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Constraint on $\bar{\rho}, \bar{\eta}$ from $B \to K^*\pi$

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A linear CKM relation, $\bar{\eta} = \tan\Phi_{3/2}(\bar{\rho} - 0.24 \pm 0.03)$, involving a 1σ range for $\Phi_{3/2}$, $20^\circ < \Phi_{3/2} < 115^\circ$, is obtained from $B^0 \to K^*\pi$ amplitudes measured recently in Dalitz plot analyses of $B^0 \to K^+\pi^-\pi^0$ and $B^0(t) \to K_S\pi^+\pi^-$. This relation is consistent within the large error on $\Phi_{3/2}$ with other CKM constraints which are unaffected by new $b \to s\bar{q}q$ operators. Sensitivity of the method to a new physics contribution in the $\Delta S = \Delta I = 1$ amplitude is discussed.

I. INTRODUCTION

Two anomalous features measured in $b \to s$ penguin-dominated processes have attracted substantial interest in recent years [1]: (i) CP asymmetries ΔS in $B^0 \to K_S X$ decays ($X = \pi^0, \phi, \eta', \rho^0, \omega, K_S K_S, \pi^0 K_S$) show a hint of systematic deviations from standard model predictions, and (ii) the pattern of direct CP asymmetries in $B \to K\pi$ decays is hard to explain using dynamical approaches based on $1/m_b$ expansion. Are these merely statistical fluctuations, a sign of our inabilities to reliably calculate the relevant observables, or are they first hints of new flavor-dependent CP-violating contributions from new physics at a TeV scale?

In order to answer this question it is important to obtain precise model-independent constraints on the CKM parameters $\bar{\rho}$ and $\bar{\eta}$ [2] using penguin dominated $\Delta S = 1$ B decays. Comparing these constraints with CKM constraints which are not affected by New Physics (NP) in $\Delta S = 1$ decays, e.g., the determination of γ from tree-dominated processes $B \to D^{(*)}K^{(*)}$ [3], may provide a test for the presence of NP in $b \to s$ penguin transitions.

In the present note we study a linear constraint in the $(\bar{\rho}, \bar{\eta})$ plane following from a combination of $B^0 \to K^*\pi$ amplitudes. The method proposed in [4] and developed further in [5] will be summarized in Section II. The necessary observables required for applying the method have been measured recently in Dalitz plot analyses of $B^0 \to K^+\pi^-\pi^0$ [6] and $B^0 \to K_S\pi^+\pi^-$ [7]. They will be used in Section III to determine the slope of the linear constraint, comparing this constraint with other CKM constraints. Section IV discusses the sensitivity of this test to New Physics effects, while Section V concludes.

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II. THE METHOD

The main idea of the method [4, 5] is studying $\Delta I = 1$ combinations of $B \to K^*\pi$ amplitudes which do not receive dominant contributions from QCD penguin operators, and thus carry a weak phase γ in the absence of electroweak penguin (EWP) terms. In the present note we focus our attention on the $I = 3/2$ final state,

$$3A_{3/2} = A(B^0 \to K^{*+}\pi^-) + \sqrt{2}A(B^0 \to K^{*0}\pi^0) \ . \quad (1)$$

In the absence of EWP terms γ would be given by

$$\gamma \ = \ \Phi_{3/2} \equiv -\frac{1}{2}\arg\left(R_{3/2}\right) \ , \quad (2)$$

$$R_{3/2} \ \equiv \ \frac{\bar{A}_{3/2}}{A_{3/2}} \ , \quad (3)$$

where $\bar{A}_{3/2}$ is the amplitude for charge-conjugated states.

The phase $\Phi_{3/2}$ can be obtained by measuring magnitudes and relative phases of $B^0 \to K^{*+}\pi^-$ and $B^0 \to K^{*0}\pi^0$ amplitudes and their charge-conjugates. The advantage of $B \to K^*\pi$ over $B \to K\pi$ decays is that $K^*\pi$ quasi–two-body states occur in Dalitz plots of $B \to K\pi\pi$, where overlapping resonances permit determining both the magnitudes and relative phases of $B \to K^*\pi$ amplitudes. In contrast, the relative phases of $B \to K\pi$ amplitudes cannot be measured directly.

The inclusion of EWP contributions modifies the ex-

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pression for $R_{3/2}$ which becomes [5]

$$R_{3/2} = e^{-2i[\gamma+\arg(1+\kappa)]}\frac{1+c_\kappa^* r_{3/2}}{1+c_\kappa r_{3/2}} , \quad (4)$$

$$\kappa \equiv -\frac{3}{2}\frac{C_9+C_{10}}{C_1+C_2}\frac{V_{tb}^*V_{ts}}{V_{ub}^*V_{us}} , \quad c_\kappa \equiv \frac{1-\kappa}{1+\kappa} , \quad (5)$$

$$r_{3/2} \equiv \frac{(C_1-C_2)\langle (K^*\pi)_{I=3/2}|\mathcal{O}_1-\mathcal{O}_2|B^0\rangle}{(C_1+C_2)\langle (K^*\pi)_{I=3/2}|\mathcal{O}_1+\mathcal{O}_2|B^0\rangle} . \quad (6)$$

Here $\mathcal{O}_1 \equiv (\bar{b}s)_{\rm V-A}(\bar{u}u)_{\rm V-A}$ and $\mathcal{O}_2 \equiv (\bar{b}u)_{\rm V-A}(\bar{u}s)_{\rm V-A}$ are the V-A current-current operators.

The straight line $\bar\eta = \bar\rho \tan\Phi_{3/2}$, in the absence of EWP terms, is shifted by these contributions along the $\bar\rho$ axis by a calculable finite amount. The actual constraint becomes [5]

$$\bar\eta = \tan\Phi_{3/2}\left[\bar\rho + C[1-2{\rm Re}(r_{3/2})] + \mathcal{O}(r_{3/2}^2)\right] , \quad (7)$$

where ($\lambda = 0.227$)

$$C \equiv \frac{3}{2}\frac{C_9+C_{10}}{C_1+C_2}\frac{1-\lambda^2/2}{\lambda^2} = -0.27 . \quad (8)$$

A finite positive shift of the straight line (7) along the $\bar\rho$ axis, given by $-C = 0.27$, is obtained using next to leading order values of Wilson coefficients C_i at $\mu = m_b$ [8]. The theoretical error in this parameter is smaller than 1%. The complex parameter $r_{3/2}$ was calculated in factorization, which gives a real result of the order of several percent, $r_{3/2} \le 0.05$ [4].

A similar but more conservative result is obtained for $r_{3/2}$ by applying flavor SU(3) to corresponding $\Delta S = 0$ decay amplitudes. Noting that the operators in the numerator and denominator in (6) transform as $\mathbf{6}$ and $\overline{\mathbf{15}}$ of SU(3), one finds [5],

$$r_{3/2} = \frac{|\sqrt{\mathcal{B}(\rho^+\pi^0)} - \sqrt{\mathcal{B}(\rho^0\pi^+)}|}{\sqrt{\mathcal{B}(\rho^+\pi^0)} + \sqrt{\mathcal{B}(\rho^0\pi^+)}} = 0.054 \pm 0.045 \pm 0.023 . \quad (9)$$

The first error is experimental. The second error is due to SU(3) breaking, small $\Delta S = 0$ penguin amplitudes and small strong phase difference between $B \to \rho\pi$ decay amplitudes which are neglected.

We have assumed that SU(3) breaking in ratios of $\Delta S = 1$ amplitudes and corresponding $\Delta S = 0$ amplitudes introduces an uncertainty of 30% in these ratios. The $B \to \rho\pi$ phase difference is expected to be suppressed by $1/m_b$ and $\alpha_s(m_b)$ [9, 10]. Indeed, evidence for a small phase difference is provided by an isospin pentagon relation obeyed by measured $B \to \rho\pi$ amplitudes [5]. The error in (7) from neglecting this small strong phase difference is negligible because ${\rm Re}(r_{3/2})$ depends quadratically on this phase. We will use the calculation (9) for $r_{3/2}$ which is more conservative than the one using factorization.

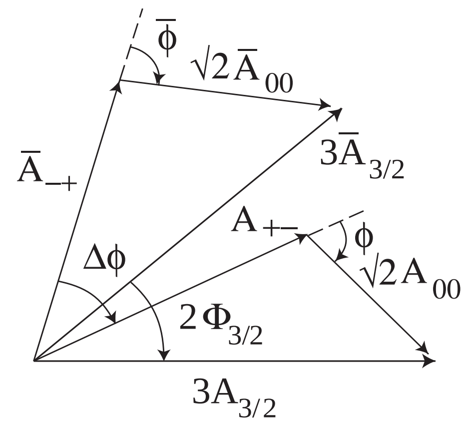

FIG. 1: Geometry for Eq. (1) and its charge-conjugate, using notations $A_{+-} \equiv A(B^0 \to K^{*+}\pi^-), A_{00} = A(B^0 \to K^{*0}\pi^0)$ and similar notations for charge-conjugated modes.

A simple test following from Eq. (4) and a real value of $r_{3/2}$ is

$$|R_{3/2}| = 1 . \quad (10)$$

This quantity is expected to be approximately one also for complex but small values of $r_{3/2}$ as predicted in the Standard Model. We expect that the approximation,

$$0.8 < |R_{3/2}| < 1.2 , \quad (11)$$

holds also in the presence of new physics contributions which we assume to be small.

Combining in quadrature the two errors in $r_{3/2}$, the constraint (7) becomes

$$\bar\eta = \tan\Phi_{3/2}\left[\bar\rho - 0.24 \pm 0.03\right] . \quad (12)$$

The dominant uncertainty in this linear constraint originates in $r_{3/2}$. It leads to an uncertainty of merely ± 0.03 in a parallel shift of the straight line along the $\bar\rho$ axis. This uncertainty is intrinsic to this method and cannot be reduced significantly without an additional input [5]. The current experimental error in the measurement of $\Phi_{3/2}$, which we discuss next, will be shown to introduce a considerably larger uncertainty.

III. DETERMINING $\Phi_{3/2}$

Using Eqs. (1) and (2), the phase $\Phi_{3/2}$ can be determined by measuring the magnitudes and relative phases of the $B^0 \to K^{*+}\pi^-$, $B^0 \to K^{*0}\pi^0$ amplitudes and their charge-conjugates. A graphical representation of the triangle relation Eq. (1) and its charge conjugate is given in Fig. 1.

The above four magnitudes of amplitudes and the two relative phases, $\phi \equiv \arg[A(B^0 \to K^{*0}\pi^0)/A(B^0 \to K^{*+}\pi^-)]$ and $\bar\phi \equiv \arg[A(\bar{B}^0 \to \bar{K}^{*0}\pi^0)/A(\bar{B}^0 \to$

Mode	Branching ratio	A_{CP}
$K^{*+}\pi^-$	10.4 ± 0.9	-0.14 ± 0.12
$K^{*0}\pi^0$	3.6 ± 0.9	-0.09 ± 0.24

TABLE I: Branching ratios in units of 10^{-6} and CP asymmetries in $B^0 \to K^*\pi$ [6, 11].

$K^{*-}\pi^+)]$, determine the two triangles separately. These quantities have been measured recently in a Dalitz plot analysis of $B^0 \to K^+\pi^-\pi^0$ and its charge-conjugate [6]. The relative phase $\Delta\phi \equiv \arg[A(B^0 \to K^{*+}\pi^-)/A(\bar{B}^0 \to K^{*-}\pi^+)]$, which fixes the relative orientation of the two triangles, has been measured in a time-dependent Dalitz plot analysis of $B^0 \to K_S\pi^+\pi^-$ [7].

Table I quotes CP-averaged branching ratios and CP asymmetries for $B^0 \to K^{*+}\pi^-$, $B^0 \to K^{*0}\pi^0$ using Refs. [6] and [11]. A value $\Delta\phi = (-164 \pm 30.7)^\circ$ was measured in $B^0(t) \to K_S\pi^+\pi^-$ [7]. The experimental situation is less clear for the phases ϕ and $\bar{\phi}$, measured recently in an amplitude analysis performed for $B^0 \to K^+\pi^-\pi^0$ and its charge-conjugate [6]. Four solutions corresponding to minima in χ^2 were found. Because of low statistics essentially all values of ϕ in the range $-180^\circ \le \phi \le 180^\circ$ and most values of $\bar{\phi}$ in this range are allowed at 1σ.

In order to calculate the χ^2 dependence on $\Phi_{3/2}$ we use the χ^2 dependence on ϕ and $\bar{\phi}$ given in Ref. [6], assuming gaussian errors for $\Delta\phi$ and for branching ratios and CP asymmetries in $B^0 \to K^{*+}\pi^-$ and $B^0 \to K^{*0}\pi^0$. Potential correlations between $\phi, \bar{\phi}$ and branching ratios and asymmetries are neglected. Two resulting χ^2 plots as function of $\Phi_{3/2}$ are shown in Fig. 2. The broken purple curve corresponds to an unconstrained $|R_{3/2}|$, while the solid blue curve is obtained by imposing the constraint (11), which is expected to hold in the Standard Model and in the presence of small new physics contributions. The latter curve defines a 1σ range,

$$20^\circ < \Phi_{3/2} < 115^\circ \ . \tag{13}$$

Thus, a large range of values is permitted for $\tan\Phi_{3/2}$, the slope of the linear constraint (12).

Fig. 3 shows the linear constraint (12) with the large range of slopes (13) overlaid on CKMFitter results following from [11, 12] $|V_{ub}|/|V_{cb}| = 0.086 \pm 0.009$, obtained in semileptonic B decays, and values $\beta = (21.5 \pm 1.0)^\circ$, $\alpha = (88 \pm 6)^\circ$ and $\gamma = (53^{+15}_{-18} \pm 3 \pm 9)^\circ$ [13], obtained in $B \to J/\psi K_S$, $B \to \pi\pi, \rho\rho, \rho\pi$ and $B^+ \to D^{(*)}K^{(*)+}$, respectively. The latter constraints are unaffected by potential NP in $\Delta S = 1$ processes. They are consistent with the new constraint obtained in $B \to K^*\pi$, which however involves a large experimental error in $\Phi_{3/2}$. The theoretical error in this constraint [± 0.03 in Eq. (12)] is very small and is described by the difference between dark and light shaded regions in Fig. 3.

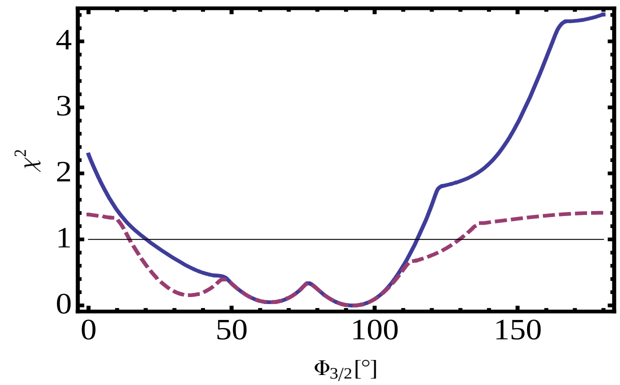

FIG. 2: χ^2 dependence on $\Phi_{3/2}$ for unconstrained $|R_{3/2}|$ (broken purple line) and for $0.8 < |R_{3/2}| < 1.2$ (solid blue line). A black horizontal line at $\chi^2 = 1$ defines 1σ ranges for $\Phi_{3/2}$.

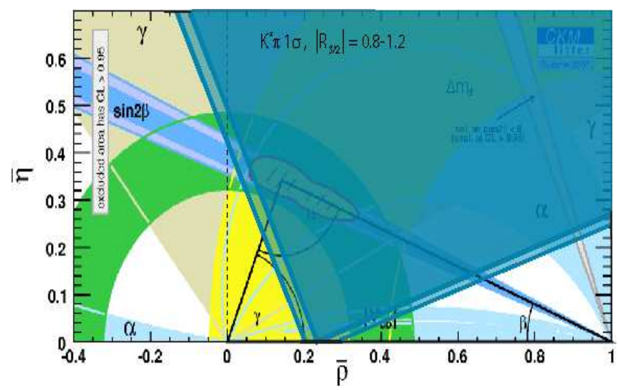

FIG. 3: Constraint in the $\bar{\rho}-\bar{\eta}$ plane following from Eqs. (12) and (13). The dark shaded region marked $K^*\pi\,1\sigma$ corresponds to the experimental error on $\Phi_{3/2}$ given by the 1σ range (13), while the light shaded region includes also the error on $r_{3/2}$ (9). Also shown are CKMfitter constraints obtained using $|V_{ub}|/|V_{cb}|, \beta, \alpha, \gamma$ and Δm_d [12].

IV. SENSITIVITY TO NEW PHYSICS

As has already been stressed, new physics (NP) $\Delta S = 1$ contributions may lead to an inconsistency between the linear constraint (7) in penguin dominated $B \to K^*\pi$ decays and values of $|V_{ub}|/|V_{cb}|, \beta, \alpha$ and γ obtained in the above-mentioned processes. The constraint (7) is affected by $\Delta I = 1$ NP operators, while NP contributions from potential $\Delta I = 0$ operators drop out. A general discussion of ways for distinguishing between NP in $\Delta I = 0$ and $\Delta I = 1$ $b \to s$ transitions can be found in Ref. [14].

In order to study the sensitivity of the Standard Model constraint (7) to new $\Delta S = 1, \Delta I = 1$ contributions, we recall the origin of this constraint. The $I = 3/2$ amplitude consists of complex tree and EWP terms, T and P_{EW}, both of which involve strong phases,

$$A_{3/2} = Te^{i\gamma} - P_{EW} \ . \tag{14}$$

The ratio [5]

$$\frac{P_{EW}}{T} = |\kappa| \frac{1 - r_{3/2}}{1 + r_{3/2}} \quad (15)$$

involves the parameter κ defined in (5), which has some dependence on CKM matrix elements whose central values correspond to $|\kappa| \simeq 0.66$.

Allowing for a NP term $A_{NP} \exp(i\psi)$, where A_{NP} involves a CP conserving strong phase while ψ is a new CP-violating phase, the $\Delta I = 1$ amplitude becomes

$$A_{3/2} = T e^{i\gamma} - P_{EW} + A_{NP} e^{i\psi} \ . \quad (16)$$

The NP term can be reabsorbed quite generally in redefined tree and electroweak penguin-like contributions, $\bar{T}$ and $\bar{P}_{EW}$, without changing the structure (14) [15],

$$A_{3/2} = \bar{T} e^{i\gamma} - \bar{P}_{EW} \ . \quad (17)$$

Here

$$\bar{T} = T + A_{NP} \frac{\sin\psi}{\sin\gamma} \ ,$$
$$\bar{P}_{EW} = P_{EW} + A_{NP} \frac{\sin(\psi - \gamma)}{\sin\gamma} \ . \quad (18)$$

(Several NP terms in (16) would translate into a sum of NP terms in (18).) The amplitudes $\bar{T}$ and $\bar{P}_{EW}$ can be used to define a parameter $\bar{r}$ in analogy to Eq. (15),

$$\frac{\bar{P}_{EW}}{\bar{T}} = |\kappa| \frac{1 - \bar{r}}{1 + \bar{r}} \ . \quad (19)$$

The effective parameter $\bar{r}$ leads to a modification of the linear constraint (7). For large values of $\bar{r}$ this constraint becomes nonlinear.

Assuming perfect measurements of $B \to K^*\pi$ amplitudes including relative phases and a given value of κ, a criterion for an observable NP amplitude is provided by requiring that $\bar{r}$ lies outside the range of values (9) allowed for $r_{3/2}$. Because of these small values, in general this criterion holds also for rather small values of A_{NP} relative to T and P_{EW}, which by themselves are subdominant to a dominant penguin amplitude in $B \to K^*\pi$. An exception is a singular case where the weak phases ψ and γ are related by

$$\frac{\sin(\psi - \gamma)}{\sin\psi} = \frac{P_{EW}}{T} \ , \quad (20)$$

for which $\bar{P}_{EW}/\bar{T} = P_{EW}/T$ is independent of A_{NP}. One expects low sensitivity to NP also in the near vicinity of these discrete values of ψ. In the following discussion we will assume a value $\gamma = 60°$.

Denoting

$$q_{NP} = \frac{A_{NP}}{P_{EW}} \quad (21)$$

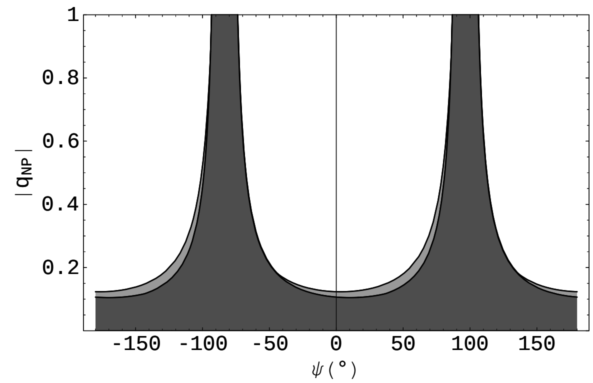

FIG. 4: Values of $|q_{NP}|$ and ψ providing a signal for NP, assuming $\gamma = 60°$. NP corresponds to points outside the dark area given by real values of $\bar{r}$ obeying Eq. (22), or beyond the extended light area given by complex $\bar{r}$ with a phase in the range (23).

and assuming real q_{NP}, we plot in Fig. 4 values of $|q_{NP}|$ as a function of ψ, for which $\bar{r}$ is real and lies in the range

$$0.054 - 0.051 < \bar{r} < 0.054 + 0.051 \ . \quad (22)$$

The points cover the dark area in Fig. 4. The region outside this area, including for most values of ψ rather small values of $|q_{NP}|$, $|q_{NP}| < 0.2$, implies a high sensitivity to an observable NP amplitude. The spikes around $\psi \sim \pm 90°$, implying very low sensitivity, correspond to solutions of (20) and nearby lying values of ψ.

A more conservative criterion for an observable NP amplitude is obtained by allowing q_{NP} and $\bar{r}$ to involve complex phases. Since the strong phase of $r_{3/2}$ is expected to be suppressed by $1/m_b$ and $\alpha_s(m_b)$, we use a range

$$-30° < \arg(\bar{r}) < 30° \ . \quad (23)$$

Combining this with the bounds (22) on $|\bar{r}|$ leads to the light area in Fig. 4 which extends only slightly beyond the dark area. The region outside this area corresponds to potentially observable NP amplitudes. The small difference between the dark and light areas follows from the fact that the sensitivity of the required value of q_{NP} is quadratic in $\arg(\bar{r})$ for small values of this phase.

V. CONCLUSION

Magnitudes and phases of $B^0 \to K^*\pi$ decay amplitudes, extracted in Dalitz plot analyses for $B^0 \to K^+\pi^-\pi^0$ and $B^0 \to K_S\pi^+\pi^-$, are used for obtaining the linear constraint (12) in the $\bar{\rho}, \bar{\eta}$ plane, where $\Phi_{3/2}$ lies in a 1σ range (13). This constraint is consistent with other CKM constraints which are unaffected by NP $\Delta S = 1$ operators. The dominant error in the slope of the straight line is purely experimental, while a much smaller theoretical uncertainty occurs in a parallel shift along the

$\bar{\rho}$ axis. This small theoretical uncertainty is shown to imply in principle a high sensitivity to a New Physics $\Delta S = 1, \Delta I = 1$ amplitude.

The method presented here has some similarity to a method for determining γ in $B^+ \to K\pi$, which is based on a complex triangle relation [16],

$$A(B^+ \to K^0\pi^+) + \sqrt{2}A(B^+ \to K^+\pi^0) = 3\mathcal{A}_{3/2}$$
$$= (T + C)(e^{i\gamma} - \delta_{EW}) \ , \ \delta_{EW} = 0.66 \pm 0.05 \ . \quad (24)$$

The determination of γ neglects a small annihilation amplitude in $B^+ \to K^0\pi^+$ and assumes flavor SU(3) for calculating δ_{EW} and for relating $T + C$ to the amplitude measured in $B^+ \to \pi^+\pi^0$. SU(3) breaking corrections, affecting γ to leading order, are treated by assuming factorization. This method determines γ up to a two-fold discrete ambiguity.

In contrast, in the method applied here to $B^0 \to K^*\pi$ SU(3) symmetry is assumed only for calculating the small parameter $r_{3/2}$, the equivalent of which vanishes in $B \to K\pi$ in the SU(3) symmetry limit. This has a very small effect on the parallel shift of the linear constraint (7) along the $\bar{\rho}$ axis. The determination of the slope of this straight line involves no discrete ambiguity.

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