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1 Introduction

In the original Generalized Geometry Holdup (GGH) model, all holdup deposits were modeled as points, lines, and areas[1, 5]. Two improvements[4] were recently made to the GGH model and are currently in use at the Y-12 National Security Complex. These two improvements are the finite-source correction CF_g and the self-attenuation correction.[1] The finite-source correction[2] corrects the average detector response for the width of point and line geometries which in effect, converts points and lines into areas. The result of a holdup measurement of an area deposit is a density-thickness[3] which is converted to mass by multiplying it by the area of the deposit. From the measured density-thickness, the true density-thickness can be calculated by correcting for the material self-attenuation. Therefore the self-attenuation correction is applied to finite point and line deposits as well as areas.

This report demonstrates that the finite-source and self-attenuation corrections also provide a means to better separate the gamma rays emitted by the material from the gamma rays emitted by background sources for an improved background correction. Currently, the measured background radiation is attenuated for equipment walls in the case of area deposits but not for line and point sources. The measured background radiation is not corrected for attenuation by the uranium material. For all of these cases, the background is overestimated which causes a negative bias in the measurement. The finite-source correction and the self-attenuation correction will allow the correction of the measured background radiation for both the equipment attenuation and material attenuation for area sources as well as point and line sources.

The self-attenuation correction was originally derived from Equation (1)[4]

$$CF(AT) = \frac{\rho_{^{235}U} \cdot x_m}{(\rho_{^{235}U} \cdot x_m)_{measured}} = \frac{\mu_m \rho_m x_m}{1 - e^{-\mu_m \rho_m x_m}} \tag{1}$$

where μ_m is the mass attenuation coefficient of the uranium material m, ρ_m is the density of the material, $\rho_{^{235}U}$ is the density of ^{235}U, and x_m is the thickness of the material. By noting that the density of ^{235}U in the material is related to the density of the material by the weight fraction, $\rho_{^{235}U} = f_{^{235}U} \cdot \rho_m$, Equation (1) can be solved for the true density-thickness of ^{235}U as shown in Equation (2) below:

$$\rho_{^{235}U} \cdot x_m = -\frac{f_{^{235}U}}{\mu_m} \ln \left[1 - \frac{\mu_m}{f_{^{235}U}} . (\rho_{^{235}U} \cdot x_m)_{measured} \right] . \tag{2}$$

[1] Both the finite-source correction and the self-attenuation correction are integrated into HMS4. In HMS3, these corrections are calculated with separate programs, the Geometric Response Correction and Self-Attenuation Correction, Ver 1.0. or in a spreadsheet.

[2] For an improvement to the finite-source correction, see Reference [2].

[3] Density-thickness is sometimes referred to as an areal density.

[4] This equation originates from Equation(6-6) in Reference [3].

Equation (2) is only correct when there is no background radiation. The problem is that the background radiation, corrected for attenuation by the true density-thickness of the material, must be subtracted to arrive at the measured density-thickness of the material, $(\rho_{^{235}U} \cdot x_m)_{measured}$. Unfortunately, the true density-thickness of the material is not known prior to performing the self-attenuation correction. In practice, for area sources, an equipment attenuated background is subtracted instead. For point and line sources an unattenuated background is subtracted. If both the background and self-attenuation are significant, the background is overstated resulting in a significant underestimate of the true density-thickness.

An example will demonstrate the problem. Suppose we measure uranium-loaded material in a small container with a collimated detector as represented in Figure 1. The container wall is labeled "Equipment, e" in the figure. Notice that the detector measures a count rate R'' that includes both the background B_0 and the gamma rays from the uranium material which are attenuated by both the material itself and a layer of equipment. Furthermore, let's assume that the uranium material is 93% enriched uranium in the form of U_3O_8. Thus, the weight fraction of grams of ^{235}U to the grams of U_3O_8, $f_{^{235}U}$, is 0.787. Suppose the area calibration constant for the detector is $K_a = 2.0392 \times 10^{-4}\,\mathrm{g \cdot s/cm^2}$. The dimensions of the uranium-loaded material layer and the equipment layer are shown in Table 1.

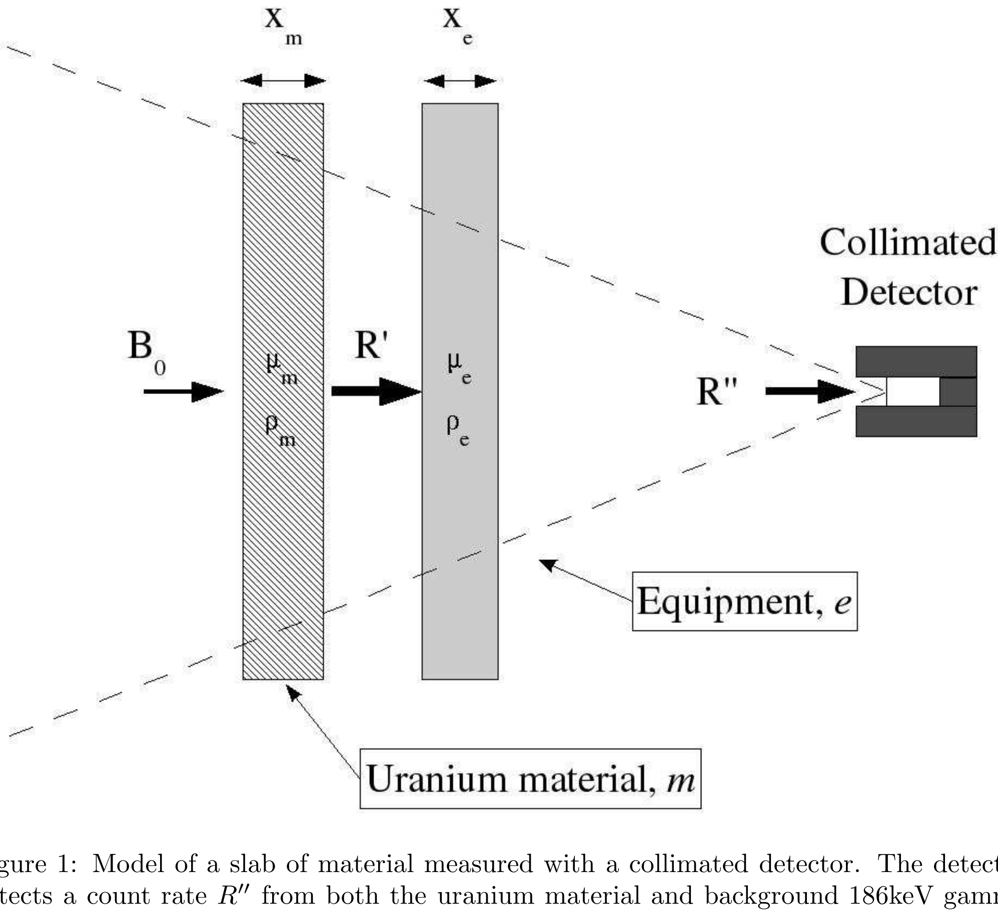

Figure 1: Model of a slab of material measured with a collimated detector. The detector detects a count rate R'' from both the uranium material and background 186keV gamma rays attenuated for both the material and equipment. A count rate R' can be calculated by correcting for equipment attenuation which is what the detector would have measured if the equipment were not present.

Using the dimensions for the equipment layer, the attenuation correction factor is $CF_e = 1.25$. Suppose we measure a net count rate from 186keV gamma rays, $R'' = 1109$ cps. The net 186keV count rate from the background is measured, $B_0 = 800$ cps. The net background-corrected count rate corrected for equipment

Table 1: Uranium and Equipment Specifications

Layer		thickness (cm)	density (g/cm^3)	mass attenuation constant (cm^2/g)
Material,	$m = U_3O_8$ $f_U = 0.787$	$x_m =?$	$\rho_m =?$	$\mu_m = 1.259$
Equipment,	$e = Fe$	$x_e = 0.2$	$\rho_e = 7.81$	$\mu_e = 0.144$

attenuation is therefore 586 cps. The measured density-thickness of ^{235}U is 0.120 g/cm^2. Applying Equation 2 for self-attenuation correction results in a corrected density-thickness of 0.134 g/cm^2.

Suppose we discover that most of the background is coming from similar containers nearby. To improve the measurement, we have this nearby material moved. Now the background reads 40 cps. The material now reads 661 cps. The corrected density-thickness of ^{235}U is now 0.186 g/cm^2, an increase of almost 40%. The fact is that we are still slightly underestimating the density-thickness of ^{235}U. We are subtracting 32 of the 40 cps background when in reality only about 24 cps are actually transmitted through both the material and equipment. We have failed to adjust the background for the attenuation of the uranium material in both cases. The true density-thickness of ^{235}U in this example is 0.189 g/cm^2.

2 Derivation of self-attenuation correction with background subtraction

2.1 Areas deposits

The solution to this problem is to include the material corrected background in Equation (1). For the slab geometry of an area source as shown in Figure 1 we measure a count rate of R'' which is related to R' by the equipment attenuation factor,

$$R'' = R' e^{-\mu_e \rho_e x_e}. \tag{3}$$

In a separate measurement we also measure the unattenuated background, B_0. Thus, the total count rate before equipment attenuation is

$$R' = \frac{\epsilon A_s I}{\mu_m} \left(1 - e^{-\mu_m \rho_m x_m}\right) + B_0 e^{-\mu_m \rho_m x_m}. \tag{4}$$

A_s is the area of the slab, ϵ is the detection efficiency, and I is the specific emission rate of the 186keV gamma rays emitted from the ^{235}U in the uranium material.

The true count rate R which would have been observed without self-attenuation or equipment attenuation is

$$R = \epsilon A_s I \rho_m x_m. \tag{5}$$

To determine the true density-thickness from the measured density-thickness, the material attenuated background must be subtracted from the measured count rate R'. Therefore, Equation (1) is rewritten as

$$\frac{R}{R' - B_0 e^{-\mu_m \rho_m x_m}} = \frac{\mu_m \rho_m x_m}{1 - e^{-\mu_m \rho_m x_m}}. \tag{6}$$

This equation is then solved to arrive at the true density-thickness of the material, uranium or ^{235}U isotope. When doing this, it is important to keep the components of the attenuation straight. The mass attenuation coefficient of the material is calculated by observing that

$$\mu_m \rho_m x_m = \sum_i \mu_i \rho_i x_m = \sum_i \mu_i f_i \rho_m x_m$$

where f_i is the mass fraction for the ith component. Therefore

$$\mu_m = \sum_i \mu_i f_i.$$

In the case of materials such as U_3O_8 or uranium metal, the mass fractions are constant and known. For uranium solutions however, the mass fractions are generally the unknown value to be determined from NDA.

The density-thickness of ^{235}U is determined from the unknown, true count rate R by

$$\rho_{^{235}U} \cdot x_m = K_a R.$$

It can be converted to a density-thickness of uranium by dividing by the fraction of ^{235}U to U. It can also be converted to a density-thickness of material by dividing by $f_{^{235}U}$, the fraction of ^{235}U to total material.

To solve Equation (6) we can convert to the density-thickness of the material

$$\rho_m x_m = \frac{K_a R}{f_{^{235}U}}$$

and solve for the true count rate R in terms of the transmission $T_m = e^{-\mu_m \rho_m x_m}$

$$R = -\frac{f_{^{235}U} \ln(T_m)}{\mu_m K_a}$$

where $\ln(T_m) = -\mu_m \rho_m x_m$. Equation (6) is then rewritten in terms of the transmission factor T_m and the true count rate R which results in

$$-\frac{f_{^{235}U} \ln(T_m)}{\mu_m K_a (R' - B_0 T_m)} = \frac{-\ln(T_m)}{1 - T_m}.$$

The calculated transmission can now be solved in several steps:

$$\begin{aligned} \frac{f_{^{235}U}}{\mu_m K_a}(1 - T_m) &= R' - B_0 T_m \\ T_m \left(B_0 - \frac{f_{^{235}U}}{\mu_m K_a} \right) &= R' - \frac{f_{^{235}U}}{\mu_m K_a} \\ T_m &= \frac{\mu_m K_a R' - f_{^{235}U}}{\mu_m K_a B_0 - f_{^{235}U}}. \end{aligned} \tag{7}$$

By equating this calculated transmission to the original definition, $T_m = e^{-\mu_m \rho_m x_m}$, the density-thickness of the material is solved:

$$\rho_m x_m = -\frac{1}{\mu_m} \ln(T_m) = -\frac{1}{\mu_m} \ln \left(\frac{\mu_m K_a R' - f_{^{235}U}}{\mu_m K_a B_0 - f_{^{235}U}} \right). \tag{8}$$

.

The true density-thickness of uranium and the isotope ^{235}U are

$$\rho_U \cdot x_m = f_U \cdot \rho_m x_m = -\frac{f_U}{\mu_m} \ln(T_m) \text{ , and} \tag{9}$$

$$\rho_{^{235}U} \cdot x_m = f_{^{235}U} \cdot \rho_m x_m = -\frac{f_{^{235}U}}{\mu_m} \ln(T_m) \text{ respectively.} \tag{10}$$

Equation (8) is equivalent to Equation (2) when there is no background. Setting the background to zero in Equation (7), the transmission becomes

$$T_m = 1 - \frac{\mu_m K_a R'}{f_{^{235}U}}.$$

Since the measured density-thickness without the self-attenuation correction is $(\rho_{^{235}U} \cdot x_m)_{measured} = K_a R'$, the transmission can be written as

$$T_m = 1 - \frac{\mu_m}{f_{^{235}U}} (\rho_{^{235}U} \cdot x_m)_{measured}$$

Now substituting back into Equation (10) we have

$$\rho_{^{235}U} \cdot x_m = -\frac{f_{^{235}U}}{\mu_m} \ln\left(1 - \frac{\mu_m}{f_{^{235}U}} (\rho_{^{235}U} \cdot x_m)_{measured}\right).$$

2.2 Extension to point and line sources

In the case of finite point and line sources, some of the measured background will be detected without equipment and material attenuation. The current holdup measurement practice is to assume that all of the background is detected without attenuation for point and line sources. For more accuracy and consistency with area sources, the background can be divided into three parts as shown in Figure 2: a portion that is not attenuated by either the material or equipment which requires no corrections, a portion that is attenuated by the equipment only which will require an equipment attenuation correction and a portion that is attenuated by both the material and equipment which will require attenuation corrections for both.

These portions of the background will be divided up by the effective areas of the detector. The effective area is the detector-response-weighted region of the detector field of view. The effective areas of a finite point or line source A_{wm},[5] of the equipment w_{we} and of an area source A are defined as follows:

$$A = \frac{1}{C_0} \sum_{i=0}^{N} a_i C_i$$

$$A_{we} = \frac{1}{C_0} \sum_{i=0}^{N} a_{we_i} C_i$$

and

$$A_{wm} = \frac{1}{C_0} \sum_{i=0}^{N} a_{wm_i} C_i$$

where a_{wm_i} is the area of the intersection of the finite point or line deposit with annulus i of the radial response as defined in Reference [2] and a_{we_i} is the area of the intersection of the equipment with annulus i of the radial response.

The part of the background that is attenuated by both the material and equipment will be proportional to the effective area of the point or line source, A_{wm}, divided by the effective area of an area source, A. The part

[5]See Reference [2] for a definition of the effective area, A_{wm}. In that reference this area is referred to as A_w

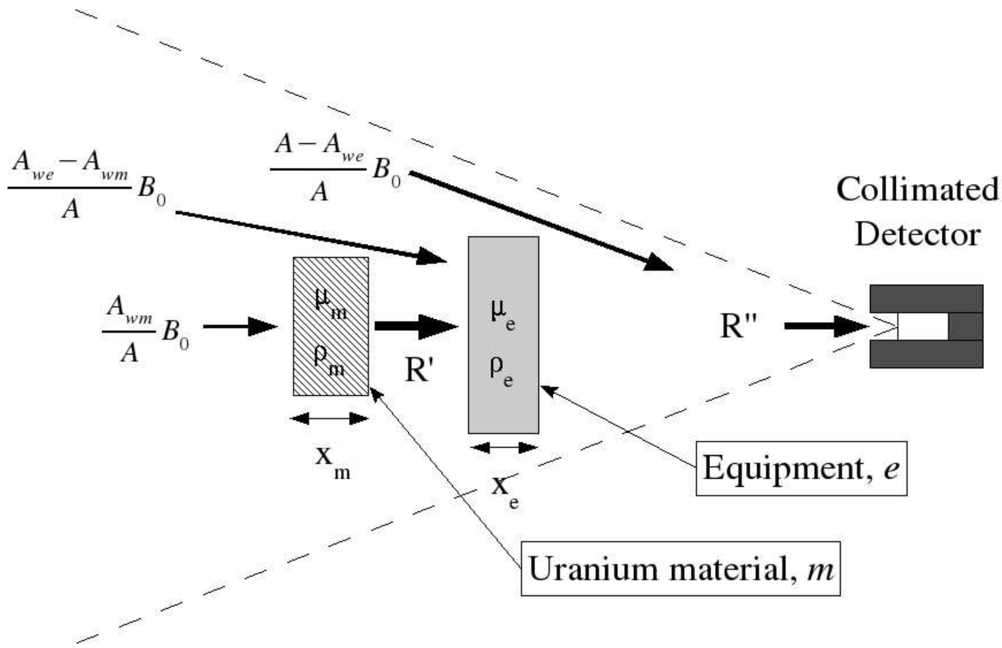

Figure 2: Model of a finite point or line deposit measured with a collimated detector. The detector detects a count rate R'' from both the uranium material and background 186keV gamma rays. Some of this background is not attenuated. Some of the background is attenuated by only the equipment and some is attenuated by both the material and equipment.

of the background that is attenuated by only the equipment will be $\frac{A_{we}-A_{wm}}{A}$. The unattenuated portion of the background is proportional to $\frac{A-A_{we}}{A}$. The measured response in terms of R' and the background B_0 is

$$R'' = \left(R' + \frac{A_{we} - A_{wm}}{A} B_0\right) e^{-\mu_e \rho_e x_e} + \frac{A - A_{we}}{A} B_0. \tag{11}$$

The measured response can be corrected for two of the components of background and equipment attenuation to arrive at R':

$$R' = \left(R'' - \frac{A - A_{we}}{A} B_0\right) e^{\mu_e \rho_e x_e} - \frac{A_{we} - A_{wm}}{A} B_0.$$

where

$$R' = \frac{\epsilon A I}{\mu_m} \left(1 - e^{-\mu_m \rho_m x_m}\right) + \frac{A_{wm}}{A} B_0 e^{-\mu_m \rho_m x_m} \tag{12}$$

is similar to Equation (4) except that for point and line deposits only a fraction of the background is attenuated by the material. Now that we have R' for point and line sources, we can again take the ratio as in Equation (1) and Equation (6)

$$\frac{R}{R' - \frac{A_{wm}}{A} B_0 e^{-(\mu_m \rho_m x_m)}} = \frac{\mu_m \rho_m x_m}{1 - e^{-\mu_m \rho_m x_m}}. \tag{13}$$

The transmission factor T_m differs between a point and line source because the density-thickness of the deposit in terms of the true count rate R differs. For a point source, the density-thickness is

$$\rho_m x_m = \frac{4 K_p C F_g R r^2}{\pi w^2 f_{^{235}U}}.$$

Again, this equation can be rewritten by making the following substitutions for the transmission factor T_m and the count rate for a point source, R:

$$T_m = e^{-\mu_m \rho_m x_m}$$

and

$$R = \frac{\pi w^2 f_{^{235}U} \ln(T_m)}{4\mu_m r^2 K_p CF_g}.$$

This gives us

$$\frac{\pi w^2 f_{^{235}U} \ln(T_m)}{4\mu_m r^2 K_p CF_g} = \frac{\ln(T_m)}{1 - T_m}\left(R' - \frac{A_{wm}}{A} B_0 T_m\right).$$

The transmission factor T_m can be solved using the following sequence of steps:

$$\frac{\pi w^2 f_{^{235}U}}{4\mu_m r^2 K_p CF_g}(1 - T_m) = R' - \frac{A_{wm}}{A} B_0 T_m$$

$$T_m\left(\frac{A_{wm}}{A} B_0 - \frac{\pi w^2 f_{^{235}U}}{4\mu_m r^2 K_p CF_g}\right) = R' - \frac{\pi w^2 f_{^{235}U}}{4\mu_m r^2 K_p CF_g}$$

$$T_m = \frac{4\mu_m r^2 K_p CF_g R' - \pi w^2 f_{^{235}U}}{4\mu_m r^2 K_p CF_g \frac{A_{wm}}{A} B_0 - \pi w^2 f_{^{235}U}} \tag{14}$$

For a line source the density-thickness is

$$\rho_m x_m = \frac{K_l CF_g R r}{w f_{^{235}U}}$$

resulting in

$$R = \frac{w f_{^{235}U} \ln(T_m)}{\mu_m r K_l CF_g}.$$

The transmission factor is then solved as before:

$$T_m = \frac{\mu_m r K_l CF_g R' - w f_{^{235}U}}{\mu_m r K_l CF_g \frac{A_{wm}}{A} B_0 - w f_{^{235}U}}. \tag{15}$$

Once the transmission factor T_m is determined for a point, line, or area source, the density-thickness of the material is calculated using Equation (8)

$$\rho_m x_m = -\frac{1}{\mu_m} \ln(T_m)$$

and the true density-thickness of uranium and the ^{235}U isotope are $\rho_U x_m = f_U \cdot \rho_m x_m$ and $\rho_{^{235}U} \cdot x_m = f_{^{235}U} \cdot \rho_m x_m$ respectively.

3 Conclusion

As shown in the previous section, it is a simple matter to solve Equation (6) and Equation (13) for the true density-thickness of the material in terms of the transmission factor T_m. The transmission factor T_m is calculated from the holdup measurement using the formulas summarized in Table 2 for point, line, and area sources. A separate transmission measurement is not required. Once T_m is determined, the density-thickness of the material is calculated from

$$\rho_m x_m = -\frac{1}{\mu_m} \ln(T_m). \tag{16}$$

The true density-thickness of uranium and the isotope ^{235}U are

$$\rho_U \cdot x_m \quad = f_U \cdot \rho_m x_m \quad = -\frac{f_U}{\mu_m} \ln(T_m) \text{ , and} \tag{17}$$

$$\rho_{^{235}U} \cdot x_m \quad = f_{^{235}U} \cdot \rho_m x_m \quad = -\frac{f_{^{235}U}}{\mu_m} \ln(T_m) \text{ respectively.} \tag{18}$$

Table 2: Transmission Factor, T_m, for point, line, and area sources.

Geometry of deposit	Transmission Factor, T_m
Point	$\frac{4\mu_m r^2 K_p CF_g R' - \pi w^2 f_{^{235}U}}{4\mu_m r^2 K_p CF_g \frac{A_{wm}}{A} B_0 - \pi w^2 f_{^{235}U}}$
Line	$\frac{\mu_m r K_l CF_g R' - w f_{^{235}U}}{\mu_m r K_l CF_g \frac{A_{wm}}{A} B_0 - w f_{^{235}U}}$
Area	$\frac{\mu_m K_a R' - f_{^{235}U}}{\mu_m K_a B_0 - f_{^{235}U}}$

K_p, K_l, K_w: point, line, and area calibration factors
R': measured count rate, corrected for unattenuated background and equipment attenuation
r: distance from front face of detector crystal to the material
μ_m: mass attenuation constant of uranium material
$f_{^{235}U}$: weight fraction of ^{235}U to material
CF_g: finite-source geometry correction factor
A: effective area for an area source
A_{wm}: effective area for a finite point or line source of width w
w: width of a point or line source
B_0: background count rate

The steps in applying the improved background subtraction are as follows:

1. The total count rate from the deposit through the equipment R'' and the unattenuated background without the equipment B_0 are measured as usual. The geometry of the measurement is illustrated in Figure 1.
2. For finite point and line sources, the unattenuated portion of the background $\frac{A-A_{we}}{A}B_0$ is subtracted from R''.
3. The total count rate R'', with unattenuated background subtracted, is corrected for equipment attenuation.
4. For point and line sources the portion of the background attenuated by only the equipment $\frac{A_{we}-A_{wm}}{A}B_0$ is subtracted to arrive at R'.
5. The transmission factor, T_m is calculated from R' and B_0 using the equations summarized in Table 2.
6. From the transmission factor T_m and Equation (18), the true density-thickness of ^{235}U is calculated.
7. The mass is then finally determined by multiplying the true density-thickness by the area of the finite point, line or area deposit.

Referring back to Figure 1 and the example problem, R'' is corrected for equipment attenuation by multiplying by $e^{+\mu_e \rho_e x_e}$ to arrive at R'. In the example problem $R'' = 1109$ cps. Therefore R' is 1391 cps. Applying the equation for an area from Table 2 we determine $T_m = 0.7388$. From Equation (18) we determine that the true density-thickness of ^{235}U is 0.189 cm^2.

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