

Small-signal stability analysis for the multi-terminal VSC MVDC distribution network; a review

 eISSN 2051-3305
 Received on 30th August 2018
 Revised 12th September 2018
 Accepted on 24th September 2018
 E-First on 7th December 2018
 doi: 10.1049/joe.2018.8842
 www.ietdl.org

Patrobers Simiyu¹ ✉, Ai Xin¹, Girmaw T. Bitew¹, Muhammad Shahzad¹, Wang Kunyu¹, Peter M. Kamunyu¹

¹North China Electric Power University, State Key Laboratory of Alternate Electrical Power System with Renewable Energy Sources, Beijing 102206, People's Republic of China

✉ E-mail: simiyupr@yahoo.com

Abstract: The stable operation and control of the proposed terrestrial multi-terminal Voltage Source Converter (VSC) medium voltage DC (MVDC) distribution network is a remarkable motivation towards developing the future universal DC grid interconnecting the low voltage DC systems on one side and the high voltage DC systems on the other. The small-signal stability analysis are important in designing these power electronic based systems (PEBS). In this study, small-signal stability approaches for PEBS are reviewed with the aim of identifying suitable methods for the proposed MVDC distribution network. It is established that the impedance-based methods are suitable for local stability analysis complementing very well with the state-space modelling-eigenvalue analysis method best for global stability analysis. The Lyapunov linearisation method offers simple, adaptable, and accurate small-signal and large-signal stability study. On the other hand, the bifurcation analysis is an emerging non-linear approach with promising efficiency and precision. In conclusion, there is no single small-signal stability criterion that encompass an acceptable level of accuracy, adaptability, and efficiency. Thus, the impedance-based methods combined with state-space modelling-eigenvalue analysis, Lyapunov linearisation, and bifurcation theory can be suitable small-signal stability approaches for the multi-terminal MVDC distribution network.

1 Introduction

Liberalisation of the power market has motivated integration of renewable energy sources (RES) such as wind, solar photovoltaic (PV), and other distributed energy resources (DERs) in the multi-terminal VSC medium voltage DC (MVDC) distribution network [1–3]. Equally, emerging end-user DC loads such as data centres [4], urban supplies [5], industrial parks [6], electric vehicle (EV) charging stations [7, 8], electric railways [9, 10], and electric ships [11–13] among others are pressing for new market requirements in favour of MVDC integration. The MVDC concept was initially explored in ship applications but with innovations in VSC technology, the focus has shifted to terrestrial electric power system applications [12, 14]. The MVDC technology is not just a scaling of voltage levels from the high voltage DC (HVDC) to low voltage DC (LVDC) systems but an innovative modern power system architecture proposal with commercial and industrial application prospects [15, 16].

The MVDC distribution network is an emerging technology with many advantages. The network can integrate the DERs and AC/DC loads with fewer power conversion stages making it more efficient than the conventional medium voltage AC (MVAC) network [17, 18]. The VSC-based network can provide lots of ancillary services such as reactive power compensation, frequency support, and voltage stability [19–21]. The VSC-based wind turbine generators participate in frequency and voltage control besides fault-ride through capability [22–24]. In solar PV integration, the VSC-based systems not only harness PV power efficiently but also provide AC network stability support [25]. Additionally, EVs – the new generation end-user DC loads – can be used as ‘distributed energy storage system’ for supplying operating and spinning reserves especially in grids with high penetration of variable RES [8, 19]. Other MVDC benefits to the distribution network are summarised by Bathurst *et al.* in [26] as efficient exploitation of the existing AC network assets, deferred investment in network reinforcement, reduced losses in other network equipment, no fault current contributed by converters in

case of faults, and feasibility of developing a multi-terminal DC (MTDC) network.

In multi-terminal VSC MVDC distribution system, energy sources and loads are fed through the power electronic converters connected directly to the DC distribution bus [27–30]. A power converter typically operate under a closed-loop control and its dynamic behaviour is largely non-linear due to the power electronic switching devices. Each converter is designed independently to be stable but interconnecting the converters on the same DC bus results in a non-linear system which adds an additional element of complexity to the overall system dynamics [20, 31]. When observing the system from the DC bus, the load converters operating under the closed-loop output voltage control exhibit a constant power load (CPL) characteristics that contrasts the voltage–current characteristic of the conventional resistive load. These CPL effect is a non-linear behaviour which causes the equivalent incremental negative input resistance that results into sub-system's interaction problem which has a destabilising effect [31–33]. Additionally, unlike the conventional power system, the power electronic based systems (PEBS) possess little inertia which makes it vulnerable to undesirable system's oscillations [20].

The VSC MVDC distribution system comprises of multi-converters with unique dynamics and stability challenges. Thus, during system design, sources of system instability should be investigated using suitable stability criteria aimed at achieving a robust system [11, 14]. In this case, system's stability is concerned mainly with keeping DC bus voltage constant in steady-state and transient operations. The DC voltage stability study under small perturbations about a given operating point is known as small-signal stability analysis. When the effect of large disturbances such as loss of generation units, large variation in load or line fault are considered, it is called large-signal stability analysis. Typically, a system is considered stable if it is able to return to its normal operating condition after a given disturbance [34–36].

The small-signal stability analysis is based on linearisation of the non-linear system around a given operating point. The linearised system is further explored using linear and non-linear

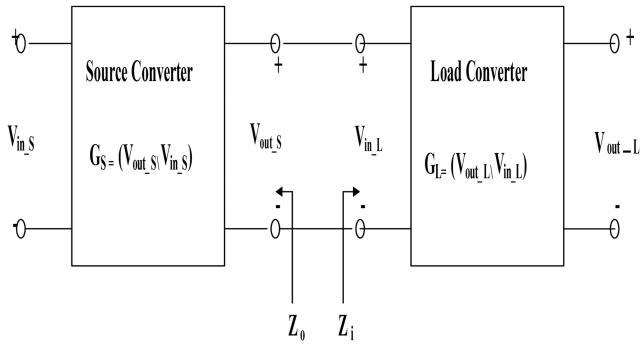


Fig. 1 Source-load converter sub-system

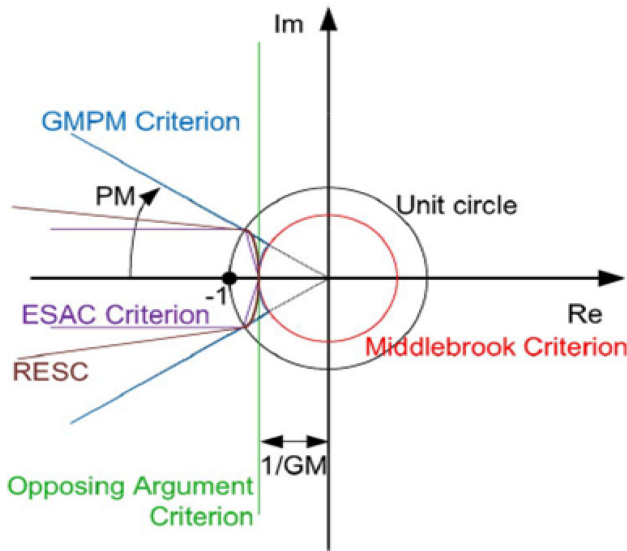


Fig. 2 Stability boundaries of Middlebrook criterion and its extensions [40]

analysis tools to assess as well as improve its stability conditions [36]. This paper reviews the various small-signal stability analysis tools with the aim of identifying the most appropriate for the multi-terminal VSC MVDC distribution system.

2 Classical control-based stability analysis

Small-signal stability studies for a DC distribution system are conducted based on the transfer function in which a system's dynamics are represented in a mathematical model of differential algebraic equations (DAEs). A variety of methods are available from classical control theory for evaluating the stability of the system namely: Routh–Hurwitz criterion, root-locus plot, and frequency-response methods. The stability of the system depends on the location of the system transfer function poles on the complex s -plane. However, further tools are employed to assess the stability margin and movement of system poles under varying operating conditions [37].

Routh's stability criterion can be used to identify whether or not there are unstable poles in the system's function without actually solving them. The necessary and sufficient condition for stability is that all poles lie in the left-half plane (LHP). This method directly reveals the absolute stability of the system from the coefficients of the characteristic equation. This is never sufficient for many practical cases hence it is limited to linear control system analysis. Additionally, this criterion can be used to investigate the effects of changing system's parameters and determining the values that cause instability but cannot study how to stabilise an unstable system. The Root-locus plot is another linear control system approach which represents the poles for the gain of the systems function. It can also be used to predict the effect of variations on the gain value or adding open-loop poles and/or zeros on the location of the closed-loop poles. The Root-locus is useful in assessing the manner in which the open-loop poles and zeros

should be modified to meet the estimated specifications of the system's performance [36, 38].

Frequency-response methods provide another encompassing approach in which a linear as well as a non-linear system can be designed with negligible effects of undesirable noise. In this method, the sinusoidal transfer function of the given system is a complex function of frequency, characterised by its magnitude and phase angle. The Bode, Nyquist, and Nichol's plots are the main forms of representing the system's sinusoidal transfer function. A Bode plot presents information on the gain and the phase margin of the system. If both the gain and the phase margin are positive, then the system is stable and the relative stability can be determined. The Nyquist plot provides information on the relative stability of the system which is usually more sufficient than absolute stability in many practical cases. The Nyquist criterion for stability condition is satisfied when the Nyquist contour does not encircle $(-1, 0)$ point in the s -plane. The Nichol's plot is typically a plot of the logarithmic magnitude against the phase margin for a given frequency range, useful when determining the relative stability of a closed-loop system [36, 38, 39].

3 Impedance-based stability analysis

In impedance-based method, the PEBS is broken into two sub-systems, i.e a source and a load converter at a given arbitrary interface as shown in Fig. 1. It is assumed that each sub-system is individually stable and a fixed power flow presumed [14, 37, 40]. Generally, stability is analysed from measurements or simulations obtained at the interface point in the system and the Nyquist criterion applied to the ratio of the source to the load impedances known as the minor loop gain (MLG). The Middlebrook criterion was the first impedance-based tool proposed in 1976. Subsequently, others evolved to offer more accurate stability assessments [41–43].

The Middlebrook criterion was proposed to investigate the stability of a feedback-controlled switching converter incorporating an input filter. This is essentially a single source and load converter DC system configuration. To ensure adequate stability for the system, the input impedance of the source converter should be greater than the output impedance of the load converter [40, 41].

The MLG, T_{MLG} is given by (1) whereby the sufficient stability condition is when the $MLG < 1$. Equation (1) defines the forbidden region in the complex s -plane outside the unit circle centred $(0, 0)$ in Fig. 2.

$$\|T_{MLG}\| = \left\| \frac{Z_o}{Z_i} \right\| \ll 1 \quad (1)$$

Given the input impedance Z_i , the input filter output impedance Z_o guarantees that the T_{MLG} lies inside the unit circle with radius equal to the inverse of the desired gain margin (GM), where $GM > 1$ as given in (2). Certainly, the Nyquist contour cannot encircle the $(-1, 0)$ point and the T_{MLG} is inside the unit circle hence the system is stable as shown in [41].

$$\|T_{MLG}\| = \left\| \frac{Z_o}{Z_i} \right\| = \frac{1}{GM} \quad (2)$$

Using the extra element theorem (EET) in [44], the output impedance of the input filter Z_o is considered as an extra element. Thus, the loop gain with the input filter T' is given in (3). The loop gain without the input filter, the null double-injection impedance, and the single injection impedance are denoted as T , Z_N , and Z_D , respectively.

$$T' = T \cdot \frac{1 + Z_o/Z_N}{1 + Z_o/Z_D} \quad (3)$$

The converter input impedance under a feedback control system is given in (4) which can be used to compute the T_{MLG} . When Z_o is designed such that the ratio $Z_o/Z_N \ll 1$ and $Z_o/Z_D \ll 1$, the stability criteria in (1) is also satisfied in (4) guaranteeing system

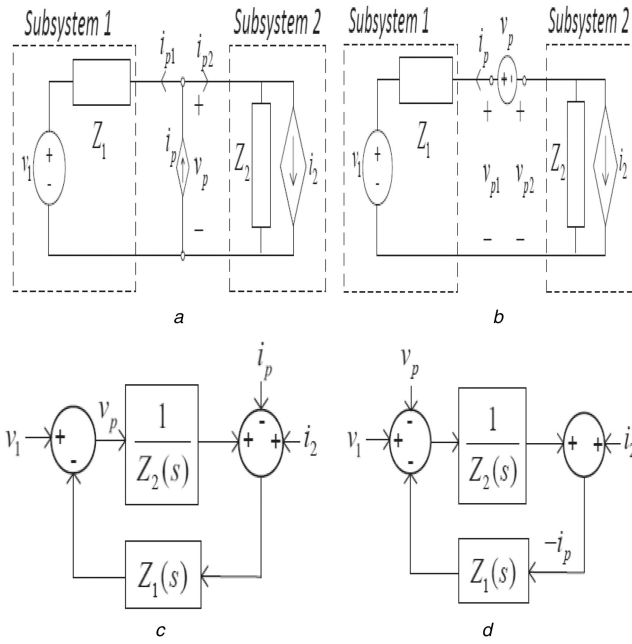


Fig. 3 DC power system injection circuit diagram

(a) Shunt sub-system, (b) Series sub-system, DC power system injection block diagram: (c) Shunt sub-system, (d) Series sub-system [46]

stability [40]. Additionally, the loop gain with and without the input filter are equal implying that the dynamic performance of the converter is not affected by the presence of the input filter. Therefore, there is no loading effect between the filter and the converter [41].

$$\frac{1}{Z_i} = \frac{T}{1+T} \cdot \frac{1}{Z_N} + \frac{1}{1+T} \cdot \frac{1}{Z_D} \quad (4)$$

$$T_{MLG} = \frac{Z_o}{Z_i} = \frac{T}{1+T} \cdot \frac{Z_o}{Z_N} + \frac{1}{1+T} \cdot \frac{Z_o}{Z_D}$$

The Middlebrook criterion is the most widely used due to its simplicity and versatility [37]. It offers more than system stability by addressing the issue of interaction effects using the EET on system dynamics which may be a design requirement for some applications. However, it is too conservative to optimise system's component sizing and costs hence a more refined stability criterion is required for large and more complex DC systems [40].

There are several criteria in literature developed from the Middlebrook criterion namely: the gain margin and phase margin (GMPM) criterion, the opposing argument criterion [40, 41], the energy source analysis consortium (ESAC) criterion [42, 43], and its extension the root exponential stability criterion (RESC) [41]. All these stability analysis approaches are based on the MLG that defines the forbidden region on the s -plane where the Nyquist contour should avoid to ensure adequate stability margin. Fig. 2 illustrates the stability boundaries of all the Middlebrook criterion and its extensions. It should be noted that these criteria define the design specifications for the load impedance for the given source impedance and offer only sufficient conditions for system stability. The method can predict sustained harmonic oscillations and discretisation is not required. However, the major drawback of these criteria is the inability to manage multiple source and load converter system where there are possibilities of changes in power flow direction. Besides, the system stability estimate is dependent on the location of the single interface measurement point of the system which influences its global stability [14, 37, 40, 41, 45].

The apparent impedance-based method is based on measurements like the conventional Middlebrook and its extensions but with a few differences. For instance, other than getting the source and load impedances separately, an equivalent impedance referred to as apparent impedance (Z_a) seen from the injection source is identified [40, 46]. Additionally, instead of applying the Nyquist criterion, the method identifies the system's

eigenvalues based on sampled values of Z_a using vector fitting (VF) and matrix fitting (MF) system identification algorithms. This method overcomes the drawbacks of the conventional method in which the stability margin depends on the location of the measurement interface point offering global system stability [41, 47].

Consider a DC power system comprising two sub-systems similar to the conventional impedance method shown in Fig. 3. The injection point applied to the sub-systems separates them into 1 and 2 represented by their frequency dependent equivalent Thevenin and Norton circuits, respectively. This representation impacts on the methodology applied as the relationship between the apparent impedance and the sub-system impedances for the shunt varies from the series injection but does not affect the stability analysis [46].

The apparent impedance (Z_a) is given by (5) as a transfer function between voltage and current, where v_p and i_p are injection voltage and current, respectively, while $Y_a(s)$ is the apparent admittance. The influence of the voltage source, v_1 and current source i_2 are neglected in the small-signal analysis.

$$Z_a(s) = \frac{v_p(s)}{i_p(s)} = \frac{1}{Y_a(s)} \quad (5)$$

The apparent impedance for the shunt and the series injection in Fig. 3 can be derived from circuit analysis as shown in (6) and (7), respectively.

$$Z_{a,\text{shunt}} = \langle Z_1 \parallel Z_2 \rangle = \frac{Z_1 Z_2}{Z_1 + Z_2} \quad (6)$$

$$Z_{a,\text{series}} = \langle Z_1 + Z_2 \rangle = \frac{Y_1 Y_2}{Y_1 + Y_2} \quad (7)$$

In small-signal stability analysis, the entire single-input-single-output system in Fig. 3 is characterised by a linear state-space model given in (8); where x is the vector for n states, u is the single input to the system, while y is its single output. A is an $n \times n$ state matrix, B is an $n \times 1$ vector, C is an $1 \times n$ vector while D and E are scalars:

$$\begin{aligned} \dot{s}x &= Ax + Bu \\ y &= Cx + (D + sE)u \end{aligned} \quad (8)$$

From control theory, a general transfer function $H(s)$ is derived as the ratio of the output y and input u in (9):

$$H(s) = \frac{y(s)}{u(s)} = C(sI - A)^{-1}B + D + sE \quad (9)$$

Accordingly, a general impedance $Z(s)$ and admittance $Y(s)$ based on some voltage v and some current I in s -domain are given in (10):

$$\begin{aligned} Z(s) &= \frac{v(s)}{i(s)} = C_z \langle sI - A_z \rangle^{-1} B_z + D_z + sE_z \\ Y(s) &= \frac{i(s)}{v(s)} = C_Y \langle sI - A_Y \rangle^{-1} B_Y + D_Y + sE_Y \end{aligned} \quad (10)$$

From Figs. 3c and d, the transfer function for shunt current injection is given as a ratio of the interface point voltage v_p to the sub-system 2 load current i_2 given in (11). Similarly, the transfer function with series voltage injection is given as a ratio of the interface point current i_p to the sub-system 1 source voltage v_1 in (12):

$$\frac{v_p(s)}{i_2(s)} = -\frac{Z_1 Z_2}{Z_1 + Z_2} = -Z_{a,\text{shunt}} \quad (11)$$

$$\frac{i_p(s)}{v_1(s)} = -\frac{1}{Z_1 + Z_2} = -\frac{1}{Z_{a,series}} = -Y_{a,series} \quad (12)$$

It can be observed that (11) and (12) are similar to (6) and (7) for shunt and series injection, respectively. Thus, the apparent impedance represents a closed-loop transfer function for the system for both shunt and series injection. The negative sign in each case does not influence the stability analysis in any way [46].

The general transfer function in (9) can be rewritten given the input and output from (11) and (12) into the state-space models (13) and (14). A_Z and A_Y are matrices that contain the system eigenvalues observable from the injection point.

$$-Z_{a,shunt}(s) = \frac{v_p(s)}{i_2(s)} = C_Z(sI - A_Z)^{-1}B_Z + D_Z + sE_Z \quad (13)$$

$$-Y_{a,series}(s) = \frac{i_p(s)}{v_1(s)} = C_Y(sI - A_Y)^{-1}B_Y + D_Y + sE_Y \quad (14)$$

Finally, the state-space models in (13) and (14) are estimated to a measured or computed transfer function based on curve fitting using the VF. A set of measured or simulated apparent impedances $Z_{a1}, Z_{a2}, \dots, Z_{an}$ for shunt-current injection ($Y_{a1}, Y_{a2}, \dots, Y_{an}$ for series-voltage injection) are taken at frequencies $f_1, f_2, \dots, f_n$ and the order of the state-space model given to the VF algorithm which yield the state-space model denoted by A, B, C, D, E in (8). ' A ' is a diagonal matrix whose eigenvalues are directly available along its diagonal and the small-signal stability analysis completed.

The apparent impedance method works in all system cases where the conventional impedance method applies as the same basic information is required. Since it does not regard where the source and load are located, it ensures a higher global stability [46, 47]. This method is proposed in [47] for meshed systems since it is not strictly required to define a source or load sub-system. The method is applicable to large systems although the dynamics not observable at the interface point can be overlooked. This problem may be overcome by investigating different injection locations. However, the main drawback is its complicated impedance measurement procedure [47].

All the above impedance-based stability criteria treat all the DC converter system components as voltage sources or current sources. However, a complex system with changing operation modes such as a PV array system, a source converter, a load converter, a bidirectional DC-DC converter, and a battery storage cannot be effectively analysed with this methods. The impedance-based local stability criteria are proposed to overcome these shortcomings. Any converter in the system is firstly classified either as a bus voltage controlled converter (BVCC) or bus current controlled converter (BCCC), secondly, the system is presented in a standard form regardless of its structure and operation mode. Finally the MLG of the system is derived and the Nyquist stability criterion applied to it as in the conventional impedance-based method to define the stability requirements [48].

4 Passivity-based stability criteria

The passivity-based stability criteria (PBSC) imposes a passivity condition on the overall equivalent bus impedance Z_{bus} rather than imposing conditions on the impedance ratio as in the Middlebrook criterion and its extensions [14, 37, 40]. The method was designed to overcome the artificial conservativeness in designs, difficulties in multi-converter systems and sensitivity to component grouping and power flow direction change in the previous stability criteria [41, 45, 49, 50]. Consider a single-bus DC power distribution system in [51] consisting of n source converters and m load converters in Fig. 4. The distribution system can be reduced into to an equivalent one-port network. In order to determine system small-signal stability, the Z_{bus} is measured or calculated by injecting a current I_{inj} from an external power device connected at the bus port and computing the ratio $Z_{bus(s)} = V_{bus(s)}/I_{inj(s)}$.

Generally, if the network is passive (absorbs energy), the small-signal stability is assured. Mathematically, passivity conditions can

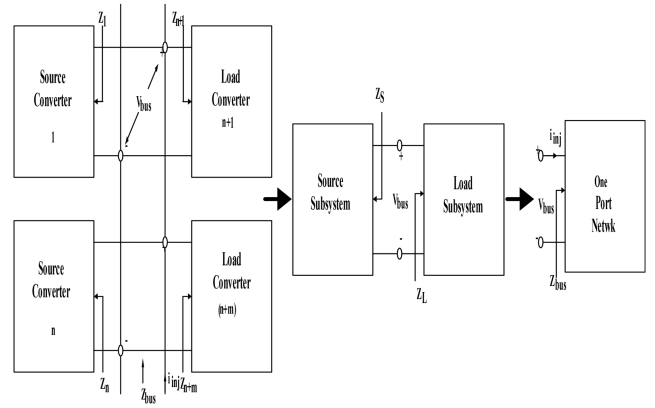


Fig. 4 Application of the PBSC

be derived considering the total energy supplied to the network using the Parseval's theorem in (15):

$$\int_{-\infty}^{+\infty} v(t)i(t) dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} v(j\omega)I^*(j\omega) d\omega \quad (15)$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} z(j\omega) \|I(j\omega)\|^2 d\omega$$

$$= \frac{1}{\pi} \int_0^{+\infty} \text{Re}\langle z(j\omega) \rangle \|I(j\omega)\|^2 d\omega \quad (16)$$

Equation (15) is computed as (16) satisfying the passivity condition that necessitates it to be non-negative for all currents meaning that $Z(s)$ which is a real and rational function of s must be positive; $\lambda_i = \text{Re}[Z(\sigma + j\omega)] \geq 0$ for $\sigma > 0$. Passivity is established when the bus impedance Z_{bus} has a positive real part at all frequencies. This also implies that the Nyquist contour of Z_{bus} lies entirely on the right half plane (RHP) [40, 41, 51, 52].

The PBSC is suitable when designing active damping system for DC networks. For instance, the positive feedforward (PFF) control offers an active approach to overall system stability and higher dynamic performance without any hardware modification [14, 37, 40, 41, 45]. Additionally, the passivity theory provides effective stability assessment for the multiple grid-connected VSC system which is challenging with impedance-based methods. However, like all the Middlebrook criteria and its extensions, the PBSC provides only sufficient stability condition [49–52].

5 State-space modelling and eigenvalue analysis

This is a global stability analysis approach that determines the system stability irrespective of the location of the source(s) of instability. The eigenvalue analysis is based on the analytical state-space model of the system and the corresponding system's matrix. In a small-signal stability analysis, the complete model is linearised around an operating point and the resulting system matrix used to derive eigenvalues [36, 45, 53–55].

Consider a dynamic system described in [34, 36], by a set of DAEs given in state space form in (17):

$$\begin{aligned} \dot{x} &= f(x, u) \\ y &= g(x, u) \end{aligned} \quad (17)$$

where x is the state variable, y the output variable and u the control variable. If the system is at equilibrium point, it is denoted as (18)

$$\dot{x} = f(x_0) = 0 \quad (18)$$

A small disturbance is introduced and changes in Δx , Δy and Δu results. Using Taylor series, the DAE is transformed to an ordinary differential in a linearised form in (19):

$$\begin{pmatrix} \dot{\Delta X} \\ \dot{\Delta Y} \end{pmatrix} = \begin{pmatrix} \frac{df}{dx} & \frac{df}{du} \\ \frac{dg}{dx} & \frac{dg}{du} \end{pmatrix} \cdot \begin{pmatrix} \Delta X \\ \Delta U \end{pmatrix} \quad (19)$$

where in the 2×2 Jacobian matrix, the state or plant matrix and the control or input matrix are in the first and the second columns of the first row, respectively. Similarly the output matrix and the feed-forward matrix are in the second row. For non-trivial solution, it is necessary that the condition in (20) is satisfied. This is the characteristic equation of the system whose zeros are the poles of the system defined by the state matrix A . The nature of the poles reveals the characteristic behaviour of the dynamic system encapsulated in the state matrix:

$$\begin{aligned} \det(sI - A) &= 0 \\ \det(A - \lambda I) &= 0 \end{aligned} \quad (20)$$

The roots of the characteristic equation are the eigenvalues of the state matrix A . Given the eigenvalues, $\lambda_i = (\alpha_i \pm j\omega_i)$ of A , the system can be assessed based on the small-signal stability criterion established for eigenvalue analysis. When the real part of all the eigenvalues are negative, i.e. on the LHP, then the system is stable. However, if any of the eigenvalues have a positive real part, i.e. on the RHP, then the system is unstable. In case all the eigenvalues have negative real parts except one complex pair with a purely imaginary value, then the system experiences an oscillatory mode [34, 36].

Each eigenvalue has two eigenvectors namely the right eigenvector (REV) and the left eigenvector (LEV). The main function of these eigenvectors is identification of mode activities of the dynamic system. The REV is an n -column vector, Φ stated as $\Phi = [\Phi_1, \Phi_2, \dots, \Phi_n]^T$. The REV, Φ_i related to an eigenvalue λ_i must satisfy the condition: $A\Phi_i = \lambda_i \Phi_i$. The LEV is the n -row vector, ψ defined as $\psi = [\psi_1, \psi_2, \dots, \psi_n]$. The LEV corresponding with the eigenvalue λ_i must satisfy the condition: $\psi_i A = \psi_i \lambda_i$. The REV and the LEV matrices are orthogonal. The two matrices are in the normalised form if $\psi \cdot \Phi = \Phi \cdot \psi = I$; where I is an identity matrix [34, 36].

The combination of the normalised REV and LEV matrices results in a participation-factors (P_{ki}). The participation factor is a measure for identifying the dynamic behaviour of the system. It measures the extent to which the system component(s) and controller(s) parameters contribute to the stable, unstable, and oscillating modes. This provides information in which the system and/or controller parameters can be maintained or corrected to achieve desirable stability conditions. The participation factors for the k th element of the state participation matrix, P_{ki} is given in (21).

$$P_{ki} = \Phi_{ki} \psi_{ik} \quad (21)$$

where, Φ_{ki} is the element of the k th row and i th column of the REV matrix while ψ_{ik} is the element of the i th row and k th column of the LEV matrix for the k th entry of the REV Φ_i and LEV ψ_i , respectively. P_{ki} is the measure of the degree of participation of the k th state variable (x_k) in the i th mode of oscillation (λ_i) [34, 36].

The trajectory of the eigenvalues when the system and controller parameters change can be evaluated to find out the sensitivity of the system. The sensitivity analysis is the measure of the association between the system parameter changes and its influence on system's stability. This sensitivity is equal to the partial derivative of eigenvalue to the system parameters given in (22).

$$\frac{d\lambda_i}{d\alpha} = \frac{v_i(dA/d\alpha)u_i}{v_i u_i} \quad (22)$$

where λ_i is the i th eigenvalue, A is the system parameter, u_i is the REV and v_i is the LEV. The effect of the parameter changes to the system stability can also be studied using the Root-locus of the

related characteristic roots. The sensitivity analysis is used mainly to optimise the design of the system and controller parameters in order to improve the small-signal stability [34, 36, 56].

The eigenvalue analysis method has been successfully applied due to its simplicity to implement a more accurate small-signal stability criteria than the impedance-based and PBSC [45, 53, 54]. However, its main setback is its complicated state-space modelling especially for higher order systems. It is also difficult to obtain accurate system's parameters [57, 58]. Additionally, a highly computational discretisation is required for eigenvalue analysis in averaged model based VSC system to overcome the inability to identify sustained harmonic oscillations [45, 54].

6 Lyapunov linearisation method

The Lyapunov linearisation evaluates the small-signal stability of a given power system based on the roots of its state-equation. If the system is time-invariant, the stability is assessed by analysing the roots of its eigenvalues. Consider a linear system described by a DAE in (23) [34, 36].

$$\begin{aligned} \dot{x} &= Ax \quad \text{for } A \in \mathbb{R}^{n \times n} \\ x(0) &= x_0 \end{aligned} \quad (23)$$

The time evolution of the state variable A has distinct eigenvalues given by (24)

$$x(t) = \sum_{i=1}^n \Phi_i \psi_i x_0 e^{\lambda_i t} \quad (24)$$

where λ_i is the i th eigenvalue of the state matrix A ; Φ_i is the REV of the state matrix A corresponding to the i th eigenvalue λ_i and ψ_i is the LEV of the state matrix A corresponding to the i th eigenvalue λ_i .

The Lyapunov stability criteria can then be applied to (24) where the origin is stable if none of the eigenvalues has positive real parts, asymptotically stable if all the eigenvalues have negative real parts, and unstable if at least one eigenvalue has a positive real part.

The Lyapunov's first method is an extension of this criterion to a general non-linear system. This can be illustrated by considering an autonomous non-linear system described by a continuously differentiable function $f(x)$ in (25) [35, 36]:

$$\dot{x} = f(x) \quad (25)$$

The system dynamics can be expressed as (26)

$$\dot{x} = \left[\frac{df}{dx} \right]_{x=0} \cdot x + f_{h.o.t.}(x) \quad (26)$$

where x is the state-vector, $f_{h.o.t.}$ indicates higher order terms (h.o.t) in x and $x=0$ defines the steady-state operation condition which is the equilibrium point of the system for linearisation.

The Lyapunov's linearisation method can be used to determine the local stability of the non-linear system. This is because linearisation is valid only in the small range about the equilibrium point. Thus, linear approximation of the original non-linear system at the equilibrium point is undertaken as in (27):

$$\dot{x} = Ax, \quad A = \left[\frac{df}{dx} \right]_{x=0} \quad (27)$$

where A denotes the Jacobian matrix of $f(x)$ at $x=0$.

From the Jacobian matrix, the small-signal stability of the equilibrium point for the non-linear system can be closely related to the linearised system based on a similar criteria applied to (27). The equilibrium point of the non-linear system is locally asymptotically stable if all the eigenvalues of the linearised system have negative real part. The equilibrium point of the non-linear system is locally unstable if at least one of the eigenvalues of the

linearised system has a positive real part. The stability of the equilibrium point of the non-linear system cannot to be defined by first approximation if there is at least one eigenvalue with a zero real part. The non-linear system can be stable, asymptotically stable, or unstable. The exact stability condition of the equilibrium point can be derived from the analysis of the h.o.t that affect the centre manifold [34–36].

In [59], the small-signal stability criterion for the equilibrium point of the cascaded system with a current-mode controlled source converter and a CPL is derived using the Lyapunov linearisation method. A similar study for aircraft power system is carried out in [60]. In this case, the Lyapunov method is simple to implement and has predictive characteristics [59]. The complicated impedance calculations or measurements in impedance-based methods are avoided [60]. Small and large-signal stability analyses can be studied using the Lyapunov linearisation and mixed-potential theory, respectively. Besides, the two can be combined into a generalised stability criterion for investigating both stability problems [59, 60]. The main challenge is its conservative proposed criterion which does not affect the validity of the system's stability [59].

7 Bifurcation theory

The bifurcation theory has proved to be an effective tool for stability analysis of non-linear dynamic phenomenon in electric power systems. It is a powerful tool for investigating the dynamic behaviour of the system under variation of parameters. A bifurcation occurs when a variation in system's parameters causes a sudden change in system's behaviour leading to instability. In small-signal stability, the study focuses on local bifurcations which occur when changes in a parameter results in the shifting of the stability of an equilibrium point [61, 62].

Consider a dynamic power system model in [63] represented by the DAE in (28):

$$\frac{dx}{dt} = F(x, \mu) \quad (28)$$

where x is the n -dimensional state-vector and μ is the time-varying system parameter. The equilibrium solutions are derived by neglecting all the time derivative terms and solving the resulting non-linear set of equations in (29):

$$F(x, \mu) = 0 \quad (29)$$

Let the solution for $\mu = \mu_0$ be x_0 and impose a small perturbation y as in (30) to determine the stability of the equilibrium solution:

$$x(t) = x_0 + y(t) \quad (30)$$

Substituting (30) into (28) we get (31)

$$\frac{dy}{dt} = F(x_0 + y; \mu_0) \quad (31)$$

Linearising (31) using the Taylor series about x_0 and retaining linear terms gives (32):

$$\begin{aligned} \frac{dy}{dt} &= F(x_0; \mu_0) + D_x F(x_0; \mu_0)y + O(|y|^2) \text{ or} \\ \frac{dy}{dt} &= D_x F(x_0; \mu_0)y = [A]y \end{aligned} \quad (32)$$

where A is the Jacobian matrix of the first partial derivatives defined at the equilibrium operating point.

The bifurcation analysis consists of three main steps as outlined in [61–65]. The first *step* involves determining the eigenvalues of the Jacobian matrix and the corresponding participation factor to assess the local stability of the equilibrium solution x_0 of the system in steady-state. The equilibrium solution x_0 is asymptotically stable if the eigenvalues of the Jacobian matrix have

negative real parts at that point. In this case, x approaches the equilibrium x_0 as $t \rightarrow \infty$. The system reaches a critical state if a real eigenvalue becomes zero or a pair of complex conjugate eigenvalues crosses the imaginary axis. When a complex conjugate pair of eigenvalues crosses the imaginary axis and moves into the RHP, the system starts oscillating with a small amplitude depicting the bifurcation phenomenon described by Hopf bifurcation theory. Hopf bifurcation of an equilibrium solution occurs in the following conditions: (i) $F(x_0; \mu_0) = 0$; (ii) the Jacobian matrix A has a pair of purely imaginary eigenvalues $\pm i\omega$, while the other eigenvalues have non-zero real parts at $(x_0; \mu_0)$; (iii) for $\mu = \mu_0$, the analytical continuation of the pair imaginary eigenvalues can be $(\lambda \pm j\omega)$, hence $(d\lambda/d\mu) \neq 0$ implying a transversal or non-zero speed crossing of the imaginary axis.

The second *step* involves numerical continuation where the equilibrium curves with selected change parameters are obtained. In this procedure, the non-linear model of the system is directly used and the values of all the corresponding equilibrium points under different parameters developed. The third *step* is based on the information obtained from the traced equilibrium curves, in which different test functions are used to detect and locate corresponding bifurcation points. Matcont in [66] is a MATLAB package for numerical bifurcation analysis and is efficient in undertaking the second and the third steps.

Several small-signal studies involving bifurcation theory and its application in power systems have been undertaken. Many aspects of small-signal stability involving qualitative changes in AC based power systems have been explored [61–64]. In DC power system, bifurcation analysis is an emerging concept that has not been widely applied. However, few cases namely, in [65], the bifurcation theory is used to examine how the control parameters affect the dynamic stability of the synchronverter based microgrid. The bifurcation theory is also applied in stability analysis and active stabilisation of the DC distribution system with multiple CPL [67] as well as in the DC PEBS in [68].

The bifurcation theory stability analysis is advantageous over the other methods because it is not based on the time series hence the exponential terms are removed and the method generalised for multi-converter systems. The possible transition of the switching command in one switching period is taken in the system model using an additional variable known as a virtual duty cycle. In addition, the system stability pattern is constructed using the discrete-time eigenvalues which can predict accurate stability margin with different controllers [68]. The main setback is the non-linear modelling and computational complexity that makes it difficult to study a large PEBS [54].

8 Conclusion

The stable operation and control of the proposed multi-terminal MVDC distribution network is a remarkable motivation towards developing the future universal DC grid interconnecting the LVDC systems on one side and the HVDC systems on the other. The small-signal stability analysis studies are important in designing this PEBS. In this paper, potential small-signal stability approaches for PEBS are reviewed with the aim of identifying suitable methods for the proposed MVDC distribution network. Table 1 gives a summary of the main strengths and weaknesses of the explored small-signal stability methods.

The impedance-based methods are suitable for local stability analysis complementing well with the state-space modelling-eigenvalue analysis best for global stability analysis. The Lyapunov linearisation is simple, accurate, and versatile for both small-signal and large-signal stability problems. The bifurcation analysis is an emerging non-linear approach with promising efficiency and precision. In conclusion, no single small-signal stability criterion encompasses an acceptable level of accuracy, adaptability, and efficiency. Thus, impedance method combined with state-space modelling-eigenvalue analysis, Lyapunov linearisation, and bifurcation theory are suitable small-signal stability approaches for PEBS that can be adopted for the multi-terminal VSC MVDC distribution network.

Table 1 Strengths and shortcomings of the small-signal stability analysis (SSSA) methods

SSSA method	Strengths	Short-comings
1 classical control based	• simple to implement • complements other stability analysis approaches	• stability margin not studied in details • not effective in large PEBS
2 impedance based	• large PEBS • source–load sub-system based hence real-time stability analysis • multi-source and load converter system • measurement based hence assess stability directly from field measurements. • predict sustained harmonic oscillations	• only a sufficient stability condition • artificial conservativeness in designs • local stability analysis • complicated impedance measurement procedure
3 passivity based	• multi-source and load converter system • large PEBS • active network damping without modification • overcomes artificial conservativeness in designs • insensitive to component grouping	• only a sufficient stability condition • cannot effectively predict the stability of the entire system
4 state-space modelling and eigenvalue analysis	• for global stability analysis • simple to implement • it very accurate • large PEBS	• require complicated system state-space modelling for higher order systems • accurate parameters not easily obtained • discretising with high computation needs • cannot predict sustained harmonic oscillations • conservative proposed criterion • complicated system state-space modelling for higher order systems
5 Lyapunov's linearisation	• simple to implement • no complicated impedance calculations or measurements • small and large signal stability problems	• complicated non-linear modelling • system stability using discrete-time eigenvalues with high computation needs • difficult for large PEBS
6 bifurcation theory	• multi-converter systems • it very accurate • detailed study of system stability margin • specific sub-system model analysis	• complicated non-linear modelling • system stability using discrete-time eigenvalues with high computation needs • difficult for large PEBS

9 Acknowledgments

This work was supported by 2016 National Key R & D Program of China to support Low-Carbon Winter Olympics of Integrated Smart-Grid Demonstration Project (2016YFB0900500), Beijing Natural Science Foundation (No. 3182037).

10 References

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