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On Nonlinear Self-interaction of Geodesic Acoustic Mode Driven by Energetic Particles

G.Y. Fu

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G. Y. Fu

Princeton Plasma Physics Laboratory, Princeton, New Jersey 08543

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Abstract

It is shown that nonlinear self-interaction of energetic particle-driven Geodesic Acoustic Mode does not generate a second harmonic in radial electric field using the fluid model. However, kinetic effects of energetic particles can induce a second harmonic in the radial electric field. A formula for the second order plasma density perturbation is derived. It is shown that a second harmonic of plasma density perturbation is generated by the convective nonlinearity of both thermal plasma and energetic particles. Near the midplane of a tokamak, the second order plasma density perturbation (the sum of second harmonic and zero frequency sideband) is negative on the low field side with its size comparable to the main harmonic at low fluctuation level. These analytic predictions are consistent with the recent experimental observation in DIII-D.

I. INTRODUCTION

Intense axisymmetric density fluctuations were recently observed in DIII-D neutral beam-heated reversed shear plasmas[1]. The instability was driven by energetic neutral beam ions[1] with its frequency near that of Geodesic Acoustic Mode (GAM)[2]. The instability was identified as the energetic particle-driven GAM or EGAM[3]. The new mode is intrinsically an energetic particle mode since both the mode frequency and mode structure is determined nonperturbatively by the energetic particle dynamics. As such, this new mode is qualitatively different from the Global Geodesic Acoustic Mode (GGAM)[4, 5] which is a pure MHD mode. The DIII-D experiment also revealed a significant second harmonic of the density fluctuation when the instability was most intensive[6]. This indicates a strong nonlinear self-interaction of the GAM oscillations.

In the past decade or so, there have been much renewed interests in GAM after its early discovery in 1968[2]. The GAM is an n=0 electrostatic mode and is a finite frequency counterpart of the zonal flow[7]. The GAM is usually driven nonlinearly by plasma microturbulence. There is much linear study on GAM's damping due to collision and ion Landau resonance[8–16] and on GAM's radial propagation due to finite thermal ion gyroradius and plasma temperature profile inhomogeneity[13, 14]. Nonlinear studies found that that GAM can be driven by plasma microturbulence[13, 17–19], consistent with experimental observation of GAM in the plasma edge region[20–22]. Recently, Sasaki et al. showed that self-interaction of turbulence-induced GAMs can drive both second harmonic[23] and zonal flow sideband[24].

In this work, we consider the self-interaction of an energetic particle-driven global GAM. We consider both fluid nonlinearity of thermal species and kinetic nonlinearity of energetic particles. First, we use fluid model to determine both the second harmonic and the zero frequency sideband driven by fluid nonlinearity via self-interaction of GAMs. In this fluid model, the primary GAM is given and is assumed to be linearly driven by energetic particles. We will show that to the leading order, the fluid nonlinearity of thermal plasma does not generate either the zero frequency sideband (zonal flow) or the second harmonic in radial electric field. However, the fluid convective nonlinearity does generate both zero frequency component and second harmonic in density fluctuation. Second, we use kinetic model to study the energetic particle effects. We will show that the energetic particle nonlinearity can indeed generate a second harmonic in the radial electric field. This second harmonic of radial electric field leads to a corresponding second harmonic in the density perturbation due to the compression of the $\mathbf{E}\times\mathbf{B}$ velocity. Finally, we will show that near the midplane, the second order density perturbation can be comparable to the size of the first order perturbation even at low level of fluctuation. It has a fixed sign being negative on the low field side of a tokamak. These analytic predictions are consistent with the experimental observations in DIII-D[6].

The paper is organized as following. Sec. II describes the fluid model for the $n = 0$ electrostatic perturbation of GAM. Sec. III and Sec. IV gives linear and nonlinear results from the fluid model respectively. Sec. V considers energetic particles' contribution to the nonlinear self-interaction. Finally, discussions and conclusions are given in Sec. VI.

II. FLUID MODEL

We start from ideal MHD equation equations:

$$\rho(\frac{\partial}{\partial t}\mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v}) = -\nabla P + \mathbf{J}\times\mathbf{B} \tag{1}$$

$$\mathbf{J} = \nabla\times\mathbf{B} \tag{2}$$

$$\frac{\partial}{\partial t}\mathbf{B} = -\nabla\times\mathbf{E} \tag{3}$$

$$\mathbf{E} + \mathbf{v}\times\mathbf{B} = 0 \tag{4}$$

$$\frac{\partial}{\partial t}\rho + \mathbf{v}\cdot\nabla\rho = -\nabla\cdot\mathbf{v}\rho \tag{5}$$

$$\frac{\partial}{\partial t}P + \mathbf{v}\cdot\nabla P = -\gamma\nabla\cdot\mathbf{v}P \tag{6}$$

where ρ is plasma mass density, $\mathbf{B}$ the magnetic field, $\mathbf{J}$ the plasma current, $\mathbf{v}$ the plasma velocity, P the plasma pressure, $\mathbf{E}$ the electric field, and γ the coefficient of specific heat.

For simplicity, we assume that the perturbation is electrostatic and axisymmetric. Then, the electric field has only radial component according to the ideal Ohm's Law (Eq. (4)) and can be written as

$$\mathbf{E} = -\nabla\Phi = E_r \nabla r \tag{7}$$

where Φ is electric potential and r is a radial flux variable. Note that E_r is a function of r only.

From the momentum equation (Eq. (1)) and the condition of $< J_r >= 0$ with the bracket $< ... >$ denoting flux average, we can derive the following equation for the radial electric field:

$$< \frac{\rho|\nabla r|^2}{B^2} > \frac{\partial}{\partial t} E_r + < \frac{\rho}{B^2} (\nabla r \times \mathbf{B}) \cdot (\mathbf{v} \cdot \nabla \mathbf{v}) >= - < \frac{(\nabla r \times \mathbf{B}) \cdot \nabla P}{B^2} > \tag{8}$$

or

$$< \frac{\rho|\nabla r|^2}{B^2} > \frac{\partial}{\partial t} E_r + < \frac{\rho}{B^2} (\nabla r \times \mathbf{B}) \cdot (\mathbf{v} \cdot \nabla \mathbf{v}) >= -2 < G(r,\theta) P > \tag{9}$$

where the function $G(r, \theta)$ is related to the geodesic curvature and is given by

$$G(r,\theta) = -\frac{B_\phi R}{JB^3} \frac{\partial B}{\partial \theta} \tag{10}$$

where B is the strength of the magnetic field, B_ϕ is the toroidal component of the magnetic field, J is the Jacobian of the coordinates (r, θ, ϕ).

The equation for the parallel fluid velocity is given by

$$\rho(\frac{\partial}{\partial t} v_\parallel + \mathbf{b} \cdot (\mathbf{v} \cdot \nabla \mathbf{v})) = -\mathbf{b} \cdot \nabla P \tag{11}$$

Note that the fluid velocity can be decomposed as

$$\mathbf{v} = v_\parallel \mathbf{b} + \frac{\mathbf{E} \times \mathbf{B}}{B^2} = v_\parallel \mathbf{b} + \frac{B_\phi R E_r}{B^2} \nabla r \times \nabla \phi \tag{12}$$

This leads to the following useful expressions for $\mathbf{v} \cdot \nabla$ and $\nabla \cdot \mathbf{v}$:

$$\mathbf{v} \cdot \nabla = \frac{v_\parallel}{B} \mathbf{B} \cdot \nabla - \frac{B_\phi R E_r}{JB^2} \frac{\partial}{\partial \theta} \tag{13}$$

$$\nabla \cdot \mathbf{v} = \mathbf{B} \cdot \nabla (\frac{v_\parallel}{B}) - 2G(r,\theta) E_r \tag{14}$$

Equations (5), (6), (9) and (11) constitute a simple fluid model for Geodesic Acoustic Modes. In the following two sections, we will examine the linear and nonlinear properties of GAM based on this model.

III. LINEAR THEORY

The linearized equations of GAM can be derived straightforwardly from our model and are given by

$$< \frac{\rho_0|\nabla r|^2}{B^2} > \frac{\partial}{\partial t}E_{r1} = -2 < G(r,\theta)P_1 > \tag{15}$$

$$\frac{\partial}{\partial t}P_1 = -\gamma\nabla\cdot\mathbf{v}_1 P_0 = -\gamma(\mathbf{B}\cdot\nabla(\frac{v_{\parallel 1}}{B}) - 2G(r,\theta)E_{r1})P_0 \tag{16}$$

$$\rho_0\frac{\partial}{\partial t}v_{\parallel 1} = -\mathbf{b}\cdot\nabla P_1 \tag{17}$$

$$\frac{\partial}{\partial t}\rho_1 = -\nabla\cdot\mathbf{v}_1\rho_0 = -(\mathbf{B}\cdot\nabla(\frac{v_{\parallel 1}}{B}) - 2G(r,\theta)E_{r1})\rho_0 \tag{18}$$

where the subscript 0 denotes equilibrium quantities and subscript 1 denotes linear perturbations.

Substitute solution of $v_{\parallel 1}$ into Eq. (16), the equation for P_1 then becomes

$$\frac{\partial}{\partial t}P_1 = 2\gamma P_0(L^{-1}G(r,\theta))E_{r1} \tag{19}$$

where L^{-1} is the inverse of the operator L with L defined as

$$L = 1 + \frac{\gamma P_0}{\rho_0\omega^2}\mathbf{B}\cdot\nabla\frac{1}{B^2}\mathbf{B}\cdot\nabla \tag{20}$$

Here ω is the mode frequency. Similarly, the equation for ρ_1 becomes

$$\frac{\partial}{\partial t}\rho_1 = 2\rho_0(L^{-1}G(r,\theta))E_{r1} \tag{21}$$

and $v_{\parallel 1}$ is given by

$$v_{\parallel 1} = \frac{2\gamma P_0}{\rho_0\omega^2}\mathbf{b}\cdot\nabla(L^{-1}G(r,\theta))E_{r1} \tag{22}$$

Combining Eq. (15) and (19), we arrive at the following dispersion relation for GAM:

$$\omega^2 = \omega_{GAM}^2 = \frac{4\gamma P_0}{\rho_0}\frac{< GL^{-1}G >}{< |\nabla r|^2/B^2 >} \tag{23}$$

In the limit of large aspect ratio tokamak equilibria with circular flux surfaces, the GAM frequency reduces to the familiar form:

$$\omega_{GAM}^2 = \frac{2\gamma P_0}{\rho_0 R^2}(1 + \frac{1}{2q^2}) \tag{24}$$

where q is the safety factor. This result reproduces the original work of Winsor et al.[2].

IV. NONLINEAR THEORY

Here we derive the leading order nonlinear response given an unstable linear mode. In particular, we aim to evaluate the second harmonic of density perturbation due to quadratic nonlinearity. It is clear that the leading order nonlinear response is quadratic in perturbation amplitude in the fluid model. Using perturbation amplitude as a small parameter, we can expand any variable u as following:

$$u = u_0 + u_1 + u_2 + ... \tag{25}$$

where the subscript 0 denotes equilibrium, subscript 1 denotes linear perturbation, and the subscript 2 denotes the second order perturbation driven by quadratic nonlinearity. Expanding Eq. (9), (11), (6) and (5) to second order, we obtain

$$< \frac{\rho_0|\nabla r|^2}{B^2} > \frac{\partial}{\partial t}E_{r2} = - < \frac{\rho_1|\nabla r|^2}{B^2} > \frac{\partial}{\partial t}E_{r1} - < \frac{\rho_0}{B^2}(\nabla r \times \mathbf{B}) \cdot \mathbf{v}_1 \cdot \nabla \mathbf{v}_1 > -2 < G(r,\theta)P_2 > \tag{26}$$

$$\rho_0 \frac{\partial}{\partial t} v_{\parallel 2} = -\rho_1 \frac{\partial}{\partial t} v_{\parallel 1} - \rho_0 \mathbf{b} \cdot (\mathbf{v}_1 \cdot \nabla \mathbf{v}_1) - \mathbf{b} \cdot \nabla P_2 \tag{27}$$

$$\frac{\partial}{\partial t} P_2 = -\mathbf{v}_1 \cdot \nabla P_1 - \gamma \nabla \cdot \mathbf{v}_1 P_1 - \gamma \nabla \cdot \mathbf{v}_2 P_0 \tag{28}$$

$$\frac{\partial}{\partial t} \rho_2 = -\mathbf{v}_1 \cdot \nabla \rho_1 - \nabla \cdot \mathbf{v}_1 \rho_1 - \nabla \cdot \mathbf{v}_2 \rho_0 \tag{29}$$

We first show that $E_{r2} = 0$ based on symmetry. From the linearized expressions in the preceding section, we observe that P_1 and ρ_1 are odd functions of θ and $v_{\parallel 1}$ is an even function. Based on the symmetry of these linearized solutions, we can deduce that the first term of the right hand side of Eq. (26) is zero because ρ_1 is odd. The second term can be shown to be zero as well because the integrant is an odd function in θ. The last term is related to P_2. . From Eq. (27) and (28), it can be shown that P_2 can be decomposed into two parts:

$$P_2 = P_{2,1} + P_{2,2}E_{r2} \tag{30}$$

where the first part comes from the quadratic nonlinearity and the second part is proportional to E_{r2}. By checking the contributions to the first part carefully, it can be shown that $P_{2,1}$ is an even function of θ. Thus, $P_{2,1}$ does not contribute to E_{r2} because the function G is odd (see Eq. (26)). Furthermore, the second part is an odd function and thus contribute to the third term of the righ hand side of Eq. (26). However, since the second part is proportional to E_{r2}, it can be combined with the left side of Eq. (26). From this analysis we can conclude that $E_{r2} = 0$, i.e., there is no quadratic nonlinear contribution to the radial electric field. Therefore, fluid nonlinearity does not generate a second harmonic in the radial electric field associated with GAM.

We now proceed to determine P_2 and $v_{\|2}$. Equation (27) can be rewritten as

$$\begin{aligned}\rho_0\frac{\partial}{\partial t}v_{\|2} &= \rho_0\mathbf{v}_1\cdot\nabla v_{\|1} - \mathbf{b}\cdot\nabla P_2\\ &= \mathbf{b}\cdot\nabla\left[2\gamma P_0\frac{B_\phi R}{JB^2}\frac{\partial}{\partial\theta}(L^{-1}G)\frac{1}{\omega^2}E_{r1}^2 - P_2\right]\end{aligned} \tag{31}$$

where we have dropped the first term in Eq. (27) because it is smaller than the second term by a factor of aspect ratio. We can write $P_2 = \tilde{P}_2 + \bar{P}_2$ where $\bar{P}_2$ is the time-averaged part of P_2(i.e., zeroth harmonic) and $\tilde{P}_2$ is the oscillative part(i.e., second harmonic). From Eq. (31), $\bar{P}_2$ can be determined as

$$\bar{P}_2 = 2\gamma P_0\frac{B_\phi R}{JB^2}\frac{\partial}{\partial\theta}(L^{-1}G)\frac{1}{\omega^2}\bar{E_{r1}^2} \tag{32}$$

Using Eq. (31), we can show that the last term in Eq. (28) can also be neglected because it is smaller than the fist term by a factor of q^2 (we assume $q^2 > 1$ here for simplicity). Then,$\tilde{P}_2$ can be obtained from Eq. (28) and the full expression for P_2 is given by

$$P_2 = 2\gamma P_0\frac{B_\phi R}{JB^2}\frac{\partial}{\partial\theta}(L^{-1}G)\left[\frac{2}{\omega^2}\bar{E_{r1}^2} - (\widetilde{\int E_{r1}dt})^2\right] \tag{33}$$

and Eq. (31) can be rewritten as

$$\rho_0\frac{\partial}{\partial t}v_{\|2} == \mathbf{b}\cdot\nabla\left[\gamma P_0\frac{B_\phi R}{JB^2}\frac{\partial}{\partial\theta}(L^{-1}G)(\frac{2}{\omega^2}\widetilde{E_{r1}^2} + (\widetilde{\int E_{r1}dt})^2)\right] \tag{34}$$

We are now ready to determine the second order density perturbation ρ_2. Equation for ρ_2 can be simplified to

$$\frac{\partial}{\partial t}\rho_2 = -\mathbf{v}_1\cdot\nabla\rho_1 - \rho_0\mathbf{B}\cdot\nabla(\frac{v_{\|2}}{B}) \tag{35}$$

where we have used E_{r2}=0 and have dropped the second term of the right hand side of Eq. (29) because it is a factor of aspect ratio smaller than the first term.

In above equation, we can also neglect the second term on the right hand side because it is smaller than the first term by a factor of $q^2 >> 1$. In this limit, the second order density perturbation is driven only by the convective nonlinearity due to $\mathbf{E} \times \mathbf{B}$ drift. From Eq. (21), we obtain an explicit expression for the first order density perturbation

$$\frac{\rho_1}{\rho_0} = 2G \int_0^t E_{r1}(r,t')dt' = -2\frac{r}{R}\sin\theta \frac{B_\phi R}{JB^2}\int_0^t E_{r1}(r,t')dt' \tag{36}$$

where we have assumed $\rho_1(t=0) = 0$ for a growing mode. Note also that $L^{-1} \sim 1$ in the limit of $q^2 >> 1$. Using this expression for ρ_1, we then obtain the second order density perturbation as

$$\begin{aligned} \frac{\rho_2}{\rho_0} &= -\frac{r}{R}\cos\theta \left(\frac{B_\phi R}{JB^2}\int_0^t E_{r1}(r,t')dt'\right)^2 \\ &\approx -\frac{r}{2R}\cos\theta\left(\frac{B_\phi R}{JB^2\omega}\hat{E}(r,t)\right)^2 (1 + cos2\omega t)) \end{aligned} \tag{37}$$

where we have also assumed $\rho_2(t=0) = 0$ and $E_{r1}(r,t) = \hat{E}(r,t)cos(\omega t)$ with $\hat{E}(r,t)$ describing a slowing-varying part of electric filed evolution (i.e., slowing growing with a small but finite growth rate γ such that $\gamma/\omega << 1$). Note that the second order density perturbation contains both zero frequency harmonic and second harmonic with equal amplitude.

The total density perturbation (up to second order) is then given by

$$\frac{\delta\rho}{\rho_0} = -\frac{r}{R}\left[2\sin\theta\frac{B_\phi R}{JB^2}\int_0^t E_{r1}(r,t')dt' + \cos\theta\left(\frac{B_\phi R}{JB^2}\int_0^t E_{r1}(r,t')dt'\right)^2\right] \tag{38}$$

Note that in arriving at Eq. (38), we have neglected the $v_{\parallel 1}$ term in the operator $\mathbf{v}_{\parallel 1}\cdot\nabla$ because it is smaller than the $\mathbf{E}\times\mathbf{B}$ term by a factor of aspect ratio.

From Eq. (37), an interesting and important property for the second order density perturbation is that it has a fixed sign. It is negative for $\cos\theta > 0$ (low field side) and positive otherwise (high field side). In contrast, the first order perturbative is oscillatory (true for any linear perturbation with finite real frequency). Furthermore, the second order perturbative is up-down symmetric and peaks at midplane whereas the linear perturbation is up-down asymmetric and is zero at the midplane. Because of this, the second order perturbation is relatively large near the midplane. In the DIII-D experiment, the density measurement was made near the midplane at the low field side. At this location, the second order perturbation can be comparable to the first order perturbation and is always negative. In particular, the ratio of ρ_2 and ρ_1 is

$$\frac{\rho_2}{\rho_1} = \frac{\cos\theta}{2\sin\theta}\frac{B_\phi R}{JB^2}\int_0^t E_{r1}(r,t')dt' = -\frac{\cos\theta}{4\sin^2\theta}\frac{R}{r}\frac{\rho_1}{\rho_0} \tag{39}$$

We observe that near midplane, the second order perturbation (both zero frequency harmonic and second harmonic) can be comparable to the first order perturbation even for low level of density perturbation. These results are consistent with the experimental observation of the beam ion-driven GAM in DIII-D plasmas[6].

V. ENERGETIC PARTICLE EFFECTS

So far, we have used fluid model to describe nonlinear self-interaction of GAMs driven by energetic particles and we have neglected energetic particle effects on the nonlinear generation of second harmonic. We find that there is no second harmonic in the radial electric field due to fluid nonlinearity of thermal plasmas. Here we investigate whether the kinetic effects of energetic particles can nonlinearly generate second harmonic in radial electric field and its impact on the plasma density perturbation.

The energetic particle effects can be included in radial electric field equations (Eq. (8), Eq. (9), Eq. (15) and Eq.(26)) by adding energetic particle pressure terms. Equation (9) then becomes:

$$< \frac{\rho|\nabla r|^2}{B^2} > \frac{\partial}{\partial t}E_r + < \frac{\rho}{B^2}(\nabla r\times\mathbf{B})\cdot(\mathbf{v}\cdot\nabla\mathbf{v}) >= - < G(r,\theta)(2P + P_{\|h} + P_{\perp h}) > \tag{40}$$

where $P_{\|h}$ and $P_{\perp h}$ is the parallel pressure and perpendicular pressure of energetic particles given by

$$P_{\|h} + P_{\perp h} = \int d^3v(mv_\|^2 + \frac{1}{2}mv_\perp^2)f \tag{41}$$

with f being the energetic particle distribution function. We use the drift-kinetic equation to solve for the distribution function:

$$\frac{\partial f}{\partial t} + (v_\|\mathbf{b} + \mathbf{v_d} + \frac{\mathbf{E}\times\mathbf{B}}{B^2})\cdot\nabla f + \frac{dE}{dt}\frac{\partial f}{\partial E} = 0 \tag{42}$$

where $v_\|$ and $\mathbf{v_d}$ is the particle parallel velocity and the magnetic drift velocity respectively, E is particle energy (not to be confused with the electric field $\mathbf{E}$). For pure radial electric field, the rate of energy change is given by

$$\frac{dE}{dt} = (mv_\|^2 + \frac{1}{2}mv_\perp^2)G(r,\theta)E_r \tag{43}$$

Similar to the expansion of fluid perturbation (Eq. 25), we expand the energetic particle distribution function by

$$f = f_0 + f_1 + f_2 \tag{44}$$

where f_0 is the equilibrium distribution, f_1 is the linear perturbed distribution, and f_2 is the second order perturbed distribution. Expanding Eq. (42) order by order, we arrive the following equations for f_1 and f_2,

$$\frac{df_1}{dt} = -(mv_\parallel^2 + \frac{1}{2}mv_\perp^2)G(r,\theta)E_{r1}\frac{\partial f_0}{\partial E} \tag{45}$$

$$\frac{df_2}{dt} = -\frac{\mathbf{E}_1 \times \mathbf{B}}{B^2} \cdot \nabla f_1 - (mv_\parallel^2 + \frac{1}{2}mv_\perp^2)G(r,\theta)(E_{r1}\frac{\partial f_1}{\partial E} + E_{r2}\frac{\partial f_0}{\partial E}) \tag{46}$$

where $\frac{d}{dt}$ is the total time derivative along the equilibrium orbit and is given by

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (v_\parallel \mathbf{b} + \mathbf{v_d}) \cdot \nabla \tag{47}$$

In order to solve Eq. (45-46), we express E_{r1} as $E_{r1} = \hat{E}\cos(\omega t)$ without loss of generality. We also consider large aspect ratio tokamak equilibria with circular flux surfaces for simplicity. In this limit, the function $G(r,\theta)$ can be written as $G = -\sin\theta/rR$. The linear distribution f_1 can then be obtained from the Eq. (45) as

$$f_{1,non-res} = \frac{mv_\parallel^2 + \frac{1}{2}mv_\perp^2}{2BR}\hat{E}\frac{\partial f_0}{\partial E}(\frac{\cos(\omega t - \theta)}{\omega - \omega_b} - \frac{\cos(\omega t + \theta)}{\omega + \omega_b}) \tag{48}$$

for the non-resonant part of the solution and

$$f_{1,res} = \pi\frac{mv_\parallel^2 + \frac{1}{2}mv_\perp^2}{2BR}\hat{E}\frac{\partial f_0}{\partial E}(\sin(\omega t + \theta)\delta(\omega - \omega_b) - \sin(\omega t - \theta)\delta(\omega - \omega_b)) \tag{49}$$

for the resonant part of the f_1. Substituting f_1 into Eq. (40), we can obtain the linear dispersion relation for the Energetic Particle-induced GAM (EGAM)[3], i.e., $\omega^2 = \omega_{EGAM}^2$.

In the preceding section, we have already shown that the second order radial electric field, E_{r2}, is zero in the fluid model. Then, finite E_{r2} can only come from the kinetic contribution of energetic particles. The equation for E_{r2} with energetic particle effects is given by

$$< \frac{\rho_0|\nabla r|^2}{B^2} > \frac{\partial}{\partial t}E_{r2} = - < G(r,\theta)(2P_2 + P_{\parallel h2} + P_{\perp h2}) > \tag{50}$$

where the second order energetic particle pressures come from the second order distribution f_2. We now solve Eq. (46) for f_2 by substituting the first order solution f_1. Here we assume that the contribution of resonant particles to the second harmonics generation can be neglected and consider only the non-resonant part of f_1. Substituting $f_{1,non-res}$ into Eq. (46), the solution for f_2 is obtained as

$$f_2 = \frac{3(mv_\parallel^2 + \frac{1}{2}mv_\perp^2)}{2B^2 rR}\hat{E}^2 \frac{\partial f_0}{\partial E}\frac{\omega\omega_b}{(4\omega^2-\omega_b^2)(\omega^2-\omega_b^2)}\sin\theta\cos 2\omega t + f_{2,lin} \tag{51}$$

where $f_{2,lin}$ is the response due to E_{r2}. It should be noted that in the first term of f_2 we have only kept the term proportional to $\sin\theta$ that have finite contribution to E_{r2} (see the right hand side of Eq. (50)). We also note that the first term comes from the convective nonlinearity. Substitute above equation into Eq. (50), we arrive at

$$E_{r2} = -\frac{\omega_h^2}{\omega_2^2 - \omega_{EGAM}^2}\frac{\hat{E}}{\omega B r}\hat{E}\sin(2\omega t) \tag{52}$$

where $\omega_2 = 2\omega$ and

$$\omega_h^2 = -\frac{3}{2\rho R^2}\int \frac{\omega^3\omega_b}{(4\omega^2-\omega_b^2)(\omega^2-\omega_b^2)}(mv_\parallel^2 + \frac{1}{2}mv_\perp^2)^2\frac{\partial f_0}{\partial E}d^3v \tag{53}$$

The corresponding second order density perturbation due to E_{r2} is given by

$$\frac{\rho_{2,h}}{\rho_0} = -\frac{\omega_h^2}{\omega_2^2-\omega_{EGAM}^2}\frac{\hat{E}}{\omega^2 B^2 R r}\hat{E}\sin(\theta)\cos(2\omega t) \tag{54}$$

where the subscript h denotes the energetic particle contribution.

From Eq. (37), the second harmonic of the density perturbation due to fluid nonlinearity can be written as

$$\frac{\rho_{2,f}}{\rho_0} = -\frac{\hat{E}}{2\omega^2 B^2 rR}\hat{E}\cos(\theta)\cos(2\omega t) \tag{55}$$

It is instructive to compare the second harmonic $\rho_{2,h}$ due to fluid nonlinearity to $\rho_{2,h}$ due to energetic particle nonlinearity. We have

$$\frac{\rho_{2,h}}{\rho_{2,f}} = \frac{\omega_h^2}{\omega_2^2-\omega_{EGAM}^2}\frac{\sin\theta}{\cos\theta} \sim \frac{P_{\parallel h}+P_{\perp h}}{P}\frac{\sin\theta}{\cos\theta} \tag{56}$$

We see that the ratio $\rho_{2,h}/\rho_{2,f}$ is proportional to the ratio of energetic particle pressure and thermal plasma pressure. We also see that the $\rho_{2,f}$ is updown symmetric whereas $\rho_{2,h}$ is updown asymmetric. Thus, near the midplane of a tokamak, the energetic particle-induced density second harmonic is negligible and our fluid results of the second order density perturbation are still valid.

VI. DISCUSSIONS AND CONCLUSIONS

In this work, we have derived perturbatively the quadratic nonlinear response of an energetic particle-driven Geodesic Acoustic Mode. A fluid model is used for the nonlinear response of thermal plasma whereas a kinetic model is used to describe the energetic particles' nonlinear interaction with GAM. Compared to the recent work of Sasaki et al[15, 23]., our result of the second harmonic of the density perturbation is similar to theirs (see Eq. (37) of this paper and Eq. (21) of Ref. 23). On the other hand, our result of the zero frequency side band in density perturbation is new. After the fluid results of this work were obtained, we became aware of a concurrent work by Zhang et al.[25] on gyrokinetic kinetic theory and simulation of the GAM's nonlinear self-interaction. They concluded that the self-interaction does not generate a second harmonic in radial electric field. It is interesting that our fluid result is similar to their gyrokinetic result in this regard.

In conclusion, we have shown that nonlinear self-interaction of energetic particle-driven GAM does not generate a second harmonic in radial electric field in the fluid limit. However, kinetic effects of energetic particles can induce a second harmonic in the radial electric field. A formula for the second order plasma density perturbation is derived. It is shown that the density second harmonic is generated by the convective nonlinearity of both thermal plasma and energetic particles. Near the midplane of a tokamak, the second order plasma density perturbation (the sum of second harmonic and zero frequency sideband) is negative on the low field side with its size comparable to the main harmonic at low fluctuation level. These analytic predictions are consistent with the recent experimental observation in DIII-D.

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[1] R. Nazikian, *et al.*, Phys. Rev. Lett. 101, 185001 (2008).

[2] N. Winsor, *et al.*, Phys. Fluids **11**, 2448 (1968).

[3] G. Y. Fu, *et al.*, Phys. Rev. Lett. 101, 185002 (2008).

[4] C.J. Boswell, *et al.*, PHYSICS LETTERS A **358** 154-158 (2006).

[5] H.L. Berk, *et al.*, Nucl. Fusion **46** S888-S897 (2006).

[6] R. Nazikian, private communication (2009).

[7] P. H. Diamond et al., Plasma Phys. Controlled Fusion 47, R35 (2005).

[8] V. B. Levedev et al., Phys. Plasmas 3, 3023 (1996).

[9] S. V. Novakovskii et al., Phys. Plasmas 4, 4272 (1997).

[10] H. Sugama and T. H. Watanabe, J. Plasma Physics 72, 825 (2006).

[11] T. Watari et al., Phys. Plasmas 14, 112512 (2007).

[12] Z. Gao et al., Phys. Plasmas 15, 072511 (2008).

[13] F. Zonca and L. Chen, Europhys. Lett. 83, 35001 (2008).

[14] X. Q. Xu et al., Phys. Rev. Lett. 100, 215001 (2008).

[15] M. Sasaki et al., Contrib. Plasma Phys. 48, 68 (2008).

[16] Z. Y. Qiu, L. Chen and F. Zonca, Plasma and Control. Fusion 51, 012001 (2009).

[17] K. Itoh, K. Hallatschek and S. I. Itoh, Plasma Phys. and Control. Fusion 47, 451 (2005).

[18] K. Miki et al., Phys. Rev. Lett. 99, 145003 (2007).

[19] N. Chakrabarti et al., Phys. Plasmas 14, 052308 (2007).

[20] McKeeG G.R. et al., Phys. Plasmas 10 1712 (2003).

[21] Conway G.D. et al., Proc. 21st Int. Conf. on Fusion Energy 2006 (Chengdu, China 2006) (Vienna: IAEA) CD-ROM file EX/2-1 and http://www-naweb.iaea.org/napc/physics/FEC/FEC2006/html/index.htm.

[22] Fujisawa A. et al., Nucl. Fusion 47 S718?26 (2007).

[23] M. Sasaki et al., Phys. Plasmas 16, 022306 (2009).

[24] M. Sasaki et al., Plasma Phys. and Control. Fusion 51, 085002 (2009).

[25] H. S. Zhang, Z. Qiu, L. Chen, and Z. Lin, Nucl. Fusion 49, 125009 (2009).

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Information Services
Princeton Plasma Physics Laboratory
P.O. Box 451
Princeton, NJ 08543

Phone: 609-243-2245
Fax: 609-243-2751
e-mail: pppl_info@pppl.gov
Internet Address: http://www.pppl.gov