of works devoted into the TV model, and it is still an active research topic in the community of image restoration, examples include [4–7], among others.

Very recently, Bayram and Kamasak [7] proposed the directional total variation (DTV) model for image denoising. The DTV model measures the oscillation along a direction specified by a constant parameter θ and enhances the diffusion amount by a parameter α. When the dominant direction in the image coincides with the specific direction θ, the DTV model works well on removing noise. However, when there is more than one dominant direction, the algorithm based on such model fails since one constant parameter θ cannot simultaneously specify several directions.

In this paper, we propose to make the DTV model able to handle several dominant directions by introducing a spatially varying parameter $\theta(x, y)$. The $\theta(x, y)$ coincides locally with the edge direction of the image (see Fig. 1*a*), so that the modified DTV model is adaptive to, and enhances the diffusion along the image edge direction. The proposed model is referred to as edge adaptive DTV (EADTV). The effectiveness of the proposed EADTV is also demonstrated by experiment and comparison.

2 EADTV model

To recover an image $f(x, y)$ from a given noisy image $I(x, y)$, for example, Rudin *et al.* [3] proposed the TV model, which minimises the following functional

$$E_{\mathrm{TV}}(f) = \lambda \int_{\Omega} |\nabla f|\, \mathrm{d}\Omega + \frac{1}{2}\int_{\Omega} (f - I)^2\, \mathrm{d}\Omega \tag{1}$$

where Ω is the image domain, ∇ is the gradient operator, λ is a positive weight and $|\cdot|$ is the L_1 norm. By introducing the concept of support function, the TV regulariser $\int_{\Omega} |\nabla f| \mathrm{d}\Omega$ is reformulated as follows [7]

$$\mathrm{TV}(f) = \sum_{i,j} \sup_{\boldsymbol{P}\in B_2} \langle \nabla f(i,\ j),\ \boldsymbol{P}\rangle \tag{2}$$

where B_2 is the unit ball of the L_2 norm. The TV regulariser is isotropic since it is invariant to rotations of the components of ∇f. In [7], Bayram and Kamasak proposed the DTV regulariser as follows

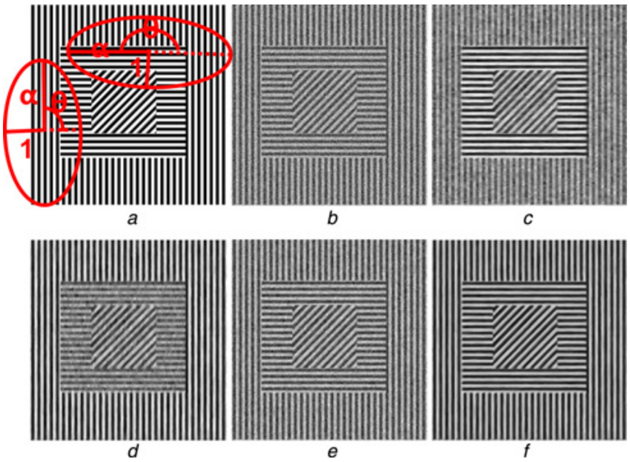

Fig. 1 *Results of the DTV, TV and EADTV models on a synthetic image*
a Noise-free image
b Noisy image with RMSE = 0.2005
c DTV, $\theta = 0$ and RMSE = 0.2545
d DTV, $\theta = \pi/2$ and RMSE = 0.1818
e TV and RMSE = 0.1245
f EADTV and RMSE = 0.0876

$$\mathrm{DTV}_{\alpha,\theta}(f) = \sum_{i,j} \sup_{\boldsymbol{P}\in B_{\alpha,\theta}} \langle \nabla f(i,\ j),\ \boldsymbol{P}\rangle \tag{3}$$

where $B_{\alpha,\theta}$ is an ellipse oriented along the direction θ, with a unit length minor axis and a major axis of length $\alpha > 1$. When $\alpha = 1$, the DTV regulariser reduces to the TV model. Larger α makes the DTV more sensitive to variations along the direction θ. The image denoising model using the DTV regulariser is

$$E_{\mathrm{DTV}} = \lambda\, \mathrm{DTV}_{\alpha,\theta}(f) + \frac{1}{2}\int_{\Omega} (f - I)^2\, \mathrm{d}\Omega \tag{4}$$

Since the DTV model enhances the diffusion along the direction θ, when the dominant direction in an image coincides with the direction θ, the DTV model will enhance the dominant structure; otherwise, destroy the structure.

It is necessary to make the parameter θ spatially varying throughout the entire image when there are several dominant directions in

This is an open access article published by the IET under the Creative Commons Attribution License (http://creativecommons.org/licenses/by/3.0/)

the image. In this paper, we propose a spatially varying $\theta(x, y)$ based on the edge direction of the image as follows

$$\left(\cos(\theta(x, y)),\ \sin(\theta(x, y))\right) = \left(n_1(x, y),\ n_2(x, y)\right) \tag{5}$$

where $(n_1(x, y), n_2(x, y))$ is the edge direction of the image. Fig. 1*a* shows that the $\theta(x, y)$ coincides locally with the image edge direction. The DTV model in (4) with $\theta(x, y)$ in (5) is referred to as EADTV in short. This way, the EADTV model is adaptive to, and enhances the diffusion along, the image edge direction. The edge direction $(n_1(x,y),\ n_2(x,y))$ plays an important role and should be estimated in advance. We suggest two choices for $(n_1(x, y), n_2(x, y))$. (i) When there is a noise-free image $R(x, y)$ as reference, the edge direction can be calculated directly from the image $R(x, y)$ as follows

$$\left(n_1(x, y),\ n_2(x, y)\right) = \left(-R_y,\ R_x\right) \Big/ \sqrt{R_x^2 + R_y^2} \tag{6}$$

where $(\boldsymbol{R}_x, \boldsymbol{R}_y)$ is the gradient vector of $R(x, y)$. (ii) When there is no reference image, the edge direction can be estimated from the noisy image $I(x, y)$ which is presmoothed using a Gaussian filter of small scale (say 1 in this Letter), therefore

$$\left(n_1(x, y),\ n_2(x, y)\right) = \left(-g_y,\ g_x\right) \Big/ \sqrt{g_x^2 + g_y^2} \tag{7}$$

where $(\boldsymbol{g}_x, \boldsymbol{g}_y)$ is the gradient vector of $g(x,y)$, and $g(x,y)$ is a smoothed version of $I(x, y)$ using a Gaussian filter. To minimise the EADTV model, the numerical method described in [7] is modified using the adaptive $\theta(x, y)$. The source code of the EADTV is available at: http://www.mathworks.com/matlabcentral/fileexchange/42238.

3 Results

We demonstrate the desirable properties of the proposed EADTV model and the performance of the TV, DTV and EADTV models are compared. We focus primarily on the images with several dominant directions and the root-mean-square error (RMSE) is employed to evaluate the performance. The noisy image in Fig. 1*b* is coined by adding Gaussian noise, and $\alpha = 5$ for DTV and EADTV, $\lambda = 0.1$ for all three models.

Fig. 1 illustrates the results of the three models on a synthetic image with three dominant directions. It is clear that the DTV can just preserve well the patterns aligned with the specified direction, see Figs. 1*c* and *d*. Fig. 1*e* shows that the TV model is able to preserve all the patterns; however, there is still an obvious noise in the result since the diffusion is not enhanced with $\alpha = 1$. The result of the EADTV is clear and all the patterns are preserved very well, see Fig. 1*f*. The RMSE values also demonstrate that the EADTV outperforms the TV and DTV models. To note, in Fig. 1, the edge direction $(n_1(x, y), n_2(x, y))$ is calculated using (6) by taking the noise-free image in Fig. 1*a* as reference.

Fig. 2*a* shows another example. It is a wood wall in dilapidated condition. Since there is no noise-free image as reference, the edge direction $(n_1(x, y), n_2(x, y))$ is calculated using (7). One can clearly see that the DTV model only preserves the patterns of one specified direction and destroys these in other directions, see Figs. 2*b* and *c*. In contrast, the EADTV effectively removes the noise and preserves the patterns well, see Fig. 2*d*. Since the TV model favours piecewise smooth image, there is a blocky effect in the result, see Fig. 2*e*.

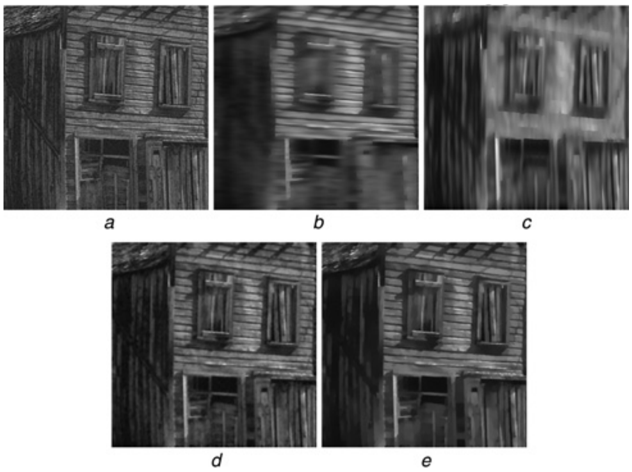

Fig. 2 *Results of the DTV, TV and EADTV models on a real image*
a Noisy wood image
b DTV and $\theta = 0$
c DTV and $\theta = \pi/2$
d EADTV
e TV

4 Conclusion

The very recently proposed DTV model favours images of one dominant direction, but cannot cope with several dominant directions for image denoising. In this paper, we propose a modification of the DTV model, that is, the EADTV, by using a spatially varying direction parameter based on the image edge direction. The edge direction can be estimated from the noisy image or from a noise-free reference image. The proposed EADTV model is able to handle images of several dominant directions and experimental results compared with the TV and DTV models have demonstrated its effectiveness in noise removal and dominant direction preservation.

5 Acknowledgments

This workwas supported by the NSFC under grant 60602050 and the program from Tianjin Commission of Technology of China under grants 11JCZDJC15600 and 10JCYBJC11000.

6 References

[1] Aubert G., Kornprobst P.: ‘Mathematical problems in image processing: partial differential equations and the calculus of variations’ (Springer, New York, 2006, 2nd edn.)

[2] Perona P., Malik J.: ‘Scale space and edge detection using anisotropic diffusion’, *IEEE Trans. Pattern Anal. Mach. Intell.*, 1990, **12**, (7), pp. 629–639

[3] Rudin L., Osher S., Fatemi E.: ‘Nonlinear total variation based noise removal algorithms’, *Physica D*, 1992, **60**, pp. 259–268

[4] Fu S., Zhang C.: ‘Adaptive non-convex total variation regularisation for image restoration’, *IEE Electron. Lett.*, 2010, **46**, (13), pp. 907–908

[5] Hu Y., Jacob M.: ‘Higher degree total variation (HDTV) regularization for image recovery’, *IEEE Trans. Image Process.*, 2012, **21**, (5), pp. 2559–2571

[6] Wang Y., Chen W., Zhou S.: ‘MTV: modified total variation model for image noise removal’, *IEE Electron. Lett.*, 2011, **47**, (10), pp. 592–594

[7] Bayram I., Kamasak M.E.: ‘Directional total variation’, *IEEE Signal Process. Lett.*, 2012, **19**, (12), pp. 781–784

This is an open access article published by the IET under the Creative Commons Attribution License (http://creativecommons.org/licenses/by/3.0/)