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A major aim of this project was the handling of dissipation and decoherence in quantum optical and quantum computational systems. Progress in the first two years has been reported before, namely, the extension of the unitary integration technique to master equations for the density matrix, together with an application to $n = 2$ and $n = 3$. This work was extended in the final years of the project, mainly in work done with a graduate student Weichang Zhao, to a complete time-dependent treatment of a general three-level system in quantum optics or quantum information, and published [1, 2]. It was also reported in talks and posters at two conferences, the AMO Divisional Meeting of the American Physical Society in Boulder, Colorado, in May 2003 [3], and an International Conference in Stockholm, Sweden, in July 2003 [4].

Another graduate student, Ganesh Selvaraj, who worked half-time on this grant for a year, set up the similar study of $n = 4$. Such a system describes two coupled qubits and is of great interest because of application to basic quantum gates. We developed the formalism and applied it to calculations that are directly relevant to experimental realizations of quantum logic gates with two coupled Josephson junctions [5]. This work, as also applications to other four-level systems, was published [6].

All of the above work amounts to solving evolution equations for the group $SU(n)$ or algebra $su(n)$. A major theme was that even though the number of coupled states and equations increases rapidly with n , judicious use of sub-algebras simplifies the problem considerably in our technique. Towards the end of the project period, a major advance made was the compact rendering of all such evolution in terms of the simplest case of $n = 2$. By considering the Hamiltonian and the evolution operator for general n as 2×2 block matrices, and writing the evolution operator as a product of three factors analogous to the three with Pauli spinors for $su(2)$, each factor built up of block matrices, the form of the procedure familiar for $n = 2$ is carried over to larger n . In particular, the final factor is block diagonal, each diagonal block then constituting a problem of lower n . This allows for a hierarchical or iterative scheme by which any $su(n)$ can be reduced in steps to a final $su(2)$ [7]. This turns out to be a powerful procedure also for determining what are called geometric phases which are of increasing interest in quantum systems.

Two other pieces of work also led to publications in areas that are peripherally related to the main body of work under this grant. In quantum optics, as elsewhere throughout physics, an embedded discrete state in a continuum leads to resonances whose general description is through the so-called “Fano formalism” [8]. Upon seeing that there are some misconceptions in the literature, a pedagogical exposition of the formalism was published [9]. The other work dealt with the concept of supersymmetry in quantum mechanics which is also relevant in quantum optics. Many examples of supersymmetry in simple quantum systems were investigated, reported in the First International Conference on Supersymmetry in Quantum Mechanics in Vallodolid, Spain, in June 2003, and were published [10].

Publications in the grant period, all of which bear an acknowledgment to the DOE Grant: [1-4], [6-7], and [9-10] in the list below.

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