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A Least Squares Method for CVT Calibration in a RLC Capacitor Discharge Circuit

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A Least Squares Method for CVT Calibration in a RLC Capacitor Discharge Circuit

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ABSTRACT

In many applications, the ability to monitor the output of a capacitive discharge circuit is imperative to ensuring the reliability and accuracy of the unit. This monitoring is commonly accomplished with the use of a Current Viewing Transformer (CVT). In order to calibrate the CVT, the circuit is assembled with a Current Viewing Transformer (CVR) in addition to the CVT and the peak outputs compared. However, difficulties encountered with the use of CVRs make it desirable to eliminate the use of the CVR from the calibration process.

This report describes a method for determining the calibration factor between the current throughput and the CVT voltage output in a capacitive discharge unit from the CVT ringdown data and values of initial voltage and capacitance of the circuit. Previous linear RLC fitting work for determining R, L, and C is adapted to return values of R, L, and the calibration factor, k. Separate solutions for underdamped and overdamped cases are presented and implemented on real circuit data using MathCad software with positive results. This technique may also offer a unique approach to self calibration of current measuring devices.

ACKNOWLEDGEMENTS

We would like to acknowledge the valuable technical discussions with Louis A. Romero and Patrick A. Smith.

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1. Introduction

The ability to monitor the outputs of Capacitive Discharge Units (CDUs) in Firing Sets or Fuzes is paramount to determining whether they are functioning reliably and accurately over time. In its simplest form, a CDU is a charged capacitor connected to a resistive and inductive load through a switch as seen in Figure 1. When testing these devices, the current profile is of the most interest and therefore a Current Viewing Transformer (CVT), as seen in Figure 2, is used.

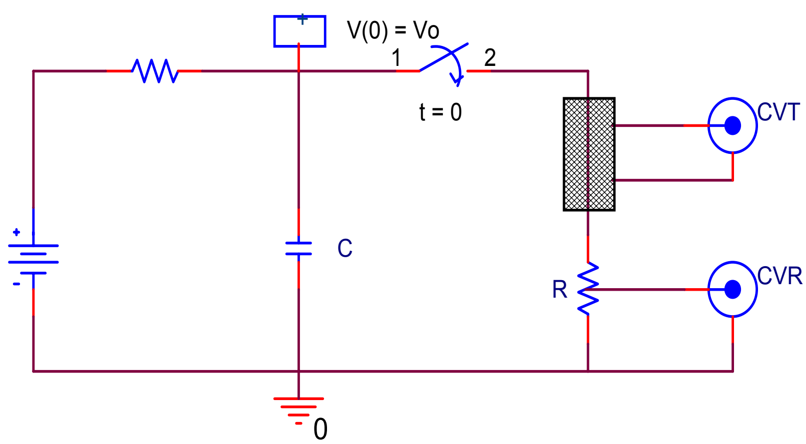


Figure 1: CDU circuit

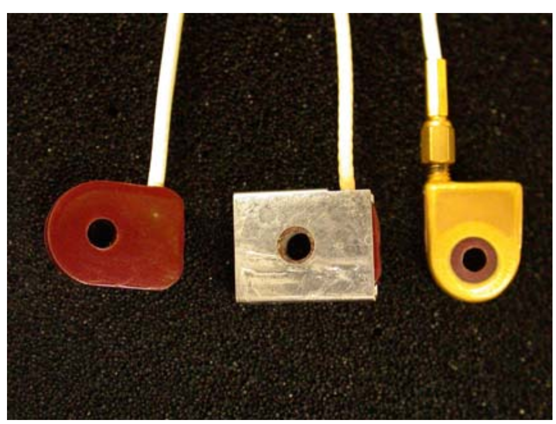


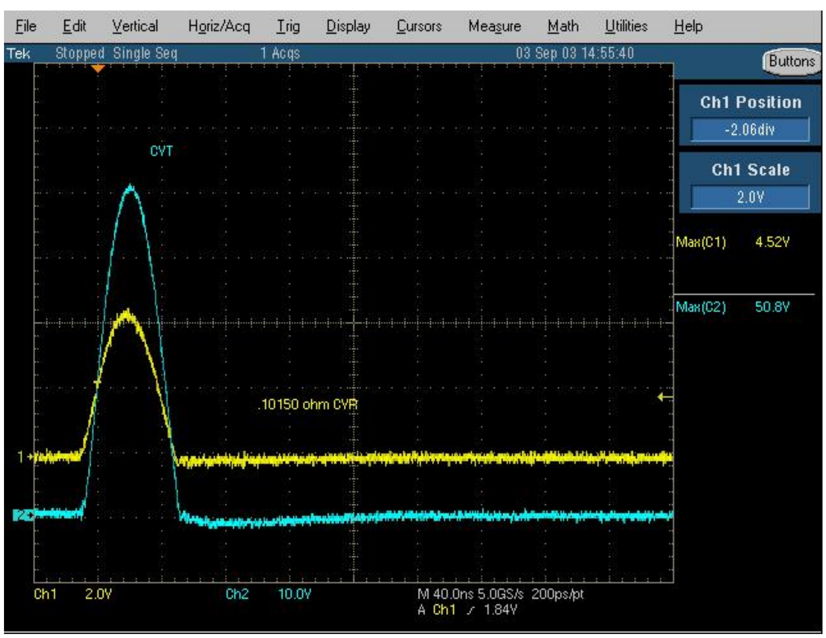
Figure 2: Shielded and Unshielded CVTs

The current process of calibrating the Volt/Amp ratio of the CVT's output is fairly straightforward. The circuit is constructed using a Current Viewing Resistor (CVR), a highly accurate low value resistor or resistor network, as the test load. The switch is closed and the current profiles are recorded by the CVR and CVT are taken. An example can be seen in Figure 3 below. The CVR provides a known value for the current peak,

$$I_{peak} = \frac{V_{peak}}{R}, \tag{1}$$

which is then used to find the Volt/Amp ratio of the CVT,

$$\frac{Volt}{Amp} ratio = \frac{V_{peakofCVT}}{I_{peak}}. \tag{2}$$



$$I_{Peak} = \frac{4.52\,V}{.1015\,\Omega} = 44.5\,A$$

$$\frac{V}{A} = \frac{50.8\,V}{44.5\,A} = 1.14$$

Figure 3: Example calculation using CVR and CVT data

The above procedure has its limitations however. The CVRs, shown in Figure 4, used are expensive and can degrade or burn open with extensive use. If too much current is put through the CVR, the resistive value of it drifts up and it will eventually become an open. Also, the values are typically so small, on the order of .005Ω, that it can be difficult to measure them accurately to verify their value.

Figure 4: Current Viewing Resistor

These features make it desirable to eliminate the CVR from the calibration procedure. Therefore, if the system is considered to be a 2nd order RLC system with an ideal switch, then just the output of the CVT could be used to calibrate the system. With a known capacitance and initial capacitor voltage, the rest of the system parameters could be determined, the theoretical current peak could be found, and the CVT could then be calibrated.

Unfortunately, because the typical resistances of the load are small, on the order of .1Ω, the resistance and inductance of the rest of the circuit need to be taken into account. The values must be either measured or calculated from other data. Measuring the characteristics of every circuit component once it is laid out is not an efficient option.

In a previous SAND report (SAND2002-1744)[1], a computational solution is presented that returned values of R, L, and C for both under and overdamped RLC circuits based on a least squares fit to circuit output waveform data. This work assumes linear circuit elements. From this work, we expand its application to CVT calibration and circuit value determination from a given input voltage, capacitance, and CVT output waveform.

A technique for determining nonlinear R, L, and C circuit components is treated in the report SAND2002-0115[2].

2. Analysis

Given specified values of C, V_0, and circuit voltage ringdown data, a calibration factor, k, is sought such that,

$$i(t) \times k = V(t) \tag{1}$$

where $i(t)$ is the transient current, and $V(t)$ is the voltage output read at the CVT element. The approach is to fit experimental voltage output data to equations of current for both the underdamped and overdamped cases. The parameters of the fit are used to determine k.

2.1 Underdamped Case

In the underdamped case, the k is found using a least squares fit to the data corresponding to the underdamped solution for current, $i(t)$,

$$i(t) = \frac{V_0}{\omega_n L\sqrt{1-\zeta^2}} e^{-\zeta\,\omega_n t} \sin\left(\omega_n \sqrt{1-\zeta^2}\, t\right). \tag{2}$$

From Eqn.(1), $V(t)$ is,

$$V(t) = \frac{k \cdot V_0}{\omega_n L \sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin\left(\omega_n \sqrt{1-\zeta^2}\, t\right), \tag{3}$$

or,

$$V(t) = \frac{V'}{\omega_n L \sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin\left(\omega_n \sqrt{1-\zeta^2}\, t\right), \tag{4}$$

where,

$$V' = k \cdot V_0,$$

$$\omega_n = \frac{1}{\sqrt{LC}},$$

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}.$$

Eqn.(4) may be re-written as,

$$V(t) = B_0 e^{-B_1 t} \sin\left(B_2 t\right) \tag{5}$$

where B_0, B_1 and B_2 are defined by the corresponding elements in Eq. (4). A fit to data with Eqn.(5) returns values of B_0, B_1, and B_2. Given values of C, and V_0; *L, R* and finally *k* may be determined in terms of these parameters.

Substituting B_2 into B_0,

$$B_0 = \frac{V'}{L \cdot B_2} \tag{6}$$

$$L = \frac{V'}{B_0 \cdot B_2} \tag{7}$$

Substituting ζ and ω_n, B_1 becomes,

$$B_1 = \frac{R}{2 \cdot L} \tag{8}$$

$$R = 2 \cdot B_1 \cdot L \tag{9}$$

The same for B_2 yields,

$$B_2 = \sqrt{\frac{1}{L \cdot C} - \frac{R^2}{4 \cdot L^2}}$$

$$B_2{}^2 = \frac{1}{L \cdot C} - \frac{R^2}{4 \cdot L^2}$$

Substituting Eqns.(7), (8),

$$B_2{}^2 = \frac{B_0 \cdot B_2}{V' \cdot C} - B_1{}^2$$

Solved for V',

$$V' = \frac{B_0 \cdot B_2}{C(B_2{}^2 + B_1{}^2)} \tag{10}$$

Thus L and R are determined by,

$$L = \frac{1}{C(B_1{}^2 + B_2{}^2)}, \tag{11}$$

$$R = \frac{2 \cdot B_1}{C(B_1{}^2 + B_2{}^2)} \tag{12}$$

From $V' = k \cdot V_0$, k is determined as,

$$k = \frac{V'}{V_0} \tag{13}$$

$$k = \frac{B_0 \cdot B_2}{V_0 \cdot C(B_2{}^2 + B_1{}^2)} \tag{14}$$

Given experimental data, B_0, B_1 and B_2 are determined from a least squares fit of the data to Eqn. (5) and thus R, L, and k are found from Eqns. (11), (12), and (14) respectively.

2.2 Overdamped Case

As before, in the overdamped case the k is found using a least squares fit to the data corresponding to the overdamped solution for current,

$$i(t) = \frac{V_0}{2 \cdot \omega_n \cdot L\sqrt{\zeta^2 - 1}}\left[e^{\left(\omega_n\sqrt{\zeta^2-1}-\zeta\cdot\omega_n\right)t} - e^{-\left(\omega_n\sqrt{\zeta^2-1}+\zeta\cdot\omega_n\right)t}\right] \tag{15}$$

From Eqn.(1), $V(t)$ is,

$$V(t) = \frac{k \cdot V_0}{2 \cdot \omega_n \cdot L\sqrt{\zeta^2 - 1}}\left[e^{\left(\omega_n\sqrt{\zeta^2-1}-\zeta\cdot\omega_n\right)t} - e^{-\left(\omega_n\sqrt{\zeta^2-1}+\zeta\cdot\omega_n\right)t}\right] \tag{16}$$

or,

$$V(t) = \frac{V'}{2 \cdot \omega_n \cdot L\sqrt{\zeta^2 - 1}}\left[e^{\left(\omega_n\sqrt{\zeta^2-1}-\zeta\cdot\omega_n\right)t} - e^{-\left(\omega_n\sqrt{\zeta^2-1}+\zeta\cdot\omega_n\right)t}\right] \tag{17}$$

as before,

$$V' = k \cdot V_0$$

$$\omega_n = \frac{1}{\sqrt{LC}},$$

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}.$$

Eqn.(17) may be re-written as,

$$V(t) = B_0\left[e^{(B_1)t} - e^{-(B_2)t}\right] \tag{18}$$

where B_0, B_1 and B_2 are defined by the corresponding elements in Eq. (17). A fit to data with Eqn.(18) returns values of B_0, B_1, and B_2. Given values of C, and V_0; *L, R* and finally *k* may be determined in terms of these parameters.

Summing and subtracting B_1 and B_2 yields,

$$B_1 + B_2 = 2 \cdot \omega_n \cdot \sqrt{\zeta^2 - 1} \tag{19}$$

$$B_2 - B_1 = 2 \cdot \zeta \cdot \omega_n \tag{20}$$

Eqn.(19) may be substituted into B_0,

$$B_0 = \frac{V'}{L(B_1 + B_2)} \tag{21}$$

Substituting ζ and ω_n, Eqn.(20) becomes,

$$B_2 - B_1 = \frac{R}{L} \tag{22}$$

The same for B_2 yields,

$$B_2 = \sqrt{\frac{R^2}{4 \cdot L^2} - \frac{1}{L \cdot C}} + \frac{R}{2 \cdot L}$$

Substituting Eqn.(22),

$$B_2 = \sqrt{\frac{1}{4}(B_2 - B_1)^2 - \frac{1}{L \cdot C}} + \frac{1}{2}(B_2 - B_1)$$

Simplified and solved for L,

$$L = \frac{1}{-C \cdot B_1 \cdot B_2} \tag{23}$$

Thus R and V' are determined from Eqns.(22) and (21) respectively,

$$R = \frac{B_1 - B_2}{C \cdot B_1 \cdot B_2}, \tag{24}$$

$$V' = \frac{-B_0(B_1 + B_2)}{C \cdot B_1 \cdot B_2} \tag{25}$$

From $V' = k \cdot V_0$, k is determined as,

$$k = \frac{V'}{V_0} \tag{26}$$

$$k = \frac{-B_0(B_1 + B_2)}{V_0 \cdot C \cdot B_1 \cdot B_2} \tag{27}$$

Given experimental data, B_0, B_1 and B_2 are determined from a least squares fit of the data to Eqn. (18) and thus R, L, and k are found from Eqns. (23), (24), and (27) respectively.

2.3 Discussion of Method

2.3.1 Linearity

This method assumes a linear system. The values of R, L, and C are treated as unique constants. In addition, it is assumed that the voltage output of the CVT scales linearly with the current throughput of the circuit. Thus, the calibration factor is a unique constant as well.

To preserve this assumption of linearity, it must be assured that the circuit behaves in a linear fashion. The two circuit elements that cause the most problems in terms of this are the switch and the CVT itself. In our case, we used a switch with a fast switching time and a constant impedance once on. Any non-linear effects on the output are negligible. The CVT however has a magnetic core inside of it, which introduces non-linearity into the circuit. By comparing the outputs of linear CVRs and the CVTs it has been observed that the CVT tracks the output of the system with a high degree of fidelity during the first cycle of an undamped output. Therefore we truncate the data after the second voltage peak to ensure that we are staying within this region.

2.3.2 Fitting the Data

As with any fitting algorithm, the input data must conform to the system it is being fitted to. The data retrieved from an experiment is usually has some noise component. Perhaps a baseline signal exists in a waveform that means the signal begins at +X volts and never damps to zero; there may be a trigger delay making t=0 of the waveform actually occur at t=X seconds. Inputting such signal data into a computational fit will yield erroneous results if not accounted for. Examples are given in Appendices A & B. In these examples, the circuit data was first examined in MS Excel. An average value was calculated from the tail end of the waveform where the signal had appeared to completely damp out and subtracted from the entire waveform as a baseline value. Next all data occurring before the point at which the baseline and the waveform coincide was deleted such that when the data is input into the MathCad worksheet, the timestamp of this new initial data point is subtracted from the timecodes of the rest of the waveform, defining the point as t=0. In addition, the data displayed in Appendix A was truncated after the second voltage peak due to inaccuracies in the signal shape after this point.

3. Experimental Results

In order to test the functionality of this method, a few capacitor discharge test circuits were assembled similarly to Figure 1, with a CVR and CVT in line. Once charged, the switch was closed and the current profiles as seen by the CVR and CVT were recorded. The peak value of the CVT voltage profile was divided by the peak value of the CVR current profile to yield the calibration factor k. Then the CVT data was processed according to section 2.3.2 and input with the values of capacitance and initial voltage into a MathCad worksheet created to fit the B parameters as discussed earlier. The parameters are solved algebraically to find V', resulting in a calibration factor of k independent of the CVR data. MathCad worksheets for the under- and overdamped cases run with experimental data are included in Appendices A & B. For the data presented in the appendices, the CVR calculated value *k* was 0.005911. The underdamped and overdamped cases returned 0.005936 and 0.00602 respectively which gives percent errors of 0.42% and 1.84% respectively in regards to the CVR calibration. These results are very promising.

4. Summary

Capacitive discharge circuits are an integral part of many systems. The development and maintenance of these systems requires the use of a current monitoring device. Expanding upon previous work in least squares fitting RLC discharge circuits, we present a method calibrating a CVT for use as a current measuring device, thus eliminating the need for a troublesome CVR in circuit development.

We specifically present Mathcad programs for least squares fitting of data to the circuit parameters for both the overdamped and underdamped case. The programs and illustrative examples are presented. The least squares fit to data approach offers the advantage of minimizing human interaction/error, minimizes experimental noise, and is quite compatible with modern laboratory instrumentation.

APPENDIX A: Underdamped Case

Underdamped Data

data :=

	0	1
0	$3\cdot 10^{-7}$	0.8
1	$5\cdot 10^{-7}$	$4\cdot 10^{-7}$
2	$4\cdot 10^{-7}$	$4\cdot 10^{-7}$
3	$2\cdot 10^{-7}$	0.4
4	$3\cdot 10^{-7}$	0.8
5	$3\cdot 10^{-7}$	$4\cdot 10^{-7}$
6	$5\cdot 10^{-7}$	0.4
7	$4\cdot 10^{-7}$	0.4
8	$2\cdot 10^{-7}$	1.2
9	$2\cdot 10^{-7}$	0.8
10	$3\cdot 10^{-7}$	1.2

Inputs:

Given:

$V_0 = 2.2\times 10^{3}$ $\quad C_0 = 1.785\times 10^{-6}$

Two-point guesses:

$R_{guess} = 0.096$ $\quad L_{guess} = 6.083\times 10^{-8}$

$\text{time} := -\text{data}_{0,0} + \text{data}^{\langle 0\rangle}$ Data column one designated as time

$\text{voltage} := 2\cdot \text{data}^{\langle 1\rangle}$ Data column two designated as voltage Note: (2X for attenuated data)

Eq.(5) and partial derivatives.
Assume *u* in place of *B* and *s* for *t*:

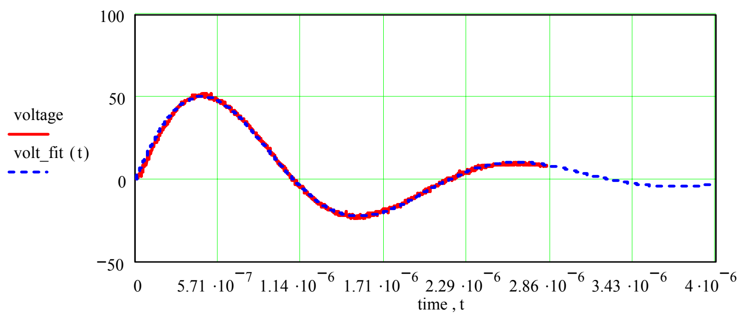
$$F(s,u) := \begin{pmatrix} u_0\cdot e^{-u_1\cdot s}\cdot \sin(u_2\cdot s) \\ e^{-u_1\cdot s}\cdot \sin(u_2\cdot s) \\ -s\cdot u_0\cdot e^{-u_1\cdot s}\cdot \sin(u_2\cdot s) \\ s\cdot u_0\cdot e^{-u_1\cdot s}\cdot \cos(u_2\cdot s) \end{pmatrix} \quad \begin{matrix} i(t) \\ \frac{\delta i(t)}{\delta B_0} \\ \frac{\delta i(t)}{\delta B_1} \\ \frac{\delta i(t)}{\delta B_2} \end{matrix}$$

Parameter Guesses (as defined by corresponding elements in Eqn (4) & (5)):

$$B = \begin{pmatrix} 1.234\times 10^{4} \\ 7.891\times 10^{5} \\ 2.93\times 10^{6} \end{pmatrix}$$

Least Squares Fit: $\quad P := \text{genfit}(\text{time}, \text{voltage}, B, F)$

Fitted Voltage Curve: $\quad \text{volt\_fit}(\text{test}) := F(\text{test}, P)_0$



Parameter Solution:

$$P = \begin{pmatrix} 71.944 \\ 7.329\times 10^{5} \\ 2.901\times 10^{6} \end{pmatrix}$$

Output:

$$V_{prime} := \frac{P_0\cdot P_2}{C_0\cdot\left[(P_1)^2 + (P_2)^2\right]} \qquad V_{prime} = 13.059$$

$$L_{fit} := \frac{V_{prime}}{P_0\cdot P_2} \qquad L_{fit} = 6.257\times 10^{-8}$$

$$R_{fit} := 2\cdot L_{fit}\cdot P_1 \qquad R_{fit} = 0.092$$

If Vfit = Vo*k, where Vfit is the intial voltage as derived from the experimental data and Vo is the intial voltage as specified (theoretical), k is expressed as:

$$k := \frac{V_{prime}}{V_0} \qquad k = 5.936\times 10^{-3}$$

APPENDIX B: Overdamped Case

Overdamped Data:

data :=

	0	1
0	$2\cdot10^{-8}$	$7\cdot10^{-7}$
1	$5\cdot10^{-8}$	0.4
2	$8\cdot10^{-8}$	0.8
3	$6\cdot10^{-8}$	0.8
4	$4\cdot10^{-8}$	0.4
5	$2\cdot10^{-8}$	1.6
6	$4\cdot10^{-8}$	1.6
7	$8\cdot10^{-8}$	2
8	$6\cdot10^{-8}$	2.4
9	$4\cdot10^{-8}$	3.2

Inputs:

Given:

$V_0 = 3.8\times10^{3}$ $\qquad$ $C_0 = 1.785\times10^{-6}$

Guesses:

$R_{guess} := .50$ $\qquad$ $L_{guess} := 6\cdot10^{-8}$

$time := data^{\langle 0\rangle} - data_{0,0}$ $\qquad$ Data column one designated as time

$voltage := data^{\langle 1\rangle}$ $\qquad$ Data column two designated as voltage

Eq.(18) and partial derivatives.
Assume *u* in place of *B* and *s* for *t*:

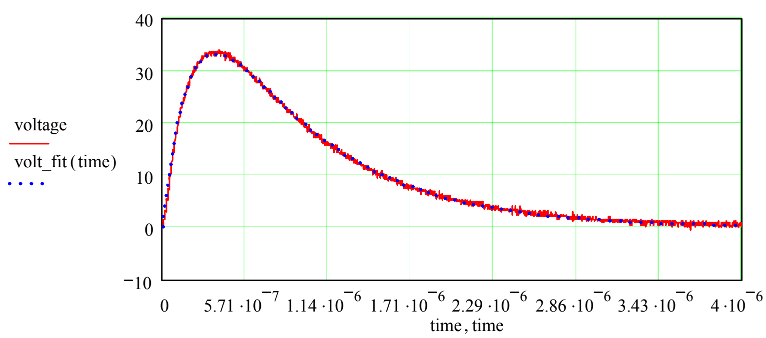
$$F(s,u) := \begin{bmatrix} u_0\cdot\left(e^{u_1\cdot s} - e^{-u_2\cdot s}\right) \\ e^{u_1\cdot s} - e^{-u_2\cdot s} \\ u_0\cdot s\cdot\left(e^{u_1\cdot s}\right) \\ u_0\cdot s\cdot\left(e^{-u_2\cdot s}\right) \end{bmatrix} \quad \begin{matrix} i(t) \\ \frac{\delta i(t)}{\delta B_0} \\ \frac{\delta i(t)}{\delta B_1} \\ \frac{\delta i(t)}{\delta B_2} \end{matrix}$$

Parameter Guesses (as defined by corresponding elements in Eqn (17) & (18)):

$$B = \begin{pmatrix} 1.118\times10^{4} \\ -1.334\times10^{6} \\ 6.999\times10^{6} \end{pmatrix}$$

Least Squares Fit: $\qquad$ $P := genfit(time, voltage, B, F)$

Generated Fit Curve: $\qquad$ $volt\_fit(time) := F(time, P)_0$



Parameter Solution:

$$P = \begin{pmatrix} 74.08 \\ -1.319\times10^{6} \\ 4.838\times10^{6} \end{pmatrix} \begin{matrix} B_0 \\ B_1 \\ B_2 \end{matrix}$$

Thus L, R and Vprime are solved for:

$$L_{fit} := \frac{1}{-C_0\cdot P_1\cdot P_2} \qquad L_{fit} = 8.776\times10^{-8}$$

$$R_{fit} := \frac{\left(P_2 - P_1\right)}{-C_0\cdot P_1\cdot P_2} \qquad R_{fit} = 0.54$$

$$V_{prime} := P_0\cdot\left(P_1 + P_2\right)\cdot L_{fit} \qquad V_{prime} = 22.877$$

k determined as: $\qquad$ $k := \dfrac{V_{prime}}{V_0}$ $\qquad$ $k = 6.02\times10^{-3}$

Difference with underdamped value from App. A: $\qquad$ $\dfrac{|k - 0.005936|}{k}\cdot100 = 1.399$

REFERENCES

[1] F. M. Dickey, S. E. Yao, L. S. Weichman, and L. A. Romero, *RLC Capacitor Discharge Circuit Evaluation*, Sandia National Labs, NM, SAND2002-1744, 2002.

[2] L. A. Romero, F. M. Dickey, and H. Dison, *A Technique for Determining Non-Linear Circuit Parameters from Ring Down Data*, Sandia National Labs, NM, SAND2002-0115, 2002.

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