Introduction to the Techniques of Interferometry and Lunar Occultation in Radio Astronomy

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Radio telescopes allow a 24×7 observation time window of those regions of the space which appear dark in the visible region. But, the major problem with a single radio telescope is its poor resolution. In this article, the method of interferometry in radio astronomy is discussed by citing the example of a two-element interferometer. Interferometry provides improved resolution.With a single telescope as well, the resolution can be improved for the radio sources along the ecliptic which are smaller in angular spread as compared to the Moon. The theoretical principle behind the technique of occultation is discussed using its optical analog of diffraction.

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1. Introduction

The Earth's atmosphere absorbs most of the incoming radiation from extraterrestrial sources. But, the window is largely open for optical and radio wavelengths of the EM spectrum. Therefore, ground-based telescopes are mainly limited to these two regions of the EM spectrum for collecting information. The regions of space that appear 'dark' in the visible region (i.e. they radiate very weakly in that part of the spectrum) may be strong radio emitters. The galactic dust clouds scatter optical radiation and obscure the view, hiding some regions of space. But since radio wavelengths are much longer than optical wavelengths, they are able to pass through such regions. Thus, radio astronomy gives a view of the regions inaccessible by optical astronomy and hence has a vital role to play.

Keywords
EM spectrum, antenna, Jansky, interferometer array, baseline, correlator, Fourier transform, lunar occultation, Fresnel diffraction, convolution.

A radio telescope is usually shaped as a parabolic dish antenna which collects the incoming radio signals bringing them to focus, and generates a corresponding voltage signal. Every receiver antenna is sensitive to signals over a certain bandwidth. With an increase in the bandwidth, the power available at the receiver increases but the resolution of the images decrease. A radio telescope measures the flux density from a certain region of the sky towards which the telescope is pointed. A parabolic dish is usually mounted such that it can be steered mechanically to different directions.

Parabolic dish brings the radio signals to focus and generates a corresponding voltage signal.

Measuring the flux from a series of directions over a particular region can then produce a map of the sky brightness. The strength of the radio signal from a distant source is usually very small. Hence to measure the spectral flux density a new unit is introduced. The unit is called the Jansky (Jy), and 1 Jy is equal to 10^{-26} W/m^{2}Hz.

2. Interferometry and Interferometer Array

Measuring the flux from a series of directions over a particular region can then produce a map of the sky brightness.

For the observations carried out at wavelength λ, the angular resolution is given by $\theta = \lambda/D$, where D is the diameter of the dish of the telescope. As the diameter of the dish increases, the total power at the receiver and the angular resolution increases. But this is accompanied by an increase in the manufacturing cost and complexity. Moreover, there is a practical limit to the size of the dish. For instance, if we consider observing the wavelength of 1 m with a dish 100 m in diameter, the corresponding θ comes out to be approximately 0.5 degrees. Hence, the angular resolution offered by a single radio telescope is insufficient in the context of point like objects such as stars which require an angular resolution of the order of few arc minutes or few arc seconds. In fact, the same order of angular resolution in both radio and optical domains is needed. To achieve such a high resolution, two or more telescopes are linked forming an interferometer arrangement.

The modified angular resolution is then given as $\theta = \lambda/B$, where B is called as the baseline[1]. A typical two-element interferometer consists of two antennae separated by a distance B and connected to a correlator which consists of a voltage multiplier and an integrator as shown in *Figure* 1. The signal received at one antenna is

[1] The distance between the two interferometer elements.

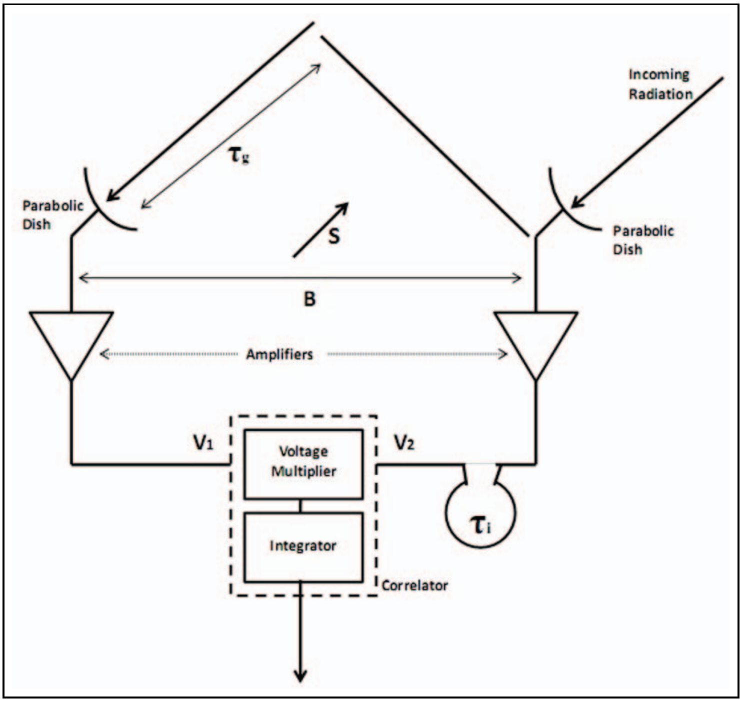

Figure 1. Correlator circuit for two-elements interferometer.

delayed by a time interval τ from the signal of the other antenna. V_1 and V_2 are the voltages at the individual receiver outputs and τ_i is the instrumental delay which offsets the geometrical delay τ_g between the two telescopes. From *Figure* 1, we can see that the geometrical delay is $\tau_g = (\mathbf{B}.\mathbf{s})/c$, c being the speed of light in air. With the rotation of the Earth, the direction of **s** changes slowly with time. The resultant delay $\tau = \tau_g - \tau_i$ will vary, and the output of the integrator is thus a series of maxima and minima corresponding to the superposition of the incoming waves from the source as shown in *Figure* 2.

In the correlator circuit, interferometer signals are input to a multiplying device followed by a low pass filter such that the output is proportional to:

$$R(\tau) = V_1\, V_2\, e^{i\omega\tau}. \quad (1)$$

Thus, the output of the correlator varies periodically with τ. This output is the mutual coherence function of the received wave. If the radio brightness distribution is given by $I_\nu(\mathbf{s})$, the power re-

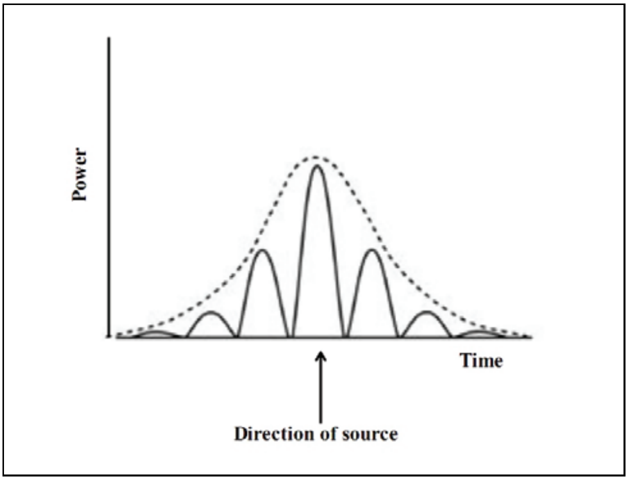

Figure 2. Interference of two signals.

ceived per bandwidth dν from the source element dΩ is given by $A(\mathbf{s})\, I_\nu(\mathbf{s})\, \mathrm{d}\nu\, \mathrm{d}\Omega$; where, $A(\mathbf{s})$ is the effective collecting area of the antenna (assumed same for both the antennae). Then the output of the correlator for radiation from the direction **s** is given by:

$$r_{12} = A(\mathbf{s})\, I_\nu\, (\mathbf{s})\, e^{i\omega\tau} \mathrm{d}\mathbf{s}\, \mathrm{d}\nu. \tag{2}$$

Therefore, the total response is obtained by integrating over the source, assuming that the effective collecting area (A) does not change over the source expanse:

$$R(\tau) = A \int I_\nu(\mathbf{s})\, e^{i\omega\tau} \mathrm{d}\mathbf{s}\, \mathrm{d}\nu. \tag{3}$$

$R(\tau)$ can be measured experimentally. Equation (3) is a Fourier transform between $R(\tau)$ and the brightness distribution $I_\nu(\mathbf{s})$. Thus, by applying Fourier transform to the mutual coherence function $R(\tau)$, we can find out the intensity distribution I_ν for the given source.

The radio sources have complicated shapes, with both large and small-scale structure. Better images of observed sources can be formed if more than two dishes are used at once. In an array of N antennae, the number of possible pairs (i.e. the number of two-element interferometers) is $\frac{N(N-1)}{2}$. More powerful computers can

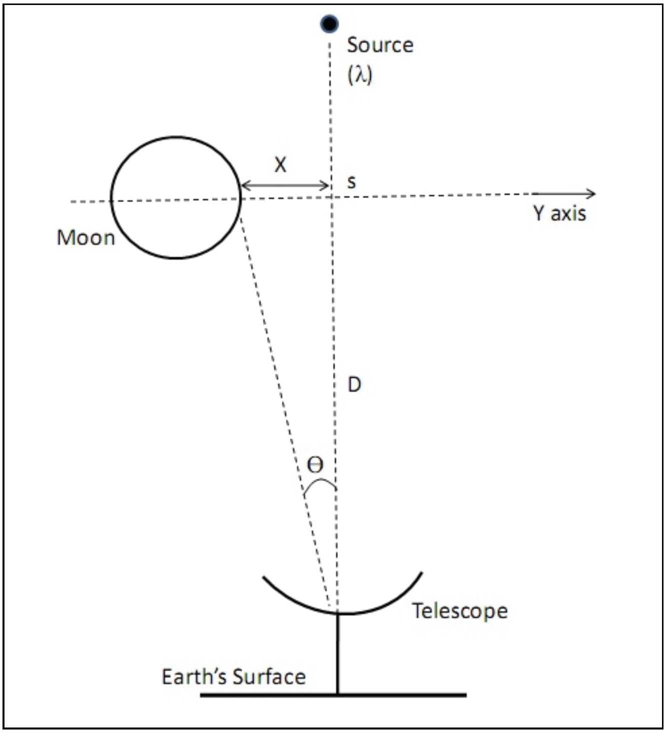

Figure 3. Lunar occultation (situation before immersion).

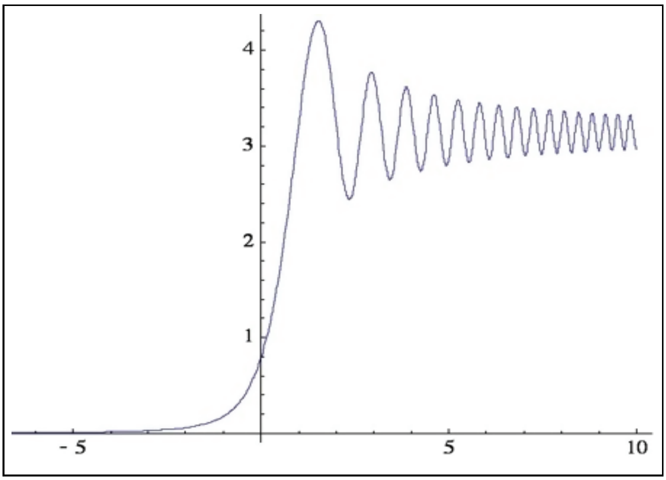

Figure 4. Fresnel diffraction at a straight edge (numerically generated with the help of Cornu spiral).

combine signals from more dishes electronically. Using many dishes together in an interferometer array allows the astronomers to arrive at more complete images of objects. The angular resolution can be increased tremendously by forming a very large baseline interferometer (VLBI) where radio telescopes separated by thousands of kilometers are used at the same time for observations. The signals are received and amplified at the participating antennae and are digitized and sent to a correlator. The correlator cross-correlates and Fourier transforms the signals from each pair of antennae, and the result of this process can be used to determine the brightness distribution of the sky at radio frequencies.

3. Lunar Occultation

To achieve a higher resolution, we thus usually opt for the interferometric arrays. However, by investigating the behavior of power received at a single antenna, as a radio source is occulted[2] by the Moon, it is possible even at meter wavelengths to achieve a resolution of a fraction of an arc sec. In principle, the beam of the antenna is simply directed to the source and the received power is recorded continuously as the source first passes behind the Moon (immersion) and then reappears (emersion). *Figure* 3 depicts the situation.

[2]In a lunar occultation, the Moon passes in front of a star or planet, thereby, hiding the view of the object.

The occultation technique is concerned only with the sources whose angular structures are much smaller than the angular size of the moon. The variation in the received power, while such a source passes behind the moon depends on the size of a Fresnel zone[3]. If the source size is much greater than the size of the Fresnel zone, diffraction effects can be neglected. In this case, the shape of the occultation curve represents the strip integral of the brightness distribution in a direction perpendicular to the limb at the point of occultation. For the sources containing angular structures measured in seconds of arc, the diffraction effects at the limb must be taken into account.

[3]In this context, Fresnel zone is a region between concentric rings in the aperture plane (at the Moon's distance) over which the phase varies at the antenna by π.

Relative to the size of a Fresnel zone, the curvature of the Moon's limb is small. Thus, for an ideal Moon[4], the occultation curves may be considered as the diffraction curves at a straight edge, provided the irregularities on the surface (the lunar mountains) are much smaller than the first Fresnel zone.

[4]Ideal Moon implies that the irregularities on the surface of the moon are neglected, and the edge of the Moon is then smooth and sharp, which is an 'ideal' situation.

The response R at the antenna is given by the equation:

$$R = \int_{-x}^{\infty} exp(\frac{\pi i y^2}{D\lambda})\,\mathrm{dy}. \quad (4)$$

Lunar occultation is useful for sources along the ecliptic, which are smaller in spread as compared to the Moon.

The power received at the antenna is $P = R^2$, which is geometrically equivalent to the Fresnel diffraction curve due to a straight edge as shown in *Figure* 4. The intensity distribution of the radio source is $I(\theta)$ = convolution of P with some pre-defined restoring function.

Suggested Reading

[1] J D Kraus, *Radio Astronomy*, Cygnus-Quasar Books, 1986.

[2] M Born and E Wolf, *Principles of Optics*, Pergamon Press, 1970.

[3] A Thompson, J Moran and G Swenson Jr., *Interferometry and Synthesis in Radio Astronomy*, Wliey-VCH, 2001.

[4] R Bracewell, *The Fourier Transform and its Applications*, McGraw-Hill, 1999.

[5] K Rohlf, *Tools of Radio Astronomy*, Springer, 2004.

[6] G Swarup, S Ananthakrishnan, V K Kapahi, A P Rao, C R Subrahmanya and V K Kulkarni, The Giant Metre-wave Telescope, *Cur. Sci.*, Vol.60, No.2, p.95, 1991.

[7] G Swarup, N V G Sarma, M N Joshi, V K Kapahi, D S Bagri, S H Damle, S Ananthakrishnan, V Balasubramanian, S S Bhave, R P Sinha, Large Steerable Radio Telescope at Ootacamund, India, *Nature Phy. Sci.*, Vol.230, p.185, 1971.

[8] A K Singhal, Ooty Lunar Occultation Survey of Radio Sources, *Astronomy and Astrophysics Supp. Series*, Vol.69, No.1, p.91, 1987.

[9] P A G Scheuer, On the Use of Lunar Occultations for Investigating the Angular Structure of Radio Sources, *Aus. J. Phy.*, Vol.15, 333, 1962.

[10] K Lang, Theory of Lunar Occultation: New Methods of Estimating Brightness Distributions and Their Widths, *Astrophys. J.*, 158, 1189, 1969.

[11] C Hazard, *Methods in Experimental Physics*, Vol.12, Part C, Chapter 4.6, p.92, 1976.

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