

High Resolution Prediction of Gas Injection Process Performance for Heterogeneous Reservoirs

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Principal Investigator: Franklin M. Orr, Jr.
Department of Petroleum Engineering
Stanford University

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Abstract

This report outlines progress in the second 3 months of the first year of the DOE project “High Resolution Prediction of Gas Injection Process Performance for Heterogeneous Reservoirs.” The development of an automatic technique for analytical solution of one-dimensional gas flow problems with volume change on mixing is described.

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1 Executive Summary

The aim of this work is to develop a set of ultra-fast compositional simulation tools that can be used to make field-scale predictions of the performance of gas injection processes. To achieve the necessary accuracy, these tools must satisfy the fundamental physics and chemistry of the displacement from the pore to the reservoir scales. Thus this project focuses on four main research areas:

- Determination of the most appropriate methods of mapping multicomponent solutions to streamlines and streamtubes in 3D,
- Development of techniques for automatic generation of analytical solutions for one-dimensional flow along a streamline,
- Experimental investigations to improve the representation of physical mechanisms that govern displacement efficiency along a streamline, and
- Theoretical and experimental investigations to establish the limitations of the streamline/streamtube approach.

In this report we briefly review the status of the research effort in each area. We then give a more in depth discussion of our development of techniques for analytic solutions along a streamline including volume change on mixing for arbitrary numbers of components.

2 Introduction

Working with Professor Tom Hewett, M.S. student Linda Chizoba Sam-Olibale is developing baseline reservoirs to compare streamtube, streamline, and conventional finite difference models. Ph.D. student Jichun Zhu and Professor Lynn Orr are extending the analytical solutions of compositional flows to include variations in temperature. Acting Professor David DiCarlo is continuing the work on measuring compositional displacements in-situ with the CT scanner. Research Associate Charles Schaefer recently left Stanford for another position after characterizing a system of fluids which will be used to study four-phase flow, and crossflow between streamlines. Post-Doctoral Associate Kristian Jessen recently joined us and has begun extending his work on the automatic generation of analytical solutions.

In this report, we extend the method of Jessen *et al.* [7] for finding solutions for displacement of multicomponent oil by a multicomponent gas to include the effects of volume change as components transfer between phases. Additional research needed for development of a three-dimensional compositional streamline simulator based on analytical one-dimensional solutions is outlined.

3 Experimental

3.1 Model

If flow is entirely along streamlines, a compositional displacement can be represented by the following 1-D problem for flow of n_c -component mixtures that partition among n_p phases,

$$\frac{\partial G_i}{\partial \tau} + \frac{\partial H_i}{\partial \xi} = 0, i = 1, n_c, \quad (1)$$

where

$$G_i = \sum_{j=1}^{n_p} x_{ij} \rho_{jD} S_j, \quad (2)$$

and

$$H_i = u_D \sum_{j=1}^{n_p} x_{ij} \rho_{jD} f_j. \quad (3)$$

G_i is the molar concentration of component i , H_i is the molar flux of i , ξ is dimensionless distance, τ is dimensionless time in pore volumes ($\tau = u_{inj} t / \phi L$), x_{ij} is the equilibrium mole fraction of component i in phase j , ρ_{jD} is the dimensionless molar density of phase ($\rho_{jD} = \rho_j / \rho_{ini}$), f_j is the volumetric fractional flow of phase j , and u is the local dimensionless flow velocity ($u_D = u / u_{inj}$, where u and u_{inj} are the dimensional local flow velocity and injection flow velocity respectively).

Eqs. 1-3 are derived based on the assumptions stated by Dindoruk, *et al.* [1]: (1) the flow takes place in a 1D porous medium with uniform properties, (2) the effects of dispersion and capillary pressure can be neglected, (3) that the phases present at any location along a streamline are in local chemical equilibrium (which is specified by the Peng-Robinson equation of state), and (4) for the purposes of the phase equilibrium calculation, the pressure along the streamline is assumed to be constant. When components can change volume as

they transfer between phases, the local flow velocity varies along the displacement length in a way that satisfies at any position, ξ , a version of the continuity equation obtained by summing Eq. 1 over the n_c components. If, on the other hand, each component has fixed partial molar volume, no matter what phase the component appears in, then the local flow velocity is fixed (see Johns *et al.* [8] for the appropriate form of the balance equations).

In the example solutions that follow, the fractional flow function of the vapor phase was taken to be

$$f_v = \frac{S_v^2/\mu_v}{S_v^2/\mu_v + (1 - S_{or} - S_v)^2/\mu_l} \quad (4)$$

where μ_v and μ_l are the viscosities of the vapor and liquid, which were calculated with the viscosity correlation of the Lohrenz *et al.* [9], and S_{or} is the residual oil saturation, assumed to be $S_{or} = 0.2$ in the example calculations. Phase equilibrium was calculated with the Peng-Robinson equation of state (Peng and Robinson [12]).

To complete specification of the 1D flow problem, initial and injection compositions and the displacement pressure and temperature (for phase equilibrium calculations) must be given. The initial composition is assumed to be constant throughout the displacement length, and the injection composition is constant for the entire displacement (a Riemann problem).

3.2 Shocks

Eqs. 1-3 can be solved by the method of characteristics (Dumoré *et al.* [3]; Monroe *et al.* [10]; Dindoruk [1], Dindoruk *et al.* [2]). In that approach, the propagation velocity of a given overall composition is determined by recasting the first-order partial differential equations in terms of an eigenvalue problem. The resulting solutions consist of continuous variations (known as rarefactions) and shocks, jumps in composition and saturation similar to the shocks that arise in a Buckley-Leverett solution for water displacing oil. In Riemann problems, the propagation velocity (which is different from the local flow velocity) of each overall composition or shock is constant throughout the displacement.

Two types of shocks are observed: (1) shocks that connect a composition in the single-phase region with a composition in the two-phase region and (2) shocks that connect two compositions within the two-phase region. Shocks of the first type must occur along the extension of a tie line (Dumoré *et al.* [3]). That is, the single-phase composition lies on the extension of a tie line. If a shock connects two tie lines, the extensions of those tie lines must intersect (Dindoruk [1], Dindoruk *et al.* [2]).

If a shock occurs, it must satisfy an integral material balance of the form,

$$\Lambda^{AB} = \frac{H_i^A - H_i^B}{G_i^A - G_i^B} = \frac{u_D^A \alpha_i^A - u_D^B \alpha_i^B}{G_i^A - G_i^B}, i = 1, n_c, \quad (5)$$

where Λ^{AB} is the propagation velocity of the shock that connects composition points A and B,

$$\alpha_i = \sum_{j=1}^{n_p} x_{ij} \rho_j D f_j. \quad (6)$$

The shocks described by Eq. 5 have two varieties. When the propagation speed of the shock matches the composition propagation velocity on one side of the shock, then $\Lambda = u_D(df/dS)$. Such shocks are known as semishocks or tangent shocks. For many shocks, however, the shock velocity is not equal to the composition velocities on the upstream and downstream sides of the shock. Shocks of this type are called genuine shocks.

3.3 Analytical Solution

The 1D flow problem with volume change was solved first by Welge [15] in a remarkable early paper that investigated the behavior of ternary condensing gas drives. Dumoré *et al.* [3] solved the ternary problem for both condensing and vaporizing ternary systems. Monroe *et al.* [10] found the first solutions for systems with four components. The behavior of four-component systems with volume change was explored in detail by Dindoruk [1], who showed that the shock balance equations of the form of Eqs. imply that the extensions of two tie lines that are connected by a shock must intersect.

Extension of these ideas to displacements with more than four components took place in stages. The analysis for four-component systems showed that there are $n_c - 1$ key tie lines that determine much of the behavior of any solution: the tie line that extends through the initial oil composition, the tie line that extends through the injection gas composition, and $n_c - 3$ additional tie lines, dubbed crossover tie lines by Monroe *et al.* [10]. Johns *et al.* [8] showed that multicontact miscible displacement results if any of the key tie lines is a critical tie line. Wang [14] developed a method for solving the tie-line intersection equations, first for injection gases containing a single component and then for multicomponent injection gases, and applied it to find minimum miscibility pressures (MMPs) for multicomponent systems (Wang and Orr [13]). Jessen *et al.* [6] developed a much more efficient algorithm for solving the tie-line intersection equations. That approach was the basis of an algorithm developed by Jessen *et al.* [7] to find the full solutions for problems with an arbitrary number of components in either the initial oil or the injection gas but without volume change. Calculations of the MMP require only that the key tie lines be found.

In the method of Jessen *et al.* [7] the key lines are found first by solving a set of equations that require that the extensions of the key tie lines intersect, and then the specific compositions that arise during the flow on each of the key tie lines are found. The algorithm used is:

1. Determine the key tie lines by the tie line intersection approach [13, 6].
2. Locate the primary (shortest) tie line.
3. Construct the upstream and downstream portions of the solution by solving sequentially the shocks balances for each adjacent pair of key tie lines. The upstream and downstream segments each begin with a shock that is tangent at the primary tie line.

In this report, we consider only fully self-sharpening displacements in which all tie lines are connected by shocks. To solve the problem for systems in which volume change plays a role, the form of the equations derived by Dindoruk [1] is used. He showed that the tie lines intersect in a molar concentration space at the intersection point X, $(\rho z_i)^X$, defined by the equations of the intersecting tie lines. Once the intersection point is known, the shock

balances can be written conveniently as (details of the derivations are given by Dindoruk [1]; Ermakov [4]; and Jessen [5])

$$\Lambda^* = \frac{\Lambda^{AB}}{u_D^A} = \frac{f^A - S^{XA}}{S^A - S^{XA}} = \frac{u_D^A f^B - S^{XB}}{u_D^B S^B - S^{XB}} = \left(\frac{df}{ds} \right)^A. \quad (7)$$

Eq. 7 is written for a tangent shock from point A to point B, in which the shock velocity matches the composition wave velocity at point A. The saturation and fractional flow, S^A and f_{or}^A , S^B and f^B , refer to the values at the shock landing points A and B on the tie lines that contain those points. The saturations S^{XA} and S^{XB} refer to the tie-line intersection point. S^{XA} is the saturation at that point measured on the tie line that contains point A, and similarly, S^{XB} is the saturation measured on the tie line that contains point B. The saturation at any point on a tie line can be calculated easily from a tie-material balance

$$z_i = V y_i + (1 - V) x_i, \quad i = 1, n_c, \quad (8)$$

where V is the vapor mole fraction, and x_i and y_i are the mole fractions of component i in the equilibrium liquid and vapor. The saturation at overall composition z_i is

$$S_i = \frac{V/\rho_v}{V/\rho_v + (1 - V)/\rho_l}. \quad (9)$$

Similar equations apply if the shock is a genuine shock. In that case, Λ^* does not equal df/dS , but the composition on one side of the shock is known.

The ratio of flow velocities on either side of the shock can be determined from Eq. 5. To determine the actual flow velocities, the velocity ratios for all the shocks between tie lines are calculated, and then the velocity ratios for the leading and trailing phase change shocks are calculated. Because the flow velocity at the inlet is $u_D = 1$, all the remaining flow velocities can be found from the ratios. For any shock in which the composition on one side of the shock is a single-phase mixture (see Dindoruk [1] or Jessen [5] for details),

$$\Lambda^* = \frac{\Lambda}{u_D^{II}} = \frac{f^{II} - S^I}{S^{II} - S^I} = \left(\frac{df}{ds} \right)^{II}, \quad (10)$$

where the superscript II refers to the two-phase side of the shock, and S^I is the saturation at the single-phase composition (greater than one or less than zero). The velocity ratio for the shock is given by

$$\frac{u_D^I}{u_D^{II}} = \frac{1 - \Lambda^*}{\rho_D^I} (\rho_{lD} + S^I(\rho_{vD} - \rho_{lD})), \quad (11)$$

where u_D^I and u_D^{II} refer to the dimensionless flow velocities on the single- and two-phase sides of the shock, and ρ_D^I is the dimensionless molar density of the single-phase mixture.

4 Results and Discussion

4.1 Example Solutions

The theory of Section 3 was used to find solutions for several gas displacement problems that illustrate the effects of volume change on mixing. Figure 1 shows the results for

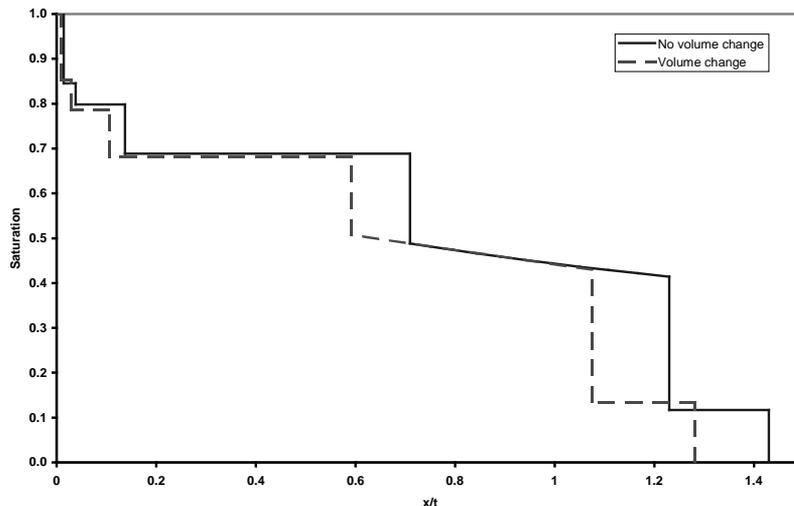


Figure 1: Saturation profiles for displacement of a five-component oil by CO_2 at 109 atm (1600psia) and 344 K (160 F).

a displacement of a five-component oil containing equal mole fractions of methane (CH_4), n-butane (C_4), decane (C_{10}), tetradecane (C_{14}), and eicosane (C_{20}) by pure CO_2 at 109 atm (1600 psia) and 71 C (160 F). Figure 1 compares the saturation profiles for solutions with and without volume change. Additional details of the solution, such as the compositions at the shock landing points and the flow velocities on each key tie line are given by Ermakov *et al.* [4]. In this example, volume change reduces the flow velocity. CH_4 from the oil present in the transition zone appears in a leading CH_4 bank, and when the injected CO_2 encounters the undisplaced oil, it dissolves, losing volume in the process. In displacements in which the pressure is high enough that the solubility of CO_2 in the oil is appreciable, but not high enough that the displacement is very efficient, the loss of volume can actually cause CO_2 breakthrough to occur after one pore volume of injection (see Orr *et al.* [11], for experimental results that show this effect and Dindoruk [1], for theoretical examples).

Figure 2 compares the analytical solution including the effects of volume change with results of a FD compositional simulation performed with single-point upstream weighting, 1000 grid blocks and $\Delta x/\Delta t$ set to 5. Also shown in Fig. 2 is a numerical solution obtained by a TVD scheme (unpublished simulator written by Marco Thiele and Michael Edwards). While the numerical solutions show some smearing of the shocks, it is clear that the numerical and analytical solutions agree very well.

The speed advantage of the analytical approach is illustrated in Table 1. The analytical solutions are self-similar, so the computation time required for any length of displacement is the same. That is not true, however, for FD compositional simulation. The computation times shown are only approximate because the simulations were run on a multiuser server, but even so, they indicate clearly that the analytical approach is orders of magnitude faster than FD simulation for grids that are fine enough to resolve the composition path well. FD simulations for coarser grids would be faster, of course, but significant adverse effects of numerical dispersion would be present in the solutions.

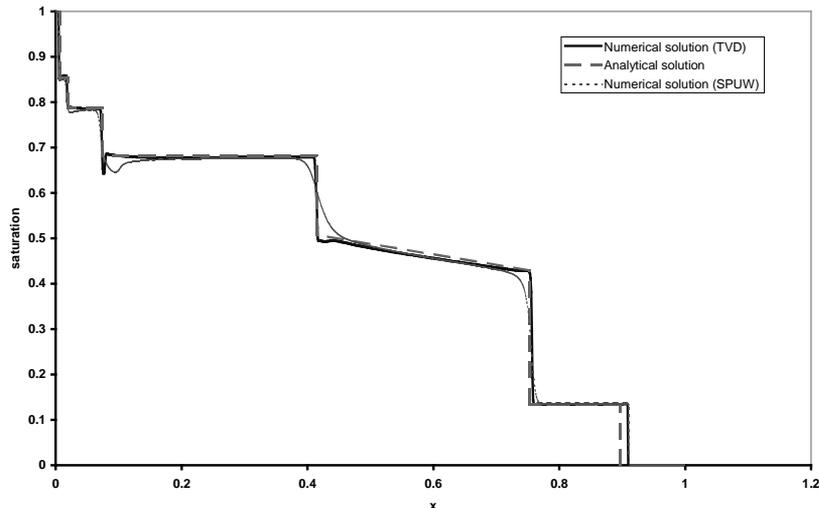


Figure 2: Comparison of analytical and numerical solutions for flow with volume change for CO₂ displacing a five-component oil (see Fig. 1) at 0.7 pore volumes injected.

Table 1: Computation times for analytical and numerical solutions for gas displacements.

Components	Analytical Solution (sec)	Simulation Length (PV)	Numerical Solution 1000 Blocks (sec)	Numerical Solution 5000 Blocks (sec)
6	1.1	0.70	300	8160
15	10	0.25	720	14820

4.2 Discussion

The examples given here demonstrate that solutions to the 1D multicomponent gas displacement problems can be found far more efficiently by analytical methods than by conventional FD compositional simulation. So far, the solutions are limited to systems that are fully self-sharpening (that is, key tie lines that make up the solutions are connected by shocks only, not by rarefactions). While many displacements are fully self-sharpening, rarefactions do occur for some systems (Jessen *et al.* [7]). Thus, extension of the method reported here to include integrations for rarefactions will be required. Even so, the fully self-sharpening solution is a reasonable approximation [5], and the existing theory can be used to calculate the performance of slim tube displacements thousands of times faster than by conventional compositional simulation.

Additional extensions of the theory will be required if these methods are to be applicable to a wide range of streamline simulations. For example, versions of the streamline approach that update the locations of the streamlines periodically will create a need for 1D solutions for problems in which the initial composition along a streamline is not constant. Similarly, changes in injection composition during the life of a project will also create

problems that are not Riemann problems. For those problems, the composition propagation and flow velocities do not remain constant in time. Instead, the solution evolves toward self-similar solutions like those presented here for some period. Finally, problems in which three-phase flow of water plus the hydrocarbon-bearing phases will also be of interest. If these more difficult problems can be solved, practical and very fast 3D compositional simulation will be possible for a wide range of problems.

5 Conclusion

The examples presented in this report establish that:

1. 1D solutions for multicomponent displacements in the absence of dispersion but including the effects of volume change as components transfer between phases can be found by an automatic algorithm.
2. Analytical solutions for 1D flow that are not influenced by numerical dispersion can be found in hundreds to thousands of times less computation time than is required for the corresponding FD compositional simulations, which are influenced by numerical dispersion.
3. The primary effect of volume change is to cause the local flow velocity to vary in space. In some systems the effects of volume change can lead to significantly earlier or later breakthrough than would occur in the corresponding displacement with no volume change.
4. In the limit as the MMP is approached, the solutions obtained with and without volume change become identical.

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