

# Reconstruction of a 4D Particle Distribution Using Underdetermined Phase-Space Data.

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## Table of Contents

Abstract .....	iii
Introduction .....	1
Statement of Problem.....	3
The Software.....	5
Discussion.....	8
Acknowledgements.....	9
References.....	9
Figures.....	11

## ABSTRACT

Reconstruction of a 4D Particle Distribution Using Underdetermined Phase-Space Data.  
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A well defined 4D distribution that describes the transverse spatial coordinates  $(x,y)$  and momenta  $(x',y')$  of the particles that make up an intense ion beam is of great value to theorists in the field of particle beam physics. If such a distribution truthfully captures the characteristic of the actual beam, it can be used to initialize an extensive simulation, and can yield insight into the processes that affect beam quality. Creating a proper representative distribution of particles is a challenge because the problem is, in general, quite underdetermined. Data is collected through a pair of “optical slit” diagnostics which provide two 3D distributions,  $f(x,y,x')$  and  $f(x,y,y')$ ; the challenge is to coalesce these into a full 4D distribution  $f(x,y,x',y')$ . Further difficulties are introduced because the data is collected at different longitudinal planes and must be “remapped” to a common plane, taking into account the convergence or divergence of the beam as well as any off-centering. This challenge was met by developing a suitable algorithm and implementing it as a “plug-in” for the popular scientific image analysis program ImageJ, written entirely in the Java programming language. The algorithm accomplishes the desired remapping and synthesizes a 4D particle distribution, using Monte-Carlo techniques. Preliminary results show that this reconstructed distribution is consistent

with actual data that was gathered from the same experiment using a different diagnostic. Also, “forward” particle-in-cell (PIC) simulations, that use the reconstructed distribution, match actual data gathered downstream in the experiment. Both these results give us some indication that the reconstruction is being done correctly. In addition to the multi-particle synthesis, the plug-in allows for the easy loading of digital data and the output of various plots that are useful to both experimenters and theorists. It also provides a framework by which its applicability can be extended to other types of experiments for which data analysis and simulation-particle synthesis are required.

## INTRODUCTION

The use of charged particle beams is prevalent throughout many areas of physics as well as in other scientific endeavors. Therefore, understanding beam dynamics in detail, including the effects of space-charge (the particles' mutual repulsion) on beam transport (movement of the particles longitudinally) in the presence of applied transverse focusing (confining) fields, is an important scientific and engineering task. In order to describe a particle beam, one can use a four dimensional distribution function  $f(x,y,x',y')$ , where  $x$  and  $y$  represent the transverse Cartesian coordinates of an individual particle, while  $x' = p_x / p_z$ , and  $y' = p_y / p_z$ , where  $p_x$  and  $p_y$  are the transverse momenta, and  $p_z$  is the longitudinal momentum; see Figure 1 and 3 for clarification of coordinate system. Using computer simulations to understand a beam's dynamics is an important capability for numerous applications; in this particular instance, it is being used for Heavy Ion Fusion (HIF) research [1].

The HIF group hopes to answer questions regarding the viability of heavy-ion beam-driven inertial fusion as an effective source of energy. Inertial fusion relies on the fuel's own resistance to movement in order to keep it intact while it is very quickly heated to temperatures needed for fusion (millions of degrees Centigrade) and then "burns" via fusion reactions. It is proposed that accelerated heavy-ion beams be used to compress and heat the fuel, in order to reach the required energy levels in a very short period of time. Thus, understanding how to best transport and steer the charged particles towards a target on the scale of a few millimeters is one aspect of the project that the HIF group is currently focusing on.

Since large scale experiments do not yet exist, the ability to feed well defined beam data, taken from small scale experiments, into powerful computer simulations is of great importance. However, it is generally not possible to directly measure a beam's characteristic quantities at all locations of an experiment. This is the problem that is encountered when trying to generate a particle distribution with data gathered from the High Current Experiment (HCX) [2]. Data is available from a pair of "optical slit" diagnostics on HCX that give the 3D projections  $f(x,y,x')$ ,  $f(x,y,y')$ [3]. By integrating these over one coordinate or the other, one can obtain 2D projections such as  $f(x,x')$ ,  $f(y,y')$ , as well as two independent measures of  $f(x,y)$ . These data are available at different longitudinal positions  $z$ , and most naturally in a "mixed" representation since the slit and scintillator for each dataset are not at the same  $z$ . However, the beam needs to be described by the 4D distribution at a common *longitudinal* position (here taken to be the position of the upstream slit) in order to be useful for simulation. Also, due to measurement discretization and errors, the two estimates of  $f(x,y)$  will differ, and need to be reconciled. It is important to note that the problem is underdetermined, and that care must be taken in synthesizing a reasonable distribution that matches the known data with acceptable fidelity while not introducing artifacts into the "projections" about which nothing is known.

This paper will give an overview of updated algorithms for analyzing the two available 3D datasets and for combining them to generate a set of particles in 4D, to be used as the initial conditions for detailed simulation. These algorithms are based on previous work done by the HIF group using an older set of diagnostics that yielded only

the 2D projections  $f(x,x')$ ,  $f(y,y')$ , and  $f(x,y)$  [4]. This paper also describes the software that was created in order to serve this purpose.

## STATEMENT OF PROBLEM

As stated earlier, the HCX diagnostics allow the experimenter to gather data that offers, for each point in the transverse  $(x,y)$  coordinate space, the distribution of beam particles along the third coordinate  $x'$  or  $y'$ . This data in itself has value in characterizing a beam, but is insufficient in describing an initial set of beam conditions to be used in a particle simulation. Therefore the task is to take two experimentally gathered datasets  $f(x,y,x')$  and  $f(x,y,y')$  and coalesce them into (effectively) one 4-dimensional dataset  $f(x,y,x',y')$ , which will describe the phase space density (likelihood of finding a particle in a small volume surrounding a point) as a function of both the  $x$  and  $y$  spatial coordinates of the particle and its momenta in the  $x$  and  $y$  direction.

As mentioned before, the problem of 4D reconstruction is underdetermined. To understand why imagine 20 coordinates in each  $x$ ,  $y$ ,  $x'$  and  $y'$  (in the real case all coordinates do not need to have the same number of points). That means the 4D distribution  $f(x,y,x',y')$  has  $20^4 = 1.6 \times 10^5$  points, while the two 3D arrays of experiment data contains only a total of  $2 \times 20^3 = 1.6 \times 10^4$  points to use in producing the distribution. There is difference by a factor of ten, therefore the reconstruction obviously does not need to be unique. Thus, the goal is to remap the data to a common plane and then use a Monte Carlo method (meaning a non-deterministic method that uses

random numbers) to load particles based on the overlap and intensity of the two datasets.

The initial data is derived from images taken of an excited scintillator as particles are sent through either a thin horizontal or vertical slit; see Figure 2. Using our knowledge of the slit position and the position at which the scintillator lights up we can compute the angle, and thus the transverse momentum, of a particle that passed through that slit position and illuminated that pixel of the scintillator (Figure 3 shows this geometry). If all system elements except for slit orientation were equal in the two datasets, one could simply take the distributions in  $x'$  and  $y'$  at a certain  $(x,y)$  coordinate and merge them together into a “product distribution”  $f(x,y,x',y')$  which assumes no correlation between the  $x'$  and  $y'$  distributions. Indeed, that is our intent, since earlier studies which imposed various ad-hoc constraints on the  $(x',y')$  distribution did not seem to improve the reconstruction for HCX problems. However, the two diagnostic slits are located at different longitudinal positions and there are factors, such as expansion or contraction of the beam, as well as tilt and a non-uniform variation of the mean transverse motion, that affect the “mapping” of the particles from the slit plane to the scintillator plane. Simply overlaying the two “raw” datasets is not sufficient. Thus, the challenge is to remap the data onto a common longitudinal position. If done correctly, the two datasets should now overlay each other more completely than they did in their unprocessed form, and we can now merge the  $x'$  and  $y'$  distributions at each  $(x,y)$  coordinate.



## THE SOFTWARE

### *The Platform*

The software developed to accomplish the aforementioned tasks is implemented in the Java programming language as a plug-in to the ImageJ image-editing and analysis software suite [5]. The advantages of writing the software as an ImageJ plug-in are several. ImageJ, which is written entirely in Java, is freely available and can be run on any operating system that has a modern Java Virtual Machine (Windows, Mac OS X, and Linux for example). Furthermore, ImageJ provides a large code base that gives the programmer of a plug-in module sophisticated and useful code to use, at no cost. The ImageJ community provides further free code and, perhaps more importantly, free advice.

### *Input*

In order to “synthesize” a set of particles, the raw data collected from the HCX experiment must first be read so that it can be analyzed. The data is stored as digital SPE files produced by the camera, which basically contain simple raw 16-bit images, for which the ImageJ community has already provided methods to read [6]. Once the data has been read in, it can be stored as a 3D array, indexed (for example) by image number, x-pixel coordinate and y-pixel coordinate. These arrays can now easily be analyzed and manipulated within the program. Each 3D array, on it's own, can provide raw x, y and x' (or y') data. It is important to note that the format of the data does not directly influence the majority of the software. For each format there must simply be a function that can translate from the given format to the 3D array.

## ***The Algorithm***

Once the two datasets have been loaded as two 3D arrays, through an abstract layer, it can be handled as purely numeric data. The first step of the algorithm is to threshold the data, or in other words remove any unwanted background noise. This is done by simply subtracting a certain value (a percentage of the maximum value, for example) from all the data.

Next both datasets are centered in all x, y and x' (or y') coordinates. This is done by simply finding the “center of mass” for each coordinate (where in this case “mass” denotes pixel intensity) and mapping it to the center of a working grid. After this transformation, the center of mass of the 3D image should be located at (0,0,0).

The final portion of the remapping must take into account the beam “tilt”. Beam tilt relates transverse velocity ( $v_x$  or  $v_y$ ) with transverse position (x or y), or, in other words, the mean rate of contraction or expansion of the beam. The mean tilt of the data gathered from the horizontally-moving slit (hereafter referred to as the “horizontal data”) is defined as  $\langle x x' \rangle / \langle x^2 \rangle$  (where  $\langle \rangle$  denotes the statistical average), and likewise for the vertical slit it is defined as  $\langle y y' \rangle / \langle y^2 \rangle$ . To remap the horizontal dataset, the mean horizontal tilt is removed uniformly from the data by remapping the y coordinate to reflect the linear expansion or contraction. The same method is used to remap the vertical dataset using the uniform vertical tilt, however, there is also an additional adjustment which also takes into account the mean *horizontal* angle as a function of y position. This additional consideration has been shown to produce significantly better remapping (Figure 4 shows the effect of this last adjustment). The asymmetry between

the treatment of the two planes reflects the fact that the separations of the two slits from the scintillator differ, and the fact that the beam is converging along one axis and diverging along the other.

Since the data has been remapped to a common upstream position, the two datasets should overlay each other very closely, and a Monte Carlo method can be used to construct a 4D  $f(x, y, x', y')$  distribution. Particles are randomly loaded into a 4D cell using the value of  $f(x, y, x', y')$  at that location as a probability that the particle will actually be loaded there. As expected, cells with a higher probability will contain more particles.

### ***Output***

The outputs of the software are presented in four main ways. The simplest and most fundamental, is as a plain ASCII file, which simply contains all the data needed to start a downstream simulation. This includes values for  $x$ ,  $y$ ,  $x'$  and  $y'$ , as well as other useful information such as the horizontal and vertical tilt and centers of mass for the different coordinates.

The rest of the output consists of visual plots which can be divided into three categories: plain images, scientific plots and a specialized SlitScanner plot. All of these visual plots are 2D and must of course choose only two of the data's dimensions to plot. The plain images are simple 16-bit (or 32-bit) images that do not allow interactivity and do not provide much quantitative value, but allows the user to quickly gain a qualitative understanding of the data (Figure 2 is an example). The pure images can also of course be easily manipulated using any of ImageJ's large list of built-in features.

For scientific plots that provide labeled axes and interactivity, the National Oceanic and Atmospheric Administration's (NOAA) Scientific Graphics Toolkit Java package was used [7]. This package allows for raster images, point plots and contours (and much more) with axes that automatically scale as the image is zoomed. Figures 4 and 5 are examples that make use of the SGT package. Finally, the SlitScanner plots are actually adaptations of another plug-in created by the HIF group that is meant to analyze specifically a beam's phase space. This view of the data shows the  $f(x,x')$  plot or the  $f(y,y')$  plot as well as numerical moments over the distribution, such as the beam emittance (a measure of phase space extent; a compact phase space, and hence a small emittance, is desired for almost all applications) and mean divergence, and provides specialized filters for the data (Figure 6).

## **DISCUSSION**

Preliminary results have been encouraging; the reconstructed phase-space particle data seems to correspond very well with phase-space plots that were generated purely from experimental data using a different diagnostic. These plots correspond to the same experiment using the same settings. The comparison can be seen in Figure 7. Furthermore, when the reconstructed particles are used as input to simulation, the simulated down-stream particles closely match actual images of the down-stream beam, as shown in Figure 8. Both of these results provide some evidence of a sufficiently accurate reconstruction of the underdetermined 4D distribution.

Building a more accurate 4D particle distribution is an ongoing endeavor. In the future, it might be beneficial to fold in further data, such as a high resolution spatial image of the beam from either the upstream or downstream position (or both). Being able to remap both datasets to a common 4D grid would allow the synthesis of a 4D distribution without the use of a Monte Carlo particle loading section. However, mapping to a common grid without losing data resolution and without making the remapping computationally inefficient is difficult at best. Furthermore, the Synthesis plug-in can be modified in order to analyze data that is not exclusively from the optical slit diagnostic. This more general version of the plug-in may become a general purpose analysis tool for both experimenters and theorists as they further their research toward better understanding the control of intense ion beams.

## **ACKNOWLEDGEMENTS**

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## FIGURES

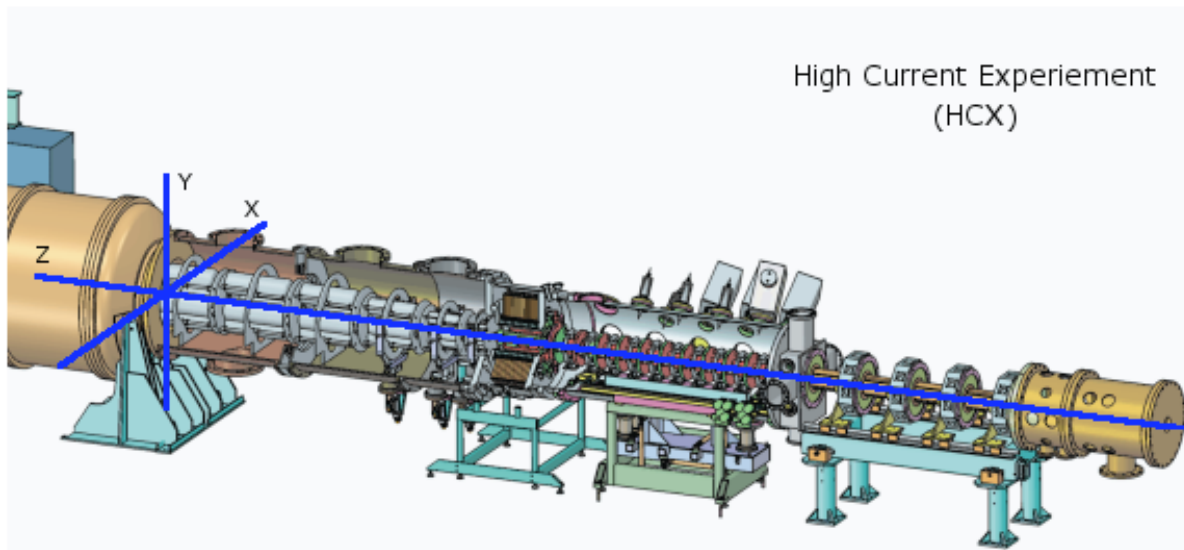


Figure 1. View of the HCX experiment, with labeled coordinate system.

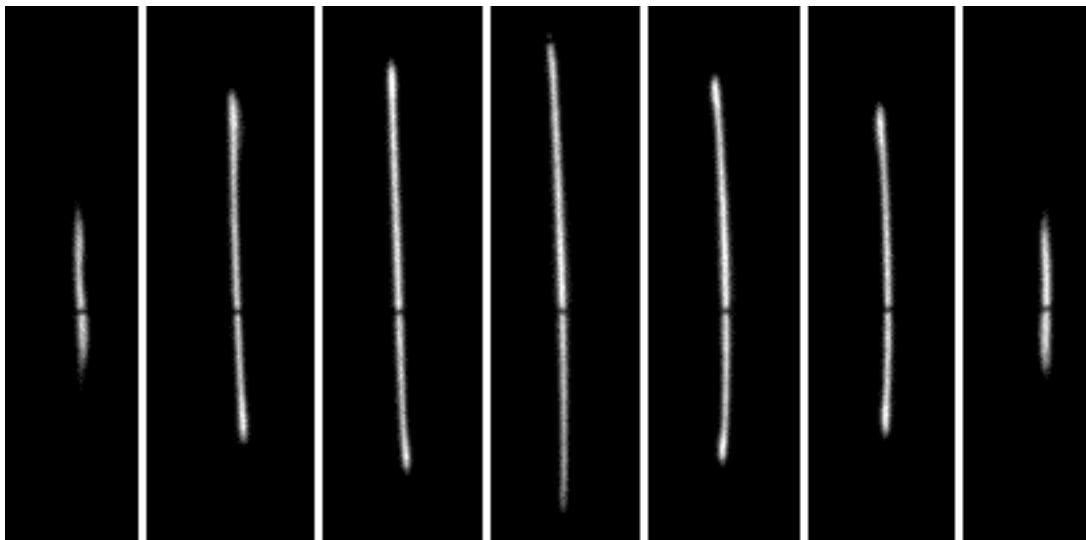


Figure 2. Seven images of the scintillator as the horizontally-moving slit progresses from left to right, over multiple “shots”.

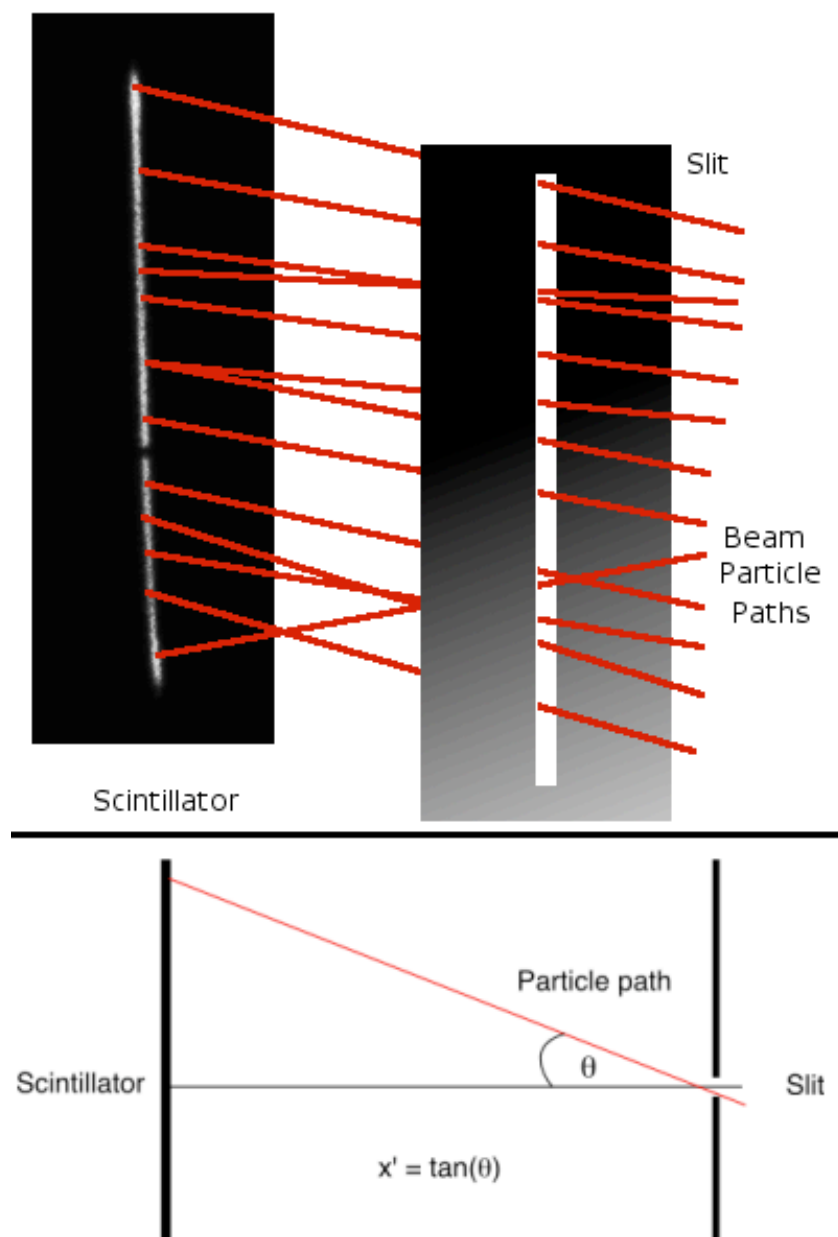


Figure 3. Simple diagram showing the geometry of the optical slit diagnostic. The upper portion shows how a beam may expand or contract as it moves through the horizontal slit. The bottom portion shows a top-down view of the experiment and the angle which we are measuring to find  $x'$  (or  $y'$ ).



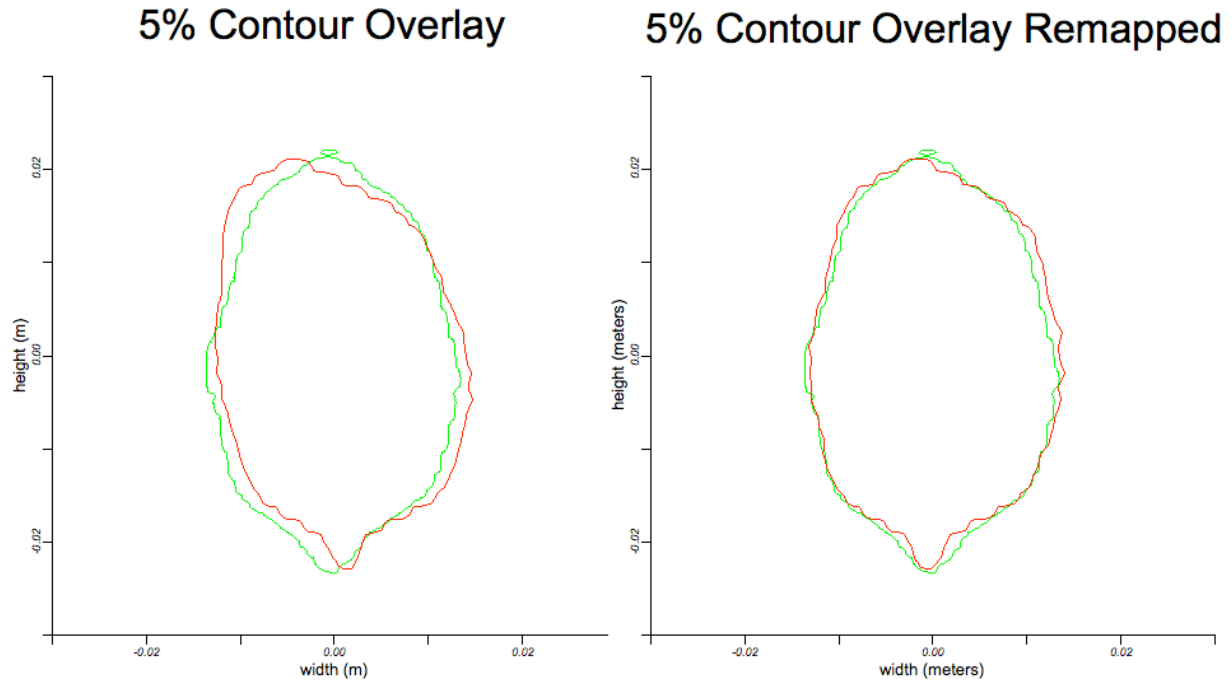


Figure 4. The panel on the left shows the results of simple re-centering and scaling using the mean “tilt”, while the panel on the right shows the effect of removing the variation of the mean  $x'$  with  $y$ , as well. In each case, the green outline represents the horizontal data that is remapped uniformly, while the red outline represents the vertical data that is remapped differently in the two panels.

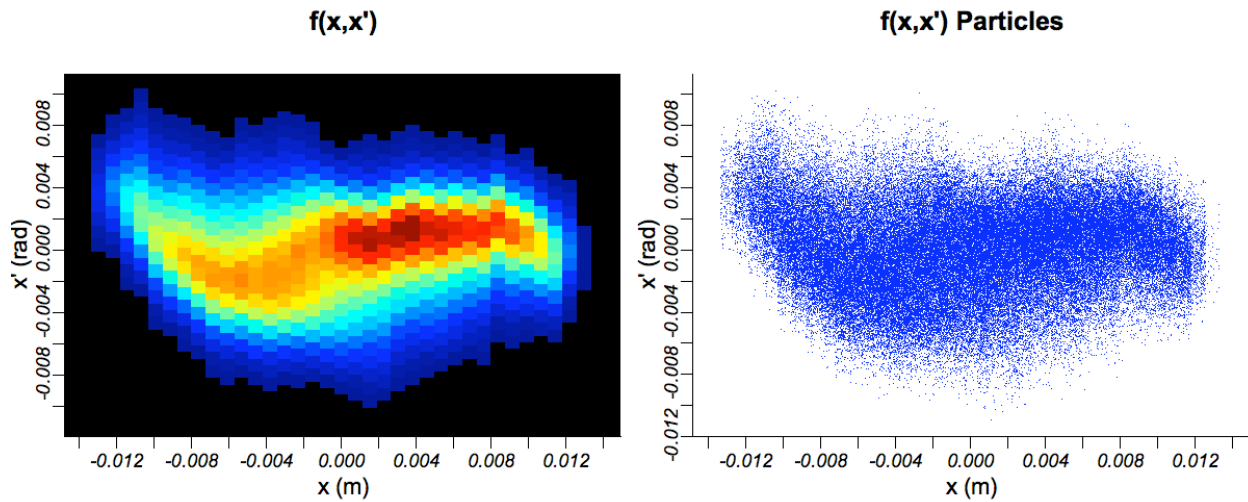


Figure 5. Two examples of SGT plots. Both showing  $f(x,x')$  data, the panel on the left showing a raster image, and the panel on the right showing generated particles.

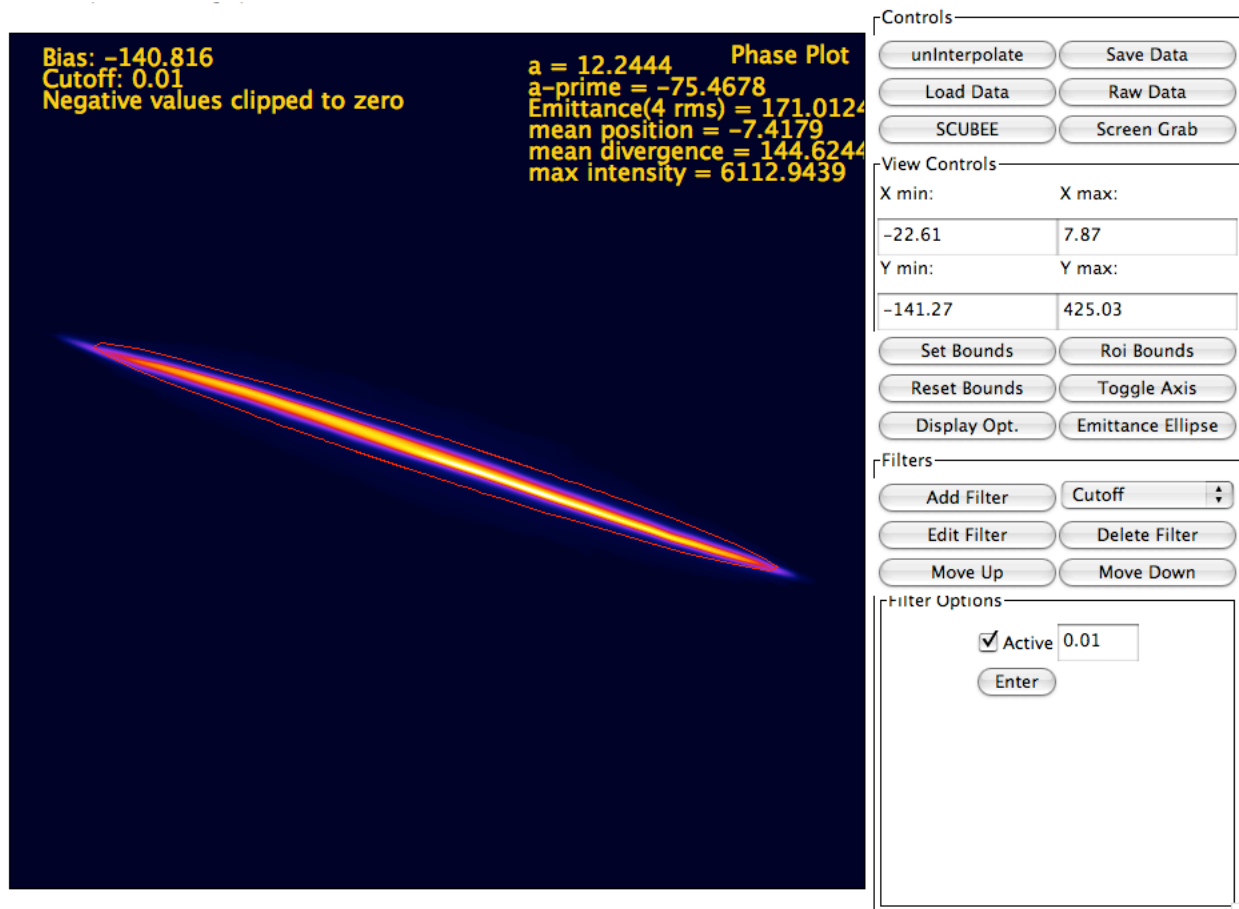


Figure 6. An example of SlitScanner output, showing an  $f(x,x')$  plot as well as the numerical values of beam moments and other quantities.

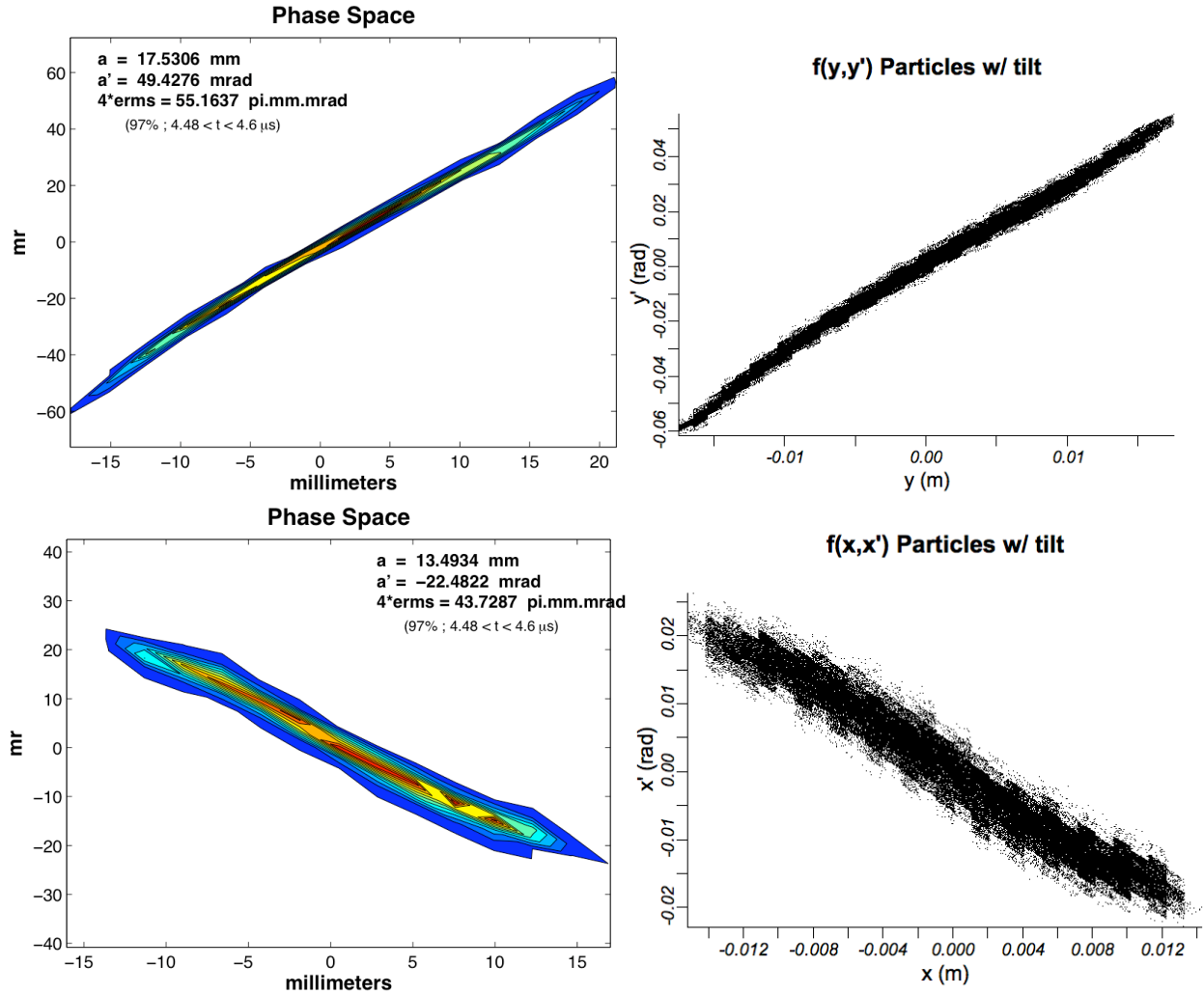
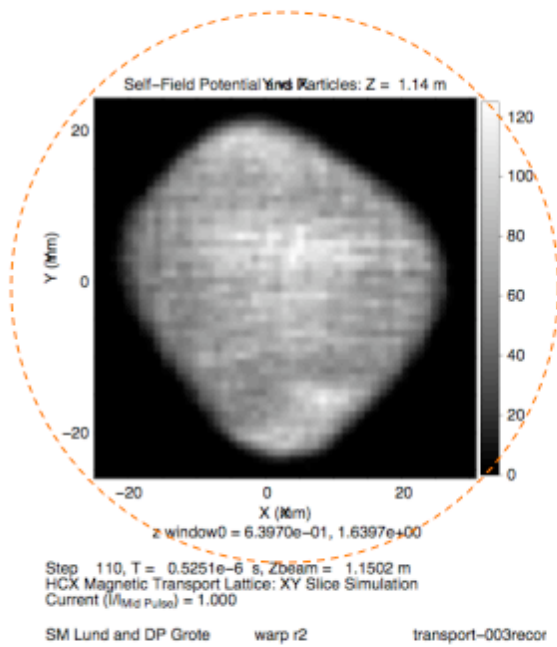
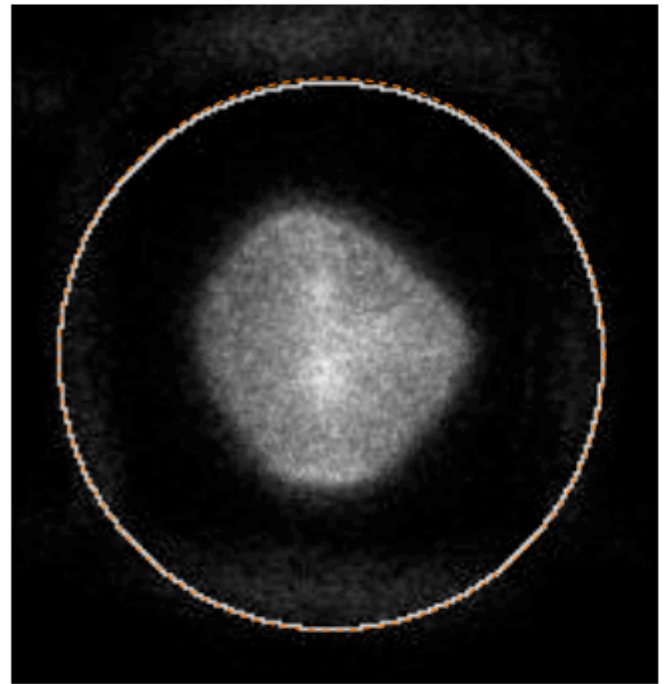


Figure 7. A comparison of synthesized particles (right panel) to phase-space plots (left panel) for the same experiment. The phase-space plots were generated purely from experimental data using a non-optical two-slit diagnostic, while the particles were synthesized using a remapping and Monte Carlo algorithm. The plots agree qualitatively and (to a certain degree) quantitatively.



Simulation Particles initialized  
with Reconstructed Particles



Experiment Scintillator Image

Figure 8. A comparison of simulated downstream particles (initialized with a reconstructed 4D particle distribution) to an image of the actual beam experiment.