**Final Report for Grant DE-FG03-98ER14891**
***Nonequilibrium Pattern Formation and Spatiotemporal Chaos in Fluid Convection***
January 1, 2002 - December 31, 2005

**PIs:** Michael Cross (Caltech), with Henry Greenside (Duke) and Paul Fischer (ANL)
**Postdocs:** Mark Paul (Caltech) and Anand Jayaraman (ANL and Duke)
**Graduate students:** Keng-Hwee Chiam and Janet Scheel (Caltech)

This is a joint grant with Henry Greenside at Duke University and Paul Fischer at Argonne National Labs. This report describes the work of the Caltech group in the collaboration.

## Overview

In the grant period of DE-FG03-98ER14891 (3 years plus a one year no cost extension), we have exploited the unique simulation capabilities developed during the preceding grant period to investigate several important issues in pattern formation and spatiotemporal chaos in Rayleigh-Bénard convection, particularly emphasizing quantitative contact with the active experimental programs (many supported by the Department of Energy). In addition, we have extended the capabilities of the numerical code to permit future investigations. Our major results are:

- The quantitative analysis of the role of mean flow in the formation of patterns and the subsequent breakdown to spatially disordered and chaotic dynamics (known as spatiotemporal chaos).
- The quantitative comparison with recent experiments by Bajaj et al. using a geometry of a cylindrical convection cell with a fluid depth that varies with radius so that the fluid is nonconvecting near the outer walls, including comparison with new analytic predictions of the mean flow in this geometry.
- The first numerical investigation of the coarsening of patterns over long times from disordered random initial conditions to patterns that are ordered over large domains. Important questions addressed are the nature of the asymptotic pattern, particularly the wave length of the locally periodic structure, and the exponents that characterize the slow decay towards this ordered pattern.
- A careful analysis of the properties of the "wall state" in rotating Rayleigh-Bénard convection systems where the thermal disturbance is localized at the sidewalls, and propagates along the sidewall.
- An analysis of transport within a spatiotemporal state, where we demonstrate an effective diffusion, due to the transport by the disordered flow, that is enormously enhanced over molecular diffusion and over the transport in ordered flows.
- The first calculation of the Lyapunov exponent in spatiotemporal chaos in experimentally realistic geometries. We showed that the leading Lyapunov exponent in domain chaos in rotating Rayleigh-Bénard convection scales linearly with the strength of driving above threshold as expected from theory, but did not show the expected independence on system size. An

investigation of the spreading of perturbations begins to address the question of how the sensitivity to initial conditions characteristic of chaotic dynamics is manifested in the very high dimensional dynamics of spatially extended systems.

- A careful analysis of the correlation of the separation of trajectories from slightly different initial conditions with observed events in the pattern dynamics and with other diagnostics of the dynamics casts doubt on the conclusion of other workers that defect dynamics are the main cause of the sensitive dependence on initial conditions.

- The calculation of the Lyapunov exponent in numerical simulations for the experimental geometries that were used in the pioneering experiments two decades ago investigating the onset of disordered dynamics in driven fluid systems (called the "origins of turbulence" in those days). Our work verified for the first time that the disordered dynamics was indeed chaotic, i.e., was characterized by a positive Lyapunov exponent.

- A numerical investigation of the scaling of the time scales of the dynamics in domain chaos in rotating Rayleigh-Bénard convection showed good agreement with the theoretical predictions of Tu and Cross in the case where the Coriolis force is the most important effect of the rotation, in contrast with experiment which yields smaller values of the scaling exponents than predicted by theory.

- A resolution of the long-standing discrepancy between theory and experiment for the scaling of spatiotemporal chaos in rotating Rayleigh-Bénard convection in terms of the importance of the centrifugal force in experiment. This work involved the combined effort of our large scale numerical simulations with the experimental work of Ahlers and Becker at Santa Barbara.

- A theoretical and numerical investigation of the glide dynamics of dislocation defects in the convection rolls in rotating Rayleigh-Bénard convection.

**Publications**

Please refer to our group web page for more information, http://www.cmp.caltech.edu/∼stchaos


1. *Rayleigh-Bénard convection with a radial ramp in plate separation*, M. R. Paul, M. C. Cross, and P. F. Fischer, Phys. Rev. **E66,** 046210 (2002).

2. *Pattern formation and dynamics in Rayleigh-Bénard convection: numerical simulations of experimentally realistic geometries*, M. R. Paul, K.-H. Chiam, M. C. Cross, P. F. Fischer, and H. S. Greenside, Physica **D184**, 114 (2003).

3. *Mean flow and spiral defect chaos in Rayleigh-Bénard convection,* K.-H. Chiam, M. R. Paul, and M. C. Cross, Phys. Rev. **E67**, 056206 (2003).

4. *Travelling waves in rotating Rayleigh-Bénard convection: Analysis of modes and mean flow*, J. D. Scheel, M. R. Paul, M. C. Cross, and P. F. Fischer, Phys. Rev. **E66**, 06621 (2003).

5. *Efficient algorithm on a non-staggered mesh for simulating Rayleigh-Bénard convection in a box*, K.-H. Chiam, M.-C. Lai, and H. S. Greenside, Phys. Rev. E, 68, 026705 (2003).

6. *Spatiotemporal chaos in Rayleigh-Bénard convection*, K-H. Chiam, Caltech Thesis (2003).

7. *Rayleigh-Bénard Convection in large-aspect-ratio domains,* M. R. Paul, K-H. Chiam, M. C. Cross, P. F. Fischer, accepted for publication in Phys. Rev. Lett. (2004).

8. *Enhanced tracer transport by the spiral defect chaos state of a convecting fluid*, K.-H. Chiam, M. C. Cross, H. S. Greenside, and P. F. Fischer, Phys. Rev. **E71**, 036205 (2005)

9. *Scaling laws for rotating Rayleigh-Bénard convection*, J. D. Scheel and M. C. Cross, Phys. Rev. **E72**, 056315 (2005)

10. *Effect of the centrifugal force on domain chaos in Rayleigh-Bénard convection,* N. Becker, J. D. Scheel, M. C. Cross, and G. Ahlers,“”, Phys. Rev. **E73**, 066309 (2006)

11. *Characterization of the domain chaos convection state by the largest Lyapunov exponent, A.* Jayaraman, J. D. Scheel, H. S. Greenside, P. F. Fischer

12. *Rotating Rayleigh-Bénard Convection*, J. D. Scheel, Caltech Thesis (2006)