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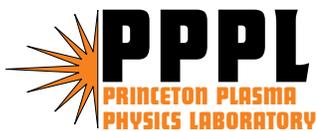
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Ion acceleration in plasmas with Alfvén waves

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Abstract

Effects of elliptically polarized Alfvén waves on thermal ions are inves-
tigated. Both regular oscillations and stochastic motion of the particles
are observed. It is found that during regular oscillations the energy of the
thermal ions can reach magnitudes well exceeding the plasma tempera-
ture, the effect being largest in low-β plasmas (β is the ratio of the plasma
pressure to the magnetic field pressure). Conditions of a low stochasticity
threshold are obtained. It is shown that stochasticity can arise even for
waves propagating along the magnetic field provided that the frequency
spectrum is non-monochromatic. The analysis carried out is based on
equations derived by using a Lagrangian formalism. A code solving these
equations is developed. Steady-state perturbations and perturbations with
the amplitude slowly varying in time are considered.

I. INTRODUCTION

The interaction of Alfvén waves and ions plays an important role in both space
and laboratory plasmas. In particular, the acceleration of the ions by Alfvén waves
is a possible source of the energetic particles in solar-terrestrial environment, various
mechanisms of such an acceleration being proposed, see, e.g., Ref. 1–3. Recently it was
shown that non-linear resonances on sub-harmonics of the ion gyrofrequency, ω_{Bi}, can
lead to the particle stochastic motion and concomitant plasma heating.[4] It was found
that the increase of the transverse energy dominates the increase of the longitudinal
energy. Therefore, it was concluded that the sub-harmonic heating can be responsible for

the fact, that the plasma temperature across the magnetic field exceeds the longitudinal temperature in the lower Solar corona. In addition, this mechanism can contribute to the plasma heating during an instability caused by the injected ions in the National Spherical Torus Experiment (NSTX)[5], in which case, however, fast magnetoacousic waves, rather then Alfvén waves, were generated by the injected ions.[6] Note that non-linear resonances on sub-harmonics of the particle gyrofrequency has been known for as long as 45 years.[7] However, it was not known till 2001[4] that they can lead to stochasticity of the particle motion.

In this work, we consider the particle motion influenced by Alfvén waves with the frequencies comparable to the ion cyclotron frequency, ω_{Bi} (the subscript "i" will be omitted below when it will not lead to misunderstanding). For these waves, the mentioned non-linear resonances are especially important. On the other hand, the dispersion relation of these waves considerably differs from the linear one (it is known that $\omega = k_\parallel v_A$, where $k_\parallel$ is the longitudinal wave number and v_A is the Alfvén velocity, only for $\omega \ll \omega_B$). Furthermore, for finite ratio of the wave frequency to the particle gyrofrequency the waves are essentially non-linearly polarized. These facts were disregarded in previous studies although they affect the particle motion and the stochasticity threshold. They are taken into account in the present work.

The structure of the work is as follows. In Sec. II, using Lagrangian formalism, we derive equations of the particle motion affected by Alfvén waves. The structure of the derived equations considerably differs from the structure of the previously studied equations. In Sec. III, we solve the obtained equations numerically. We build Poincare maps for the case of a monochromatic wave and investigate temporal evolution for the case when several waves are present. In addition to the previously studied case of a steady-state perturbation, we study the particle behavior in the presence of the waves with slowly varying amplitudes. In Sec. IV we summarize the results obtained. In the Appendix the features of Alfvén waves with finite ω/ω_B are considered.

II. BASIC EQUATIONS AND NON-LINEAR RESONANCES

We begin with a derivation of equations of the particle motion in the electromagnetic field. With this purpose, we use a Lagrange equation, with the Langrangean

$$L = \frac{m\dot{r}_j^2}{2} + \frac{e}{c}\dot{r}_j A_j - e\Phi(\mathbf{r}), \tag{1}$$

where $\mathbf{r} = (x, y, z)$, $\mathbf{A}$ is the vector potential of the electromagnetic field, Φ is the field

scalar potential, two repeating subscripts (j) in a product mean the summation, $\dot{r}_j^2 = \dot{\mathbf{r}} \cdot \dot{\mathbf{r}}$, and $\dot{r}_j = dr_j/dt$. The Lagrange equation with L given by Eq. (1) can be written as

$$m\ddot{r}_i + \frac{e}{c}\dot{A}_i = \frac{e}{c}\dot{r}_j\frac{\partial A_j}{\partial r_i} - e\frac{\partial \Phi}{\partial r_i}. \tag{2}$$

We assume that the electromagnetic field consists of an equilibrium homogeneous straight magnetic field, $\mathbf{B}_0 = (0, 0, B_0)$ and waves propagating in the (x, z) plane. In this case $\mathbf{A}_0 = (-B_0 y, 0, 0)$ and $\Phi_0 = 0$, where the subscript "0" denotes the equilibrium (unperturbed) quantities and the waves are described by the perturbed vector potential $\widetilde{\mathbf{A}}$ and scalar potential $\widetilde{\Phi}$. We take a perturbed quantity, $\widetilde{X}$, in the form $\widetilde{X} = \sum_k X_k = \sum_k \hat{X}_k exp(i\Psi_k)$ with $\Psi_k = k_x x + k_z z - \omega_k t$, where $\mathbf{k}$ is the wave vector and ω_k is the corresponding wave frequency. Then Eq. (2) yields:

$$m\ddot{x} - \frac{eB_0}{c}\dot{y} + \frac{e}{c}\dot{\widetilde{A}}_x = \sum_{\mathbf{k}}\frac{e}{c}ik_x(\dot{x}A_{kx} + \dot{y}A_{ky}$$
$$+\dot{z}A_{kz}) - \sum_{\mathbf{k}}eik_x\Phi_k, \tag{3}$$

$$m\ddot{y} + \frac{e}{c}\dot{\widetilde{A}}_y = -\frac{eB_0}{c}\dot{x}, \tag{4}$$

$$m\ddot{z} + \frac{e}{c}\dot{\widetilde{A}}_z = \sum_{\mathbf{k}}\frac{e}{c}ik_z(\dot{x}A_{kx} + \dot{y}A_{ky} + \dot{z}A_{kz}) - \sum_{\mathbf{k}}eik_z\Phi_k. \tag{5}$$

Alfvén waves are known to be linearly polarized, $\widetilde{\mathbf{E}} = (\widetilde{E}_x, 0, 0)$ for $\widetilde{B}_z = 0$, $k_y = 0$, in the ideal MHD approximation. Vanishing $\widetilde{B}_z$ impies that the terms of the order ω/ω_B are neglected in the dispersion relation and polarization. For these waves one can take $\widetilde{\mathbf{A}}_\perp = 0$ and use the equation

$$\widetilde{E}_z = -\frac{1}{c}\frac{\partial \widetilde{A}_z}{\partial t} - \nabla_z\widetilde{\Phi} = 0$$

to connect $\widetilde{A}_z$ and $\widetilde{\Phi}$. On the other hand, when $\widetilde{\mathbf{A}}_\perp = 0$, Eqs. (3)-(5) lead to Eq. (12) of Ref. 4. However, we are interested in waves with finite ω/ω_B. For these waves, $\widetilde{B}_\parallel \neq 0$. Therefore, only one component of the perturbed vector potential (or $\widetilde{\Phi}$) is arbitrary. We use the gauge $\widetilde{A}_x = 0$ in order to describe elliptically polarized Alfvén waves. The longitudinal component of the electric field of these waves is vanishing in ideal MHD. In kinetic MHD, it is small, $E_{kz}/E_{kx} \sim \beta\omega^2/\omega_B^2$ for $k_\perp \sim k_\parallel$ (see Appendix), where β is the ratio of the plasma pressure to the magnetic field pressure, and will be neglected in our analysis.

Let us express all the perturbed quantities through the y component of the wave magnetic field. This can be done by using the following equations (see Appendix):

$A_{ky} = -\alpha_k B_{ky}/k_z$, with $\alpha_k = iE_{ky}/E_{kx}$, $A_{kz} = -B_{ky}/(ik_x)$, $\Phi_k = i\omega B_{ky}/(ck_z k_x)$. Then integrating Eq. (4) once and taking into account that $\dot{A}_{kz} = i\dot{\psi}_k A_{kz} = i(k_x\dot{x}+k_z\dot{z}-\omega)A_{kz}$ we obtain:

$$\frac{d^2\hat{x}}{d\tau^2} + \hat{x} - \hat{x}_0 - \left.\frac{d\hat{y}}{d\tau}\right|_{\tau_0} = \sum_k \left[\frac{\hat{\omega}+\alpha_k}{k_z\rho_*} - \frac{d\hat{z}}{d\tau}\right]\hat{b}_k \sin\Psi_k$$

$$+ \left\{\hat{x} - \hat{x}_0 - \left.\frac{d\hat{y}}{d\tau}\right|_{\tau_0} - \sum_k \frac{\alpha_k\hat{b}_k}{k_z\rho_*}(\sin\Psi_k - \sin\Psi_{k0})\right\}$$

$$\times \sum_k \frac{k_x}{k_z}\alpha_k\hat{b}_k \cos\Psi_k - \sum_k \frac{\alpha_k}{k_z\rho_*}\hat{b}_k \sin\Psi_{k0}, \tag{6}$$

$$\frac{d\hat{y}}{d\tau} \equiv \frac{v_y}{v_*} = \hat{x}_0 - \hat{x} + \sum_k \frac{\alpha_k}{k_z\rho_*}\hat{b}_k(\sin\Psi_k - \sin\Psi_{k0}) + \left.\frac{d\hat{y}}{d\tau}\right|_{\tau_0}, \tag{7}$$

$$\frac{d^2\hat{z}}{d\tau^2} \equiv \frac{1}{v_*}\frac{dv_z}{d\tau} = \frac{d\hat{x}}{d\tau}\sum_k \hat{b}_k \sin\Psi_k - \frac{d\hat{y}}{d\tau}\sum_k \alpha_k\hat{b}_k \cos\Psi_k. \tag{8}$$

Here

$$k_\parallel^2\rho_*^2 = \frac{\hat{\omega}_k^2}{(1-\hat{\omega}_k^2)}\frac{N_\epsilon^2}{\hat{v}_A^2}, \tag{9}$$

$$N_\epsilon^2 = \frac{1 + k_\perp^2/(2k_z^2) + \sqrt{D}}{1 + k_\perp^2/k_z^2} \quad \text{with} \quad D = \frac{k_\perp^4}{4k_z^4} + \hat{\omega}_k^2\left(1 + \frac{k_\perp^2}{k_z^2}\right), \tag{10}$$

$$\alpha_k = i\frac{E_{ky}}{E_{kx}} = -\frac{\hat{\omega}_k}{1 - N_\epsilon^2(1 + k_\perp^2/k_z^2)}, \tag{11}$$

the normalized quantities are introduced: $\hat{x} = x/\rho_*$, $\hat{y} = y/\rho_*$, $\hat{z} = z/\rho_*$, $\hat{\omega}_k = \omega_k/\omega_B$, $\rho_* = v_*/\omega_B$ is a characteristic Larmor radius, v_* is a characteristic particle velocity, $\tau = \omega_B t$, $\hat{v}_A = v_A/v_*$, $\hat{b}_k = \hat{B}_{ky}/B_0$, $\Psi_k = k_x\rho_*\hat{x} + k_z\rho_*\hat{z} - \hat{\omega}_k\tau$. Three last equations determine the Alfvén wave dispersion relation and polarization shown in Fig. 1.

These equations differ from the corresponding equations of Refs. 4, 8, where the same problem was studied. In particular, it was assumed in Ref. 8 that $\omega = k_\parallel v_A$, which is not true for $\hat{\omega} \gtrsim 1/2$, see Fig. 1. The derived equations differs from those of Ref. 4 not only by the presence of additional terms proportional to the wave amplitude but also by the presence of a term proportional to the square of the wave amplitude. The latter term affects the wave-particle interaction. To see it we linearize Eqs. (6)-(8) by following the procedure of Ref. 4. First of all, we note that due to the fact that $v_z \approx const$, the phase of the wave-particle interaction can be written as $\Psi_k \equiv k_x x + k_z z - \omega_k t \approx s + 2\hat{t}$, with

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$s = k_x(x - x_0)$, $2\hat{t} = k_x x_0 + k_z z_0 - \omega'_k t$, $\omega'_k = \omega_k - k_z v_z$. Then assuming $s \ll 1$ we obtain an equation of the following type:

$$\frac{d^2 s}{d\hat{t}^2} + \left[a + \epsilon \exp(2i\hat{t}) \right] s = p\epsilon \exp(2i\hat{t}) + q\epsilon^2 \exp(4i\hat{t}), \tag{12}$$

where $\epsilon \propto \hat{b}_k$, a, p, q are constant coefficients. We solve Eq. (12) perturbatively by using a small parameter ϵ. Writing $s = \sum_i s_i$, with $s_i \propto \epsilon^i$ and $i = 0, 1, 2...N..$, we obtain the resonances $a_N^{(1)} = N^2$, which corresponds to the Mathieu equation resonances[4] and, in addition, the resonances $a_N^{(2)} = 2N^2$. Taking into account that $a = 4\omega_B^2/\omega'^2$ we have the following resonance frequencies: $\omega'^{(1)}/\omega_B = 2/N$ (Mathieu resonances) and $\omega'^{(2)}/\omega_B = 1/N$. We conclude that the presence of the quadratic term provides additional resonances for given N. This term, however, does not produce new resonant frequencies because $\omega'^{(1)}$ for even N coincides with $\omega'^{(2)}$.

When $k_\perp = 0$, Eq. (6) is reduced to

$$\ddot{x} + \omega_B^2 x = \epsilon \exp\left[-i \int (\omega - k_z v_z) dt \right]. \tag{13}$$

Equation (13) contains only Cherenkov resonance, $\omega = k_z v_z$, which selects the resonant particles with a special magnitude of the longitudinal velocity $v_\parallel/v_* = \hat{v}_A \sqrt{1 - \hat{\omega}}$. For this reason, the waves propagating along the magnetic field do not lead to stochasticity of the particle motion.[4] However, this conclusion is relevant only to monochromatic waves. The situation changes in the presence of non-monochromatic waves. To see it we write the following equation obtained from Eqs. (3)-(5) for the case of $k_\perp = 0$, $x_0 = z_0 = 0$:

$$\frac{dv_\perp^2}{d\tau} = -2iv_y \sum_k \left(\frac{\omega}{k_z} - v_z \right) \hat{b}_k \exp(i\Psi_k), \tag{14}$$

where $v_\perp = \sqrt{v_x^2 + v_y^2}$. Taking into account that v_y can be written as $v_y = v_\perp \cos\tau + v_x^{(1)}$ with $v_y^{(1)} \propto \sum_k \hat{b}_k \exp(i\Psi_k)$ we can conclude that when there are two waves with the frequencies ω_1 and ω_2 and the longitudinal wave numbers k_1 and k_2, a particle can interact with the waves through the non-linear resonance

$$\omega_2 - \omega_1 = (k_2 \pm k_1)v_\parallel^{res}, \tag{15}$$

where $k_{1,2}\rho_* \hat{v}_A = \pm\hat{\omega}_{1,2}/\sqrt{1 - \omega_{1,2}}$ according to Eq. (A11). Depending on ω_2/ω_1, this resonance can provide a wave-particle interaction for various magnitudes of the ratio $v_\parallel^{res}/v_A$, see Fig. 2. When there are many waves with the frequencies close to each other, the particle motion can become stochastic. Our calculations in the next section confirm this possibility. It is clear that these resonances exist for any $k_\perp$ rather than for $k_\perp = 0$ only and can decrease the stochasticity threshold when $k_\perp \neq 0$.

III. ANALYSIS OF THE PARTICLE MOTION

Below we investigate the particle motion by solving numerically Eqs. (6)-(8). The parameters of the mentioned equations are the wave amplitudes ($\hat{b}_k$), the normalized wave frequencies ($\hat{\omega}_k$), the direction of the wave propagation ($k_\perp/k_\parallel$), the normalized Alfvén velocity ($\hat{v}_A$), and the wave spectrum. Alfvén velocity is normalized to v_*. We take $v_* = v_{Ti}$, where $v_{Ti} = \sqrt{2T_i/M_i}$ and T_i is the ion temperature, in which case $v/v_* = 1$ for the thermal particles, whereas $v/v_* \gg 1$ for the superthermal particles.

Let us consider first the ion motion in the presence of a monochromatic wave. In this case, a Poincaré map can be used to show the character of the particle motion. We build Poincaré sections of $v_\perp^2/v_*^2$, ψ_k and $v_\parallel^2/v_*^2$, ψ_k, by taking points where $v_y = 0$, $\dot{v}_y > 0$ during the particle Larmor rotation. Note that, in contrast to the case of the linearly polarized waves, the x-coordinate is not conserved after a cyclotron period, i.e., in general, $x_{j+1} \neq x_j \neq x_0$ with j the integer. This follows from Eq. (7).

We take $\hat{\omega} = 1/2$, $k_\perp = k_\parallel$, $\hat{v}_A = 10$ (this corresponds to $\beta_i \equiv 8\pi p_i/B^2 = 1\%$) and vary the wave amplitude in order to observe the transition from regular motion to the stochastic motion. The results for the elliptically polarized waves [with α given by Eq. (11)] are shown in Figs. 3, 4. We observe that the particle motion is regular for $\hat{b} = 0.1$, whereas it becomes stochastic for $\hat{b}_k = 0.2$.

This result implies that when $\hat{v}_A = 10$, plasma heating by a monochromatic wave with $\hat{\omega} = 1/2$ and $k_\perp = k_\parallel$ is possible provided that $\hat{b}_k$ exceeds 0.1. To demonstrate this effect, we consider the same wave but assume that its amplitude slowly varies in time as

$$\hat{b}(t) = \hat{b}_0 \exp\left[-\frac{(t - 3t_*)^2}{t_*^2}\right], \tag{16}$$

where $t_* \gg \omega^{-1}$, i.e., the perturbation is slowly switched on and then, after its amplitude reached a maximum, it is slowly switched off. Note that although Eqs. (6)-(8) are derived for the steady-state perturbation amplitude, they are valid also for sufficiently slowly varied amplitudes. Under regular motion ($\hat{b}_0 = 0.1$) particle energy is conserved by the perturbation because the process is adiabatic, see Fig. 5a. In contrast to this chaotic motion changes the energy irreversibly, as shown in Fig. 5b for $\hat{b}_0 = 0.2$.

One can expect that the stochasticity will be possible for lower wave amplitudes when the wave spectrum is non-monochromatic, in which case the resonance given by Eq. (15) will work. Let us see how strongly the stochasticity threshold can be reduced. Because different waves have different Ψ_k, we investigate the temporal evolution of the particle energy instead of building Poincaré maps. We decrease wave amplitudes by a factor of

ten in comparison to the amplitude considered in Fig. 5. We find that adding one wave with $\hat{\omega} = 2/3$ is sufficient to produce stochasticity, but the particle energy increases by less than by a factor of two for the considered time $t_* = 100 t_B$, with $t_B = 2\pi/\omega_B$ (the wave amplitude is about its maximum value for $t_* \sim 300 t_B$), Fig. 6a. Adding waves with $1/2 < \omega < 2/3$ strongly enhances the effect. Figure 6b shows that the energy of a thermal particle increases by a factor of forty when there are many waves in the considered frequency interval, i.e., when $\Delta\omega = \omega_{n+1} - \omega_n$, with n integer, is very small. On the other hand, calculations show that the heating rate does not necessarily grow with the number of waves. The rate grows provided that the increase of the number of waves is accompanied by the decrease of $\Delta\omega$.

The elliptically polarized wave can be considered as a superposition of two linearly polarized waves. Therefore, one can expect that the stochasticity threshold will be higher for the linearly polarized waves. The calculation confirms this, see Fig. 7.

A peculiarity of the regular motion is that the particle energy can strongly oscillate. According to Fig. 2, the transverse energy of a thermal particle reaches the magnitude $\mathcal{E}_\perp \gtrsim 10T$ during its motion. The maximum energy should grow with $\hat{v}_A$ because the term proportional to the wave amplitude in Eq. (6) is proportional to $\hat{v}_A$, $[1/k_\parallel \propto v_A$, see Eq. (9)]. On the other hand, taking larger $\hat{v}_A$ we decrease $k_\perp$ for given $\hat{\omega}$ and $k_\perp = k_\parallel$. This fact in unfavorable for stochasticity. For instance, let us consider a monochromatic wave with $\hat{b} = 0.2$, $k_\perp = k_\parallel$ and take $\hat{v}_A = 50$. We obtain a picture with the regular motion (in contrast to the results shown in Figs. 4, 5b for $\hat{b}_k = 0.2$) and a very strong enlargement, by a factor of 250, of the wave amplitude.

IV. SUMMARY AND CONCLUSIONS

In conclusion, we have derived equations of the particle motion in the presence of elliptically polarized Alfvén waves. The equations are valid for arbitrary frequencies of Alfvén waves, which is of importance because non-linear resonances are most efficient when ω is comparable to ω_{Bi}. It is found that, in addition to the Mathieu resonances $\hat{\omega} = 2/N$ determining the interaction of the ions and linearly polarized waves, there exist the resonances $\hat{\omega} = 1/N$ when the waves are elliptically polarized. Although this does not lead to new resonance frequencies as compared to Mathieu resonances, it provides some additional interaction leading to additional frequencies in a given order of the perturbation theory (i.e., for given N).

Solving the obtained equations numerically we obtained the following results. First,

we showed that the stochasticity threshold is lower by a factor of two in comparison to the case of the linearly polarized weaves. Second, we confirmed the result of Ref.[9] that the stochasticity threshold can be strongly reduced (by several orders of magnitude) in the presence of many waves, ω_n with $n = 1, 2, 3...n_{max} \gg 1$. However, we found that this will be the case only when $\Delta\omega \equiv |\omega_{n+1} - \omega_n| \ll \omega_n$, the heating efficiency growing with $(\Delta\omega)^{-1}$. Third, we found that the oscillation of the particle energy when the wave amplitude is below threshold value can be significant, especially in low-β plasmas. In particular, in the considered case of $\hat{b}_k = 0.2$ the amplitude of the energy oscillations of thermal ions reached $\mathcal{E}_\perp^{max}/T = 300$ for $\hat{v}_A = 50$ ($\beta_i = 0.04\%$).

APPENDIX A: FEATURES OF ALFVÉN WAVES WITH FINITE ω/ω_{BI}

The dispersion relation and polarization of Alfvén waves are well known, but typically the approximation $\omega \ll \omega_{Bi}$ is used. Below we consider Alfvén waves without making this approximation. We consider the waves in the frame with $k_y = 0$ assuming that the following conditions are satisfied:

$$k_\perp\rho_i \ll 1, \quad k_\parallel\rho_i \ll 1, \quad \omega \gg k_\parallel v_{Ti}, \tag{A1}$$

where $\rho_i = v_{Ti}/\omega_{Bi}$, $v_{Ti} = \sqrt{2T_i/M_i}$. Eliminating the magnetic field from the equations $\omega\mathbf{B} = c[\mathbf{k} \times \mathbf{E}]$ and $c[\mathbf{k} \times \mathbf{B}]_\alpha = -\omega\epsilon_{\alpha\beta}E_\beta$, where $\epsilon_{\alpha\beta}$ the dielectric permeability tensor, and taking into account that $\epsilon_{xz} \approx 0$ due to Eq. (A1) we have:

$$\left(\epsilon_{xx} - \frac{k_z^2 c^2}{\omega^2}\right)E_{kx} + \epsilon_{xy}E_{ky} + \frac{k_z k_x}{\omega^2}c^2 E_{kz} = 0, \tag{A2}$$

$$-\epsilon_{xy}E_{kx} + \left(\epsilon_{yy} - \frac{k^2 c^2}{\omega^2}\right)E_{ky} + \epsilon_{yz}E_{kz} = 0, \tag{A3}$$

$$\frac{k_x k_z}{\omega^2}c^2 E_{kx} - \epsilon_{yz}E_{ky} + \left(\epsilon_{zz} - \frac{k_x^2 c^2}{\omega^2}\right)E_{kz} = 0. \tag{A4}$$

Here the components of $\epsilon_{\alpha\beta}$ are as follows:

$$\epsilon_{xx} = -\frac{\omega_{pi}^2}{\omega^2 - \omega_{Bi}^2}, \quad \epsilon_{xy} = -\epsilon_{yx} \approx i\frac{\omega}{\omega_{Bi}}\epsilon_{xx}, \quad \epsilon_{xz} = \epsilon_{zx} = O(k_\perp^4 \rho_i^4),$$

$$\epsilon_{yy} \approx \epsilon_{xx} \quad \text{for} \quad k_\perp^2 \rho_i^2 \ll \omega^2/\omega_{Bi}^2, \tag{A5}$$

$$\epsilon_{yz} = -\epsilon_{zy} = \begin{cases} i\frac{\omega_{pi}^2}{\omega\omega_{Bi}}\frac{k_x}{k_z}\frac{Z_i}{2} & \text{for} \quad \omega \gg k_z v_{Te} \\ i\frac{\omega_{pi}^2}{\omega\omega_{Bi}}\frac{k_x}{k_z} & \text{for} \quad \omega \ll k_z v_{Te} \end{cases} \quad \epsilon_{zz} = \begin{cases} -\frac{\omega_{pe}^2}{\omega^2} & \text{for} \quad \omega \gg k_z v_{Te} \\ \frac{2\omega_{pe}^2}{k_z^2 v_{Te}^2}(1 + i\sqrt{\pi}\frac{\omega}{k_z v_{Te}}) & \text{for} \quad \omega \ll k_z v_{Te} \end{cases}$$

where Z_i is the ion charge number.

Using Eq. (A5) and combining Eqs. (A3), (A4) we obtain:

$$\frac{E_{kz}}{E_{kx}} = -\frac{k_x k_z v_{Ts}^2}{2\omega_{Bi}^2}\left[\frac{k_z^2 v_A^2}{\omega^2} + \frac{1}{1 - (k^2 v_A^2/\omega^2)(1 - \omega^2/\omega_{Bi}^2)}\right], \tag{A6}$$

where $v_{Ts} = \sqrt{2T_e/M_i}$. Due to Eq. (A1), $E_{kz}/E_{kx} \ll 1$. The terms propotional to E_{kz} in Eqs. (A2), (A3) are small provided that $k_z^2 k_x^2 \rho_i^4 \ll \beta_i$. Assuming that this is the case we obtain from Eqs. (A2), (A3) the ratio of E_{ky}/E_{kx} and the dispersion relation as follows:

$$\frac{E_{ky}}{E_{kx}} = \frac{\epsilon_{xy}}{\epsilon_{yy} - c^2 k^2/\omega^2}, \tag{A7}$$

$$(1 - N_\epsilon^2)\left[1 - N_\epsilon^2\left(1 + \frac{k_\perp^2}{k_\parallel^2}\right)\right] - \frac{\omega^2}{\omega_{Bi}^2} = 0, \tag{A8}$$

where N_ϵ is defined as

$$N_\epsilon^2 \equiv c^2 k_\parallel^2/(\omega^2 \epsilon_{xx}). \tag{A9}$$

The last equation leads to Eq. (9). Equations (A7), (A8) lead to Eqs. (11), (10), respectively. Note that if we select another root of Eq. (A8) we would obtain the dispersion relation of the fast magnetoacoustc waves [instead of Eq. (10)]. In the limited case of $\omega/\omega_B \ll 1$, Eqs. (A6)-(A9) yield:

$$\omega = k_\parallel v_A, \quad \frac{E_{ky}}{E_{kx}} = -i\frac{\hat{\omega} k_z^2}{k_\perp^2}, \quad \frac{E_{kz}}{E_{kx}} = -\frac{k_x k_z v_{Ts}^2}{2\omega_{Bi}^2}\left[1 - \frac{1}{k_\perp^2/k_z^2 - k^2 v_A^2/\omega_{Bi}^2}\right]. \tag{A10}$$

For the waves propagating along the magnetic field we obtain:

$$k_\parallel^2 v_A^2 = \frac{\omega^2}{1 - \omega/\omega_{Bi}}, \quad \frac{E_{ky}}{E_{kx}} = i, \quad E_{kz} = 0. \tag{A11}$$

Because the equations of the particle motion [Eqs. (6)-(8)] are written in terms of the vector potential $\widetilde{\mathbf{A}}$ and scalar potential $\widetilde{\Phi}$, let us express $\widetilde{\mathbf{A}}$ and $\widetilde{\Phi}$ through a component

of the perturbed magnetic field. Using x, y- components of the equations $\mathbf{B} = i\mathbf{k} \times \widetilde{\mathbf{A}}$ and $c\mathbf{k} \times \widetilde{\mathbf{E}} = \omega \widetilde{\mathbf{B}}$ and taking $i\widetilde{E}_y/\widetilde{E}_x \equiv \alpha_k$, we find:

$$\widetilde{A}_y = -\frac{\alpha}{k_z}\widetilde{B}_y, \quad \widetilde{A}_z = -\frac{1}{ik_x}\widetilde{B}_y. \tag{A12}$$

Then taking into account that $\widetilde{E}_z \approx 0$ and $\widetilde{E}_z = -ik_z\widetilde{\Phi} + i\omega\widetilde{A}_z$ and using Eq. (A12) we obtain

$$\widetilde{\Phi} = \frac{i\omega}{k_z k_x}\widetilde{B}_y. \tag{A13}$$

Other useful relationships are

$$\widetilde{B}_x = -\frac{ck_z}{\omega}\widetilde{E}_y, \quad \widetilde{B}_y = \frac{ck_z}{\omega}\widetilde{E}_x, \quad \widetilde{B}_z = \frac{ck_x}{\omega}\widetilde{E}_y. \tag{A14}$$

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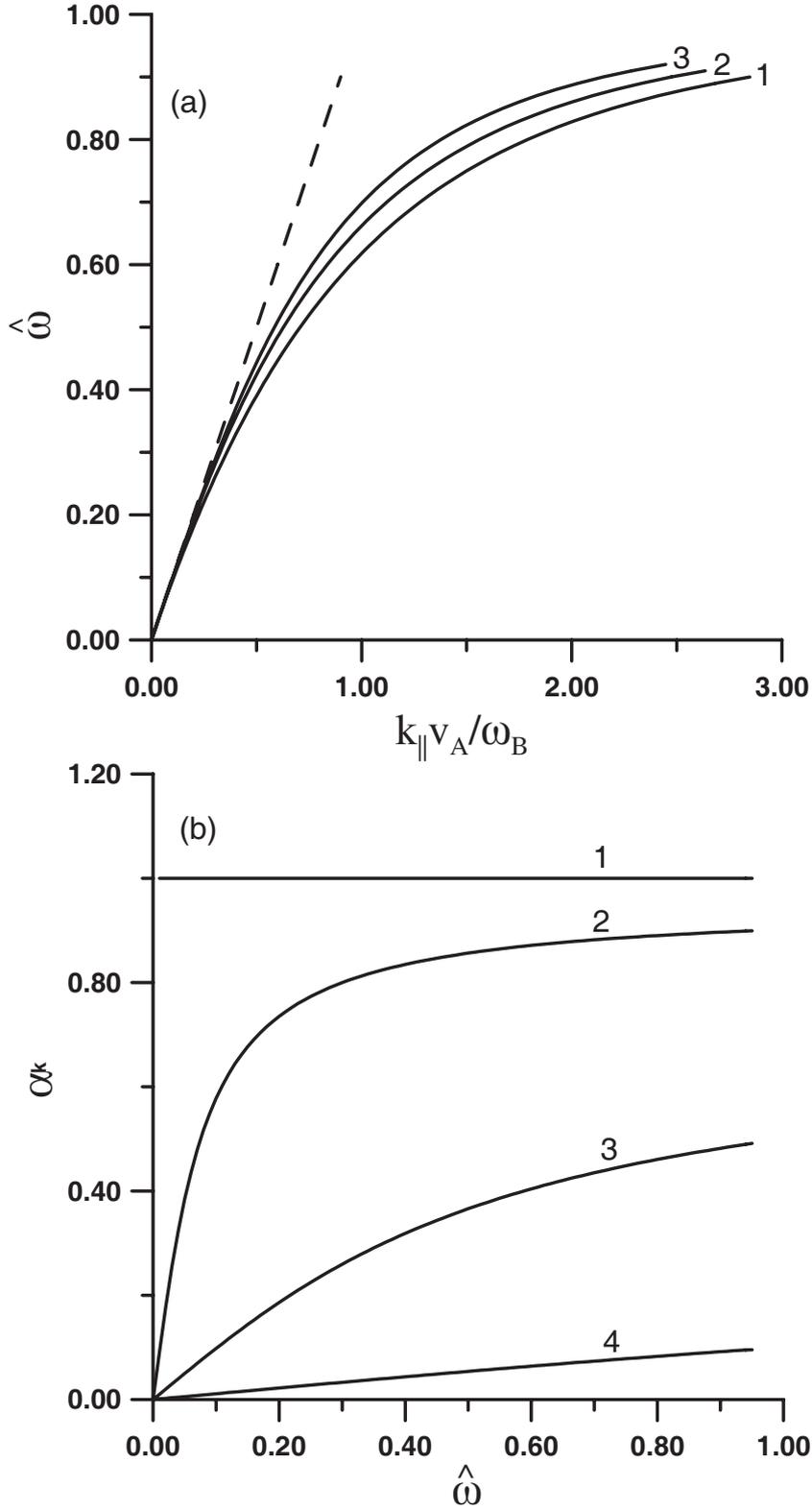
FIG. 1. Effect of the finite ratio of ω/ω_B on features of Alfvén waves: a, wave dispersion for $k_\perp/k_\parallel = 0, 1, 3$ (curves 1, 2, 3, respectively; the dotted line corresponds to linear dispersion); b, wave polarization for $k_\perp/k_\parallel = 0, 1/3, 1, 3$ (curves 1, 2, 3, 4, respectively).
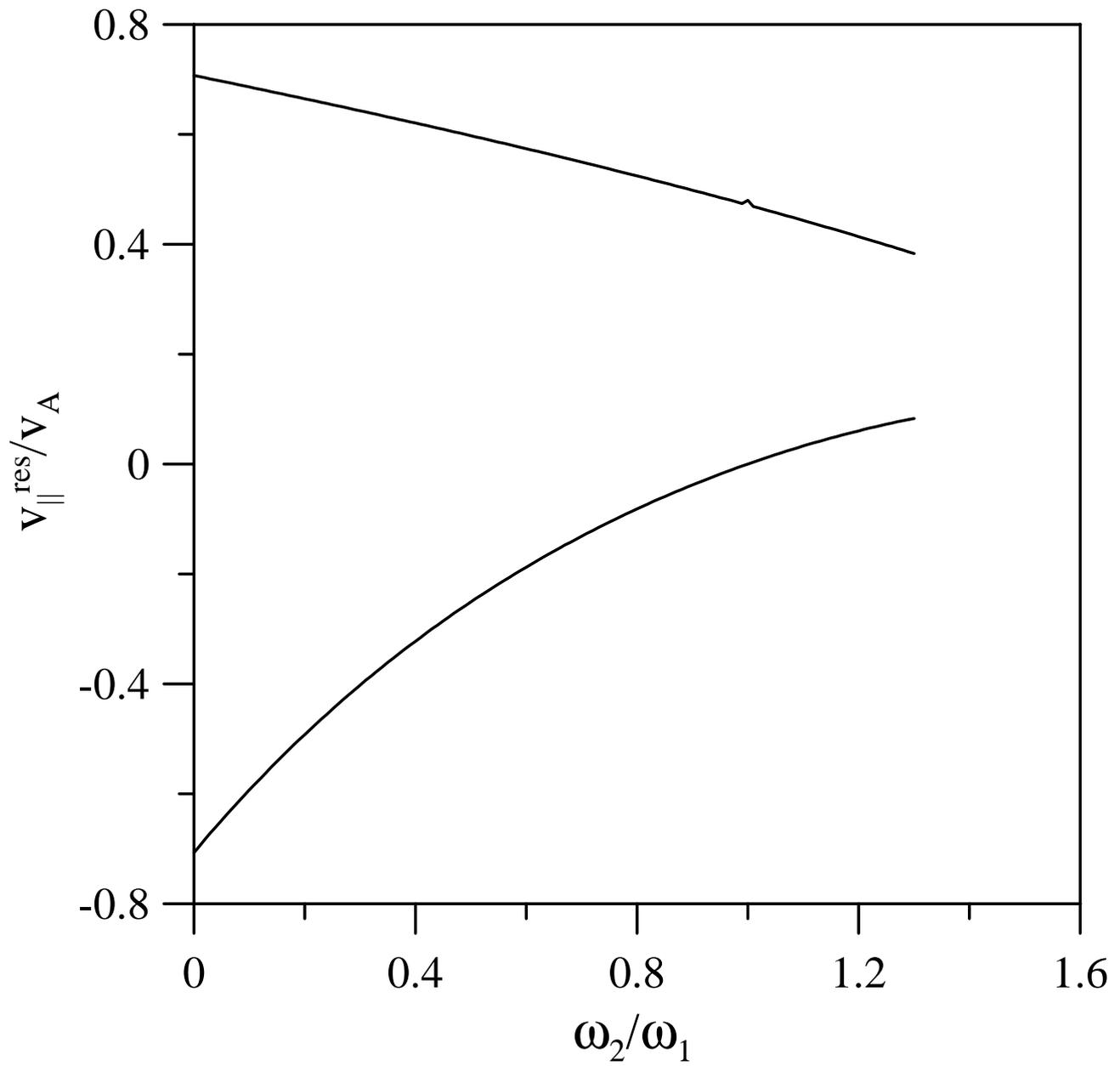
FIG. 2. Velocities of particles interacting with two waves through the non-linear resonance given by Eq. (15).
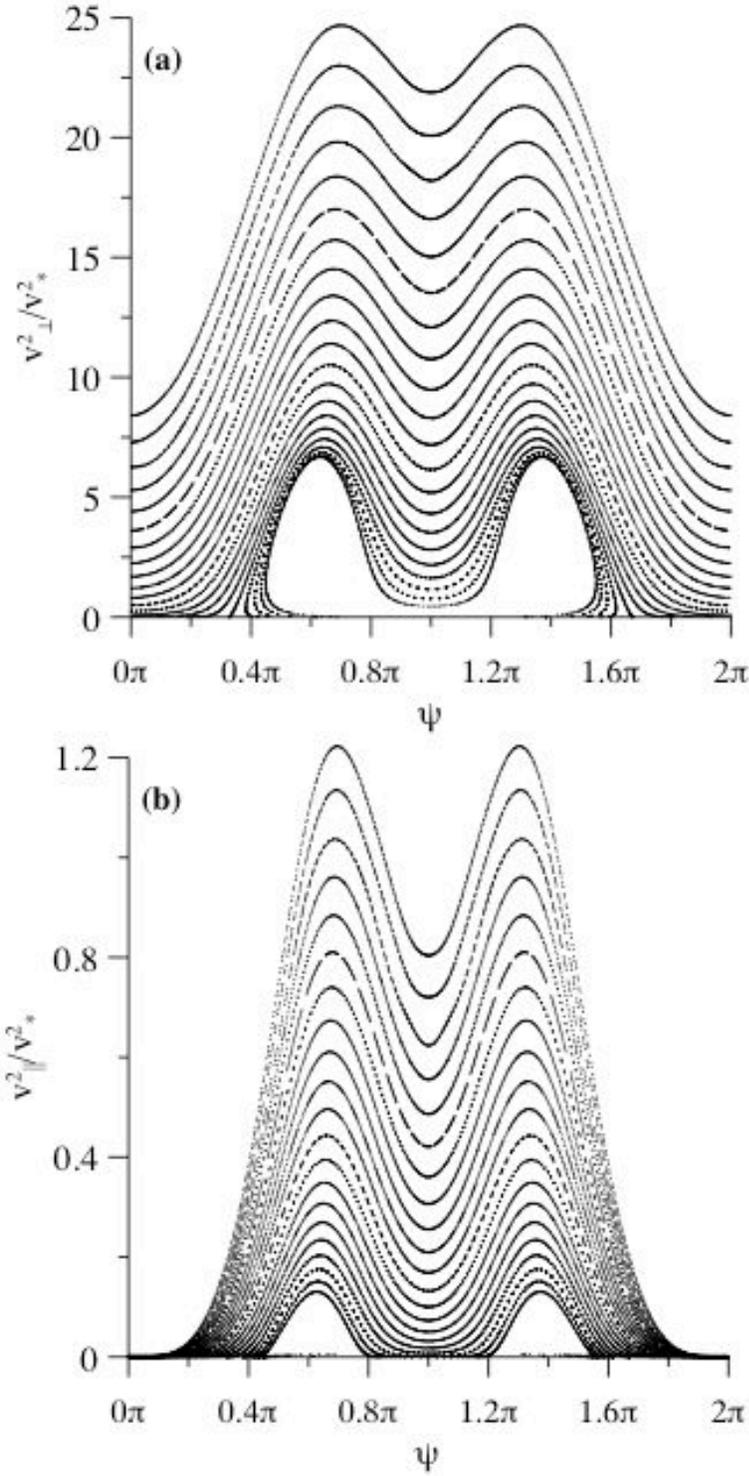
FIG. 3. Poincare map showing the change of the particle energy across and along the magnetic field in the presence of a monochromatic elliptically polarized wave for $\hat{v}_A = 10$, $k_\perp/k_\parallel = 1$, $\hat{\omega} = 1/2$, $\hat{b}_k = 0.1$.
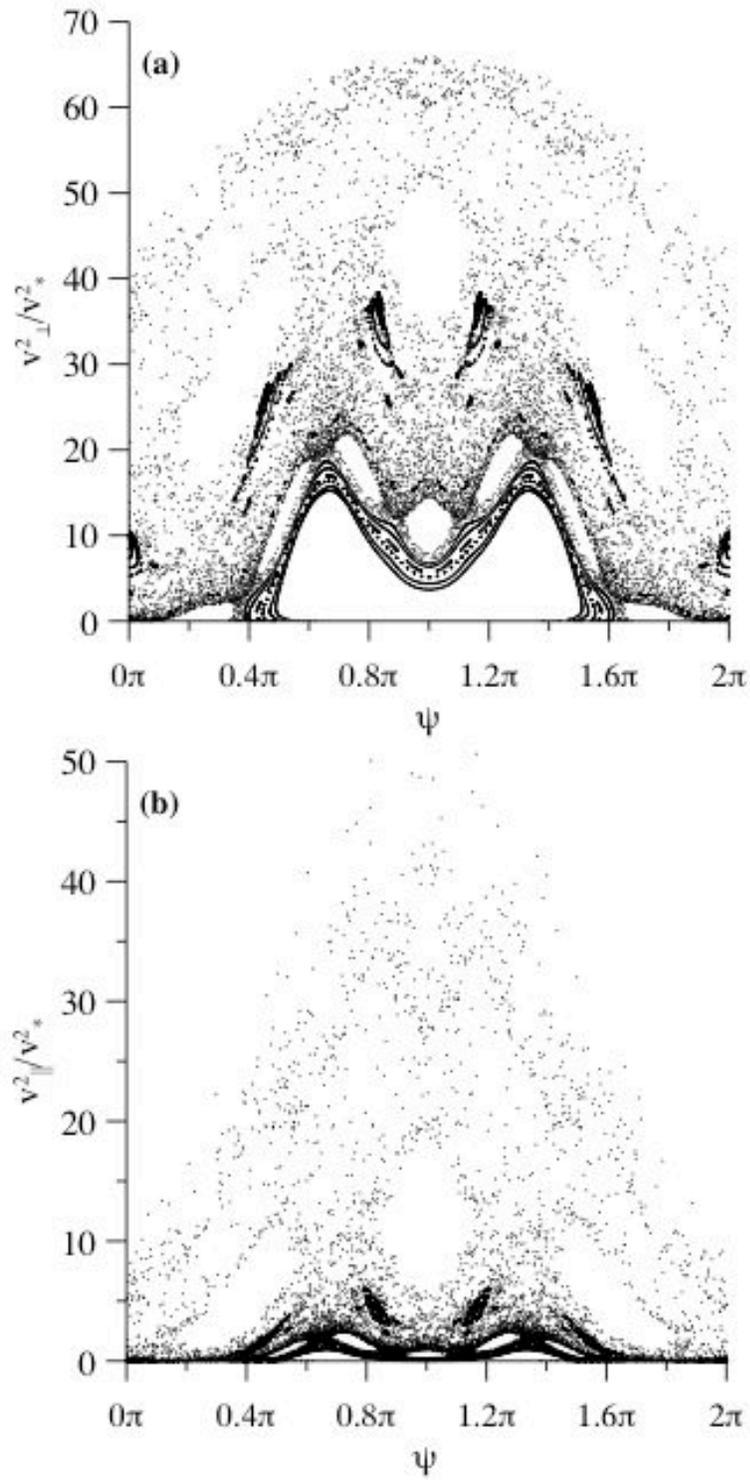
FIG. 4. The same as in Fig. 3, but for $\hat{b}_k = 0.2$.
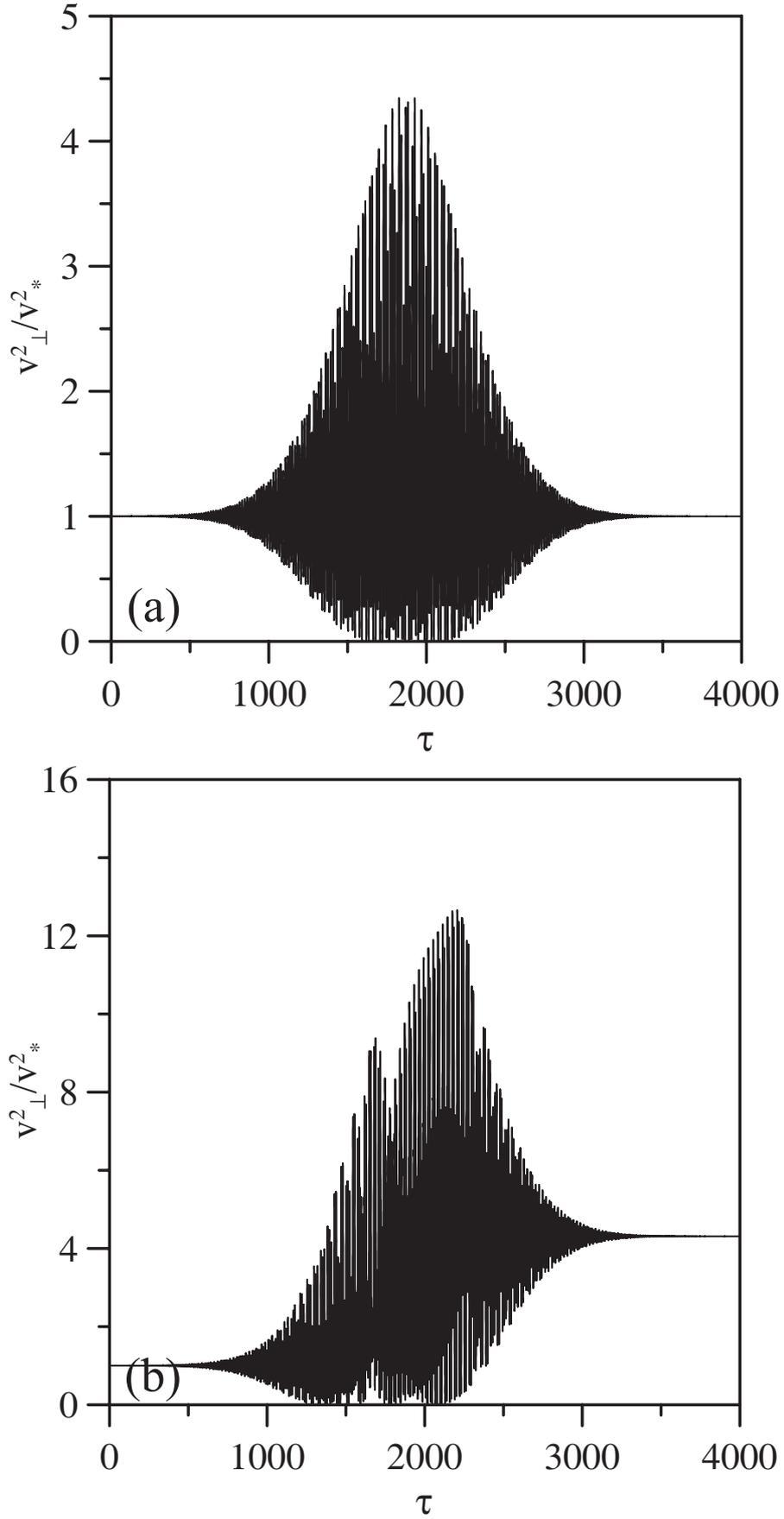
FIG. 5. Temporal evolution of the transverse particle energy when the perturbation is slowly varying as $\hat{b}(t) = \hat{b}_0 \exp[(t - 3t_*)^2/t_*^2]$ with $t_* = 100t_B$, $\tau_B = 2\pi/\omega_B$, for the same parameters as in Fig. 3. The perturbation is maximum at $\tau \approx 1880$: a, $\hat{b}_0 = 0.1$; b, $\hat{b}_0 = 0.2$.
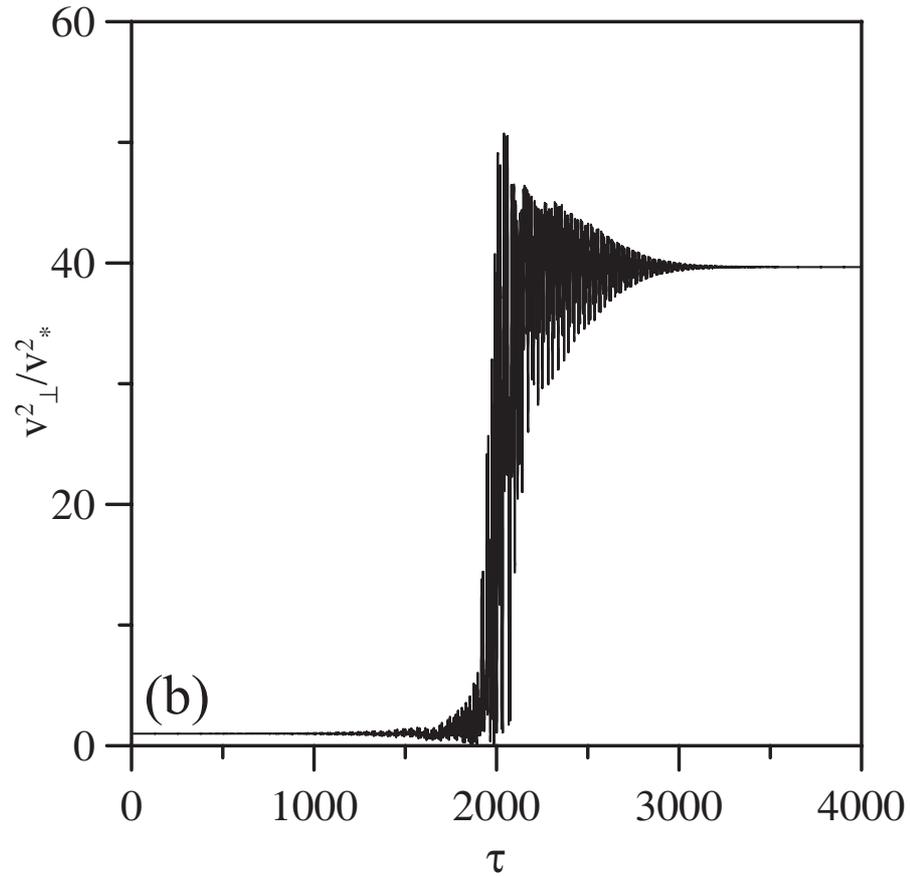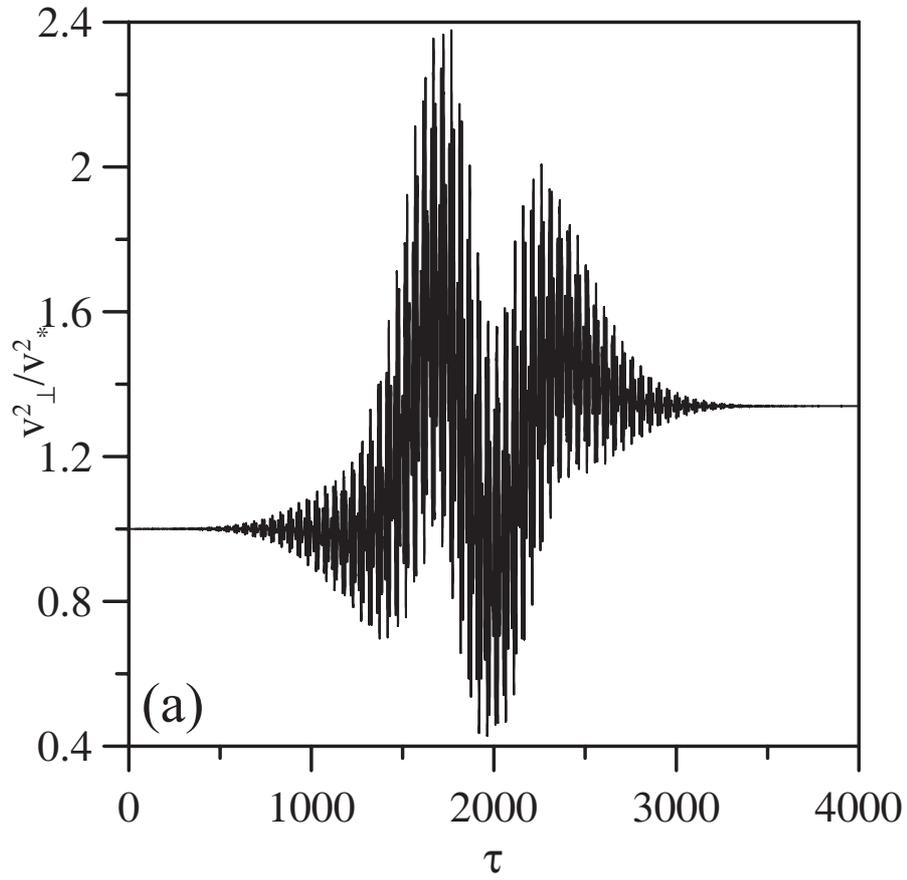
FIG. 6. The same as in Fig. 5, but for several waves with $b_0 = 0.02$: a, $\omega_1 = 1/2$, $\omega_2 = 2/3$; b, $\omega_j = 0.5 + 0.004j$, $j = 0, 1, 2..50$;
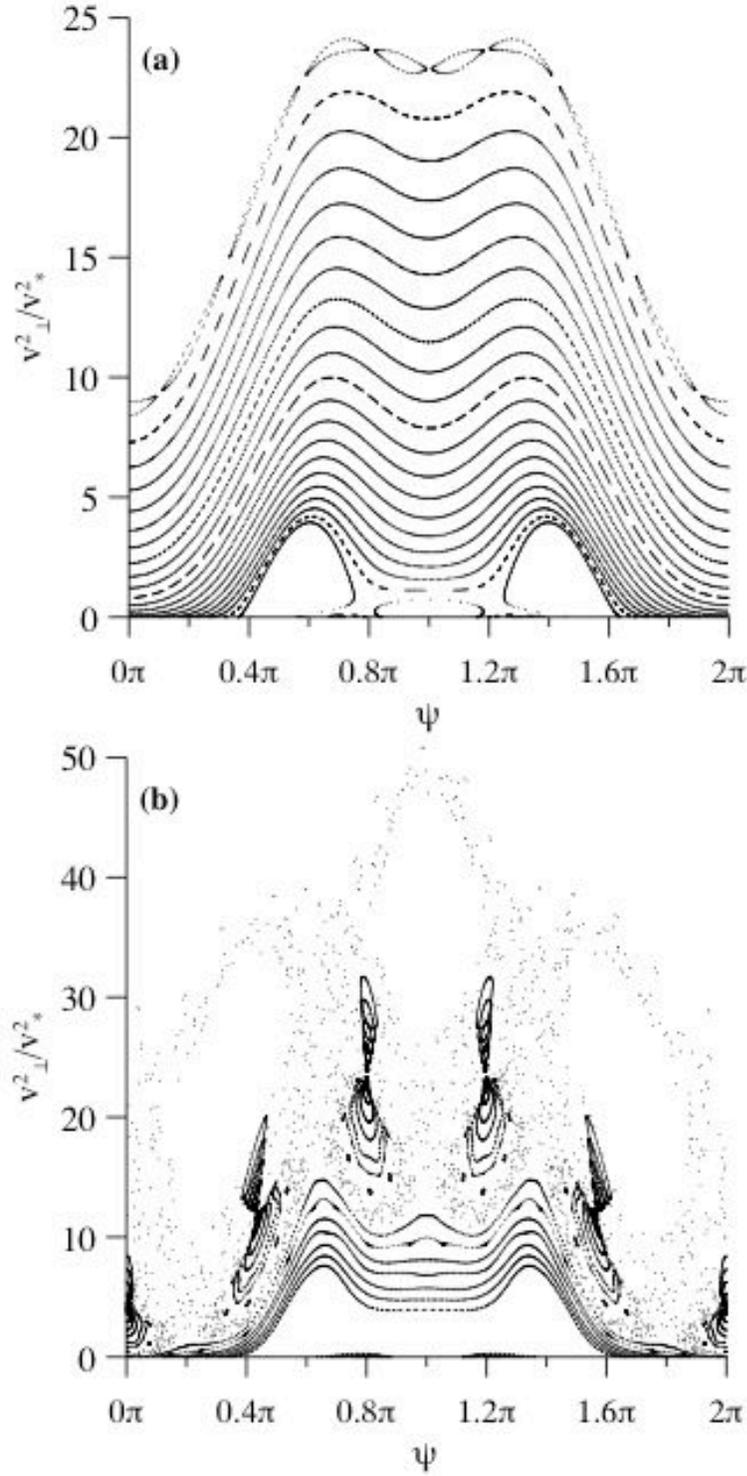
FIG. 7. Poincaré map showing the change of the transverse particle energy in the presence of a linearly polarized monochromatic wave ($\alpha_k = 0$) with $\hat{\omega} = 1/2$, $k_\perp = k_\parallel$ when $\hat{v}_A$: a, $\hat{b}_k = 0.2$; b, $\hat{b}_k = 0.4$.
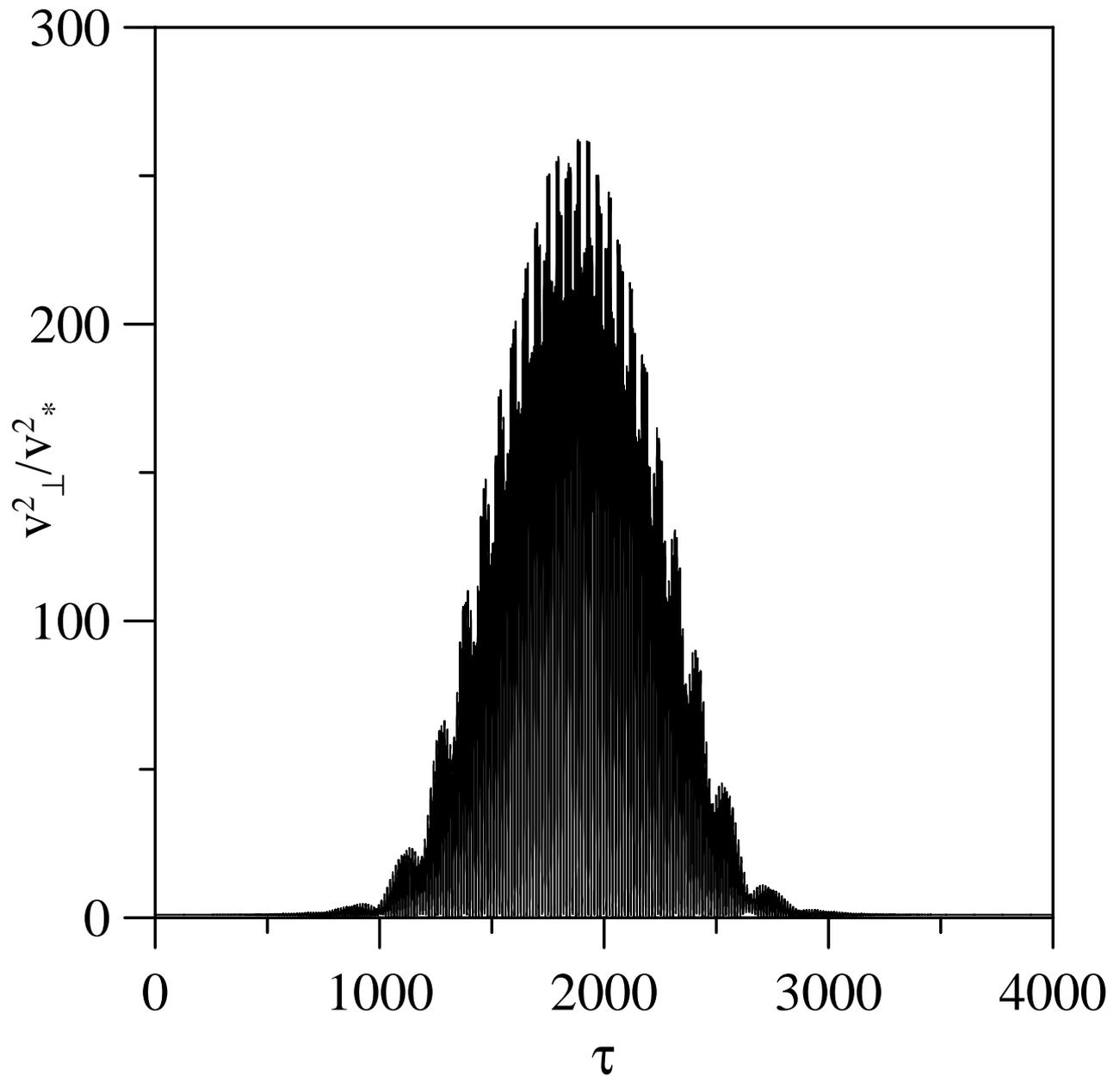
FIG. 8. Strong acceleration of a thermal particle $[v_\perp(\tau = 0) = v_*]$ below stochasticity threshold in the presence of a monochromatic wave with $\hat{b}_k = 0.2$, $k_\perp = k_\parallel$, $\hat{v}_A = 50$.

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