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On the Use of Rossi Alpha Critical Assembly Measurements for Validating and Constraining Nuclear Data

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ABSTRACT

Critical assemblies are exquisitely sensitive to details of the microscopic nuclear reactions that govern neutron multiplication. For this reason experimental studies of critical assemblies represent a cornerstone in the process of validating nuclear data. Several different characteristics of a critical system can be measured. The most commonly considered is the so-called effective k eigenvalue, $k_{\rm eff}$. Another well-measured property of these systems is α_0, the inverse e-folding time of the neutron population in the absence of β-delayed neutrons. Through Monte Carlo calculations and appeal to perturbation theory we show that for fast critical systems α_0 and $k_{\rm eff}$ can be viewed as lying on a single straight line for any reasonable assumptions about the underlying nuclear data. This means that the two quantities provide the same constraint on nuclear data. In principle, α_0 could be associated with a very small uncertainty, and this would make the measurements for neutron multiplication rates more useful than $k_{\rm eff}$ for constraining nuclear data. In practice, though, uncertainties in $k_{\rm eff}$ and α_0 are dominated by uncertainties in the representation of the critical system, and not by pure measurement errors for these quantities. This, together with the linear relation between α_0 and $k_{\rm eff}$, implies that the two measured quantities provide exactly the same constraint on nuclear data. We do not consider other measured or inferred characteristics of critical assemblies, such as neutron generation times or spectral indices, that may be valuable in the validation process.

1. Introduction

Here we consider the value of using Rossi-α measurement for critical assemblies to validate and improve nuclear data libraries. There is a long history of simulating critical systems for code and data validation (see Frankle (1999,b) for discussion of a comprehensive test suite). By far, most effort in this area has gone toward understanding implications of measurements for effective k eigenvalues ($k_{\rm eff}$). Measurements of the neutron multiplication rate (α_0) have seen comparatively little use.

Roughly speaking, $k_{\rm eff}$ measurements provide integral constraints on certain data that are about a factor of ten better than constraints provided by laboratory scattering experiments. However, since critical assemblies are sensitive to many different reactions, $k_{\rm eff}$ measurements do not uniquely pin down a best set of nuclear data. In principle Rossi Alpha measurements of multiplication rates could provide additional constraints.

To illustrate how measurements of α_0 might be useful, suppose that $k_{\rm eff}$ were sensitive to a ratio of two cross sections while α_0 was sensitive mostly to a single cross section. If this were the case, then the two quantities would provide independent information and constraints that were in some sense orthogonal. Even if $k_{\rm eff}$ and α_0 had exactly the same functional dependence on nuclear data, it could still be the case that α_0 was associated with a very small experimental uncertainty. This would make the multiplication rate useful for data validation (while making $k_{\rm eff}$ superfluous).

Our basic finding is that, at least for fast systems in which neutrons do not efficiently thermalize, α_0 does not provide any constraints not already provided by $k_{\rm eff}$. To support this we show in section 2 that α_0 and $k_{\rm eff}$ are linearly related.

Table 1
Data Varied in Exploring the Relations Between α_0 and $k_{\rm eff}$ for Jezebel

reaction	quantity	range of variation
^{239}Pu(n,el)	cross section	$\pm 20\%$
^{239}Pu(n,γ)	cross section	$\pm 50\%$
^{239}Pu(n,f)[a]	cross section	$\pm 4\%$
^{239}Pu(n,f)[a]	prompt neutron spectrum	$\approx \pm 150$keV[b]

[a]For both the fission cross section and the prompt neutron spectrum we assumed an energy-dependent variation. The cross section was varied at four separate energy nodes and a correlation length of 1.5 MeV was assumed in connecting the variations between nodes. The spectrum was varied at 5 separate energy nodes and the correlation length was again 1.5 MeV.

[b]This is approximately the range of variation in the mean energy of prompt fission neutrons.

By this we mean that if we independently vary the different pieces of nuclear data describing neutron interactions on a fissile isotope, and calculate $k_{\rm eff}$ and α_0 for each set of varied data, then a plot of the results is a straight line. Of course, the $\alpha_0/k_{\rm eff}$ curve won't be a straight line for any arbitrary choice about the nuclear data. And the line isn't exactly straight. But for plausible data variations - 10% changes in the different cross sections and changes in the mean energy of post-fission neutrons of order 150 keV - the line is effectively straight. In section 3 we note that when uncertainties in the configuration of a critical assembly dominate the quoted uncertainties in $k_{\rm eff}$, the uncertainty in α_0 just scales as the uncertainty in $k_{\rm eff}$. The proportionality constant between the two is roughly the slope of the $\alpha_0/k_{\rm eff}$ curve, so the two quantities give the same information about nuclear data. [h]

2. The relation between $k_{\rm eff}$ and α_0 for modest changes to the nuclear data

2.1. Perturbation Theory

Bell & Glasstone (1970) (see also Hansen & Maier (1960)) describe the influence of small changes in cross sections on the multiplication rate and k eigenvalue for a system. Their analysis is based on first order perturbation theory and a consideration of the adjoint transport equation. We will simply quote the main results. For small changes to any of the cross sections (including fission, absorption, elastic, and so on) the change in multiplication rate of a nearly critical system is proportional to the change in the k eigenvalue for the system. This is true even for position-dependent changes to the cross sections.

2.2. Results from Monte Carlo Simulations

To support the perturbation theory result we ran many different simulations for the Jezebel critical assembly. Each simulation used a different set of nuclear data. Several different characteristics of reactions for ^{239}Pu, the main fuel in the Jezebel assembly, were varied. These include cross sections for elastic scattering, absorption and fission. The spectrum of prompt fission neutrons was also varied. We tried to choose the largest plausible range of variations for the different reaction quantities. The fission cross section was varied in a range of $\pm 4\%$ about the nominal ENDL99 value, the elastic cross section by $\pm 20\%$, the (n,γ) cross section by $\pm 50\%$, and the mean energy of prompt fission neutrons was varied by about ± 150 keV. For fission we assumed fairly complicated energy-dependent variations defined by a correlation length in energy. But for the present arguments these details are not so important. All quantities were varied at once and assumed to be uncorrelated. Table 1 summarizes the different pieces of nuclear data that we varied.

In all we calculated prompt K Jezebel eigenvalues (here K represents the k eigenvalue calculated without considering the contribution of β-delayed neutrons) for some 500 different data libraries. About fifty of these give prompt K values that are less than two standard deviations from the experimentally determined value of 0.998±0.001 (Spriggs & Busch 2002). For this smaller number of data sets we also calculated α_0 for Jezebel. Accurate Monte Carlo calculations of α_0 are modestly expensive.

Figure 1 gives results of the simulations. All of the (α_0, K) pairs fall nearly on a straight line. There is a little spread away from the line. Some or most of this may come from uncertainties in the calculations. In the next section we'll discuss the importance of small deviations from a linear relation. For now we note that for a broad range of assumptions about the underlying nuclear data, arguably the broadest plausible range, the greatest deviation in α_0 from the mean linear trend is $\Delta\alpha_0 \approx 0.1\text{gen}/\mu\text{sec}$.

3. The uncertainty in α_0

Critical assemblies are associated with two qualitatively different types of uncertainties. First, there are uncertainties in the values of α_0, k_{eff} and other parameters describing the system that is actually studied in the laboratory. These uncertainties arise from measurement errors and are typically very small. The measurement error in α_0, for example, is about 1% (Spriggs & Busch 2002).

For validating data through calculations of critical systems a second type of uncertainty is important. Here we don't necessarily care so much about measurement errors for the experimentally studied system. Instead we have to determine the uncertainty in α_0 or k_{eff} for a system that has the characteristics assumed in a simulation of the critical system. The distinction between the experimentally studied system and the simulated system is important. Dimensions and masses of different pieces, isotopic ratios and levels of trace impurities are all associated with uncertainties. These uncertainties, as well as approximations made in idealizing the experiment, introduce errors in the values of k_{eff} and α_0 associated with the modeled system. These errors dominate the uncertainties important for data validation.

The overall uncertainty in k_{eff} for a modeled representation of Jezebel is about 0.001 (Spriggs

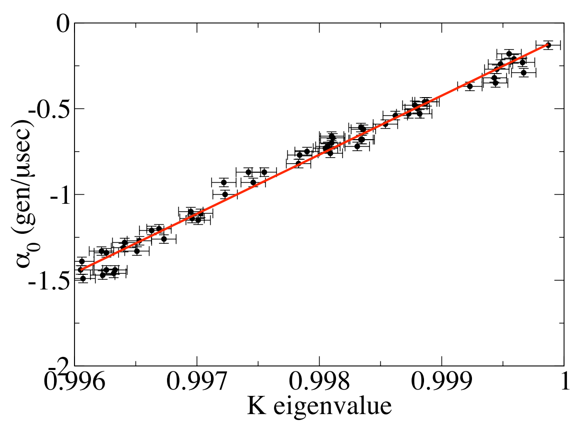

Fig. 1.— Results of the simulations for α_0 and K. Each point in the plot corresponds to a different choice for the nuclear data. Table 1 shows the different reaction quantities that were varied and the range of variation for each. Also shown in this plot is a simple linear fit to the data. Note that for a very broad range of changes to the nuclear data the maximum deviation in α_0 away from the linear fit is smaller than the change in α_0 resulting from a change in k_{eff} of 0.001, which is approximately the uncertainty in k_{eff}. The uncertainty in α_0 shown in this figure is from the statistical uncertainty in the Monte Carlo calculation. The K eigenvalue results shown in this figure are from a deterministic calculation. We estimate the uncertainty in these calculations to be about 0.0001.

& Busch 2002; NEA Nuclear Science Committee 1998). This uncertainty results mostly from a 100 gram uncertainty in the critical mass. As a first approximation we could use this and the linear relation between α_0 and $k_{\rm eff}$ to define the uncertainty in α_0 for modeled representations of Jezebel. This seems reasonable because the perturbation theory estimate holds for spatially-dependent changes to the cross sections. These simple approximations imply an uncertainty for the multiplication rate of

$$\delta(\alpha_0) \approx 0.35\,\mathrm{gen}/\mu\mathrm{sec}. \tag{1}$$

In the previous section we saw that the largest deviations from an exactly linear $\alpha_0/k_{\rm eff}$ curve are about half as big as this. This implies that when errors associated with the modeled representation of the experiment dominate uncertainties, α_0 and $k_{\rm eff}$ provide the same constraint on nuclear data.

4. Summary

We have considered the possibility of strengthening data validation through use of Rossi-α measurements for fast critical systems. Our basic finding is that α_0 and $k_{\rm eff}$ effectively give the same information about underlying nuclear data. At least from the standpoint of data validation, there doesn't seem to be a good reason to consider both quantities. We have not considered using the neutron generation time Λ to improve data libraries. The generation time is sensitive to very small changes in $k_{\rm eff}$ and might provide strong independent constraints on the data.

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