

**A Simple Drift-Diffusion Model for
Calculating the Neutralization Time of
 H^- in Xe Gas for Choppers placed in the LEBT**

Cheng-Yang Tan

Accelerator Division/Tevatron

ABSTRACT: The neutralization of H^- beam with a gas like Xe is an important part of low energy beam transport (LEBT). It is well known that choppers which use an electric field when placed in the LEBT strongly affects the neutralization of H^- . The question then naturally arises as to whether a magnetic chopper has a better neutralization time than an electric chopper. To answer this question, a simple 1-space, 1 time drift-diffusion model of H^- beam in Xe gas has been used to calculate the neutralization times for the following scenarios: (a) a region initially cleared of Xe^+ ions with an electric field but partially neutralized outside, (b) a region within and outside the chopper which is initially partially neutralized.

INTRODUCTION

We are designing a new H^- injector which will have a chopper in the low energy transport line (LEBT).[†] We have placed the chopper here rather than in the medium energy transport line (MEBT) because of space constraints.[‡] At first glance, it may seem to be easier to chop the beam at low energy rather than at high energy, but it turns out that there is one major problem which needs to be addressed for chopping the beam transversely at low energy to be successful: we require “neutralization” of the H^- beam not be spoilt when it is chopped. The concept of neutralization is explained below.

The usual method which improves H^- beam transport in the LEBT is to neutralize the H^- beam to overcome space charge forces. Neutralization requires a low pressure background gas like Xe which has a mass of 131 amu. The H^- collides with the Xe gas which produces Xe^+ ions. These Xe^+ ions are slow moving and are attracted to the H^- ions. As the Xe^+ builds up, it forms a potential well which attracts the H^- ions, i.e. gives focusing, as well as cancelling out the charge of the H^- beam. In effect, the H^- beam becomes neutralized because of the Xe^+ ions. And because the beam is now neutral, H^- beam blow up from space charge is no longer a problem. Experiments done at BNL (Brookhaven National Laboratory) have shown that neutralization can increase H^- transport by 30%.¹

Unfortunately, choppers which employ an electric field creates a problem for neutralization because when we turn the chopper on, the E-field is strong enough to sweep the Xe^+ ions from the H^- beam.² This causes the H^- beam to be de-neutralized and thus blow up within the chopper. When we turn the chopper off for the H^- beam to be transported

[†] The LEBT is defined to be the transport line from the source to the entrance of the RFQ. The energy of the H^- beam is 35 keV.

[‡] The MEBT is defined to be the transport line between the RFQ and the first drift tune linac. Energy of H^- is 750 keV.

into the RFQ, the first part of the H^- beam is of the blown up variety and is therefore not fully accepted by the RFQ, i.e. lower beam current at the end of the RFQ. In the mean time, as the H^- is being neutralized, its emittance changes and the beam current fluctuates at the end of the RFQ for some μs (or perhaps even tens of μs).

The neutralization problem with the electric chopper has made us consider a magnetic or a hybrid electric-magnetic chopper instead. The advantage of a magnetic chopper is that it does not destroy neutralization because the magnetic force on the Xe^+ ions is small because its speed is small. However, like the electric chopper, there is a region of space between the chopper and the RFQ which is only partially neutralized when we turn the chopper off. For the magnetic chopper to be of any advantage over the electric chopper, we have to show that the neutralization time in this case is substantially smaller than the neutralization time when an electric chopper is used.

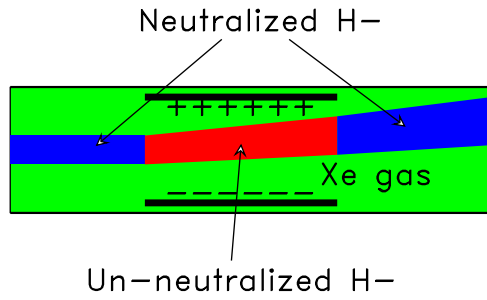
In order to obtain the neutralization times for the various chopper configurations discussed above, we have come up with a simple 1-space, 1-time drift-diffusion model for this purpose. The details of the three chopper configurations which we have mentioned here will be discussed in greater detail in the *The Systems* section. The *Theory* section shows how we derived the drift-diffusion model. The *Numerical Results* section shows the solution of this model for the various scenarios necessary to compare the electric and magnetic choppers. And finally the *Summary* section which tells us what to expect for neutralization time with an electric chopper and a magnetic chopper.

THE SYSTEMS

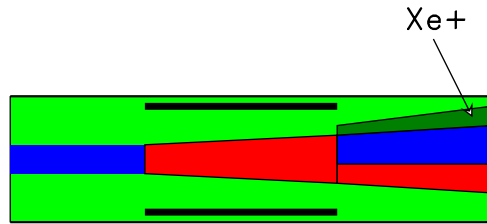
The systems which we have discussed in the *Introduction* are shown in Figures 1, 2 and 3. Figure 1 shows how an electric chopper affects neutralization. BNL experiments have shown that the H^- ions in the region between the chopper plates are essentially unneutralized. At the instant when the chopper is turned off, the Xe^+ ions are left behind. It is only after some time that the beam is fully neutralized in this system. Clearly, the neutralization time will depend on the length of the chopper.

Figure 2 shows the effect on the beam *outside* the chopper when it is turned off. This situation is applicable to both the electric or magnetic chopper. After the chopper is turned off, the H^- beam is suddenly separated from the Xe^+ ions. It will take some time for the H^- to be neutralized again. The success or failure of the magnetic chopper critically depends on whether this neutralization time is much smaller than the neutralization time within the electric chopper.

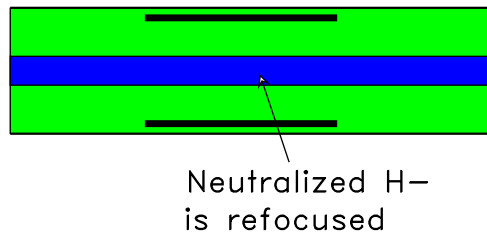
Figure 3 shows how the hybrid electric-magnetic chopper system works. The reason why we are considering a hybrid system is because it is difficult to make a system with only one magnetic chopper because high voltage charging networks cannot turn on-off-on again within $10\ \mu s$.



H⁻ neutralized outside
electrostatic chopper



When chopper is off
H⁻ is suddenly moved
Xe⁺ left behind



After some time,
H⁻ is neutralized again
There may be some Xe⁺
diffusion from outside
the chopper

Figure 1 The electric chopper region does not have any Xe⁺ ions. Outside the chopper the H⁻ is neutralized. In conjunction with the Xe⁺ ions created by the H⁻ ions from collisions, there is also drift-diffusion from outside the chopper when it is off.

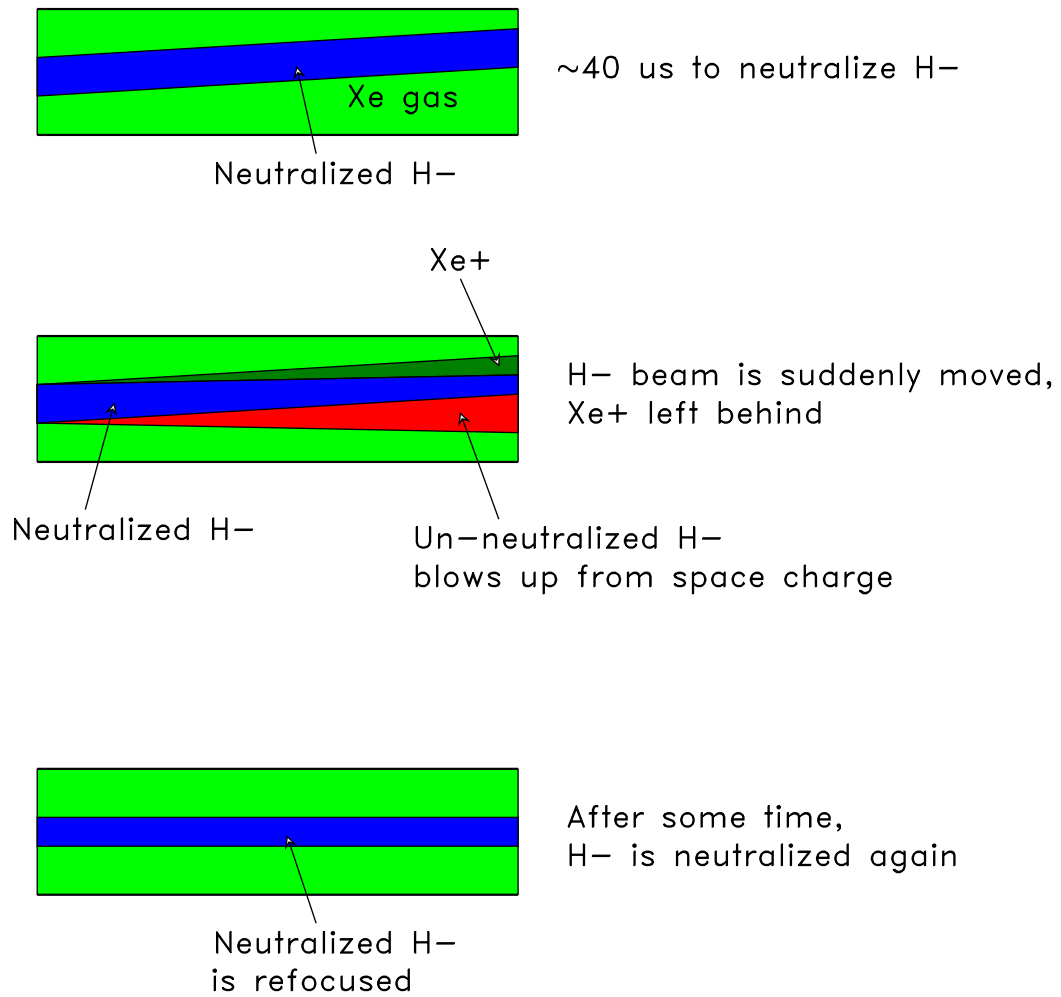


Figure 2 When the neutralized H⁻ beam is suddenly moved, some of the Xe⁺ ions are left behind and the H⁻ beam is now only partially neutralized. Some of the Xe⁺ will migrate into the H⁻ beam by drift-diffusion and some will be created from the H⁻ ionization of Xe gas. The H⁻ beam will be neutralized some later time when there is sufficient Xe⁺ ions.

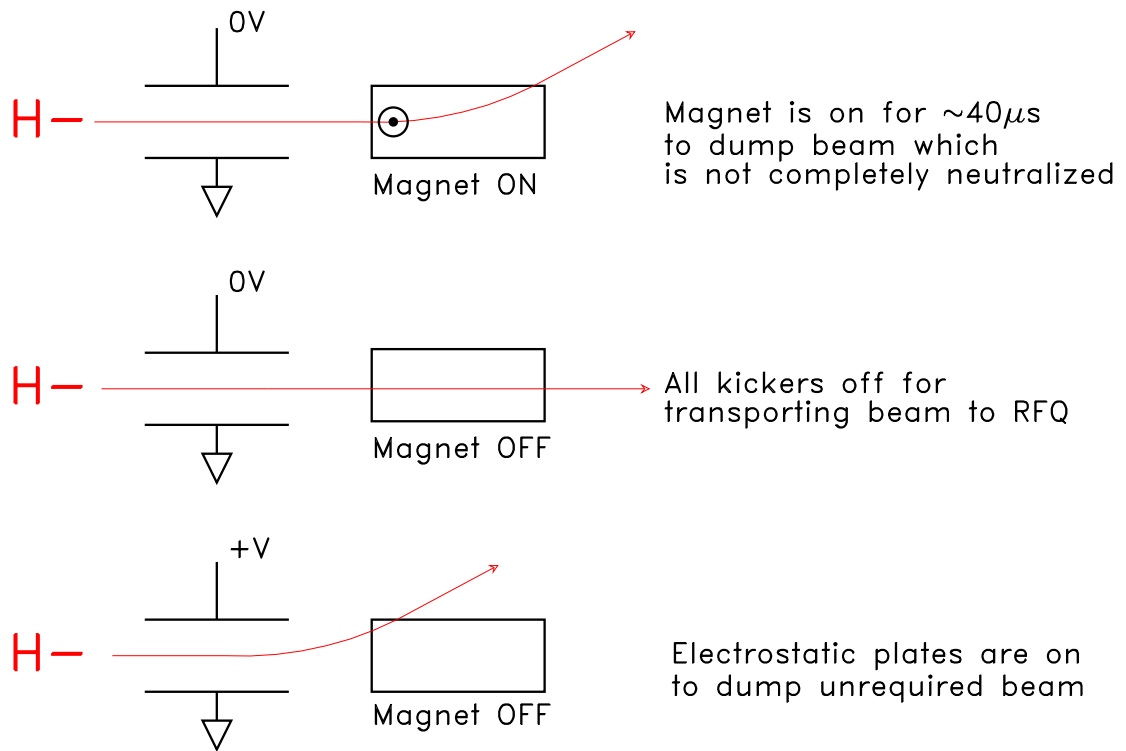


Figure 3 This is a hybrid electric-magnetic chopper system. The system is feasible if the H^- neutralization time with a magnetic chopper is much shorter than the neutralization time with an electric chopper.

THEORY

There are three species which are involved in the drift-diffusion process of the Xe^+ : Xe gas, electrons and H^- ions. The electrons and the H^- create an E-field which causes the Xe^+ to drift; while the differences in concentration of the Xe^+ in the medium consisting of these three species causes the Xe^+ to diffuse.

For simplicity, we are going to assume that we have a 1-D problem in the longitudinal direction and the H^- beam is *non-relativistic* so that non-retarded time can be used in the derivation.

We will first concentrate on the diffusion part of the problem. The flux $J_{x+,D}$ of Xe^+ from diffusion is simply related to the number density gradient n_{x+} of the Xe^+ ions

$$J_{x+,D} = -D_{x+} \frac{\partial n_{x+}}{\partial z} \quad (1)$$

where D_{x+} is the diffusion coefficient of the Xe^+ ions in Xe gas. We have made the simplifying assumption that D_{x+} is fully described by the diffusion of Xe^+ ions in pure Xe gas only.

The drift of Xe^+ in the E-field is more complicated. There are three regimes to consider:³

- (i) When $e|E_z|\lambda \ll kT$ where e is the electronic charge, k is the Boltzmann constant, T is the temperature of the ion, E_z is background longitudinal E-field, λ is the mean free path of the ions. In this case, the drift velocity v_d of the ion is related to the E-field by

$$v_d = \mu_{x+} E_z \quad (2)$$

where μ_{x+} is the mobility of the Xe^+ ion.

- (ii) When $e|E_z|\lambda \sim kT$. In this case there is no simple relationship between the electric

field and the drift velocity that we are aware of. Therefore, the usual Newtonian equation $F = ma$ must be used.

(iii) When $e|E_z|\lambda \gg kT$. The average drift velocity is

$$\langle v_d \rangle = \text{sign}[E_z] \times \sqrt{\frac{2e|E_z|\lambda}{\pi m_x}} \quad (3)$$

where m_x is the mass of the Xe atom. Unlike (i), the drift velocity is proportional to the square root of the background field.

Using the parameters of our LEBT from Table 1, $kT = (1.38 \times 10^{-23}) \times 273 = 0.024$ eV, and $\lambda = 636$ cm. This means that the equivalent $|E_z| = 4 \times 10^{-5}$ V/cm when $|E_z|\lambda = 0.024$ eV. And so, unless $|E_z|$ is very close to zero, we will make the simplification that (iii) is adequate for every case and use it for the Xe^+ flux and ignore the other two cases. When we do this, the flux from the drift of Xe^+ ions in an E-field is simply

$$J_{x+,d} = \mu_E n_{x+} \times \text{sign}[E_z] \times |E_z|^{1/2} \quad (4)$$

where $\mu_E = \sqrt{2e\lambda/\pi m_x}$. And so we can write the flux equation as

$$\left. \begin{aligned} J_{x+} &= J_{x+,D} + J_{x+,d} \\ &= -D_{x+} \frac{\partial n_{x+}}{\partial z} + \mu_E n_{x+} \times \text{sign}[E_z] |E_z|^{1/2} \end{aligned} \right\} \quad (5)$$

Now, from the continuity equation, we have

$$\frac{\partial n_{x+}}{\partial t} + \nabla \cdot \mathbf{J}_{x+} = 0 \quad \Rightarrow \quad \frac{\partial n_{x+}}{\partial t} = -\frac{\partial J_{x+}}{\partial z} \text{ for a 1-D problem.} \quad (6)$$

and so when we substitute (5) into (6), we get p.d.e. which we can in principle solve for n_{x+} when the appropriate boundary and initial conditions are given

$$\frac{\partial n_{x+}}{\partial t} = +D_{x+} \frac{\partial^2 n_{x+}}{\partial z^2} - \mu_E \frac{\partial}{\partial z} \left(\text{sign}[E_z] n_{x+} |E_z|^{1/2} \right) \quad (7)$$

In the simplification of (7), we have assumed that both μ_{x+} and D_{x+} are independent of z . The boundary and initial conditions which are needed for solving (7) will be specified for each scenario which we will be considering.

(7) is still incomplete because we have not included the production of Xe^+ from the ionization of Xe gas atoms by the H^- ions. The rate of production of Xe^+ by this mechanism is given by⁴

$$\frac{dn_{x+}}{dt} = n_h n_x \sigma_x v_h \quad (8)$$

This equation says that the rate of production of Xe^+ is only dependent on the number density n_h of H^- , Xe gas number density n_x , ionization cross section σ_x of Xe and the velocity v_h of the H^- . Interestingly, it does not depend on the concentration of Xe^+ .

The complete production rate of n_{x+} is thus the sum of (7) and (8) and is

$$\frac{\partial n_{x+}}{\partial t} = +D_{x+} \frac{\partial^2 n_{x+}}{\partial z^2} - \mu_E \frac{\partial}{\partial z} \left(n_{x+} \text{sign}[E_z] |E_z|^{1/2} \right) + n_h n_x \sigma_x v_h \quad (9)$$

We can divide the above by n_h , which in our model is assumed to be constant (see *Weaknesses of Method* section), and introduce the variable $f = n_{x+}/n_h$ for the fractional neutralization of H^- (clearly when $f = 1$, the system is completely neutralized) to get

$$\frac{\partial f}{\partial t} = +D_{x+} \frac{\partial^2 f}{\partial z^2} - \mu_E \frac{\partial}{\partial z} \left(f \text{sign}[E_z] |E_z|^{1/2} \right) + n_x \sigma_x v_h \quad (10)$$

This is the p.d.e. which has to be solved after the boundary and initial conditions have been specified. Note:

$$\frac{\partial}{\partial z} \left(f \text{sign}[E_z] |E_z|^{1/2} \right) = \text{sign}[E_z] |E_z|^{1/2} \frac{\partial f}{\partial z} + \frac{f}{2|E_z|^{1/2}} \frac{\partial E_z}{\partial z} \quad (11)$$

Boundary Conditions

The boundary conditions for both the electric and magnetic chopper systems are listed below. The boundary conditions which we are using are analogous to a metal rod with one end placed in a constant temperature water bath and the other end insulated.

(i) $f(0, t) = 1, \forall t$. i.e. we have complete H^- neutralization at the upstream end of the chopper. This boundary condition is analogous to placing one end of a metal rod into a water bath which is held at a constant temperature.

(ii) $J_{x+,D}(L, t) + J_{x+,d}(L, t) = 0, \forall t$. i.e. nothing flows in or out at the end of the system. This boundary condition is analogous to insulating one end of a metal rod. This boundary condition when put into the flux equation (5) tells us that $\left(-D_{x+} \frac{\partial f}{\partial z} + \mu_E f \text{sign}[E_z] |E_z|^{1/2}\right) \Big|_{z=L} = 0$. Notice that $f|E_z|^{1/2} \Big|_{z=L}$ can never be zero unless $E_z(L, t) = 0$ because the production rate of Xe^+ from collisions of Xe gas atoms with H^- is not zero. This simplifies the boundary condition to $\frac{\partial f}{\partial z} \Big|_{z=L} = 0, \forall t$.

E-field inside a finite line charge

E_z at position z inside a finite line charge of length L can be calculated by vectorially summing the contributions from all the charges from $z' = 0$ to $z' = L$. See Figure 4. We will neglect the charge contribution from the electrons because they escape easily from the potential well, and we will collapse all the charges in the volume $\pi R_0^2 L$ (where R_0 is the radius of the beam pipe) to a line charge. Therefore, the E-field E_z which comes from the H^- and the Xe^+ ions is

$$\left. \begin{aligned} E_z(z, t) &= E_{\rightarrow} + E_{\leftarrow} \\ &= \frac{qR_0^2}{4\epsilon_0} \left(\int_z^L dz' \frac{n_h - n_{x+}(z', t)}{|z' - z|^2} - \int_0^z dz' \frac{n_h - n_{x+}(z', t)}{|z - z'|^2} \right) \\ &= \frac{qR_0^2 n_h}{4\epsilon_0} \left(\int_z^L dz' \frac{1 - f(z', t)}{|z' - z|^2} - \int_0^z dz' \frac{1 - f(z', t)}{|z - z'|^2} \right) \end{aligned} \right\} \quad (12)$$

where n_h is the number density of the H^- ions, and ϵ_0 is the permittivity of free space.

We notice that there can be a singularity at either $z = 0$ or $z = L$. For example if

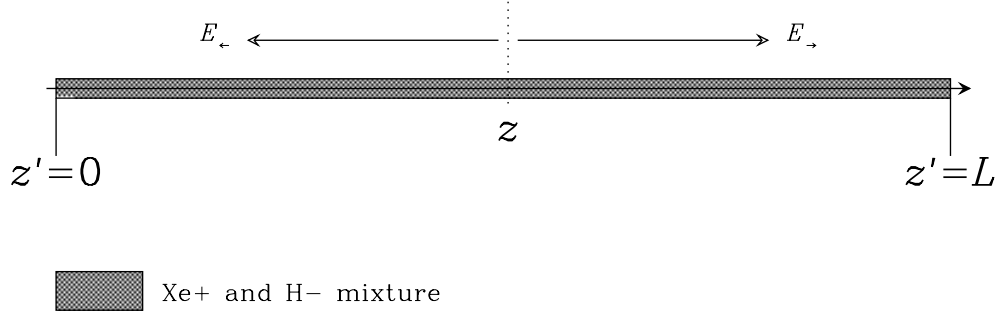


Figure 4 The E-field at z is the contribution of E-fields E_{\leftarrow} and E_{\rightarrow} which in turn depends on the number of Xe^+ and H^- ions to the left and right of z .

$f(z', t) = 0$, then (12) is easily integrated

$$e_z(z, t) = \frac{qR_0^2 n_h}{4\epsilon_0} \left(\frac{1}{z} + \frac{1}{z-L} \right) \quad (13)$$

(13) is clearly singular at either $z = 0$ or $z = L$. **There is no possibility of getting rid of this type of singularity at the ends of the finite length line charge.** The consequence of this type of singularity means that when we do our numerical integration of E_z , we have to reduce the step size in the z -direction until we do not see a change in the results.

Weaknesses of Method

There are at least three obvious weaknesses in this 1-space, 1-time model of the LEBT. They are:

- (i) There is no transverse transport of the Xe^+ ions because our model is only in the longitudinal direction. We can see from the middle pictures of Figure 1 and 2 for the partially neutralized H^- beam that in this model, we do not take into account the transverse flow of Xe^+ ions back into the H^- . This flow will probably shorten

the neutralization time calculated by our model.

- (ii) There is no loss of Xe^+ to keep the H^- beam from becoming over-neutralized, i.e. there is no mechanism to stop $f > 1$. For example in Reiser⁴ eq. (4.293), he forces $f = 1$, $\forall t$ after it is neutralized.
- (iii) There is no loss of H^- beam from under-neutralization or ionization. For under-neutralization, the H^- blows up transversely and can be scraped at the walls of the beam pipe. For ionization of the Xe atom, the weakly bound electron of the H^- ion can be stripped in the process, and so $\text{H}^- \rightarrow \text{H} + e$ and is lost. In fact, in our model, we have assumed that n_h is constant.

NUMERICAL RESULTS

We will first calculate the LEBT neutralization time and compare the result to experiment. The next step is to calculate the neutralization time for the electric and magnetic choppers. The initial conditions of these two scenarios will be discussed in *Electric Chopper System* and *Magnetic Chopper System* sections.

LEBT Neutralization Time

We can calculate the neutralization time of the LEBT when the H^- beam is first turned on. It is essentially the solution to (8).

$$\frac{dn_{x+}}{dt} = n_h n_x \sigma_x v_h \quad \Rightarrow \quad \frac{df}{dt} = n_x \sigma_x v_h \quad (14)$$

The neutralization time τ_N is found by integrating (14) once with the limits 0 to 1 on the lhs and 0 to τ_N on the rhs

$$1 = n_x \sigma_x v_h \tau_N \quad \Rightarrow \quad \tau_N = \frac{1}{n_x \sigma_x v_h} \quad (15)$$

We can substitute the parameters of the LEBT from Table 1 into (15) to find that

$$\tau_N = \left[(1.31 \times 10^{11}) \times (8 \times 10^{-16}) \times 0.00864 \times (3 \times 10^{10}) \right]^{-1} = 37 \mu s \quad (16)$$

The measured value is 40 μs which is close to what we have calculated.

Electric Chopper System

The initial f_{init} distribution of the electric chopper system is shown in Figure 5. We have constructed f_{init} for the situation where the chopper has just been turned off *after* the H^- has been neutralized. See Figure 1. We have set f_{init} between the chopper plates

to zero and we have assumed that the amount of neutralization falls off linearly from the end of the plates to an inch before the entrance of the RFQ. The value of f_{init} at the end of the chopper plates comes from assuming that $f = 1$ at the mid-point of the plates. The boundary conditions at $z = 0$ and $z = L$ are exactly those which we have discussed in the *Boundary Conditions* section.

Diffusion and Ionization Only

We can compare the neutralization time for the case when drift is turned off i.e. the p.d.e. is

$$\frac{\partial f}{\partial t} = +D_x + \frac{\partial^2 f}{\partial z^2} + n_x \sigma_x v_h \quad (17)$$

When we integrate (17), we can see from Figure 6(a) that the time required for the entire LEBT to have $f \geq 1$ is about $32 \mu\text{s}$. Notice that although $f > 1$ at this time, it is probably not reality because the Xe^+ ions will escape the potential well to keep $f \approx 1$. This is a limitation of our model which was discussed in section *Weaknesses of Method*.

A quick back of the envelope calculation shows that $32 \mu\text{s}$ is not too far off because at $t = 0$, about 22% of the LEBT has already been neutralized. Therefore, for complete neutralization by ionization alone and assuming *uniform* ionization within the LEBT, we have $(1 - 0.22) \times 37 = 29 \mu\text{s}$ where the $37 \mu\text{s}$ comes from the *LEBT Neutralization Time* section. The extra few μs predicted by (17) can be explained by noting from Figure 6(a) that the process of neutralization is not uniform in the LEBT and parts of it became over neutralized.

Drift, Diffusion and Ionization

The neutralization time calculated with the p.d.e. shown in (10) is summarized in

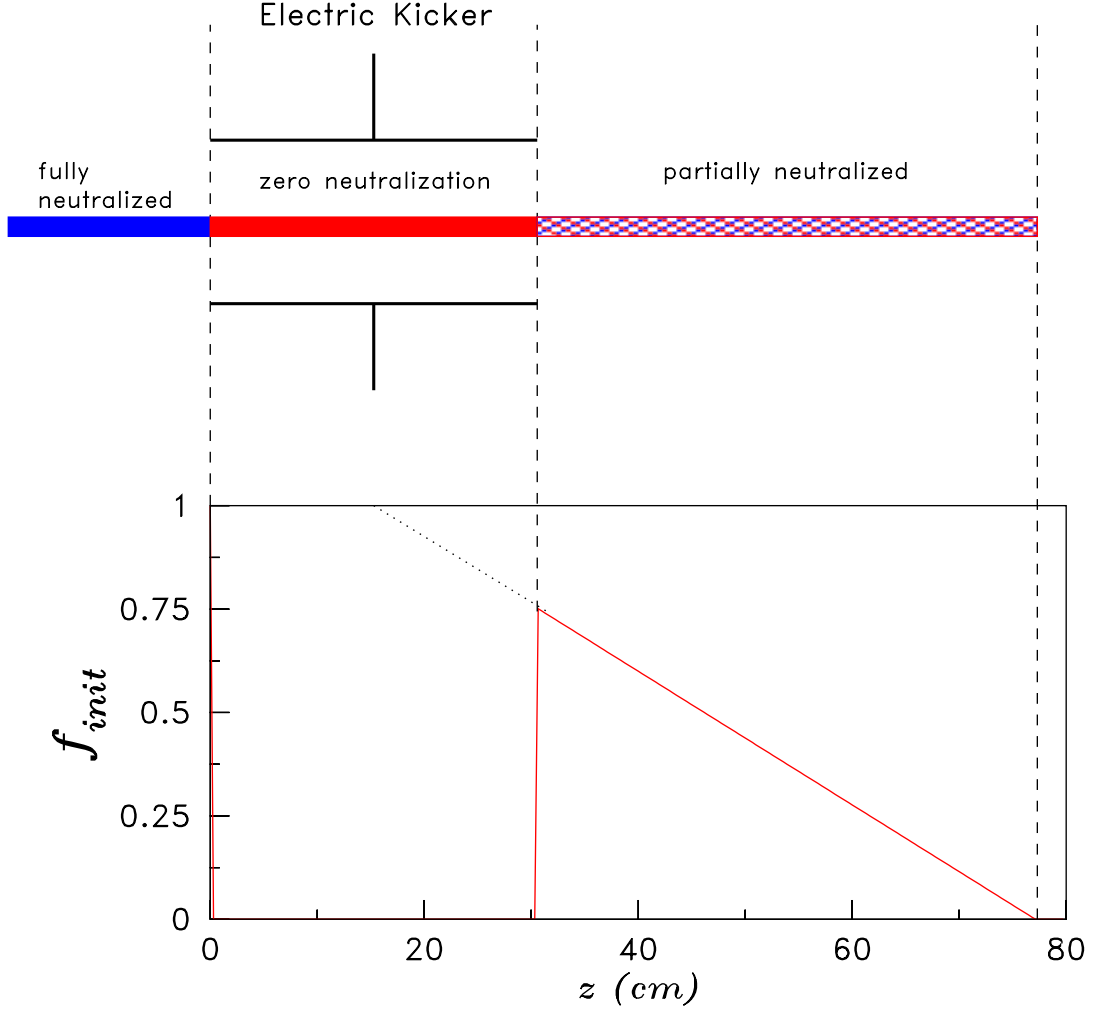


Figure 5 f_{init} for the electric chopper system just after the chopper is turned off. This corresponds to the middle picture of Figure 1. $f_{\text{init}} = 1$ (dotted line) when projected back to the middle of the chopper.

Table 2 for different step sizes in z . Although the neutralization time t_E is decreasing as the step size is halved, we have not found the asymptotic value for t_E . We decided to stop at $\Delta z = 0.08$ cm because the computing time took more than 1 day to complete on a core 2 duo running at 2.33 GHz.

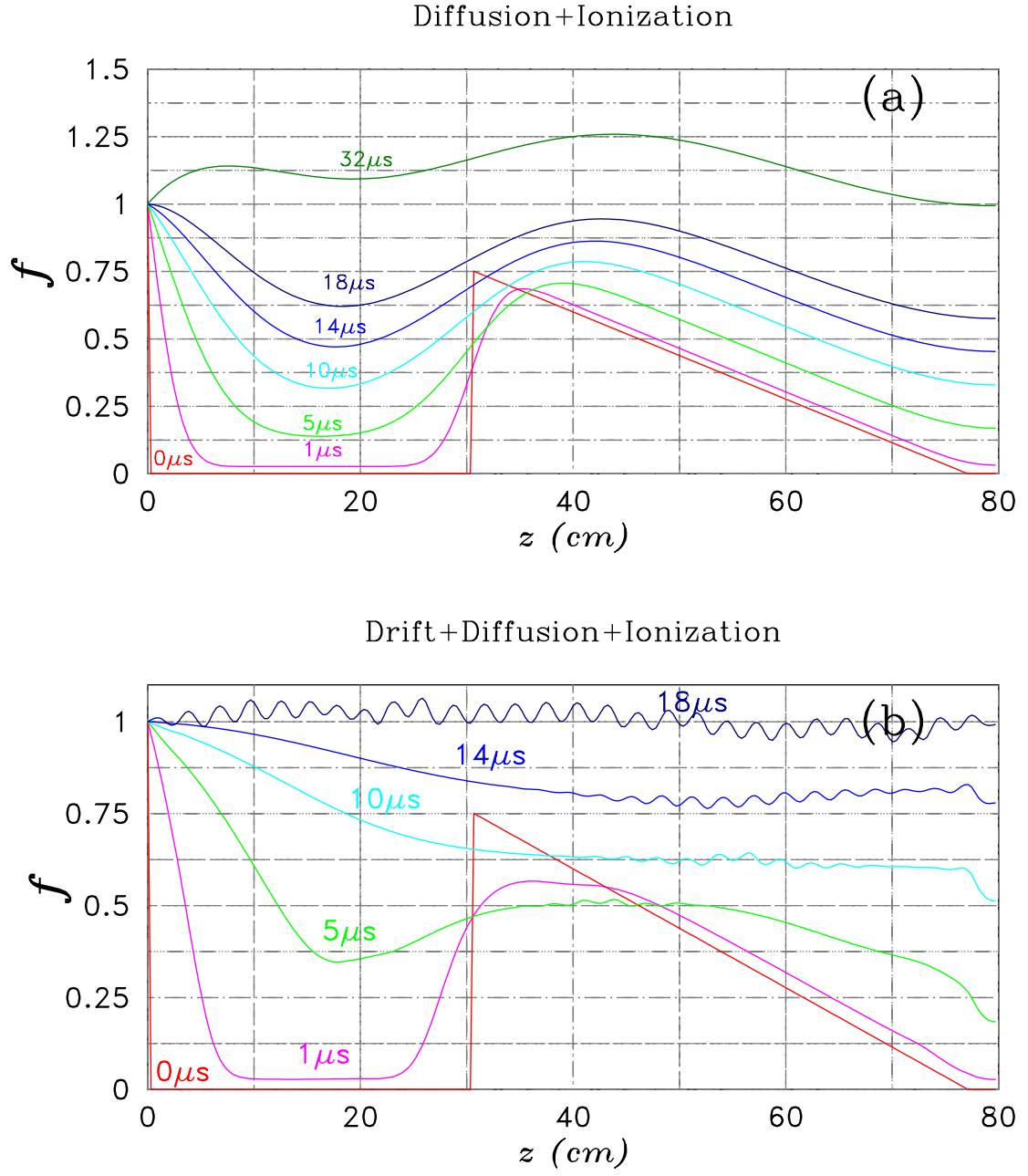


Figure 6 The time evolution of f for the electric chopper system for the cases where there is (a) diffusion and ionization only (b) drift, diffusion and ionization. The wiggles starting at $14 \mu s$ comes from the deficiency of the numerical integrator.

An example of how f evolves as a function of time is shown in Figure 6(b) for $\Delta z = 0.32$ cm. It is interesting to see that at $z = 30$ cm, which is the downstream edge of the electric chopper plate, f decreases at the start of the simulation because the higher concentration of Xe^+ initially flows into the Xe^+ starved region first. Then as time evolves, f slowly flattens out in the region $z \geq 30$ cm and then lifts up towards $f = 1$.

It is clear from Figure 6 that the effect of drift speeds up the re-distribution of Xe^+ substantially when added to the diffusion and ionization processes. Therefore, drift is an important part in the neutralization process.

Magnetic Chopper System

The f_{init} distribution for the magnetic chopper system is shown in Figure 7. The major difference between f_{init} for this system and the electric chopper system is the absence of a $f_{\text{init}} = 0$ portion within the magnetic chopper. We have assumed that the amount of neutralization falls off linearly from the middle of the magnet to an inch before the entrance of the RFQ. The boundary conditions are exactly those discussed in the *Boundary Conditions* section applied to the partially neutralized part of the beam (marked with green arrows) in Figure 7.

Diffusion and Ionization Only

Like in the electric chopper case, we can first turn the drift off to calculate the neutralization time. The results are shown in Figure 8(a). The neutralization time is about $32 \mu\text{s}$, which surprisingly, is about the same as the electric chopper case. This time is substantially different from the back of the envelope calculation with ionization only because at $t = 0$, the LEBT is already 57.5% neutralized. This means that the LEBT can be neutral-

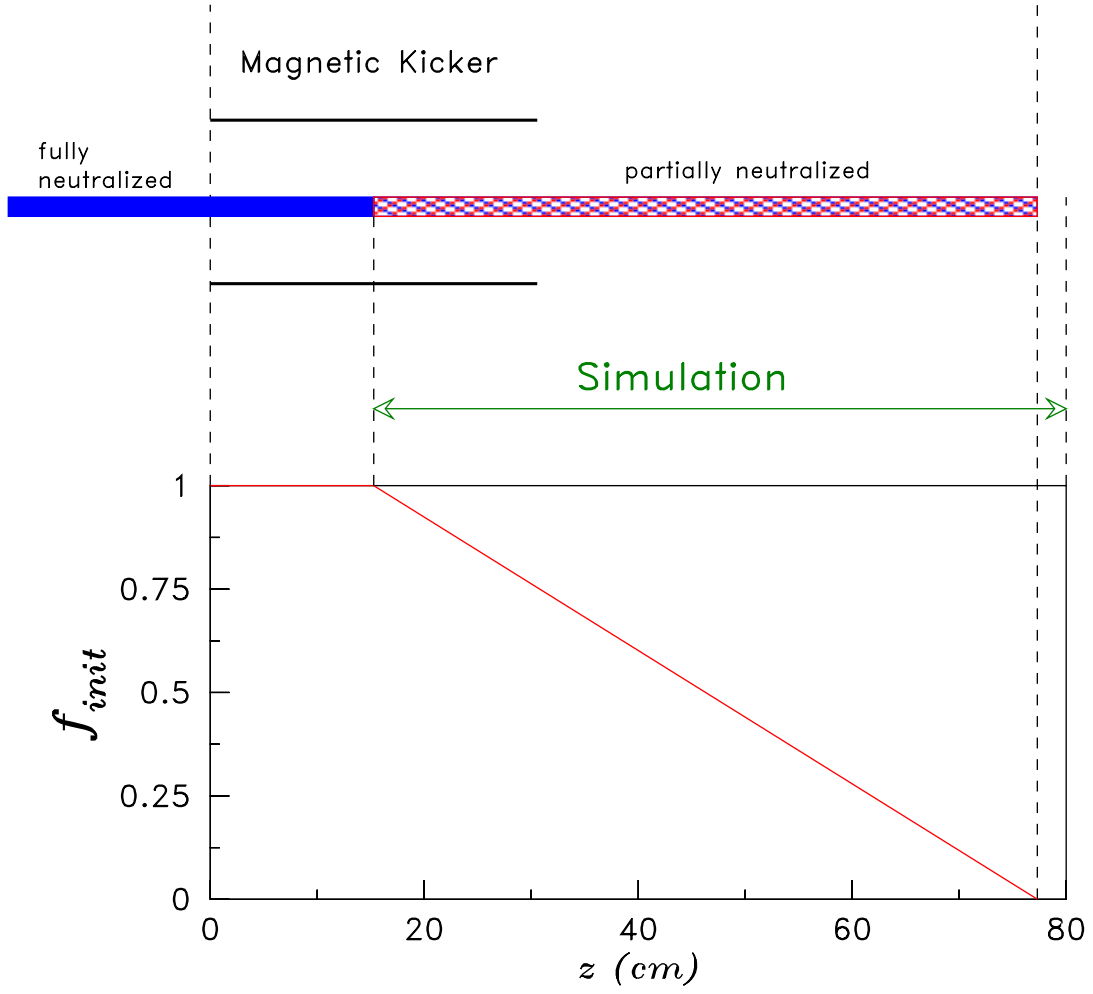


Figure 7 f_{init} for the magnetic chopper system just after the chopper is turned off. This corresponds to the middle picture of Figure 2. The simulation only considers the partially neutralized portion of the beam which have been marked with green arrows.

ized by ionization alone in $(1 - 0.575) \times 37 = 15.7 \mu\text{s}$. We can explain this discrepancy by noting that in Figure 8(a), from $t = 5 \mu\text{s}$, the LEBT is beginning to get over-neutralized around $z = 20$ cm. And at $t = 32.25 \mu\text{s}$, more than half the LEBT is over-neutralized and only the last few centimetres are at $f = 1$. The non-uniform neutralization process has increased the overall neutralization time of the LEBT.

The neutralization time calculated with the p.d.e. shown in (10) is summarized in Table 2 for different step sizes in the numerical integration of (10). Like the electric chopper case, the neutralization time t_M decreases as Δz is halved. We did not try to get to the asymptotic value of the neutralization time because it took more than a day to integrate (10) for $\Delta z = 0.08$ cm.

An example of how f evolves as a function of time is shown in Figure 8(b) for $\Delta z = 0.32$ cm. Unlike the electric chopper case, $f \neq 0$ within the magnetic chopper. As time evolves, f approaches 1. And like the electric chopper case, drift is an important part in the neutralization process because it speeds up the re-distribution of Xe^+ when added to the diffusion and ionization processes.

Table 2. Neutralization Time vs step size

Δz (cm)	t_E (μs)	t_M (μs)	$(t_E - t_M)/t_E$ (%)
0.32	18	16	11
0.16	14.5	12.5	14
0.08	12	11	8

SUMMARY

When we examine the neutralization times t_E and t_M shown in Table 2, we see that $t_E > t_M$ no matter the step size Δz . We can say that t_M is only faster by about 10% compared to t_E . This means that there is no advantage in using the magnetic chopper over the electric chopper. This is our major result: Substituting a magnetic chopper for an electric chopper does not fix the de-neutralization problem posed in the *Introduction*.

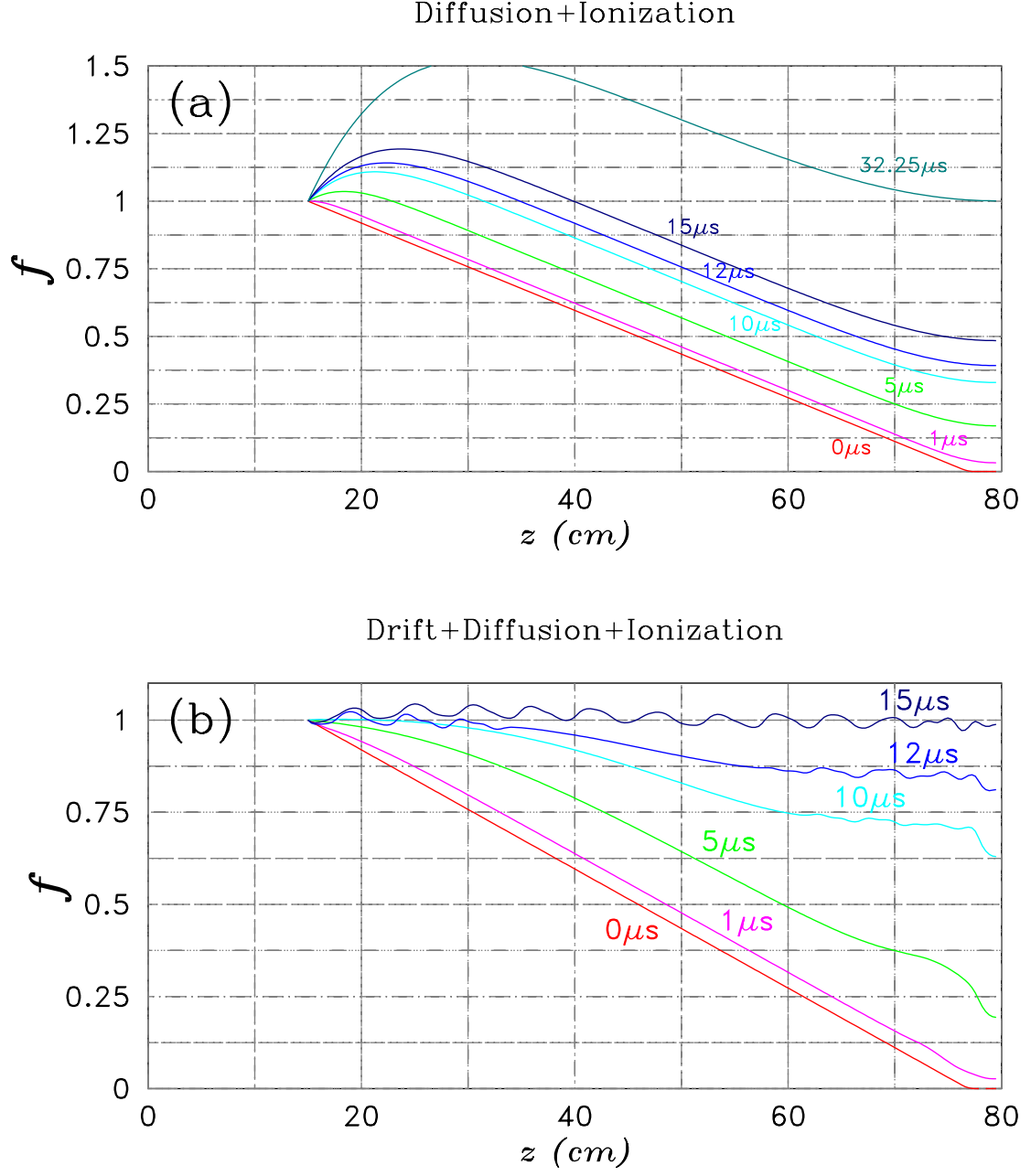


Figure 8 The time evolution of f for the magnetic chopper system. (a) is for the case when only diffusion and ionization are turned on, (b) is when drift, diffusion and ionization are turned on. The wiggles starting at $10\mu s$ comes from the deficiency of the numerical integrator.

ACKNOWLEDGEMENTS

This work is supported by Fermi Research Alliance, LLC under contract No. DE-AC02-07CH11359 with the U.S. Department of Energy.

APPENDIX I

The parameters used in the calculations are summarized in Table 1. We will show where all of these numbers come from here.

p_x is the value used at BNL.

σ_x comes from ref. 1.

n_x is calculated by assuming that Xe is an ideal gas. We can use the relationship between Loschmidt's number $n_0 = 2.69 \times 10^{19} \text{ cm}^{-3}$ ($= 2.69 \times 10^{25} \text{ m}^{-3}$) defined for pressure $p_0 = 760 \text{ torr}$ and temperature $T_0 = 273 \text{ K}$ for calculating n_x as follows

$$\begin{aligned} n_0 &= \frac{P_0}{kT_0} \Rightarrow \frac{n_x}{n_0} = \frac{p_x T_0}{p_0 T_x} \\ \Rightarrow n_x [\text{cm}^{-3}] &= \frac{p_x T_0}{p_0 T_x} n_0 = (3.54 \times 10^{16}) \times p_x [\text{torr}] \end{aligned} \quad (18)$$

Therefore, for $p_x = 3.6 \times 10^{-6} \text{ torr}$, $n_x = 1.31 \times 10^{11} \text{ cm}^{-3}$.

n_h is calculated from the H^- current $I_h = 50 \times 10^{-3} \text{ A}$ and speed of the H^- ion at 35 keV. We used the following formula

$$n_h = \frac{I_h}{\pi R_0^2 q v_h} = 1.49 \times 10^7 \text{ cm}^{-3} \quad (19)$$

D_{x+} The diffusion constant comes from ref. 5, and is $D_{x+} = 10.5/p_x = 10.5/(3.7 \times 10^{-6}) = 2.84 \times 10^6 \text{ cm}^2\text{s}^{-1}$.

λ We can calculate the mean free path of the Xe^+ in Xe gas with the following formula

$$\lambda = \frac{1}{n_x \sigma_{\text{res}}} = \frac{1}{(1.31 \times 10^{11}) \times (1.2 \times 10^{-14})} = 636 \text{ cm} \quad (20)$$

where σ_{res} comes from ref. 5.

Table 1. LEBT Parameters

Parameter	Value	Units	Comments
c	3×10^{10}	cm s^{-1}	speed of light
k	1.38×10^{-23}	J K^{-1}	Boltzmann's constant
ϵ_0	8.854×10^{-12}	$\text{C}^2\text{N}^{-1}\text{m}^{-2}$	permittivity of free space
R_0	2	inches	radius of beam pipe
T_x	273	K	temperature of Xe gas
p_x	3.7×10^{-6}	torr	pressure of Xe gas
m_x	131.29	amu	mass of Xe atom
n_x	1.31×10^{11}	cm^{-3}	# density of Xe ions at T and P
n_h	1.49×10^7	cm^{-3}	# density of H^- in 50 mA beam
D_{x+}	2.84×10^6	cm^2s^{-1}	diffusion constant of Xe^+
σ_x	8×10^{-16}	cm^2	ionization cross section of Xe
v_h	$0.00854c$	cm s^{-1}	speed of 35 keV H^- ions
σ_{res}	1.2×10^{-14}	cm^2	resonant exchange cross section of Xe
λ	636	cm	mean free path of Xe^+ at p_x and T_x

REFERENCES

- [1] D. Raparia *et al*, *Results of LEBT/MEBT Reconfiguration at BNL 200 MeV Linac*, Part. Acc. Conf., 2009.
- [2] J.G. Alessi *et al*, *H⁻ Source and Low Energy Transport for the BNL RFQ Preinjector*, AIP Conf. Proc., Vol. 210, pg. 711-716, 1990.
- [3] B.M. Smirnov, *Physics of Ionized Gases*, Chapt. 11, John Wiley & Sons, 2001.
- [4] M. Reiser, *Theory and Design of Charged Particle Beams, 2nd Ed.*, pg. 244, Wiley-VCH, 2008.
- [5] R. Hakam, *Mobility and Diffusion Cross Section of Atomic Xenon Ion in Xenon*, J. Phys. B, Ser. 2, Vol. 2, pg. 790-797, 1969.