LAWRENCE
LIVERMORE
NATIONAL
LABORATORY

LLNL-TR-400126

SUPERTHERMAL ELECTRON DISTRIBUTION

R. Kauffman

December 28, 2007

Disclaimer

This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

Interdepartmental letterhead

Mail Station L- 379

Ext: 27857

XRM-78-43

March 16, 1978

TO: Distribution

FROM: R. L. Kauffman *RLK*

SUBJECT: Superthermal Electron Distribution

This memo discusses the analysis of the high-energy x-ray distribution from a laser-induced plasma to determine the superthermal electron distribution. The methods of deconvolution outlined in I are similar to formulae derived in the literature not including[1] and including[2] effects due to electron stopping. In II the methods are applied to an x-ray spectrum from an Au disc irradiated by ARGUS.

I. Solutions for n(E)

If the cross section for emission of bremsstrahlung having energy, $h\nu$, by an electron of energy, E, is given by $\sigma(E, h\nu)$, the energy radiated by the electron along a pathlength, dx, through a medium having an ion density, N_i, is

$$d\varepsilon_{rad}(E,h\nu) = N_i\ h\nu\ \sigma(E,h\nu)\ dx \tag{1}$$

The total energy per electron is obtained by integration along the path length. For electron stopping power given by dE/dx, the expression becomes

$$\varepsilon_{rad}\ (E,h\nu) = N_i \int_{h\nu}^{E} \frac{h\nu\ \sigma(E',h\nu)}{\frac{dE}{dx}\ (E')}\ dE' \tag{2}$$

The electron flux has the form $\sqrt{\frac{2E}{m}}\ n(E)dE$ where n(E) is the number density of electrons having energy in the interval from E to E + dE. By folding the electron flux with the radiated energy per electron, the x-ray fluence, $F(h\nu)$ is given by

$$F(h\nu) = A\tau N_i \int_{h\nu}^{\infty} dE \sqrt{\frac{2E}{m}} \; n(E) \int_{h\nu}^{E} \frac{h\nu \, \sigma(E',h\nu)}{\frac{dE}{dx}(E')} dE' \tag{3}$$

where A is the area of the electron flux and τ is the characteristic plasma time. The expression is, in general, not integrable, but if suitable approximations are made for $\sigma(E', h\nu)$ and $\frac{dE}{dx}(E')$ analytic results can be obtained.

The bremsstrahlung cross-section is given in the Bethe-Heitler approximation[3] as

$$\sigma_{BH}(E,h\nu) = \frac{\sigma_o Z^2}{E \, h\nu} \frac{\sqrt{3}}{\pi} \ln \left[\frac{E}{h\nu}\left(1 + \left(1 - \frac{h\nu}{E}\right)^{1/2}\right)^2\right] , \tag{4}$$

where

$$\sigma_o = \frac{8}{3} \frac{e^6}{\hbar mc^3} = 7.90 \times 10^{-22} \text{ cm}^2 - \text{eV}.$$

By comparison the classical expression for bremsstrahlung is[4]

$$\sigma_{class}(E,h\nu) = \frac{\sigma_o Z^2}{E \, h\nu} \tag{5}$$

The ratio of the Bethe-Heitler cross-section to the classical expression is

$$\frac{\sigma_{BH}(E,h\nu)}{\sigma_{Class}(E,h\nu)} = \frac{\sqrt{3}}{\pi} \ln \left[\frac{E}{h\nu}\left(1 + \left(1 - \frac{h\nu}{E}\right)^{1/2} \right)^2\right] \tag{6}$$

and the ratio of the total energy radiated is

$$\frac{\int_o^E \sigma_{BH}(E, h\nu) \cdot h\nu \; d(h\nu)}{\int_o^E \sigma_{Class}(E, h\nu) \cdot h\nu \; d(h\nu)} = \frac{2\sqrt{3}}{\pi} \tag{7}$$

For the electron energies considered here, the stopping power can be expressed using the Bethe-Bloch formula:[5]

$$- \frac{dE}{dx} = \frac{2\pi e^4 N_i}{E} Z \ln \left[\frac{2E}{I}\right] \tag{8}$$

The ratio under the second integral in Eq. (3) is then

$$\frac{h\nu\, \sigma(E,h\nu)}{\frac{dE}{dx}(E')} = \frac{\sigma_o Z}{2\pi e^4 N_i} \; \frac{\frac{\sqrt{3}}{\pi} \ln \left[\frac{E}{h\nu}\left(1 + (1 - \frac{h\nu}{E})^{1/2}\right)^2\right]}{\ln \left[\frac{2E}{I}\right]} \tag{9}$$

Both logarithms in Eq. (9) are slowly varying functions of E. If their ratio is replaced by a constant, α, Eq. (3) can be written as

$$F(h\nu) = \frac{A \tau Z \alpha \sigma_o}{2^{1/2} \pi e^4 m^{1/2}} \int_{h\nu}^{\infty} E^{1/2} n(E) (E - h\nu) \, dE. \tag{10}$$

Differentiating Eq. (10) with respect to $h\nu$ twice, Eq. (10) becomes

$$\frac{d^2F(h\nu)}{d(h\nu)^2} = \frac{A \tau Z \alpha \sigma_o}{2^{1/2} \pi e^4 m^{1/2}} (h\nu)^{1/2} n(h\nu), \tag{11}$$

or solving for n(E),

$$n(E) = \frac{2^{1/2} \pi e^4 m^{1/2}}{A \tau Z \alpha \sigma_o} E^{-1/2} \left. \frac{d^2 F(h\nu)}{d(h\nu)^2} \right|_{h\nu=E} \tag{12}$$

where

$$\frac{2^{1/2} \pi e^4 m^{1/2}}{\sigma_o} = 2.78 \text{ eV}^{3/2}\text{- sec/cm.}$$

An estimate for α can be obtained for a given problem. The electron energies of interest are from 10 keV to 40 keV, and for Au I $\approx$ 797 eV.[6] The ln $(\frac{2E}{I})$ term varies from 3.2 to 4.6. Taking the value to be 4 and the numerator to be $\frac{2\sqrt{3}}{\pi}$ (to normalize to the total energy radiated by the electron),

$$\alpha \sim \frac{\sqrt{3}}{2\pi} = 0.28.$$

If electron stopping is not important, for example in targets thin compared with the electron range, the second integral in Eq. (3) can be replaced by $h\nu\sigma(E,h\nu)\Delta x$ where Δx is the thickness of the material. If the logarithm term is set equal to $\frac{2\sqrt{3}}{\pi}$ the equation can be solved for n(E). The expression for the thin target is

$$n_{Thin}(E) = -\frac{m^{1/2}\,\pi}{3^{1/2}\,2^{3/2}\,A\tau\,N_i\Delta x\,Z^2\sigma_o}\,E^{1/2}\left.\frac{dF(h\nu)}{dh\nu}\right|_{h\nu=E} \tag{13}$$

It is seen that $n_{Thin}(E)$ is proportional to only the first derivative of the x-ray spectrum compared with the second derivative for a thick target. The absolute magnitude though is inversely proportional to $A\Delta xN_i$, which is the number of scattering centers contributing to the bremsstrahlung. In contrast, N(E) for a thick target depends only on A which is the area over which the electron flux is incident. This in principle is easier to determine and should provide better estimates of the electron density. The energy dependence of the electron distribution is much more difficult to determine precisely because the shape depends upon the second derivative of the x-ray fluence.

II. Au Disc Spectrum

The formulae derived in Section I can now be applied to data from a Au disc spectrum. In the analysis it is assumed that the bremsstrahlung is emitted from the cold Au material produced by electrons from the laser produced plasma. It is also assumed that the plasma is optically thin, since no absorption effects are included.

The results from the DANTE measurement from shot #38011309 are shown in Figure 1. This shot had a 1 ns pulse width at 10^{15} watts/cm^2 focused to a 150 μm diameter spot. The three high energy points have been fit to an exponential, $F(h\nu) = ae^{-h\nu/b}$. The resultant fit, the solid line in Figure 1, has the parameters $a = 2.5 \times 10^{13} \frac{keV}{keV}$ and $b = 6.3$ keV. Substituting this fit into Eq. (12), the electron density can be expressed as

$$n(E) = n_o(\frac{\beta}{E})^{1/2} e^{-E/\beta} \tag{14}$$

where $n_o = 5.7 \times 10^{18}$ cm^{-3} keV^{-1} and $\beta = 6.3$ keV.

The distribution is not Maxwellian

$$n(E) = \frac{2n_o}{\pi^{1/2}} \frac{E^{1/2}}{(kT)^{3/2}} e^{-E/kT}$$

for a Maxwellian distribution . For n(E) to be Maxwellian, $F(h\nu)$ must have the form $ae^{-h\nu/b}$ $(h\nu/b +2)$. A curve with $b = 5.13$ keV and $a = 9.4 \times 10^{12} \frac{keV}{keV}$ is plotted as the dashed curve in Figure 1. It is seen to deviate from the exponential fit only at energies above 30 keV. The functional form of the flux would imply a Maxwellian distribution for the superthermal electrons with $kT = 5.1$ keV and $n_o = 1.6 \times 10^{19}$ cm^{-3}.

For comparison Eq. (13), which gives the thin target deconvolution results, can be used. For the exponential fit, a Maxwellian distribution is obtained. Assuming $\Delta x \sim 1$ μm, $n_o = 5.3 \times 10^{18}$ cm^{-3} and $kT = 6.3$ keV which is in reasonable agreement with the thick target results.

From the above example it can be seen that an estimate of the superthermal electron density can be obtained from the high energy x-ray spectrum. The method probably gives a good estimate of the number density, but can only give gross spectral information from the present data. A more detailed x-ray intensity measurement could produce a better deconvolution, but considering the approximations used to derive Eq. (12), the better detail would probably not improve the confidence in the results. For a more precise estimate of the spectral distribution of the superthermal electrons, a numerical fit to the x-ray data can be done using Eq. (3) in which the deconvolution is used as an initial guess.

REFERENCES

[1] M. H. Key, J. Phys. B, 8 674 (1975).

[2] K. A. Brueckner, Phys. Rev. Letters 36, 677 (1976).

[3] H. W. Koch and J. W. Motz, Rev. Mod. Phys. 31, 920 (1959).

[4] Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High Temperature Hydrodynamic Phenomena (Academic Press, New York, 1966) p. 251.

[5] N. F. Mott and H. S. W. Massey, The Theory of Atomic Collisions (Oxford University, London) 3rd ed., p. 516.

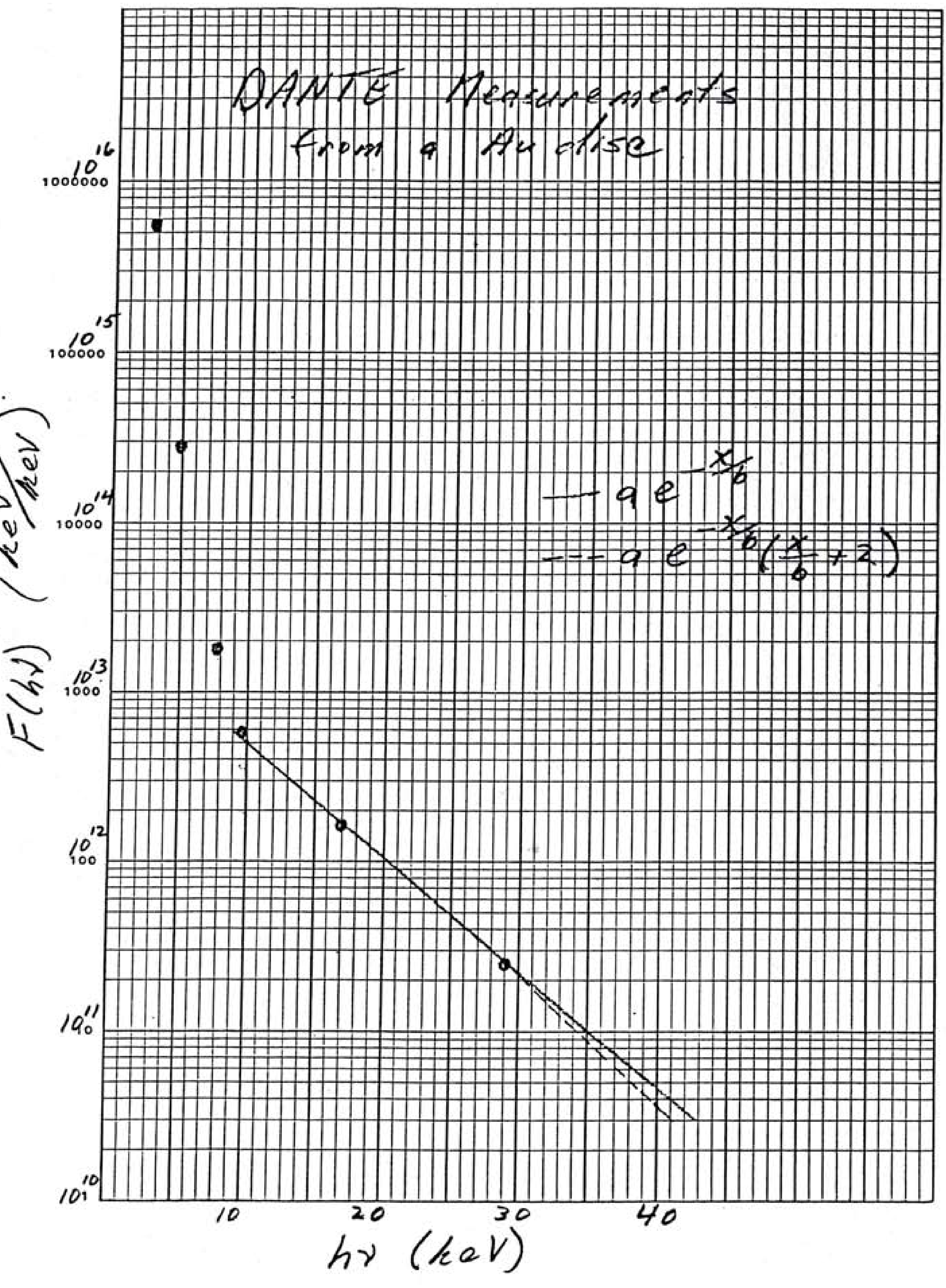
[6] M. J. Berger and S. M. Seltzer, "Tables of Energy Losses and Ranges of Electrons and Positrons," NASA report SP-3012, 1964.

RK·lh

DISTRIBUTION

R. Woodruff
P. Ebert
W. Slivinsky
R. Neifert
W. Mead
J. Nuckolls
Yu-Li Pan
K. McCullom - 40 copies

MODEL DATE


DANTE Measurements
from a Au disc
— a e^-x/b
- - - a e^-x/b (x/b + 2)
F(hν) (keV/keV)
hν (keV)
10^16
10^15
10^14
10^13
10^12
10^11
10^10
1000000
100000
10000
1000
100
10
10
20
30
40