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## **Finite Element Calculations Illustrating a Method of Model Reduction for the Dynamics of Structures with Localized Nonlinearities**

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# Finite Element Calculations Illustrating a Method of Model Reduction for the Dynamics of Structures with Localized Nonlinearities

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## Abstract

A technique published in SAND Report 2006-1789 *Model Reduction of Systems with Localized Nonlinearities* is illustrated in two problems of finite element structural dynamics. That technique, called here the Method of Locally Discontinuous Basis Vectors (LDBV), was devised to address the peculiar difficulties of model reduction of systems having spatially localized nonlinearities. Its illustration here is on two problems of different geometric and dynamic complexity, but each containing localized interface nonlinearities represented by constitutive models for bolted joint behavior.

As illustrated on simple problems in the earlier SAND report, the LDBV Method not only affords reduction in size of the nonlinear systems of equations that must be solved, but it also facilitates the use of much larger time steps on problems of joint macro-slip than would be possible otherwise. These benefits are more dramatic for the larger problems illustrated here.

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# Acknowledgment

Wil Holzmann of Sandia National Laboratories is thanked for providing to the authors a Matlab formatted Component Mode Synthesis model for the three-legged structure discussed here. Mr. Holzmann also shared all mesh and response data for the high fidelity finite element model necessary to perform the comparison calculations on this object.

The experiments on the three-legged mock AF&F structure were performed by Dan Gregory and Brian Resor.

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# **Finite Element Calculations Illustrating a Method of Model Reduction for the Dynamics of Structures with Localized Nonlinearities**

## **Chapter 1**

### **Introduction**

Though the weapons program has access to massively parallel computers and has finite element code that can employ many processors simultaneously, the resulting numerical predictions often are difficult to interpret physically. Additionally, when one incorporates nonlinear joint models in the structural models and considers loads that bring the joints to macro-slip, very high frequency responses are elicited from the model and the system response becomes so nonlinear that extremely small time-steps become necessary. The resulting requirement on computing resources can become intractable.

Though it would be natural to pursue model reduction strategies, conventional Galerkin model reduction using basis vectors natural to the reference linear system converge very slowly in these problems [7]. The difficulty was eventually isolated to a Gibb's type phenomena having to do with stiffness nonlinearity of the joint.

A solution was proposed in Reference [7] augmenting the original basis vectors with ones having local discontinuities at the joint. This method of Locally Discontinuous Basis Vectors (LDBV) was shown to accelerate greatly the convergence of a Galerkin model reduction on small (toy) problems. The benefits of that technique demonstrated on those smaller problems were

- The low- to mid-frequency dynamics of complex jointed structures can be modeled with relatively few degrees of freedom.
- In problems of micro-slip, the generalized degrees of freedom associated with the locally discontinuous basis vectors remain small relative to those obtained from the

reference linear system. This facilitates the qualitative discussion of structural dynamics in terms of linear eigen modes - a feature very helpful in understanding system dynamics.

That formulation has very natural application in finite element analysis of the transient dynamics of jointed structures and the purpose the work reported here is to explore such applications.

## Chapter 2

# The Method of Locally Discontinuous Basis Vectors

The first natural step in model reduction of a structure containing local nonlinearities would be to employ a Galerkin procedure using as basis functions the eigen modes of a reference linear system (obtained by linearizing the nonlinear system about zero load.) In problems containing stiffness nonlinearities, such model reductions may converge very slowly in the sense of requiring very large sets of basis functions to approximate the behavior of the full nonlinear systems.

The difficulty with the above straight-forward approach can be understood by considering the structure's response to very large impulses. The resulting free vibrations will involve large deformations at the joints and the stiffness nonlinearities will result in configurations that are not easily represented as linear combinations of eigen modes of the reference linear system. In fact, in [7] proper orthogonal decomposition (POD) [4] was used to demonstrate that, even when the eigen modes of the reference linear system are continuous, the POD modes derived from the nonlinear response can be discontinuous at the joints. This demonstrates the need for basis functions other than eigen modes of the reference linear system.

One can compare the convergence difficulties in this problem to those of trying to represent a curve having a discontinuity with a Fourier series. A very large number of *sine* and *cosine* terms are required to obtain an even approximately adequate representation - a feature of Gibbs phenomena. Just as a Fourier series approximation is facilitated by augmenting the orthogonal polynomials by a function with a discontinuity at the appropriate location, model reduction of structures with localized nonlinearities is facilitated by including basis vectors with an appropriate discontinuity at the location of the nonlinearity.

In [7] two classes of basis functions with discontinuities were examined. Those classes appeared to manifest identical benefits in terms of model reduction and convergence, but one was much easier to implement. It is that more convenient class of basis vectors that was employed in this study.

Those basis functions are found by a statics solution on the reference linear system where equal and opposite loads are applied at the location of the joint and co-linear with the joint displacement. The resulting deformation field, which is the solution of the statics problem on the reference linear system, is used to augment the basis vectors obtained by eigenanalysis of the reference linear system. These basis functions, apparently identical to basis vectors employed by Milman and Chu ([2], [5]) in their optimization of linear structural dampers, are referred to in the following as joint modes.

It should be noted that in the problems being addressed here, the joint kinematics have been simplified by the “whole joint” approximation([6]), where all nodes on each side of the joint interface are slaved to one representative node. The joint kinematics are now described by the relative motion of those two representative nodes.

# Chapter 3

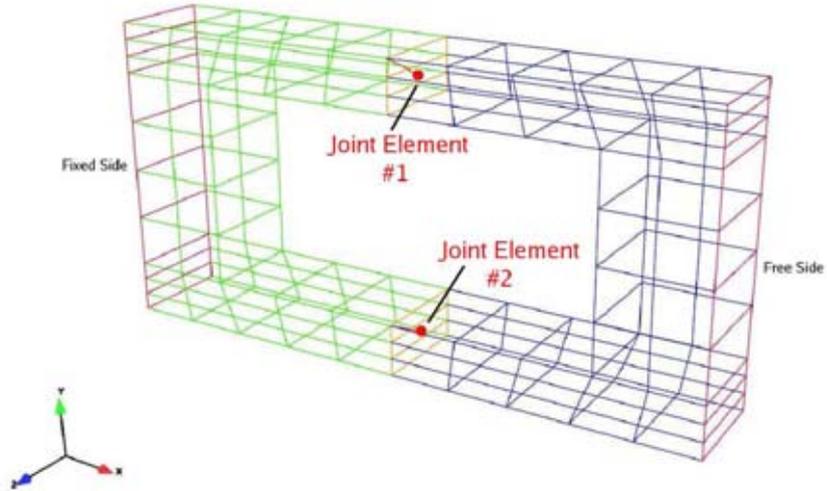
## Transient Analysis of Structures Containing Mechanical Joints

### 3.1 Application to a Simple Structure with Two Joints

We consider a structure represented by the finite element mesh shown in Figure 3.1. The structure contains two joints at locations indicated in the figure. Each joint is capable of deformation in only the indicated  $x$  direction. This system contains 722 nodes or 2166 total degrees of freedom; however, due to boundary and MPC constraints the model possesses only 1803 active degrees of freedom. We consider the analysis of this 1803 degree of freedom model to be the full order system. Dimensions, material properties, and joint properties are specified in Appendix A. The joint model is one discussed in Reference [6], manifesting hysteretic loss that roughly follows a power-law dependence on force amplitude and macro-slip at high load.

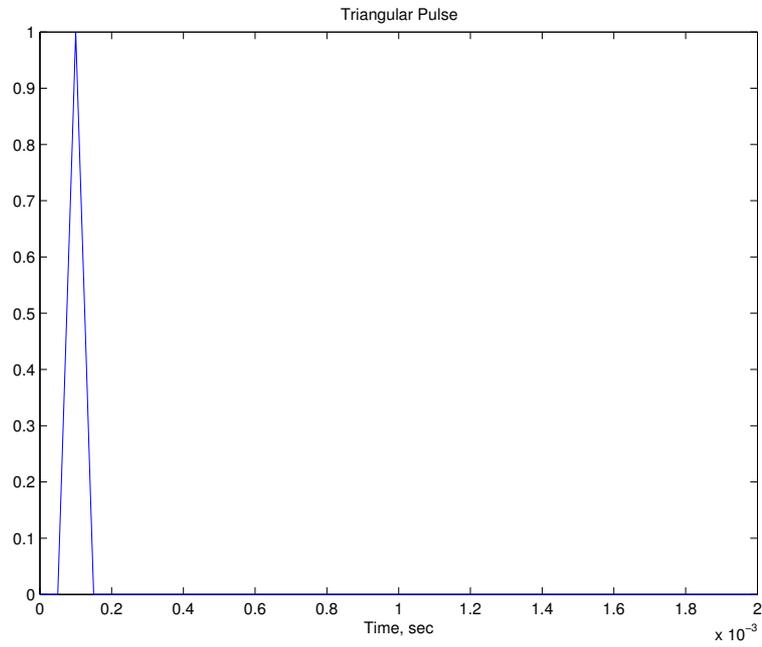
The chosen loading for this simulation is a uniform traction in the  $y$  direction on the free side of the structure as noted in Figure 3.1 modulated by a triangular pulse of  $1e-4$  second duration, a normalized version of which is shown in Figure 3.2. Cases of two very different load amplitude were examined. In each case, three analyses were performed:

1. Transient analysis of the full nonlinear finite element model. This will be our truth model.
2. Transient analysis of a nonlinear Galerkin model using twenty eigen modes of the reference linear system. In the following, we refer to these analyses of the modally truncated system as the reference model reduction.
3. Transient analysis of a nonlinear reduced model using eighteen eigen modes of the reference linear system and one joint mode appropriate for each of the two system joints. (The first eighteen eigen modes of the reference linear system include all

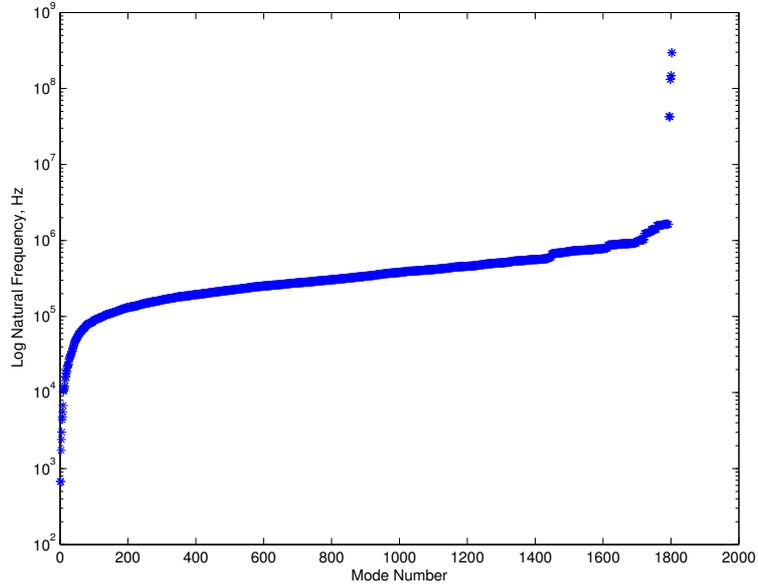


**Figure 3.1.** *Mesh for Two Joint Structure.*

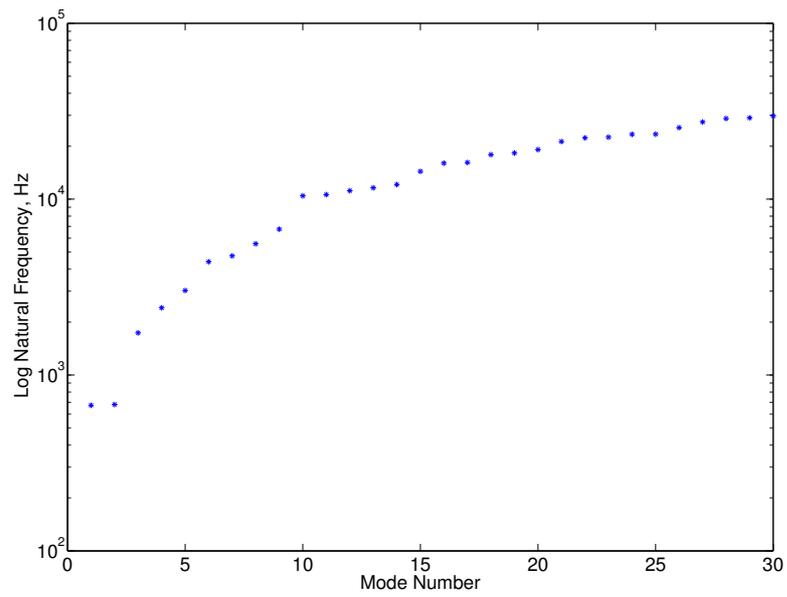
those with frequencies below 20 kHz. See Figures 3.3 and 3.4 for plots of all eigen frequencies and a close up of the first 30 eigen frequencies, respectively). In the following, we refer to this LDBV analysis as the augmented model reduction.



**Figure 3.2.** *Normalized Force Input. A triangular pulse input is applied to the free side of the structure.*



**Figure 3.3.** *Natural Frequencies of Full Order System. The natural frequencies of the full order system are plotted to show the modal density for the model being studied.*



**Figure 3.4.** *Natural Frequencies of Full Order System. The natural frequencies for the first 30 modes are plotted here. The first 20 modes include frequency content up to 20 kHz.*

### 3.1.1 Case of Very Small Loading

The amplitude of the triangular impulse traction was set at a low value ( $100 \frac{lbf}{in^2}$ ) such that very little nonlinearity is manifest in the structure.

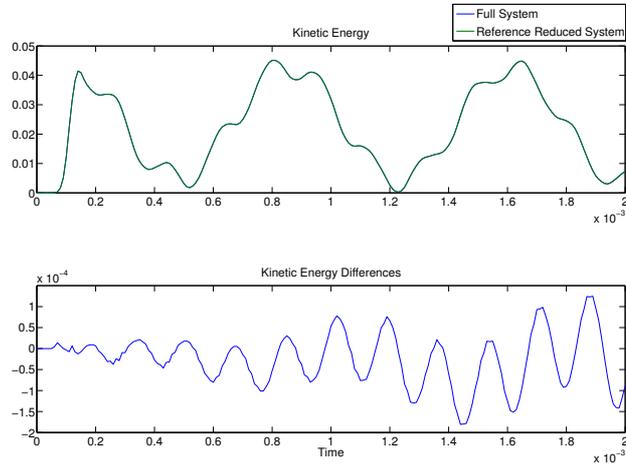
To evaluate the reference reduced system, we compare the histories of kinetic energy and joint force with those of the truth model. The kinetic energy of the reference reduced system along with that of the truth model is plotted in the upper portion of Figure 3.5 and the difference between the two is plotted in the lower portion of Figure 3.5. The force of the upper joint predicted by the reference reduced system along with that of the truth model is plotted in the upper portion of Figure 3.6 and the difference between them is plotted in the lower portion of Figure 3.6. Due to the symmetry of the geometry and loading condition, the joint force history for the second joint is identical to that of the first modulo a sign difference. Unless otherwise noted, energy is presented with units of in-lbf and force is presented with units of lbf throughout this report.

In the above analysis, the maximum error in kinetic energy is about 0.25% and the maximum error in joint force is about 1%. The quality of agreement should not be surprising since it is only the first mode ( 672 Hz) that is excited and there is very little nonlinearity to couple this mode with higher modes.

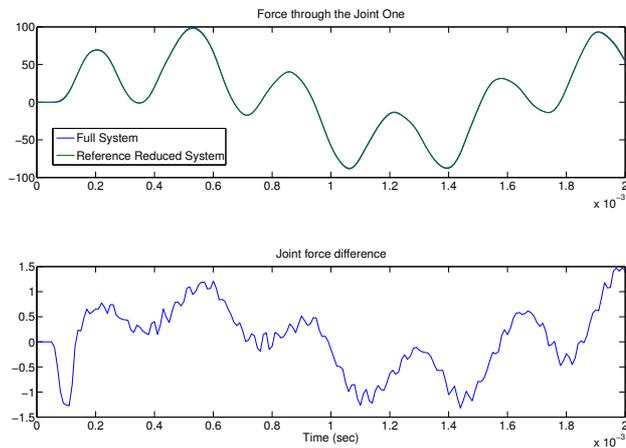
Similarly, to evaluate the augmented reduced system, we compare the histories of kinetic energy and joint force with those of the truth model. The kinetic energy of the augmented reduced system along with that of the truth model is plotted in the upper portion of Figure 3.7 and the difference between the two is plotted in the lower portion of Figure 3.7. The force of the upper joint predicted by the augmented reduced system along with that of the truth model is plotted in the upper portion of Figure 3.8 and the difference between them is plotted in the lower portion of Figure 3.8.

In the above analysis, the maximum error in kinetic energy is about 0.063% and the maximum error in joint force is about 0.5%. Though these results are slightly better than those of the reference reduced system, the nonlinearity of this minutely excited structure is too small to require the benefits of the joint modes.

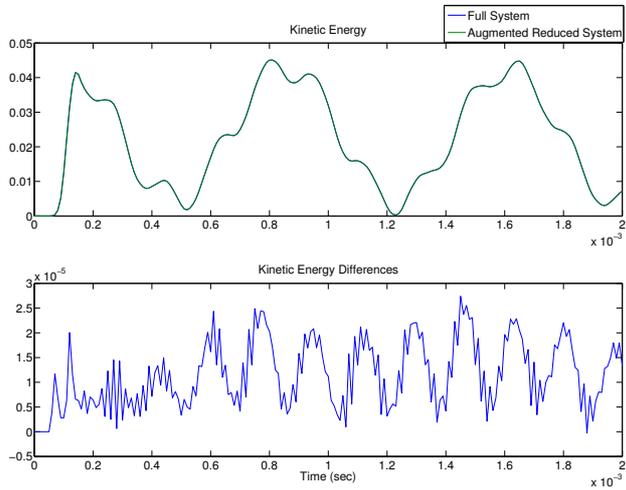
It should be observed that point-wise comparison of history variables is an extremely stringent test of model reduction because accumulated phase errors can result in very large error in point-to-point comparison.



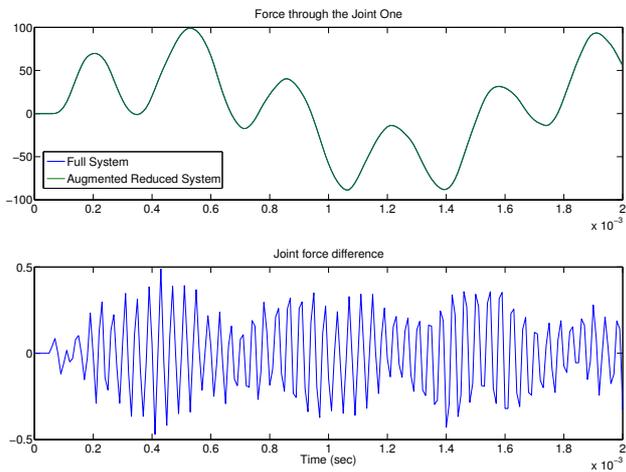
**Figure 3.5.** The Kinetic Energy is plotted versus time for the case of small loads in the micro-slip regime for the case of a model truncated with the 20 lowest frequency eigen modes – the reference reduced model.



**Figure 3.6.** The Force history of joint one is plotted for the case of small loads in the micro-slip regime for the case of a model truncated with the 20 lowest frequency eigen modes – the reference reduced model.



**Figure 3.7.** The Kinetic Energy is plotted versus time for the case of small loads in the micro-slip regime for the case of a model truncated with the 18 lowest frequency eigen modes augmented with 2 joint modes – the augmented reduced model.



**Figure 3.8.** The Force history of joint one is plotted for the case of small loads in the micro-slip regime for the case of a model truncated with the 18 lowest frequency eigen modes augmented with 2 joint modes – the augmented reduced model.

### 3.1.2 Case of Large Loading

Here the amplitude of the triangular impulse traction was set at a sufficiently large value ( $1000 \frac{lbf}{in^2}$ ) to drive the joints into macro-slip. We use the reference reduced model and the augmented reduced model employed in the previous section.

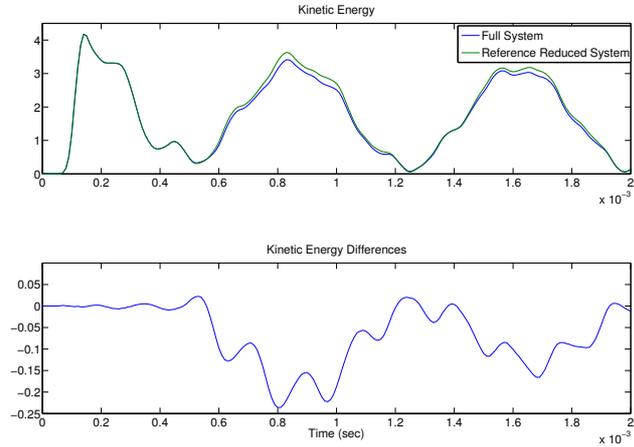
For the reference reduced model, the kinetic energy histories and the error with respect to the truth model are plotted in Figure 3.9. The joint force histories and its error with respect to the truth model are plotted in Figure 3.10. Here we see substantially more error than was the case when low loads were applied: 5% error in kinetic energy and 7% error in joint force. This deficiency is expected for reasons discussed above.

Similar plots are made for the predictions of the augmented reduced model (LDBV analysis). The kinetic energy history and the error with respect to the truth model are plotted in Figure 3.11. The joint force and its error with respect to the truth model are plotted in Figure 3.12. We see substantially less error than was the case with the reference reduced model: 0.05% error in kinetic energy and 0.7% error in joint force. We also note that the character of the joint force errors for the augmented reduced model are zero mean Gaussian type errors for both the small and large loading cases, which indicates that the discontinuous basis vectors (joint modes) generally capture the correct physics of the joint response for all load levels experienced. Thus, we see that for small and large load, joint modes offer significant improvement in accuracy, especially in capturing the nature of the response near the joints.

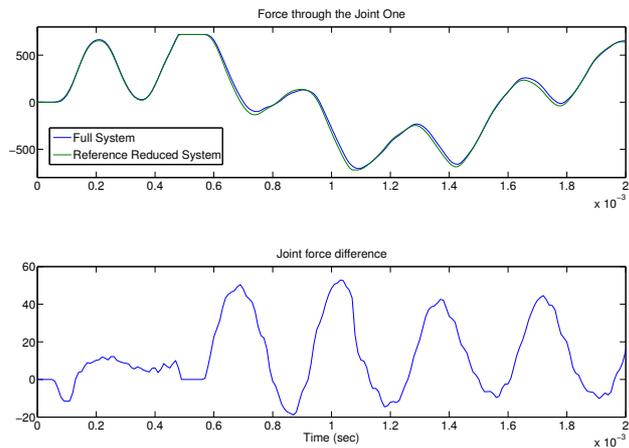
The timing summary for the full and reduced order systems solved in Matlab are given in Table 3.1. As can be seen, the solution to the augmented reduced order system, having 20 degrees of freedom, is much faster than that of the full order system with 1803 degrees of freedom, especially for loads which excite macro-slip. In this case, not only does each iteration of the full model require solving much larger systems of equations, but the full system requires more iterations to converge in each time step.

**Table 3.1.** Timing Summary for Two Joint Structure Nonlinear Model Reduction.

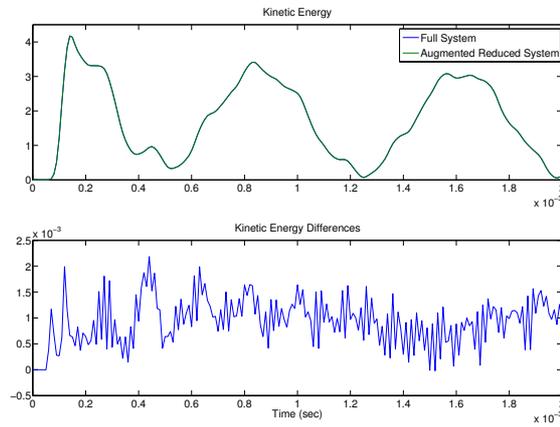
	Full System(sec) 1803 DOF	Reduced System(sec) 20 DOF
Nominal Load	426.5	0.4
10x Nominal	2281.2	0.4



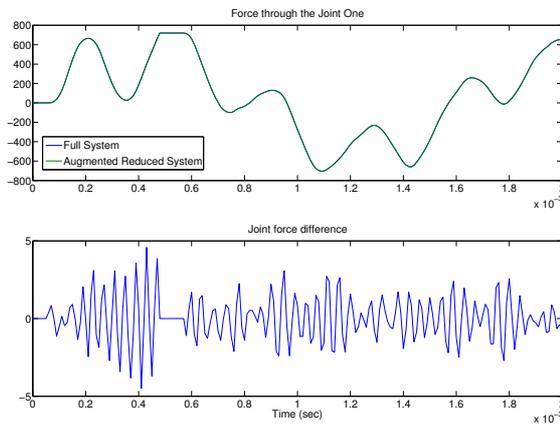
**Figure 3.9.** The Kinetic Energy is plotted versus time for the case of loads in the macro-slip regime for the case of a model truncated with the 20 lowest frequency eigen modes – the reference reduced model.



**Figure 3.10.** The Force history of joint one is plotted for the case of loads in the macro-slip regime for the case of a model truncated with the 20 lowest frequency eigen modes – the reference reduced model.



**Figure 3.11.** *The Kinetic Energy is plotted versus time for the case of loads in the macro-slip regime for the case of a model truncated with the 18 lowest frequency eigen modes augmented with 2 joint modes – the augmented reduced model.*



**Figure 3.12.** *The Force history of joint one is plotted for the case of loads in the macro-slip regime for the case of a model truncated with the 18 lowest frequency eigen modes augmented with 2 joint modes – the augmented reduced model.*

## 3.2 Application to a Three-Legged Mock AF&F Structure with Component Mode Synthesis of Linear Degrees of Freedom

In Reference [7] a mode of application was outlined for reduced order modeling of large structures with local nonlinearities. The strategy employed Component Mode Synthesis (CMS) and was outlined as follows.

Consider a structure  $\mathcal{B}$  consisting of a number of substructures  $\mathcal{B}_k$  with joint models connecting some of the interface degrees of freedom. The kinematics of each substructure is characterized by the values of interface degrees of freedom  $\{u_{k,n}\}$  and modal degrees of freedom  $\{\phi_{k,n}\}$ . The development of the reduced order model proceeds much as discussed earlier in this report:

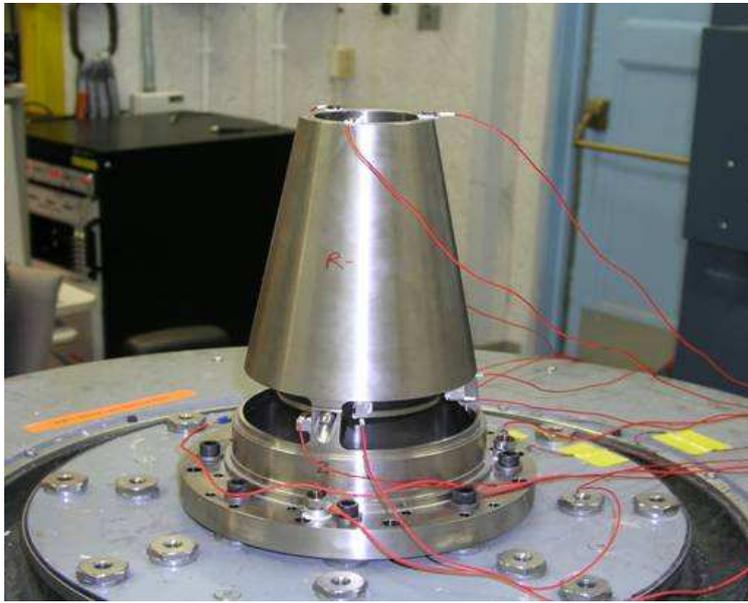
- eigen analysis is performed on the linearized component mode representation for  $\mathcal{B}$ .
- the joint vectors are calculated by placing self equilibrating loads on nodes on the interfaces between substructures and performing system level statics solutions.
- the numerical results are in terms of vectors whose support is the whole structure.

These basis vectors are used in a reduced order Galerkin formulation for the nonlinear transient dynamics of the structure. The number of elastic eigen modes and joint modes necessary for application to a particular problem can be estimated in a manner similar to that employed in modal truncation of linear systems. In the simplest implementation, one employs all elastic modes corresponding to frequencies below an appropriately chosen cut-off frequency and one uses a joint mode for each joint degree of freedom.

### 3.2.1 The Three-Legged Structure

The above strategy is employed to study the transient dynamics of the three-legged structure indicated in the photograph of Figure 3.13 and the finite element mesh of Figure 3.14. Each leg is connected to the upper section by a bolted joint oriented approximately  $45^\circ$  radially outward from the vertical.

The study reported here grew out of an earlier one whose objective was to improve non-linear response prediction in existing structural dynamics models [3]. A focus of the study was bolted joints modeled using Iwan constitutive models. This study revealed that 3D high-fidelity transient simulations as well as Component Mode Synthesis (CMS) analysis based on the high-fidelity model tend to produce a large amount of hashy high frequency response when a joint modeled with an Iwan element goes into macro-slip. Indeed, this behavior was seen in the experimental tests, though to a much lesser extent. A CMS model

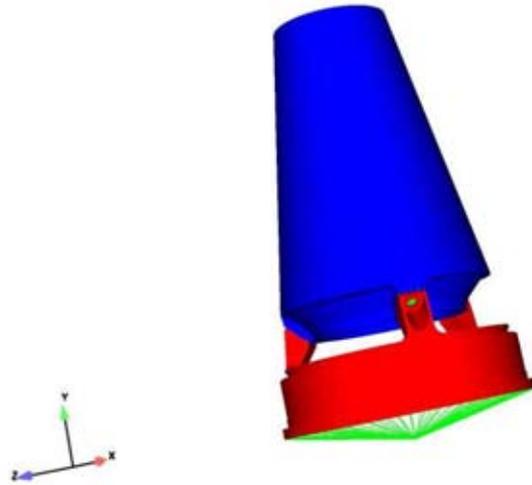


**Figure 3.13.** *Test Photo of Mock AF&F Structure .*

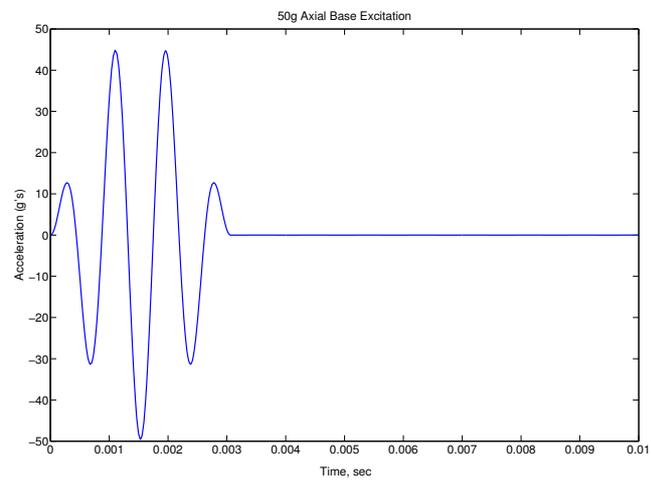
was created to mitigate this effect as observed in the 3D high-fidelity model, yet the high frequency hash persisted. Stiffness proportional damping and filtering were used to reduce the high frequency content of the response and predictions closer to the experimentally measured accelerations were achieved.

The full system solution suffered the related deficiencies that the desired low frequency behavior was obscured by high amplitude “hash” and very small time steps were necessary for the nonlinear solver to converge. Examination of simpler problems indicates that the high frequency hash resulting as joints move into macro slip is mathematically correct, manifesting the response of high frequency modes to a discontinuous change in the tangent stiffness. The CMS approach can suffer the same deficiencies because including enough subsystem modes to invest the structure with appropriate static compliances also invests it with high frequency resonances.

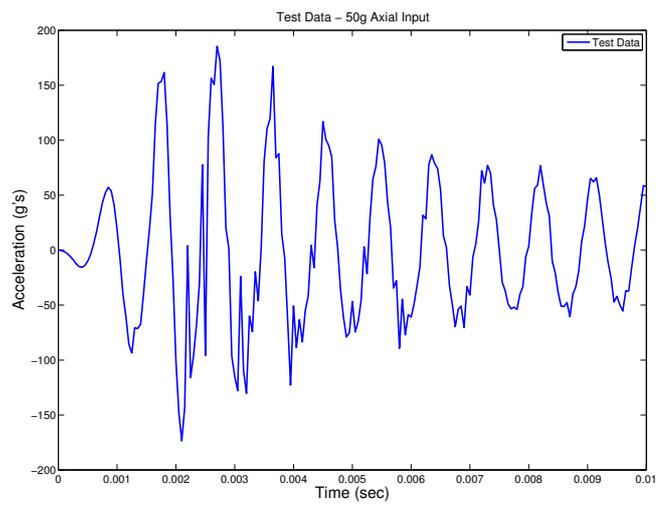
The method of Locally Discontinuous Basis Vectors (LDBV) is demonstrated on this problem using parameters employed in the earlier study. Joints are represented by a four parameter Iwan model for the two principal directions in the slip plane and by a very stiff spring in the direction normal to the slip plane for each leg. Parameters for the joints and material parameters are itemized in the Appendix of this report. The acceleration history shown in Figure 3.15 was applied to the base of the structure and acceleration was measured on the  $270^\circ$  leg. Of the responses of nine specimens (all sampled at 20 kHz), the acceleration shown in Figure 3.16 is typical.



**Figure 3.14.** *FEM Model of Mock AF&F Structure.*



**Figure 3.15.** *The base excitation for the axial input case is a shaped sine wave. The peak acceleration for this case is 50g, which is sufficient to excite macro-slip in the joints.*

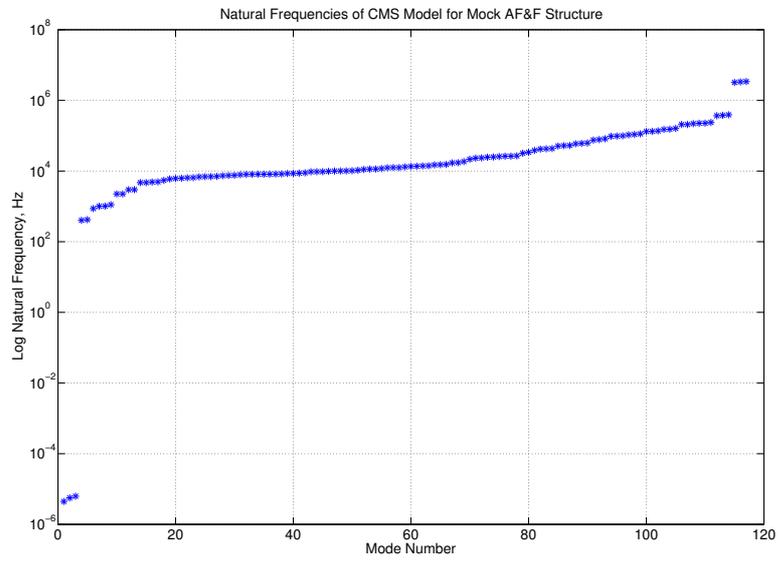


**Figure 3.16.** *Measured acceleration for 50g axial input case on 270 degree leg of structure.*

### 3.2.2 Simulation Results

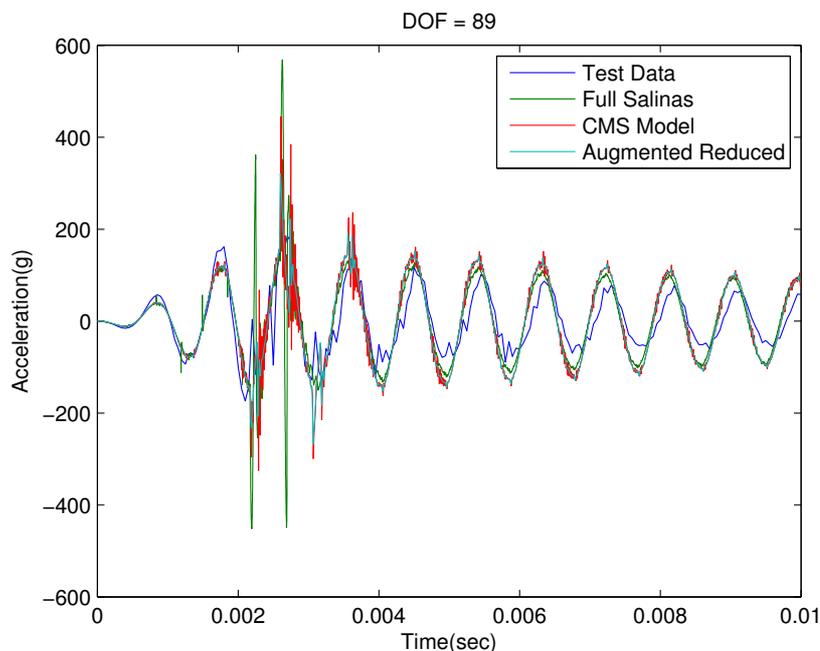
In evaluating this application of the LDBV method, we compare four calculations:

1. The predictions of the full nonlinear finite element model (206,343 active degrees of freedom) solved using Salinas [1]. We call this the Full Salinas model.
2. The predictions of a nonlinear CMS model. In this application the kinematics of the upper substructure is represented by its first 30 eigen modes and 27 interface degrees of freedom while the lower substructure is represented by its first 30 eigen modes and 30 interface degrees of freedom. Each of the three bolted joints (one on each leg) are represented by two Iwan elements (one for each principal axis in the slip plane) and one linear spring in the direction normal to the slip plane. The lower section is attached to ground by constraints applied to all nodes around the holes of the attachment bolts. This model, referred to as the CMS model, has 117 degrees of freedom.
3. The predictions of a model based on only the fifteen lowest frequency eigen modes of the reference linear system. In this case the reference linear system is the CMS model where the joints are represented by springs of stiffness equal to the tangent stiffness of the joints at zero load. We call this modally truncated CMS model the reference reduced model.
4. The predictions of the LDBV method using nine eigen modes of the reference linear system augmented with six joint modes. Again, the reference linear system is the CMS model where the joints are represented by springs of stiffness equal to the tangent stiffness of the joints at zero load. These eigen modes are sufficient to capture the resonances of the linear system to 2000 Hz (See Figure 3.17). The joint modes are deduced from the CMS system in the manner outlined above. We refer to this LDBV analysis of the CMS model as the augmented reduced model.



**Figure 3.17.** *Natural Frequencies for CMS Model of Mock AF&F Structure. The first 10 modes capture frequencies up to a 2000 Hz cutoff.*

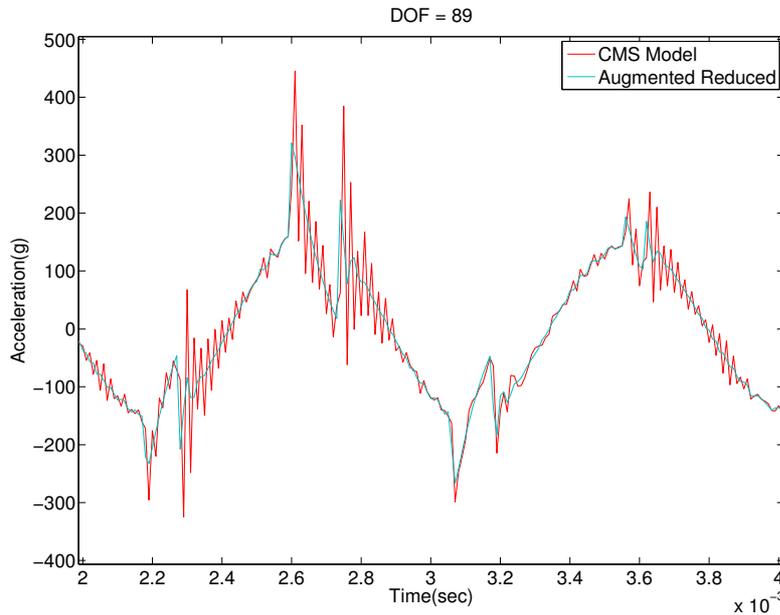
Figure 3.18 shows the experimental acceleration and those of three analyses including the Full Salinas, CMS, and augmented reduced model. We see that the full finite element nonlinear analysis prediction is so hashy as to obscure the responses at the frequencies that dominate the experimental results. The CMS model predictions are a little less hashy, but the desired aspects of the response are still obscured by hash. The predictions of the augmented reduced model are closest to the experimental accelerations, though there are some systematic differences that can be ascribed to limitations of the joint model.



**Figure 3.18.** *Measured acceleration plotted with predictions of Full Salinas model, CMS model, and augmented reduced model.*

Figure 3.19 shows a closeup of the CMS model response along with the augmented reduced system both from Figure 3.18 for the time between 2 and 4 micro-seconds. This plot shows in detail the high frequency hash associated with the CMS model predictions. A natural remedy for the suppression of this high frequency hash is a reduction in the time step size. This exercise was performed by dividing the step size by ten, and indeed it was found that the high frequency response was suppressed in the CMS model predictions. On the other hand, as shown in Figure 3.19, the high frequency hash is suppressed by augmenting the basis of eigen modes with joint modes with the added benefits of being solved at much larger time steps with fewer degrees of freedom.

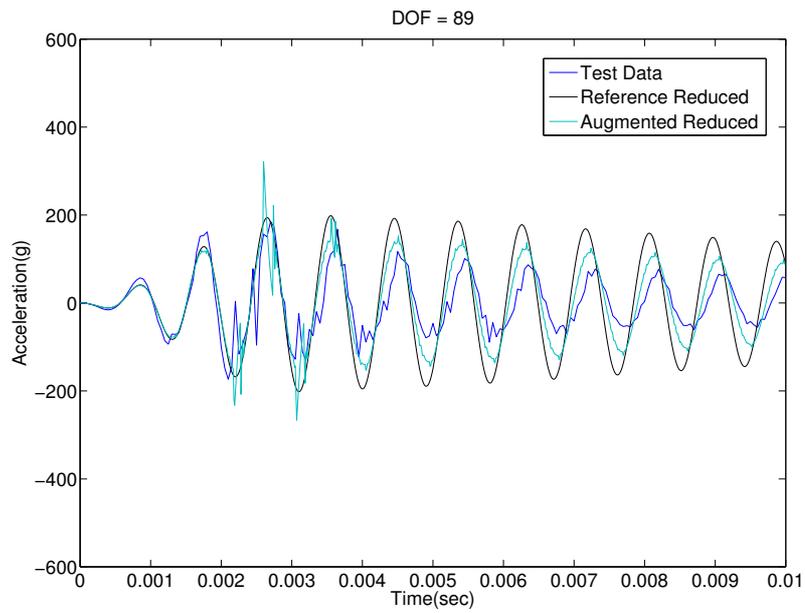
Figure 3.20 shows the test data plotted with the reference reduced model (the modally truncated CMS model) and the augmented reduced model predictions. The plot shows that without joint modes, the reference reduced model does not predict any of the hashy response that is observed in the experiment. On the other hand, the reduced model with



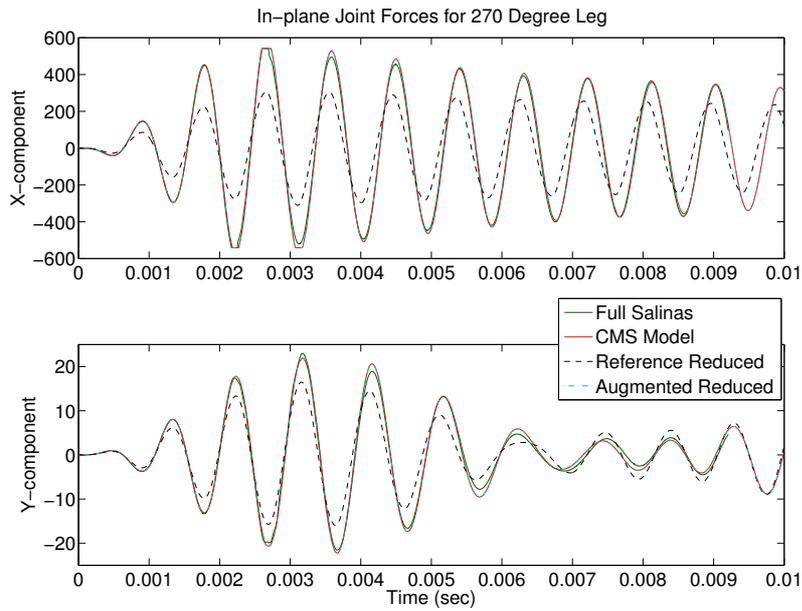
**Figure 3.19.** Closeup showing CMS model response and augmented reduced model response.

joint modes (augmented reduced model) are closer to the experimental accelerations. It is noted that the inclusion of joint modes in the basis results in a better prediction of the dissipation in the joints. While the reference reduced model appears to act as a low-pass filter in removing high frequency response, the augmented reduced model can be described best as applying a “low-pass physics” cutoff to the model in allowing some of the high frequency response.

Figure 3.21 shows the predicted forces in the local  $x$  and  $y$  directions for the  $270^\circ$  leg. These results show that the augmented reduced model is capable of predicting the macro-slip behavior of the force in the joint as also observed in the Full Salinas and CMS models, while the reference reduced model is not. Joint modes are clearly very valuable in capturing the proper forces in the joints which is very important to the overall system response. The forces in the local  $y$  direction for the  $270^\circ$  leg are small as a result of the loading direction; therefore, we can consider using only the three joint modes corresponding to the dominant response direction of each leg. In fact, this exercise has been performed and the effect of replacing these three joint modes with the next three lowest frequency eigen modes is minimal. Although one may, in some cases, find an advantage in ignoring some joint modes in favor of eigen modes, in general it is desirable to include a joint mode for each joint in the model in order that proper response due to those joints is predicted for a different loading condition which may tend to excite those degrees of freedom.



**Figure 3.20.** Measured acceleration plotted with reduced order model predictions.



**Figure 3.21.** *In-plane joint forces for the 270 Degree Leg. The augmented reduced model captures the macro-slip behavior of the joint as do the Full Salinas and CMS model. The reference reduced model using only eigen modes does not capture the macro-slip behavior.*

Table 3.2 compares the compute times for all analyses. The full nonlinear finite element analysis was performed using the Salinas finite element method in parallel mode. This run required approximately 4 hours of CPU time using 10 processors. The nonlinear CMS analysis was performed in Matlab using subsystem matrices computed earlier in Salinas. The LDBV analysis was also performed in Matlab using the Matlab formatted CMS model subsystem matrices. We see a vast improvement in compute time from the full finite element analysis to the nonlinear CMS and a further 74% improvement from the nonlinear CMS to the augmented reduced model (LDBV). The reference reduced model runs faster because fewer iterations are needed on some time steps. The time step size for the CMS and both reduced order CMS models is the same for these runs. However, the plot of Figure 3.20 shows that high frequency hash associated with joint macro-slip in the Full Salinas and CMS models is not evoked in the reduced system.

**Table 3.2.** Timing Summary for Mock AF&F Structure Nonlinear Reduced Models.

Model	No. DOF	CPU Time (sec)
CMS	117	36.3
Reference Reduced	15	2.8
Augmented Reduced	15	9.3
Full Salinas	206,343	40 hours (approximate)



# Chapter 4

## Conclusions

The advantages demonstrated for the Locally Discontinuous Basis Vector method in problems of structures with local nonlinearities in terms of reduction in model size and in terms of accuracy for simple problems appear to be retained when the technique is exercised in a finite element context. It is the high frequency artifacts that prevent nonlinear solvers of the full system model from converging except at very small time steps. The suppression of those responses in the LDBV method makes possible calculation of correct responses in the regimes of interest using far fewer degrees of freedom and much longer time steps than is possible in either the full system model or the nonlinear CMS model.

It should be mentioned that the success of this method in suppressing spurious high frequency hash associated with transition into macro-slip would not be seen in problems that contained significant rate dependence. Still the problems of structural dynamics generally have localized nonlinearities of the sort treated here.

There is an additional strategy that could be employed to reduce further the system sizes associated with the LDBV method. One could employ the LDBV method over short times and identify those elastic eigen modes among the basis vectors for which the corresponding generalized accelerations are not excited. The set of basis vectors could then be reduced appropriately and the simulations run over long times. Of course, this presumes that the excitations seen over that longer time interval are very similar to those seen over the short time interval during which the generalized accelerations were scrutinized. This approach has not been explored.

The efficiencies demonstrated here were all associated with scalar calculations. The next major challenge in the development of this technique will be its implementation in the context of massively parallel computation.



# References

- [1] *SALINAS – An Implicit Finite Element Structural Dynamics Code Developed for Massively Parallel Platforms*, 41st AIAA/ASME/ASCE/AHS/ASC/ Structures, Structural Dynamics, and Materials Conference, Atlanta, GA, April 3-6 2000.
- [2] C. Chu and M. H. Milman. Eigenvalue error analysis of viscously damped structures using a ritz reduction method. *AIAA Journal*, 30:2935–2944, December 1992.
- [3] Wil Holzmann. *Summary of W76-0 Forward Mount/AF&F Joint Study (AD2004-01) FY05*. Sandia National Laboratories, Albuquerque, New Mexico 87185, September 22, 2005.
- [4] G. Kerschen, J. C. Golinval, A. F. Vakakis, and L. A. Bergman. The method of proper orthogonal decomposition for dynamical characterization and order reduction of mechanical systems: An overview. *Nonlinear Dynamics*, 41(1-3):147 – 169, August 2005.
- [5] M. H. Milman and C. C. Chu. Optimization methods for passive damper placement and tuning. *Journal of Guidance, Control, and Dynamics*, 17(4):848 – 56, Jul-Aug 1994.
- [6] Daniel Joseph Segalman. A four-parameter iwan model for lap-type joints. *ASME Journal of Applied Mechanics*, 72(5):752–760, September 2005.
- [7] Daniel Joseph Segalman. Model reduction of systems with localized nonlinearities. Technical Report SAND2006-1789, Sandia National Laboratories, PO Box 5800, Albuquerque, NM, March 2006.



# Appendix A

## Two Joint Structure Material and Joint Properties

Table A.1 lists the exterior dimensions and material properties for the simple structure containing two joints.

**Table A.1.** Dimensions and Isotropic Material Properties for Two Joint Structure.

Parameter	Value
X dimension	6 inches
Y dimension	3 inches
Z dimension	1 inch
E	2.8572e7 psi
$\nu$	0.28
$\rho$	0.289 $\frac{\text{lbf}}{\text{in}^3}$

The joint parameters used for each of the two joints are given in Table A.2. These parameters correspond to a slip force ( $F_S$ ) of 720 lbf and a tangent stiffness ( $K_T$ ) at zero load of  $5.0\text{e}5 \text{ lbf/in}$ .

**Table A.2.** Joint Parameters for Two Joint Structure.

Parameter	Value
R	9.50519e4
S	3.64864e5
$\phi_{max}$	1.8586e-3
$\chi$	-0.8



# Appendix B

## Mock AF&F Structure Joint Properties

The joint parameters used for each of the joints of the mock AF&F structure are given in Table B.1. These parameters correspond to a slip force ( $F_S$ ) of 541.2 lbf and a tangent stiffness ( $K_T$ ) at zero load of  $8.55e6$  lbf/in.

**Table B.1.** Joint Parameters for mock AF&F Structure.

Parameter	Value
R	5.5050e6
S	2.1097e6
$\phi_{max}$	1.75e-4
$\chi$	-0.82

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