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Understanding the Physics of a Possible Non-Abelian Fractional Quantum Hall Effect State

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Understanding the Physics of a Possible Non-Abelian Fractional Quantum Hall Effect State

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Abstract

We wish to present in this report experimental results from a one-year Senior Council Tier-1 LDRD project that focused on understanding the physics of a possible non-Abelian fractional quantum Hall effect state. We first give a general introduction to the quantum Hall effect, and then present the experimental results on the edge-state transport in a special fractional quantum Hall effect state at Landau level filling $\nu=5/2$ – a possible non-Abelian quantum Hall state. This state has been at the center of current basic research due to its potential applications in fault-resistant topological quantum computation. We will also describe the semiconductor “Hall-bar” devices we used in this project.

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1. INTRODUCTION

Electron physics in low dimensional systems has been one of the most exciting fields in condensed matter physics for many years. This is especially true of quantum Hall effect (QHE) physics [1,2], which has seen its intellectual wealth applied in and has influenced many seemingly unrelated fields, such as the black hole physics, where a fractional QHE-like phase has been identified [3]. Two Nobel prizes have been awarded for discoveries of quantum Hall effects: in 1985 to von Klitzing for the discovery of integer QHE [4], and in 1998 to Tsui, Stormer, and Laughlin for the discovery of fractional QHE [5]. Today, QH physics remains one of the most vibrant research fields, and many unexpected novel quantum states continue to be discovered and to surprise us, such as utilizing an exotic, non-Abelian FQHE state at $\nu=5/2$ [6] for fault resistant topological computation. Below we give a briefly introduction of the quantum Hall physics.

1.1. Two-dimensional electron systems

At the present time, high quality two-dimensional electron gas (2DEG) can be routinely achieved in modulation doped AlGaAs/GaAs heterostructures [7]. Figure 1 shows a schematic of such a structure. MBE technique is used for growth. In this structure, the GaAs is undoped and its Fermi level is roughly at the middle of its band gap. The AlGaAs is deliberately doped (n-type as shown), and its Fermi level moves closer to its conduction band. Due to this non-equilibrium condition, electrons migrate from the AlGaAs to GaAs and populate the GaAs conduction band. When the system reaches the thermal equilibrium state, a thin layer of 2DEG is formed at the interface.

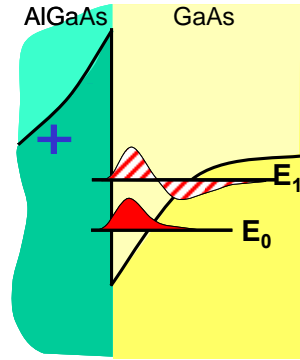


Figure 1: Modulation doped GaAs/AlGaAs heterostructures.

Due to the so-formed potential well near the interface, the energy levels in the z -direction (or the growth direction) are quantized. The difference between the lowest energy level E_0 and the second lowest level E_1 is around 20 meV or 200K, much larger than the typical temperatures, i.e., $< 4K$, at which quantum transport studies are carried out. In this regard, it is a good approximation to neglect the electron motion in the z -direction. Only the motion in the x - y plane matters to quantum transport. This gives rise to the term we frequently use in this article, the two-dimensionality.

The later inventions of δ -doping scheme and quantum well structures further improve the sample quality. Extremely high electron mobility, e.g., $\mu=38\times10^6$ cm²/Vs has been achieved by Loren Pfeiffer now at Princeton University. In this highest specimen, the mean free path of electron exceeds 0.3 mm.

Recently, a theoretical paper by Hwang and Das Sarma [8] explored the possibility of achieving even high mobility, up to 100×10^6 cm²/Vs, in modulation doped quantum well structures.

1.2. Integer quantum Hall effect

The Hall effect was discovered by Edwin Hall in 1879 while doing his thesis work at John Hopkins University [9]. In his original experiment, a current was passed from one end of a conductor, for example a slab of metal, to the other end in a magnetic (B) field perpendicular to the slab of metal. He observed a voltage buildup in the plane of metal at right angles to the current path. The so-defined Hall voltage is linear with B. The measurement setup and the linear B field dependence is shown in Figure 2. Later on, the Hall effect was explained by invoking the so-called Lorentz force which is generated on moving charged carriers by the applied magnetic field. The Hall effect has now widely used to determine the density of charge carriers in materials of conductor and semi-conductor.

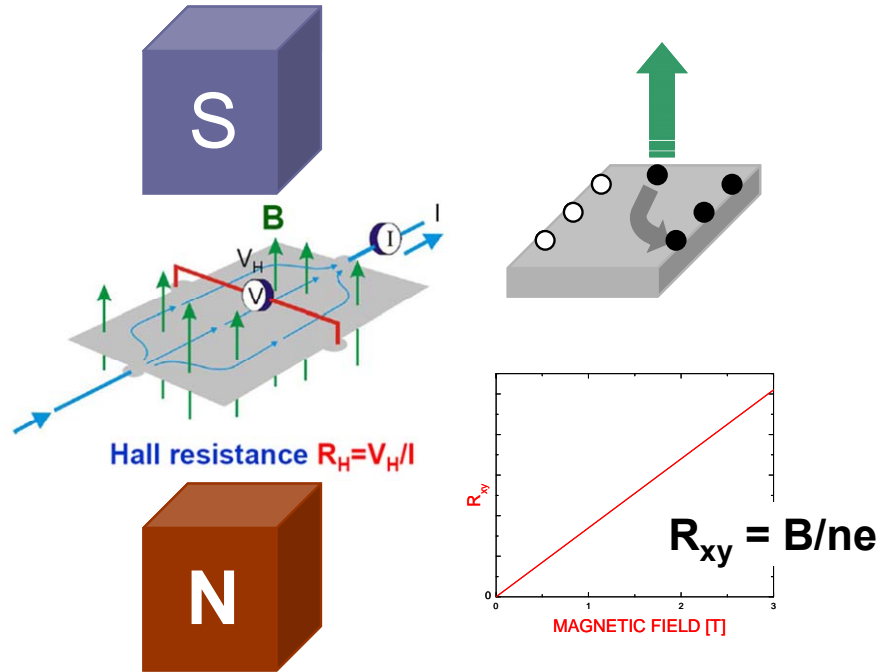


Figure 2: Hall measurement setup and classical Hall effect.

With the advent of high quality two-dimensional electron systems, the Hall effect was re-examined by various groups in later 70's of the last century. In 1980, Dr. von Klitzing and his colleagues discovered the integer quantum Hall effect in a high quality Si-MOSFET [1]. In this new quantum regime, the Hall resistance would display a precisely quantized Hall plateau and the diagonal resistance a vanishingly small value whenever the Landau level filling factor, defined by $\nu=n\hbar/eB$, is an integer. The quantized Hall value is given by $\hbar/e^2/\nu$.

The physical origin of IQHE can be in general understood by invoking the Landau level quantization and disorder broadening. For a two-dimensional electron system, at zero magnetic field, the density of states (DOS), $D(E)$, is constant in the k -space, and $D(E) = m/\pi\hbar^2$. With the application of an external magnetic field perpendicular to the plane, this continuous DOS breaks into discrete, δ -shaped Landau levels at the energies $E_n = (n+1/2)\hbar\omega_c$. Here $n = 0, 1, 2, \dots$ is the Landau level number and $\omega_c = eB/m$ is the cyclotron energy. A gap thus forms between two adjacent two Landau levels, given by $\Delta = \hbar\omega_c$, i.e., the cyclotron energy. In real samples, however, disorder exists. As a result, the δ -shaped Landau level is broadened into a Landau band, with delocalized states in the middle of the band and elsewhere localized, as shown in Fig.3.

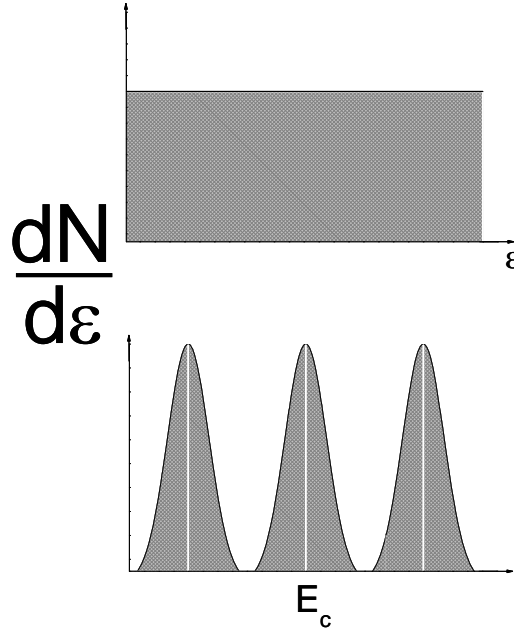


Figure 3: density of state at $B = 0$ (top) and in magnetic fields (bottom).

Now, let's assume that the Fermi level is in the center of energy gap and at an integer filling factor $\nu = nh/eB$ (the Landau level filling can also be written as $\nu = n/n_B$, where $n_B = eB/h$ is the degeneracy of each Landau level. In other words, ν can be viewed as a counting number of how many Landau levels are filled below the Fermi level). At this position, the occupied Landau levels are separated from the empty levels by an energy gap, which is much larger than $k_B T$. According to the Pauli principle, under this condition, no scattering processes are possible. Under this condition, $R_{xx} = 0$, $R_{xy} = B/ne = h/e^2/\nu$ is quantized. Next, when the Fermi level is moved into the delocalized states in the middle of Landau band, the R_{xx} assumes finite resistance and the R_{xy} jumps from one quantized value to another.

One important feature associated with a quantum Hall effect is its energy gap, i.e., the energy to excite a quasiparticle from the ground state to the next un-occupied state. Its value can be determined by measuring the temperature dependence of R_{xx} . At $T = 0$, all the electrons are in their ground state and there are no excitations. As T is increased, a certain amount of quasiparticles $N \propto \exp(-\Delta/2 k_B T)$ are thermally excited from the ground state to the previously

un-occupied up-empty state. Due to the existence of these quasiparticles, R_{xx} assumes a finite value, and under the Drude model, $R_{xx} \propto N \propto \exp(-\Delta/2 k_B T)$. In other word, fitting the temperature dependence of R_{xx} then would yield a measurement of energy gap associated with a particular quantum Hall state.

1.3. Edge states

The integer quantum Hal effect, or in general, the quantum Hall effects, can also be understood under the edge transport picture [10]. Under this picture, at the edges of the sample, the energy eigenvalues increase near the edges. Figure 4 shows the energy levels for several Landau quantum numbers with infinite potentials at the edges [11].

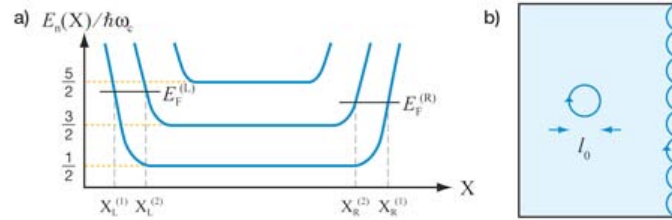


Figure 4: edge channel and electron skipping motion, adopted from Ref. [11].

The current-carrying states at the edges are called edge channels and are shown schematically as simple lines with arrows, where the arrow marks the direction of the edge transport velocity. Figure 5 gives an example of the edge states or edge channels in a Hall-bar geometry with two levels occupied. The edge channels are one-dimensional channels as shown above. Transport in one-dimensional channels has been treated by Büttiker [12], and is depicted by a general description in terms of global conductances.

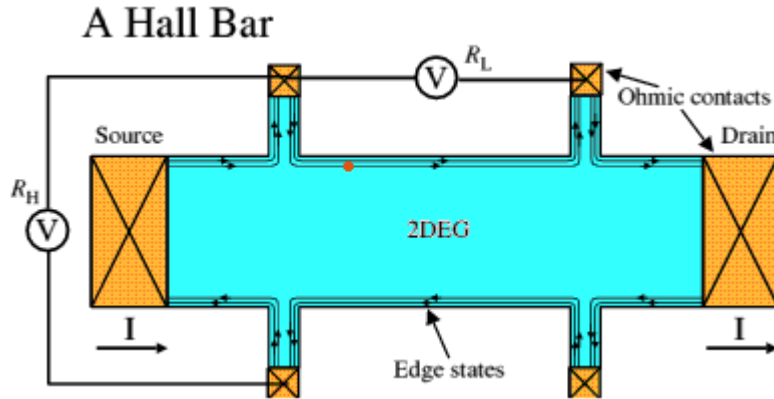


Figure 5: Edge channels in a Hall bar device. Figure copied from the website www.sp.phy.cam.ac.uk/SPWeb/research/QHE.html

Within this edge-state picture, a single elastic scattering event at one edge does not lead to a backscattering due to the chiral cyclotron motion, if the sample width is large enough. This is the reason why the IQHE is so robust against disorder.

1.4. Fractional quantum Hall effect

Research on the fractional quantum Hall effect in two-dimensional electron systems (2DESs) really took off after the inventions of molecular beam epitaxy (MBE) and the modulation doping technique for the growth of GaAs/AlGaAs heterostructures. For the first time, 2DES electron mobilities could exceed $100,000 \text{ cm}^2/\text{Vs}$. In 1982, Tsui, Stormer, and Gossard reported the discovery of the fractional quantum Hall effect (FQHE) state at Landau level filling factor $\nu=1/3$ [2], when the lowest Landau level is only partially filled.

This new QH state was totally unexpected. It soon became clear that its origin is purely due to electron-electron interactions and cannot be understood under any single particle picture. Its driving force is the reduction of Coulomb interaction between the like-charged electrons. Laughlin's wavefunction [13] ingeniously identified that the $\nu=1/3$ is solely due to the formation of a new incompressible many-body liquid, with quasiparticles bearing a $1/3$ fractional charge.

In the following, we cite a physical picture [14] of the FQHE at $\nu=1/3$ by H.L. Stormer, one of the three laureates awarded the Nobel Physics prize in 1998 for the discovery of the fractional quantum Hall effect. Here, Stormer et al discussed the picture of the attachment of magnetic vortices to electrons that has become the unifying principle underlying the multiple many-particle states of the FQHE.

'The presence of the magnetic field requires the many electron wave function to assume as many zeroes within a unit area as there are magnetic flux quanta penetrating it. Each zero "heals" on the scale of a magnetic length and, limiting ourselves to the lowest Landau level, each such "hole" in the electron sheet represents an overall charge deficit of e . Since the magnetic field also imparts a 2π phase twist to the wave function at the position of each such zero, these objects are termed vortices. In a certain sense, vortices are the embodiment of flux quanta in an electron system. A tiny coil threaded through the plane of the electrons and energized to generate just one magnetic flux quantum through its core would create one such vortex. Therefore, loosely speaking, vortices are often equated with flux quanta. Just like electrons, vortices are delocalized in the plane. However, since electrons represent a charge accumulation and vortices a charge deficit, they attract each other. Considerable Coulomb energy can be gained by placing vortices onto electrons. At $\nu=1/3$ there exist three times as many vortices as there are electrons, each vortex representing a local charge deficit of $e/3$. Each electron must carry at least one vortex equivalent to one zero in the wave function to satisfy the Pauli principle. Additional vortex attachment is "optional," driven by Coulomb gain. Vortex attachment to an electron, representing a local depletion of companion electrons, is always energetically beneficial. The situation is somewhat reminiscent of the screening cloud around an electron in a regular metal, although in the case of the FQHE, such "screening" is very rigid and quantized in units of vortex charge. The attachment of exactly three vortices to each electron is at the origin of the prominent $\nu=1/3$ FQHE state expressed by Laughlin's wave function.'

Continued efforts to improve MBE growth conditions resulted in dramatic advances in sample quality, with electron mobility reaching $500,000 \text{ cm}^2/\text{Vs}$ by 1984. These samples, the highest quality attained at the time, lead to the discovery of a hierarchy of FQHE states at Landau level filling factors $\nu=p/(2p+1)$ (p is integer) and later, at $\nu=1/5$ in even higher mobility samples [15].

This trend has continued to this day. Figure 6 shows the most current status of known FQHE states, from an extremely high quality quantum well sample [16]. Fragile FQHE states are observed at Landau level fillings as high as $\nu=10/21$ and $10/19$ around $\nu=1/2$, and at $\nu=6/25$ and $6/23$ around $\nu=1/4$. All these newly discovered FQHE states arise from the condensation of electrons into highly-correlated many-body states, and most of them are very accurately described by Laughlin's wavefunctions and a hierarchical model. In recent years, the successful composite fermion (CF) model [17-19] has mapped the FQHE states of electrons onto the integer quantum Hall effect (IQHE) states of CFs, providing a unified view of the FQHE and IQHE. In detail, it has been shown [17, 18] that a useful way to understand the system theoretically is to consider the dynamics of composite particles each consisting of one electron and two flux quanta. These composite particles obey fermion statistics, earning them the name “composite fermions”, and it can be shown that at the mean-field level, the $\nu=1/2$ system is equivalent to a system of composite fermions in zero magnetic field. When B field is away from the B field at $\nu=1/2$, the Landau levels of CFs are formed. This gives rise the IQHE of CFs. A simple mapping shows that the IQHE state of CFs at $\nu^*=p$ corresponds to the FQHE state of electrons at Landau level filling $\nu=p/(2p+1)$.

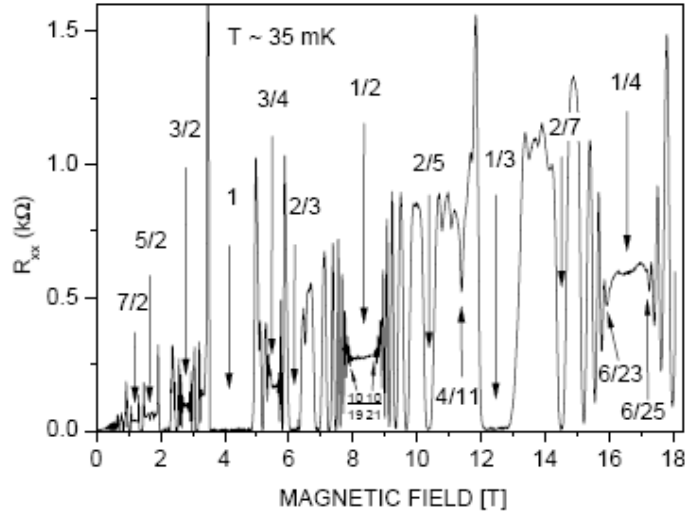


Figure 6: Diagonal resistance of an extremely high quality 2DES sample, showing a large number of FQHE states. States as high in the hierarchy as $10/19$, $10/21$, $6/23$, and $6/25$ can be seen.

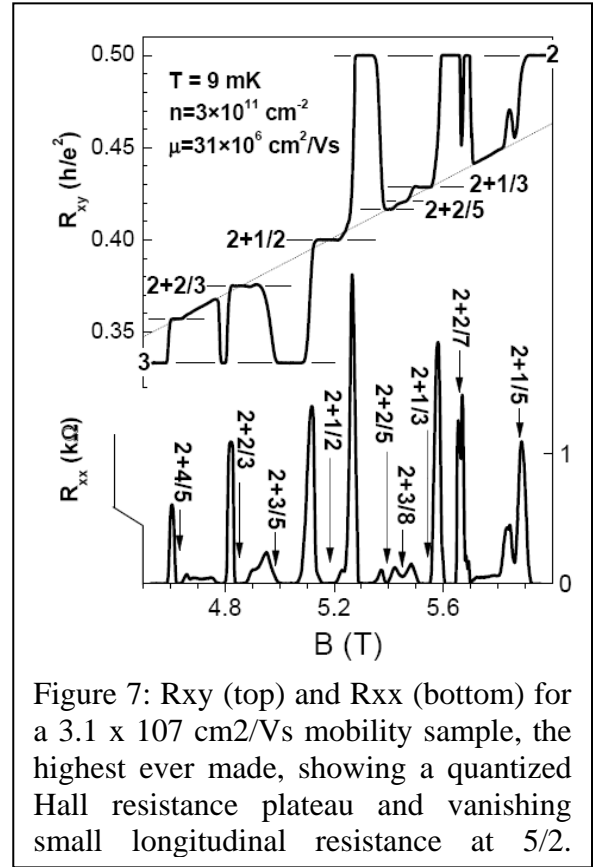
In the following, we will focus on the so-called $5/2$ state in the second Landau level. The FQHE state at $\nu=5/2$ is exotic. Unlike conventional FQHE states, for example, at $\nu=1/3$ and $1/5$, this $5/2$ state may belong to the so-called *non-Abelian QH states* [6]. In this new state of quantum matter, when a quasiparticle makes a full circle around another, the quantum state vector changes its direction in Hilbert space [20] and the order of exchange of particles is relevant. Quasiparticles which behave this way are said to obey *non-abelian statistics*, and never before been definitely observed. It is now widely believed that the non-abelian topological phases may hold the key for a future highly fault-resistant topological quantum computation [21].

1.5. Even-denominator quantum Hall state at $\nu=5/2$

Sometime between 1986-1987, the first GaAs/AlGaAs heterostructure samples surpassing 1 million mobility ($10^6 \text{ cm}^2/\text{Vs}$) were produced, ushering in the new era of non-abelian quantum Hall physics. In a highly quoted paper, Willett et al [22] reported the first evidence of an even-denominator FQHE state at $\nu=5/2$ in the first excited Landau level. In contrast to the featureless Hall resistance (R_{xy}) and largely temperature independent magneto-resistance (R_{xx}) observed at $\nu=1/2$ in the lowest Landau level, $\nu = 5/2$ showed a developing plateau in R_{xy} and a deep temperature-dependent minimum in R_{xx} , unmistakable signatures of a FQHE state. This FQHE state disobeys the usual odd-denominator rule set by the Laughlin's wavefunction and thus cannot be explained by either Laughlin's theory or the hierarchical model. With the advent of composite fermion model a new interpretation of the states at $\nu = 5/2$ (and at $\nu=7/2$, its particle-hole conjugate state) has arisen [23]. Very recent numerical calculations seem to favor a BCS like ground state of p-wave pairs of CF's [24].

In the earliest experiment by Willett et al [22], although R_{xx} showed a deep minimum at $\nu=5/2$, it was far from vanishingly small, even at the lowest experimental temperature of 20 mK. Further, the developing Hall plateau was far from fully and precisely quantized. Consequently, there had been some doubt about whether the $\nu=5/2$ state was a true quantum Hall state. It was only 12 years later with the advent of a very clean sample with an electron mobility of 17 million cm^2/Vs , that the $\nu=5/2$ state could be confirmed as a true quantum Hall state [25]. Pan et al. showed that for this sample, at an electron temperature of $\sim 4 \text{ mK}$, R_{xx} is vanishingly small and R_{xy} is fully and precisely quantized [25]. Furthermore, R_{xx} at $\nu = 5/2$ showed a true activated behavior with an energy gap of $\sim 110 \text{ mK}$. Later, in one of the highest quality samples with an electron mobility of 31 million cm^2/Vs , the $\nu=5/2$ state, as shown in Figure 7, displayed a much stronger energy gap of $\sim 0.5 \text{ K}$ [26].

The FQHE state at $\nu=5/2$ are special. Unlike conventional QH states at $\nu=p/(2p\pm1)$, it is believed that they belong to the so-called *non-abelian QH states*. In this new state of matter, when a quasiparticle makes a full circle around another, the quantum state vector changes its direction in Hilbert space [27]. In other words, *the quasiparticles obey non-abelian statistics*. It has been shown theoretically that non-abelian topological phases may hold the key for the future fault-tolerant topological quantum computation [21].



At filling fraction $\nu=5/2$, the first Landau level is completely filled (both spin up and spin down electrons) and the second Landau level is at filling $1/2$ so that there are two flux quanta per electron. We ignore the details of the first Landau level, and assume that those electrons simply renormalize interactions among the second Landau level electrons. In other words, a CF sea state is arrived. As supported by numerical results on finite systems, in the second Landau level, the composite fermions can form Cooper pairs and Bose condense into a superfluid. If the latter scenario occurs, excitations of the $\nu=5/2$ system are vortices in the composite fermion picture, similar to those in a BCS superconductor. Each vortex provides a zero-energy Majorana mode, has fractional charge $e/4$, and behaves like a σ anyon in the Ising anyon theory. Two vortices together have charge $e/2$ and behave as a 1 or ψ -type anyon depending on the presence or absence of the neutral fermion mode resulting from the two Majorana modes. Thus, we expect that Ising anyons exist as quasiparticle excitations in the bulk of a $\nu=5/2$ FQHE state. The low-energy edge physics of the $\nu=5/2$ state are described by a conformal field theory which we do not detail here. The primary result germane to this work is that along the $\nu=5/2$ edge there are gapless current carrying $e/4$ (σ) modes and $e/2$ (1 and ψ) modes. [We thank Erik Nielsen for this paragraph]

1.6. Current experiments on the $\nu=5/2$ state

1.6.1. Spin polarization of the $5/2$ state

Under the above picture of pairing of composite fermions (CFs), the ground state at $\nu=5/2$ is expected to be spin-polarized. Yet, even this relatively simple property has not been totally substantiated experimentally. The earlier effort on examining the spin polarization of the $5/2$ FQHE state has been on using the tilt magnetic field technique. In the first experiment of this kind [27], the strength of the $5/2$ state and its quasi energy gap were observed to decrease with increasing tilt angle, similar to that in an unpolarized FQHE state [28,29]. Consequently, this result was taken as evidence for a spin singlet $5/2$ state. Since 1991, however, a spin-polarized ground state at $\nu=5/2$ has gained more and more support from numerical calculations [24]. Yet, the apparent contradiction between a spin-polarized ground state and the fact that the $5/2$ state can be destroyed by a tilt magnetic field was unresolved, until in 1999 when two groups studied the influence of in-plane magnetic field on the $5/2$ state and observed that the disappearance of R_{xx} (diagonal resistance) minimum at $\nu=5/2$ under tilt magnetic fields was associated with the formation of an anisotropic phase [30,31]. A theoretical study by Rezayi and Haldane [32] confirmed this transition and they further showed that this phase transition can be viewed as evidence of supporting a spin-polarized ground state at $\nu=5/2$ and should be first order.

Alternatively, the spin-polarization of the $5/2$ state can also be pursued by investigating the competition between Coulomb energy E_c and Zeeman energy E_z through a two-dimensional electron system (2DES) density dependent study. Since $E_c \propto n^{1/2}$, whereas $E_z \propto n$, increasing electron density modifies the ratio of E_z to E_c . This approach is equivalent to tilting the sample, but it cannot cause a tilt-field induced transition to the anisotropic phase. An earlier study [33] on the $5/2$ state was conducted in a GaAs transistor device or heterojunction insulated-gate field-effect transistor (HIGFET) that was of moderate quality with a peak mobility $\mu = 5.5 \times 10^6$ cm²/Vs. In this earlier study, it was observed that the quasi-energy gap at $\nu=5/2$ varies smoothly with the 2DES density and shows a weak electron mobility dependence. These results were taken

as evidence as supporting a spin polarized $5/2$ ground state. However, due to the usage of quasi-energy gap, the conclusion was questioned. In recent years, sample quality of HIGFET has been improved greatly. In a recently fabricated one an electron mobility $\mu \sim 14 \times 10^6 \text{ cm}^2/\text{Vs}$, more than a factor of two increase compared to the sample used in the earlier study, was achieved. With this high mobility specimen, the spin polarization of the $\nu=5/2$ state was re-examined. In detail, the density dependence of the activation energy gap at Landau level filling $\nu=5/2$ was measured in a large density range. It is observed that the true activation energy gap, $\Delta_{5/2}$, obtained from the temperature dependence of the R_{xx} minimum at $\nu=5/2$, increases with increasing density from $n = 1.2 \times 10^{11}$ to $4 \times 10^{11} \text{ cm}^{-2}$. Fitting to this density dependence shows that this density dependent result is consistent with a spin polarized $5/2$ state.

Resistively detected – nuclear magnetic resonance (RD-NMR) technique has been utilized to examine the spin polarization at $\nu=5/2$ by a few groups. The conventional RD-NMR, however, is now deemed non-suitable for the $5/2$ state due to a small gap of this state and possible electron heating induced by applying continuous microwave irradiation. Recent, a group in Japan has applied the so-called pump-probe RD-NMR. This technique seems to reduce the electron heating to its minimum. Consequently, this group was able to measure the Knight Shift at $\nu=5/2$ and thus to determine the spin polarization of the $5/2$ state.

1.6.2. Interferometry experiments

Interferometer experiments have also been performed by a few groups to examine the non-Abelian statistics of the $5/2$ state. So far, no concrete evidence has been agreed upon by the whole community. In the following, we briefly describe the measurements carried out by various groups.

Willett et al at Bell Labs of Alcatel-Lucent, Inc. [34]: this group has been focusing on the interference measurements in quantum interferometers fabricated on ultra-high mobility 2DES grown by Dr. Loren Pfeiffer and his group now at Princeton University. Around $\nu=5/2$, by sweeping magnetic field, two quantum oscillations, with periods of $e/4$ and $e/2$, were observed. This result has received wide interests.

Heiblum group at Weitzman Institute of Technology [35]: They carried out short noise measurements in quantum point contact device structures. This technique can give information such as the quasiparticle charge of a fractional quantum Hall state. However, it does not provide information on the statistics of quasiparticles.

A recent paper by this group [36] reported the observation of the neutral mode of the $5/2$ edge state.

Marcus's group at Harvard University [37]: they have shown that they could tune their devices into two different regimes: A-B and Coulomb blockade regimes. However, this group so far has not seen the $e/4$ and $e/2$ oscillations.

Goldman's group at The State University of New York at Stony Brook [38]: This group concentrates on the interferometer measurements in the conventional fractional quantum Hall regime. No results on the $5/2$ state have been reported by this group.

Woowon Kang's group at the University of Chicago [39]: this group recently reported the observation of the so-called telegraph noise at $\nu=5/2$ as a function of time. Their results seem to be consistent with a theoretical model that predicts this type of noise features under the non-Abelian statistics picture. No $e/4$ and $e/2$ oscillations have been reported by this group.

1.7. Other possible non-Abelian quantum Hall states

Up to date, theoretical and experimental work on the topological FQHE states mostly focuses on the $5/2$ state, due to its relatively large energy gap. Work on another possible non-Abelian state, the $12/5$ state, is considerably less. This, partially, is due to its extremely small energy gap. Even in the highest quality sample of 31 million cm^2/Vs , the energy gap (Δ) of the $12/5$ state is only ~ 70 mK [xia04]. Consequently, at operation temperatures of ~ 10 mK, the error rate [40] is $\sim T/\Delta \times \exp(-\Delta/T) \sim 10^{-4}$, unacceptably high. Thus, to be able to ultimately manipulate the $12/5$ state in topological quantum computation, its energy gap has to be made larger. It is clear that only 2DES material with mobility substantially higher than 31 million cm^2/Vs has any hope of meeting this goal.

Beyond the $5/2$ and $12/5$ states, there are a few studies on other possible non-Abelian FQHE states, for example, at $\nu=4/11$ [41] and $1/4$ in the lowest Landau level in high quality wide quantum well systems [42, 43]. Further detailed studies on these possible candidates would require much high sample quality.

2. PROPOSED RESEARCH IN THIS PROJECT

The ultimate goal of this project is to understand a special class of quantum Hall effect states, a possible non-Abelian fractional quantum Hall effect (FQHE) state at the Landau level filling $\nu=5/2$.

It has been a daunting challenge to measure and examine this exotic non-Abelian statistics. So far, there is no concrete evidence showing that the quasiparticles of the $5/2$ state do obey non-Abelian statistics. Several device structures have been proposed in tackling this challenge. Among them, edge tunneling in a confined geometry (for example, a quantum point contact shown in Fig.8a) is a promising one for detecting both the (non-integer) charge of the quasiparticle and the possible non-Abelian statistics. For a correlated electron state (in our case, the $5/2$ state), charge excitations within the confined region (for example, edge tunneling) provide an experimentally accessible avenue for measuring quasiparticle properties.

These kinds of experiments have been carried out by very few groups [47,35,34]. However, their data were not definitive. More data with less ambiguity are needed to determine unequivocally the nature of the $5/2$ state. To our best knowledge, this ambiguity may be due to a lack of knowledge of the degree of coupling between the bulk state and the edge state at $\nu=5/2$. If there is a coupling, what is its quantitative strength? So far, neither of these two questions has been answered nor even studied.

In this proposed project, we study in detail the coupling between the edge states and the bulk states at Landau filling factor $\nu=5/2$, through two types of experiments.

First, we study magneto-resistance fluctuations in narrow channels (as shown in Fig.8b). It has been shown [46] that in these narrow channel devices conductance fluctuations are due to quasiparticle interference when the sample size is on the order of the so-called phase coherence length, and the amplitude and period of the fluctuations can give a measure of the charges of the quasi-particles.

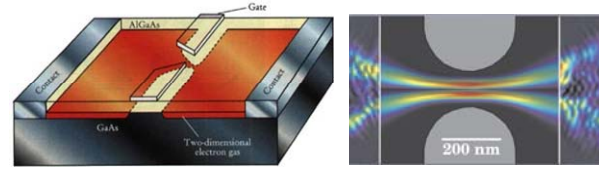


Figure 8(a) shows a schematic cross-sectional view of a quantum point contact (QPC). QPC is formed when a negative voltage is applied to the gate electrodes (Adapted from Ref. [44]). Right panel shows simulations and experimental images of the spatial pattern of coherent electron flow through a QPC (Adapted from Ref. [45]). In the quantum Hall regime, this electron flow would represent QH edge channels.

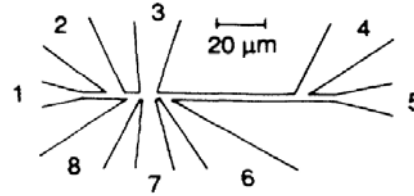


Figure 8(b) shows sample geometry of a narrow Hall bar. The width of the center narrow channel is $2.5\mu\text{m}$. (Adapted from Ref. [46])

Second, we study the non-local and non-linear magneto-resistance around $\nu=5/2$. We will follow the methodology developed in a past experiment on the non-local, non-linear magneto-resistance in the regime of usual FQHE states, which clearly showed a picture of electronic transport via co-existing edge and bulk states [48]. We believe that this study will allow us to obtain a good understanding of the electronic transport at $\nu=5/2$, especially, on the scattering of edge current, and the distribution of edge current into the bulk at Ohmic contacts on the sample periphery. We will apply the bulk-edge transport model in analyzing our data. The temperature dependence of the scattering between edge and bulk currents will also be studied.

With a better understanding of the edge state at $\nu=5/2$ from the above proposed two experiments we hope to differentiate the different contributions to the signal, be they from edge states or from the bulk. In so doing, we might be able to remove the ambiguity seen in the past experiments by other groups.

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3. EXPERIMENTAL RESULTS AND DISCUSSIONS

3.1. Magneto-resistance fluctuations in narrow channel

We follow the methodology developed by Simmons et al [46], and fabricated a narrow channel Hall bar of similar geometry, as shown in Figure 9. The width of Hall bar in this particular device is $20\text{ }\mu\text{m}$.

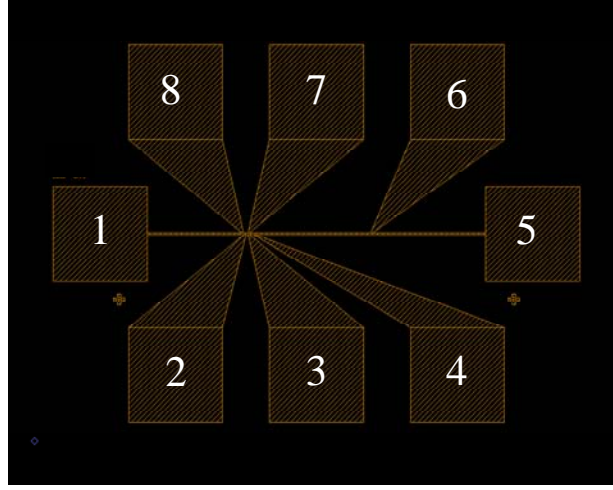


Figure 9: Layout of a narrow Hall bar device for measurements of magneto-resistance fluctuations in the quantum Hall regime.

The sample was cooled down to 15 mK. Figure 10 shows a high temperature ($T \sim 1.2\text{ K}$) obtained in this device. Pronounced minima are observed, corresponding to the integer quantum Hall states at Landau level filling $\nu=1, 2$, and 4. From the position of these minima, we deduce an electron density of $n \sim 0.9 \times 10^{11}\text{ cm}^{-2}$.

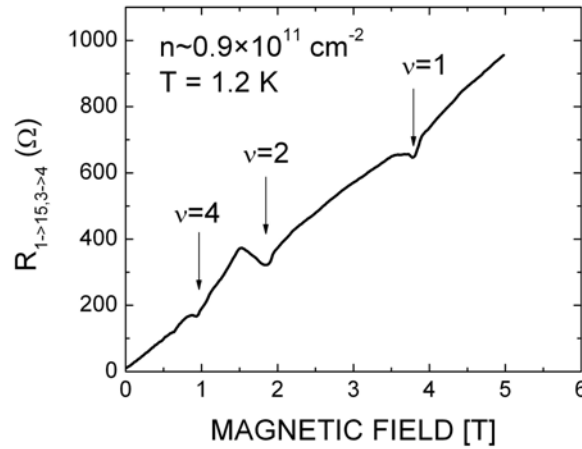


Figure 10: R_{xx} trace measured in a narrow Hall bar device at a relatively high temperature of $\sim 1.2\text{ K}$. The arrows mark the position of integer quantum Hall states at $\nu=1, 2$, and 4. The electron density is deduced from the positions of these minima.

In Figure 11, we show the R_{xx} traces taken at different places in the Hall bar at a fridge temperature of 15 mK, measured around $\nu=4$. In both traces, the current runs from contact 1 to contact 5. For the top trace ($R_{1 \rightarrow 5, 7 \rightarrow 6}$), the voltage leads are separated by 500 μm , while 20 μm for the bottom trace ($R_{1 \rightarrow 5, 2 \rightarrow 3}$). It is clearly seen that pronounced resistance fluctuations are observed in the bottom trace while such fluctuations do not exist in the top trace. Moving to higher magnetic field, for example, around $\nu=2$, again, oscillations are observed in the bottom trace while not in the top trace, as shown in Figure 12.

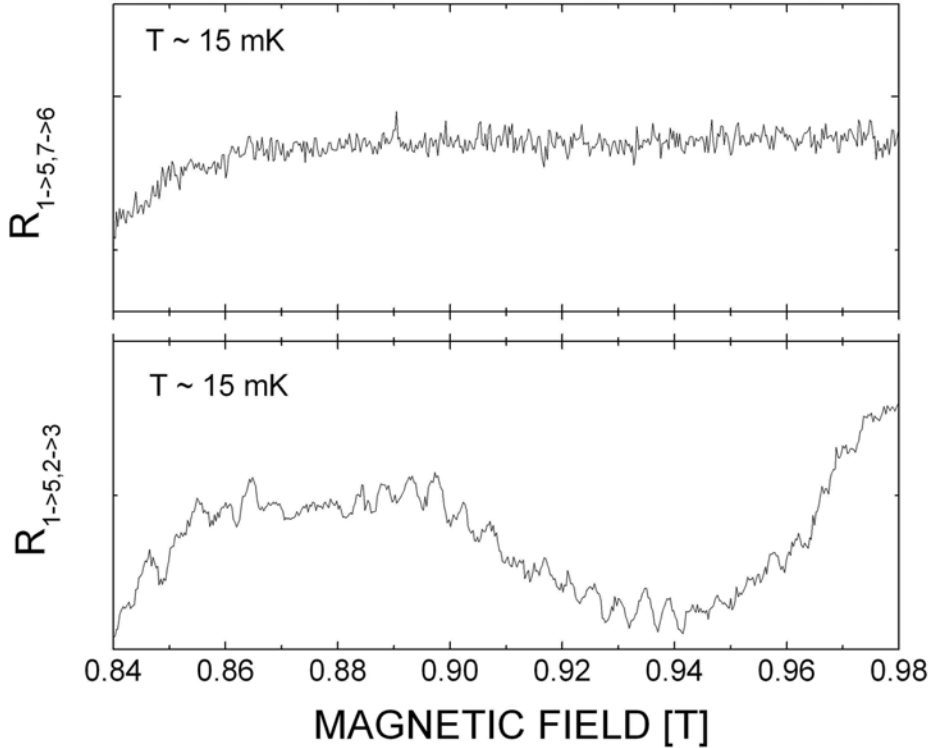


Figure 11: Magneto-resistance in R_{xx} around Landau level filling $\nu=4$. Data were taken at a fridge temperature of $T \sim 15$ mK. In the top trace, the two voltage leads are separated by 500 μm ; in the bottom trace, by 20 μm .

Resistance fluctuations in narrow channels were originally reported by Simmons et al [46]. The model of resonant reflection through magnetically bound states as a mechanism for the breakdown dissipationless transport in narrow channels was invoked by the authors. Under this model, by comparing the fluctuation periods at integer (for example, at $\nu=1,2,3,4$) and that at the $\nu=1/3$ fractional filling, the authors were able to obtain direction evidence of fractional charge in the $1/3$ fractional quantum Hall effect.

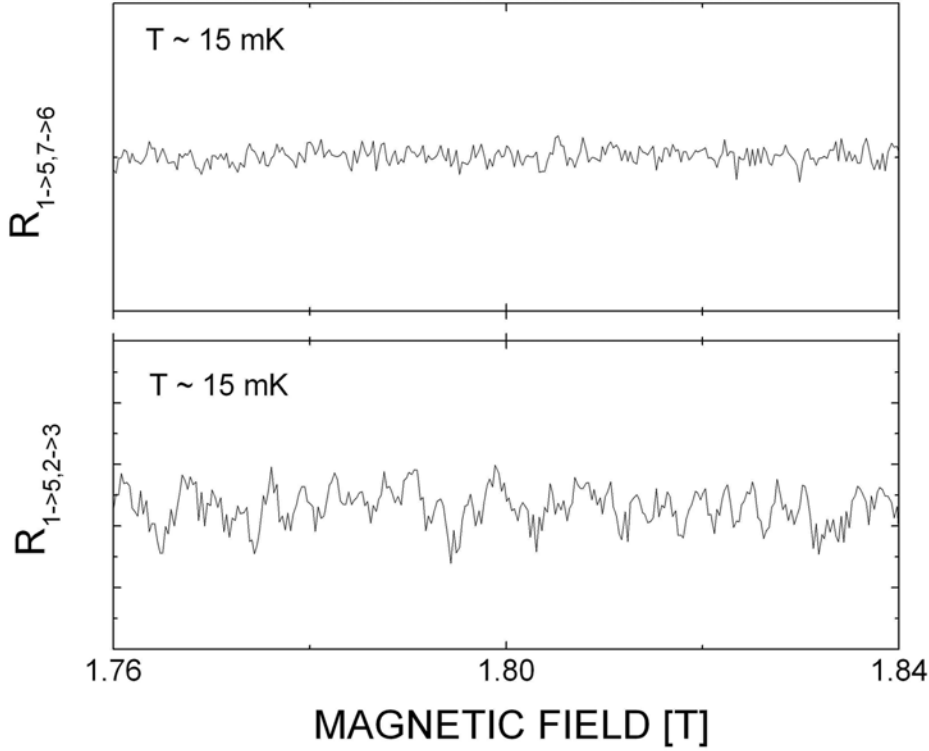


Figure 12: Magneto-resistance in R_{xx} around Landau level filling $\nu = 2$. Data were taken at a fridge temperature of $T \sim 15$ mK. In the top trace, the two voltage leads are separated by $500 \mu\text{m}$; in the bottom trace, by $20 \mu\text{m}$.

Due to the degradation in the sample quality after Hall bar fabrication, the $5/2$ state was not observable in this sample and we were not able to deduce the quasiparticle charge at this filling. Nevertheless, a detailed comparison to the previous study reveals some interesting feature. For example, in the previous study, the period of fluctuations is roughly the same at different integer fillings, with an uncertainty 30%. In our device, the period around $\nu = 4$ is about $\Delta B \sim 0.0046\text{T}$, while at $\nu = 2$ $\Delta B \sim 0.0070\text{T}$. More over, ΔB seems to scale with $1/l_B$, where l_B is the magnetic length. Indeed, $\Delta B(\text{at } \nu = 2)/\Delta B(\text{at } \nu = 4) = 0.0070/0.0046 = 1.52 \sim l_B(\text{at } \nu = 4)/l_B(\nu = 2) \sim 1.41$.

At the present time, it is not known to us whether this scaling behavior is accidental or bears a deep physics meaning. More experiments in high quality and high density samples are needed.

3.2. Non-local magneto-resistance around $\nu=5/2$

In this section, we will address the issue of edge and bulk states coupling at $\nu=5/2$ through non-local transport measurement. This kind of study is expected to shed light on the uncertainties related to previous work in quantum confined structures.

We follow the methodology developed in the earlier 90's of the last century on non-local transport in the regime of integer and fractional quantum Hall effects that clearly demonstrate edge-state conduction over macroscopic distance [49,48,50].

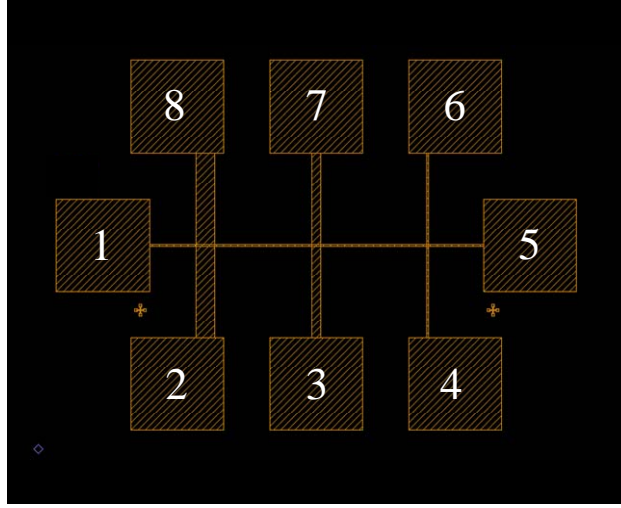


Figure 13: Schematic of a Hall bar device with 8 Ohmic contacts.

Figure 13 shows the schematic of the device used in this project. Simply connected Hall bar pattern was defined by wet chemical etching of a high quality AlGaAs/GaAs/AlGaAs quantum well grown by Loren Pfeiffer at Princeton University. Hall bars with various width were fabricated. The device we measured has a width of $20\text{ }\mu\text{m}$. A brief low temperature illumination by a red light emitting diode was used to achieve an electron density $n\sim 3\times 10^{11}\text{ cm}^{-2}$ and mobility $\mu\sim 20\times 10^6\text{ cm}^2/\text{Vs}$.

For 2D transport measurements, the conventional four-terminal technique is used. The notion of $R_{i\rightarrow j, k\rightarrow l}$ denotes the magneto-transport resistance coefficient in that current is passed through contacts i and j and the potential difference is measured between contacts k and l . For conventional “local” transport, we usually use $R_{1\rightarrow 5, 8\rightarrow 6}$ (R_{xx}) and $R_{1\rightarrow 5, 3\rightarrow 7}$ (R_{xy}). For non-local transport, we use $R_{7\rightarrow 3, 8\rightarrow 2}$.

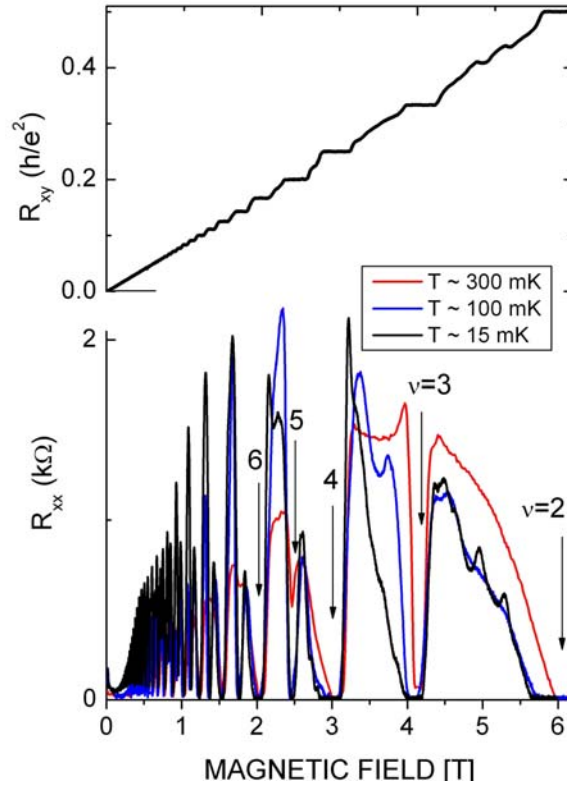


Figure 14: conventional local magneto-transport coefficients of R_{xx} and R_{xy}

Figure 14 shows $R_{1 \rightarrow 5,8 \rightarrow 6}$ (R_{xx}) at three different temperatures, and $R_{1 \rightarrow 5,3 \rightarrow 7}$ (R_{xy}) at $T \sim 15$ mK. Pronounced Shubnikov-de Haas oscillations are seen at small magnetic fields in R_{xx} . In high magnetic fields, well developed integer quantum Hall states are seen at $\nu = 2, 3, 4, 5, 6$, etc, where R_{xx} is vanishingly small and R_{xy} is quantized. The fragile $\nu = 5/2$ is visible in R_{xx} , and a corresponding Hall plateau is clearly seen at the same filling factor.

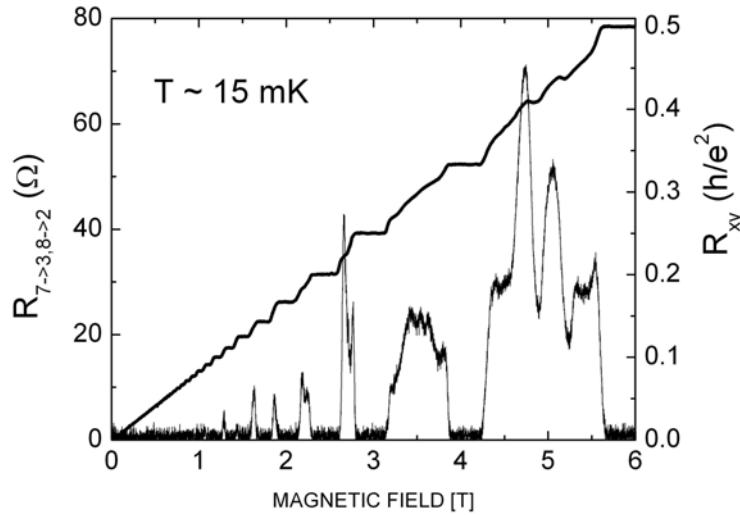


Figure 15: non-local resistance $R_{7 \rightarrow 3,8 \rightarrow 2}$ at $T \sim 15$ mK. Hall resistance is also shown.

For non-local transport, we measured $R_{7 \rightarrow 3,8 \rightarrow 2}$ at a current of 10 nA. The $T \sim 15$ mK data is shown in Figure 15, together with the R_{xy} trace in Fig.14. Overall, similar quantum Hall features are observed, except that in this non-local transport trace, the resistance is almost zero at small magnetic fields. The non-zero resistance around $\nu = 5/2$ and other fillings in the second Landau level clearly demonstrates edge-state conduction associated with the $\nu = 5/2$ fractional quantum Hall state, and this edge-state conduction persists over a macroscopic distance of several mm.

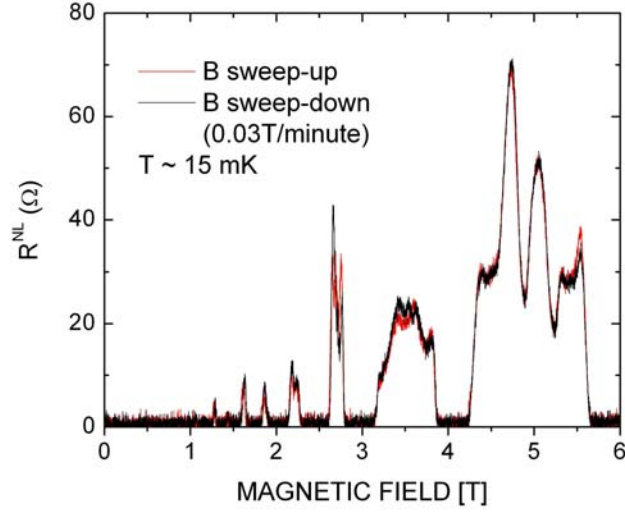


Figure 16: non-local resistance at $T \sim 15$ mK with magnetic field swept up and down.

In Figure 16, we show the results of non-local resistance for magnetic field sweeping up and down. The ramping rate for each direction was 0.03T/minute, or 5×10^{-4} T/sec. Except for the minor difference around $B \sim 2.7$ T, overall, the B-up trace and B-down trace overlap perfectly. This is in contrast to a previous study [50], in which it was observed that the non-local resistance peak heights in the fractional quantum Hall effect regime were unusually sensitive to the finite sweep rate (the same rate as we used) of magnetic field.

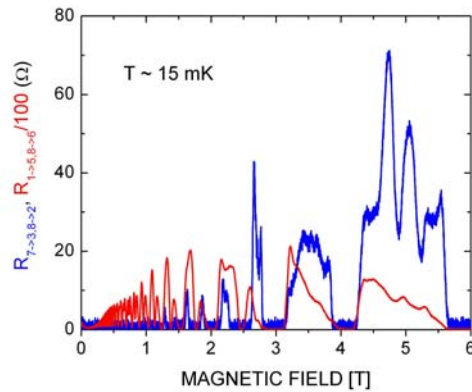


Figure 17: Comparison of local and non-local resistance. The local resistance is scaled down by a factor of 100.

At zero magnetic field, $R_{7 \rightarrow 3,8 \rightarrow 2} \approx 0.01 \times R_{1 \rightarrow 5,8 \rightarrow 6}$. Assuming this geometric constant is independent of magnetic field, a classical, dissipative, homogeneous 2D transport would render in the high magnetic field regime the same ratio between $R_{7 \rightarrow 3,8 \rightarrow 2}$ and $R_{1 \rightarrow 5,8 \rightarrow 6}$. This, however, is not true in our device. As shown in Figure 17, $R_{7 \rightarrow 3,8 \rightarrow 2} \gg 0.01 \times R_{1 \rightarrow 5,8 \rightarrow 6}$, especially around $\nu = 5/2$.

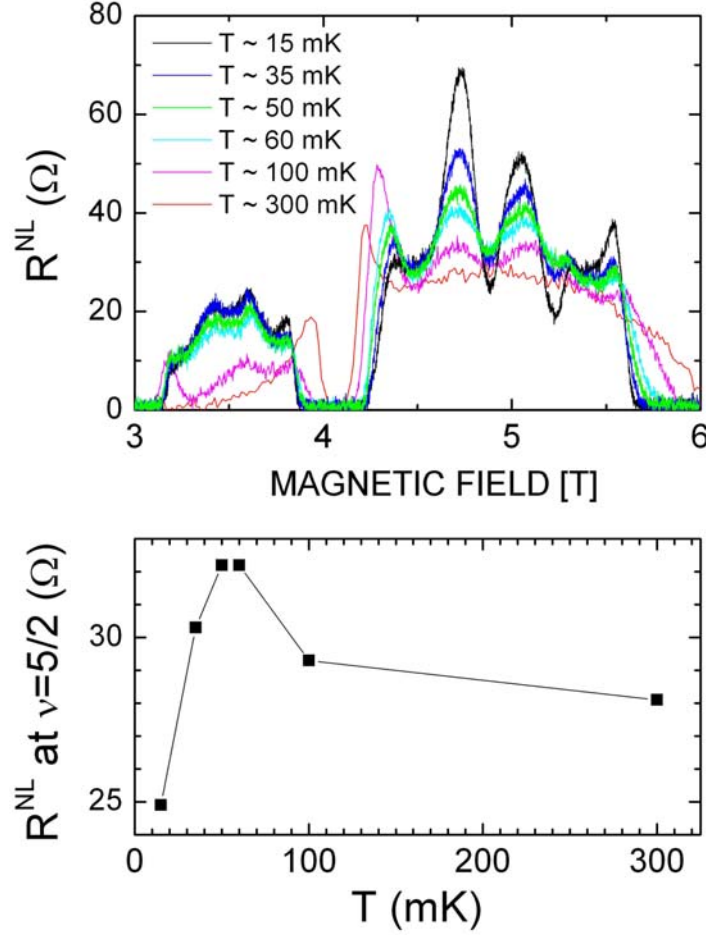


Figure 18: Temperature dependence of non-local resistance.

Figure 18 shows the temperature dependence of $R_{7 \rightarrow 3,8 \rightarrow 2}$ at several selected temperatures. It is surprising to observe that the $5/2$ state shows an activated transport behavior at low temperatures even in this non-local transport. At higher temperatures, the resistance at $\nu = 5/2$ decreases with increasing temperature. Similar temperature dependent behavior is also observed at $\nu = 7/3$.

The occurrence of non-zero R_{xx} (or $R_{1 \rightarrow 5,8 \rightarrow 6}$) imply existence of dissipative transport in the sample bulk and thus generated potential differences [48]. Non-zero non-local resistance of comparable or even larger magnitude implies that at certain ν potential differences extend unattenuated for macroscopic distance of several mm away from dissipative bulk current paths [48]. In this regard, our non-local transport results show that dissipationless edge states do exist in the fractional quantum Hall regime in the second Landau level.

3.3. Spin-filtering phenomenon in long, narrow Hall bar device

Carefully examining Figure 19, we find that the peak between an even-integer IQHE state and an odd one (for example, between $\nu = 8$ and 7) is always larger than that between an odd IQHE state and an even one (for example between $\nu = 7$ and 6). This is different from the transport features in a square sample without Hall bar pattern.

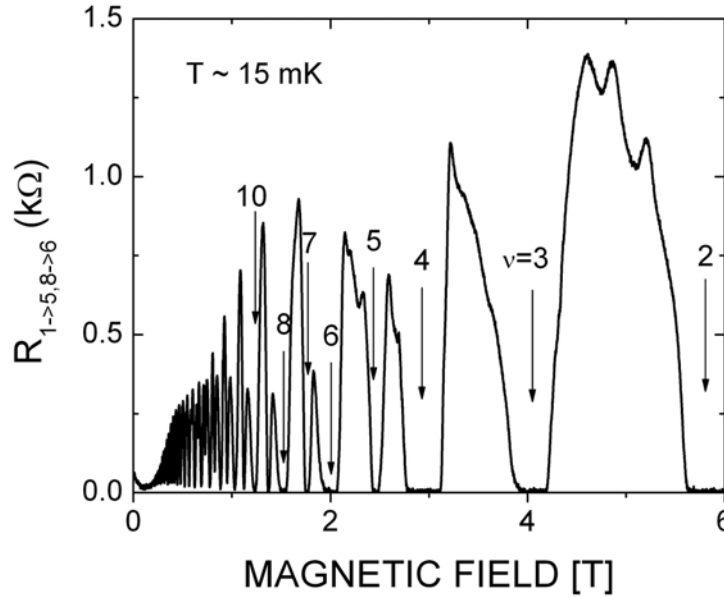


Figure 19: Conventional local resistance at $T \sim 15$ mK.

This transport feature has been observed before [51,52,53] in GaAs/AlGaAs single heterojunctions and in AlAs, and Si/SiGe quantum wells. A spin-filtering mechanism based on spin Hall effect was proposed in Li's thesis [51], which we will adopt in this report.

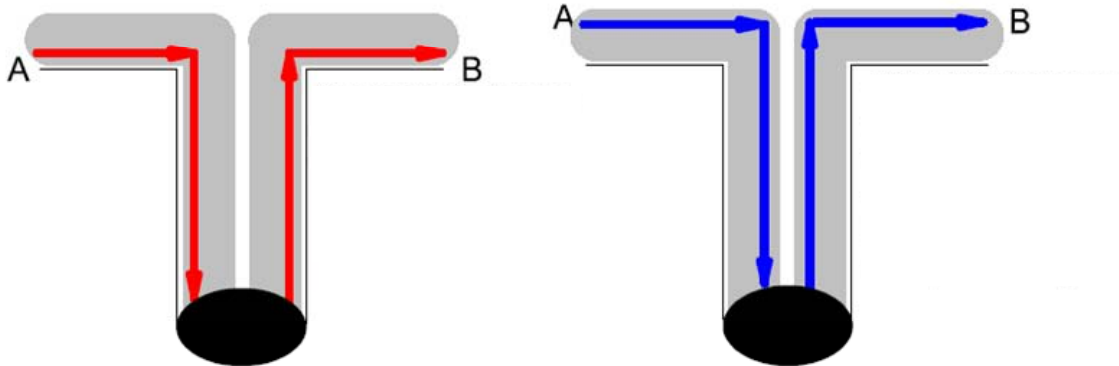


Figure 20: Schematic of edge state transport for spin up and spin down electrons.

It is known that in the quantum Hall regime, 2D electrons travel via dissipationless edge channels (schematically shown in Figure 20). Between two quantum Hall states ν and $\nu - 1$, the

dissipative transport (non-zero R_{xx}) occurs as current is carried both by extended bulk states of the partially occupied topmost Landau level (i.e., ν) and by the extended edge states (of up to $\nu - 1$). Due to a screening effect [54], the width of the inner edge channel in high mobility samples can be widened to an order of tens of micrometers, which is much larger than the magnetic length. In the narrow leg, if the two opposite-propagating inner edge currents A and B on the two sides of contact legs are very close to each other, the backscattering events between them effectively are expected to depress the edge transport and thus reduce the corresponding edge channel's conductivity.

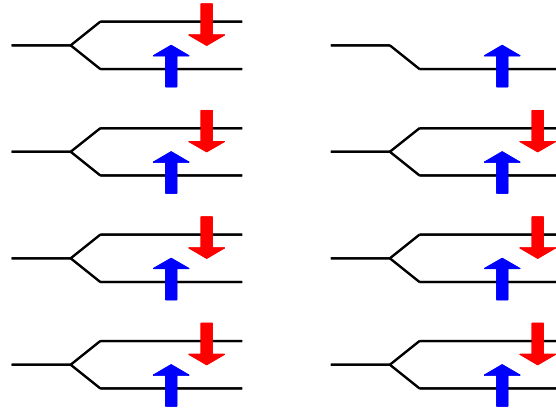


Figure 21: Schematic of Landau levels with spin.

Now, assuming the current direction on side A is down. Between $\nu = 8$ and 7, the electron spin in the topmost Landau level (partially occupied) is down and the spin of the edge channel of the highest occupied Landau level is up, as shown in the left panel of Fig.21. In the presence of a spin-orbital coupling, though weak in GaAs, the electrons are pushed to the direction defined by $-\mathbf{S} \times \mathbf{J}$ [55]. Here \mathbf{S} is the spin vector and \mathbf{J} is the current vector. In other word, the electrons in the highest occupied Landau level are pushed toward the edge of contact leg (as shown in the left panel of Figure 20). On the B side, the electrons are also pushed to the edge of contact leg. As a result, the scattering between the edge channels A and B is weak, and a high peak can be expected in R_{xx} between $\nu = 8$ and 7. On the other hand, between $\nu = 7$ and 6, the spin in the topmost Landau level (partially occupied) is up and the spin of the edge channel of the highest occupied Landau level is down (right panel Fig.21). As shown in the right panel of Fig.20, the edge currents A and B now both accumulate towards the center of the leg. Therefore, the scattering between the two edge channels, and the scattering between the edge channels and the bulk states are stronger. This, in turn, reduces the transition rate of edge channel transport. As a result, the R_{xx} peak between $\nu = 7$ and 6 is smaller. The same mechanism also applied for other combinations such as between $\nu = 10$ and 9 and between $\nu = 9$ and 8.

In the above discussions, we have omitted the Landau levels below the highest occupied Landau level, and treat them as inert.

4. SUMMARY AND OUTLOOK

This LDRD was successful in studying the physics of a possible non-abelian fractional quantum Hall state at Landau level filling $\nu = 5/2$.

In summary, we discovered that the non-local transport over macroscopic distances of several mm can exist at the Landau level filling factor $\nu = 5/2$ in narrow Hall bar samples fabricated in high quality GaAs/AlGaAs quantum wells. It clearly demonstrates edge-state conduction around $\nu = 5/2$. This result will eventually help to build a qubit device structure using the $\nu = 5/2$ state.

We observed the conductance fluctuations in narrow channels around $\nu = 5/2$. This behavior is probably due to the breakdown of dissipationless transport in narrow channels due to resonant reflection through magnetically bound states. Detailed studies in high quality devices in the future are expected to allow us to obtain the information on the quasiparticle charge of the $\nu = 5/2$ state, a necessary step towards confirming the exotic non-abelian statistics at $5/2$.

To achieve the above results, during the course of this research, we designed photo-mask for narrow Hall bar samples. Multiple device structures with different dimensions are included in the design. This is necessary since different quantum Hall samples may need devices of different dimensions. More over, we perfected device fabrication procedures so that the sample quality can be maximally maintained. We developed a recipe for chemical etching of Hall bar so that the sample quality can be sustained ever after fabrication. This is very important for eventually dissipationless edge state transport around $5/2$, a very fragile quantum Hall state. Finally, we set up a cryogenic system, a dilution refrigerator, for ultra-low temperature measurement. The lowest temperature reached in this system is 10mK. This is by far the lowest temperature ever achieved at Sandia. Due to the operation of this system, we were able to conduct a detailed, systematic study of edge-bulk transport in the $5/2$ fractional quantum Hall regime.

So far, the most promising approach to construct a qubit structure in the quantum Hall topological quantum computation scheme is to utilizing the edge state of the so-called $5/2$ fractional quantum Hall effect state. Before this great challenge can ever be achieved, however, one has to gain a deep understating of the $5/2$ edge state – whether it is stable against disorder; whether it is stable against rising temperatures; whether it can travel over a macroscopic distance. All these need more, detailed experimental investigation. The results we obtained in this project show that indeed, the dissipationless edge state at $\nu = 5/2$ can extend over a distance of mm length. This result is significant as it supports the possibility of using the $5/2$ edge state for constructing a qubit structure.

5. REFERENCES

- [1] K. von Klitzing, M. Pepper, and G. Dorda, Phys. Rev. Lett. 45, 494 (1980).
- [2] D.C. Tsui, H.L. Stormer, and A.C. Gossard, Phys. Rev. Lett. 48, 1559 (1982).
- [2] A.P. Balachandran, L. Chandar, A. Momen, Nucl. Phys. B 461, 581 (1996).
- [4] http://nobelprize.org/nobel_prizes/physics/laureats/1985/
- [5] http://nobelprize.org/nobel_prizes/physics/laureats/1998/
- [6] See, for example, S. Das Sarma, M. Freedman, and C. Nayak, Phys. Today, July (2006).
- [7] H.L. Stormer, et al, Solid State Communications **29**, 705 (1979).
- [8] E.H. Hwang and S. Das Sarma, Phys. Rev. B 77, 235437 (2008).
- [9] http://en.wikipedia.org/wiki/Hall_effect
- [10] B.I. Halperin, Phys. Rev. B 25, 2185 (1982).
- [11] H.A. Fertig, Physics 2, 15 (2009).
- [12] M. Büttiker, Phys. Rev. B 38, 9375 (1988).
- [13] R.B. Laughlin, Phys. Rev. Lett. 50, 1395 (1983).
- [14] Rev. Mod. Phys. 71, 875 (1999).
- [15] H.W. Jiang, R.L. Willett, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. 65, 633 (1990).
- [16] W. Pan, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, K.W. Baldwin, and K.W. West, Phys. Rev. Lett. (2002).
- [17] J.K. Jain, Phys. Rev. Lett. 63, 199 (1989).
- [18] B.I. Halperin, P.A. Lee, and N. Read, Phys. Rev. B 47, 7312 (1993).
- [19] for a review of composite fermion theory, see, Composite Fermions, J.K. Jain, Cambridge University Press, 2007.
- [20] M. Freedman, C. Nayak, K. Walker, preprint, cond-mat/0512066.
- [21] C. Nayak, et al, Rev. Mod. Phys. 80, 1083 (2008).
- [22] R.L. Willett, J.P. Eisenstein, H.L. Stormer, D.C. Tsui, A.C. Gossard, and J.H. English, Phys. Rev. Lett. 59, 1776 (1987).
- [23] G. Moore and N. Read, Nucl. Phys. B 360, 362 (1991).
- [24] R.H. Morf, Phys. Rev. Lett. 80, 1505 (1998).
- [25] W. Pan, J.S. Xia, V. Shvarts, E.D. Adams, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, K.W. Baldwin, and K.W. West, Phys. Rev. Lett. 83, 3530 (1999).
- [26] J.S. Xia, W. Pan, C.L. Vicente, E.D. Adams, N.S. Sullivan, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, K.W. Baldwin, and K.W. West, Phys. Rev. Lett. 93, 176809 (2004).
- [27] J. P. Eisenstein, R. Willett, H. L. Stormer, D. C. Tsui, A. C. Gossard, and J. H. English, Phys. Rev. Lett. **61**, 997 (1988).
- [28] R. G. Clark, S. R. Haynes, A. M. Suckling, J. R. Mallett, P. A. Wright, J. J. Harris, and C. T. Foxon, Phys. Rev. Lett. 62, 1536 (1989).
- [29] J. P. Eisenstein, H. L. Stormer, L. Pfeiffer, and K. W. West, Phys. Rev. Lett. 62, 1540 (1989).
- [30] W. Pan, R.R. Du, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, K.W. Baldwin, and K.W. West, Phys. Rev. Lett. **83**, 820 (1999).
- [31] M.P. Lilly, K.B. Cooper, J.P. Eisenstein, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. **83**, 824 (1999).
- [32] E.H. Rezayi and F.D.M. Haldane, Phys. Rev. Lett. 84, 4685 (2000).
- [33] W. Pan, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, K.W. Baldwin, and K.W. West, solid State Communications **119**, 641 (2001).
- [34] R.L. Willett, L.N. Pfeiffer, K.W. West, preprint, arXiv:0911.0345.
- [35] M. Dolev, et al, Nature 452, 829 (2008).
- [36] A. Bid, et al, Nature 466, 585 (2010).
- [37] Y.M. Zhang, et al, Phys. Rev. B 79, 241304 (2009).

- [38] F.E. Camino, W. Zhou, and V.J. Goldman, Phys. Rev. Lett. 98, 076805 (2007).
- [39] M.D. Godfrey, P. Jiang, W. Kang, S.H. Simon, K.W. Baldwin, L.N. Pfeiffer, K.W. West, preprint, arXiv:0704.0368.
- [40] S. Das Sarma, M. Freeman, and C. Nayak, Phys. Rev. Lett. 94, 166802 (2005).
- [41] W. Pan, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Phys. Rev. Lett. 90, 016801 (2003).
- [42] D. R. Luhman, W. Pan, D. C. Tsui, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Phys. Rev. Lett. 101, 266804 (2008).
- [43] J. Shabani, T. Gokmen, Y. T. Chiu, and M. Shayegan, Phys. Rev. Lett. 103, 256802 (2009).
- [44] H. van Houten and C. Beenakker, *Quantum point contacts*, Physics Today, July 1996, page 22.
- [45] M.A. Topinka, R.M. Westervelt, and E.J. Heller, Physics Today, December 2003.
- [46] J.A. Simmons, et al, Phys. Rev. Lett. **63**, 1731 (1989).
- [47] J.B. Miller, et al, Nature Physics **3**, 561 (2007).
- [48] J.W. Wang and V.J. Goldman, Phys. Rev. Lett. **67**, 749 (1991).
- [49] P.L. McEuen, et al, Phys. Rev. Lett. **64**, 2062 (1990).
- [50] J.W. Wang and V.J. Goldman, Phys. Rev. B **45**, 13479 (1992).
- [51] Wanli Li, Ph.D. Thesis, Princeton University, 2007.
- [52] K. Vakili, Y. P. Shkolnikov, E. Tutuc, N. C. Bishop, E. P. De Poortere, and M. Shayegan, Phys. Rev. Lett. **94**, 176402 (2005).
- [53] K. Lai, T. M. Lu, W. Pan, D. C. Tsui, S. A. Lyon, J. Liu, Y. H. Xie, M. Muhlberger, and F. Shaffer, Phys. Rev. B **73**, 161301 (2006)
- [54] D. B. Chklovskii, B. I. Shklovskii, and L. I. Glazman, Phys. Rev. B **46**, 4026 (1992).
- [55] J.E. Hirsch, Phys. Rev. Lett. **83**, 1834 (1999).

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