

High Resolution Prediction of Gas Injection Process Performance for Heterogeneous Reservoirs

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Abstract

This report outlines progress in the second quarter of the second year of the DOE project “High Resolution Prediction of Gas Injection Process Performance for Heterogeneous Reservoirs”. A three-dimensional streamline simulator, developed at Stanford University, has been modified in order to use analytical one-dimensional dispersion-free solutions to multicomponent gas injection processes. The use of analytical one-dimensional solutions in combination with streamline simulation is demonstrated to speedup compositional simulations of miscible gas injection processes by orders of magnitude compared to a conventional finite difference simulator. Two-dimensional and three-dimensional examples are reported to demonstrate the potential of this technology. Finally, the assumptions of the approach and possible extensions to include the effects of gravity are discussed.

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1. Executive Summary

Performance evaluation of miscible and near-miscible gas injection processes can be assessed through conventional finite difference (FD) compositional simulation. However, low-resolution compositional simulation is adversely affected by numerical dispersion and may fail to represent geological heterogeneities adequately, and high-resolution simulation is too expensive in computation time. The number of components can be reduced but at the price of less accurate representation of phase behavior. Hence, the use of FD simulators in such studies is subject to limitations that can be quite significant in some field settings.

To close this gap, we propose a method combining the use of an analytical one-dimensional (1D) dispersion-free solution for multicomponent gas injection with a representation of flow along streamlines to capture the effects of heterogeneity. The 1D analytical solver allows any number of components to be present in the injected gas as well as in the reservoir fluid and includes the effects of volume change on mixing.

The current work is based on a 3D field-scale streamline simulator (3DSL) developed at Stanford University¹. 3DSL has been modified to use dispersion-free analytical 1D solutions to propagate compositions along the streamlines. Simulation examples in 2D and 3D heterogeneous porous media are reported to demonstrate the potential speed-up of compositional simulation by the use of analytical solutions in combination with streamlines. We demonstrated that the CPU requirement is reduced by several orders of magnitude compared to conventional FD compositional simulation, even for problems with modest grid resolution.

2. Introduction

It is now well established that injection of gases such as CO₂, methane, enriched hydrocarbon gases or nitrogen into an oil reservoir can lead to efficient displacement of the reservoir oil if the displacement pressure is sufficiently high. In such processes, transfer of components from the injection gas mixture to the oil in place in the reservoir and from the oil to the flowing gas phase creates hydrocarbon mixtures that can displace the oil much more efficiently than does water in the portion of the reservoir that is swept by the injected gas. When injection pressures are sufficiently high, these component transfers between phases cause compositions in the transition zone between the injected gas and the displaced oil to pass close to a critical point. When that happens, the local displacement efficiency is high, and the residual oil saturation is small. The high displacement efficiency that results is a consequence of the transfer of components between phases.

At field scale, local displacement efficiency is only part of the story, however. Injected gas will have lower viscosity than the oil being displaced, and the reservoir rocks that contain the oil will have a wide range of permeabilities. As a result, injected gas will flow preferentially through high permeability zones. It is the combination of local displacement efficiency, determined primarily by compositional phenomena, and sweep

efficiency determined primarily by reservoir heterogeneity and gravity segregation, that controls overall process performance. Accurate prediction of process performance at field scale requires, therefore, that the combined effects of component transfers due to phase equilibrium and flow through heterogeneous reservoir rocks be represented adequately. Standard finite difference compositional simulation methods can do just that, of course. Permeability heterogeneity can be represented at the level of detail allowed by the number of grid blocks, and phase equilibrium calculations are performed for each grid block. Very large scale, three-dimensional compositional simulations are rarely attempted, however, for two reasons: (1) computations with enough grid blocks to give a high-resolution representation of permeability heterogeneity require too much computation time, and (2) very large numbers of grid blocks are required to control adverse effects of numerical dispersion. Computations with coarser grid resolutions are possible, but accuracy may be reduced significantly. If the effects of zones of low and high permeability are not represented accurately displacement efficiency is likely to be seriously in error because sweep efficiency is often determined by the extremes of permeability, not average quantities assigned to large grid blocks. In addition, the effects of numerical dispersion frequently reduce estimates of local displacement efficiency for flows that are above or near the minimum miscibility pressure. Thus, for field-scale computations, conventional finite-difference compositional simulations are usually not feasible. Thus, what is needed for accurate, three-dimensional, field-scale displacements is a very fast simulation method that allows both high resolution representation of heterogeneity and accurate calculation of the effects of phase equilibrium that are not adversely affected by numerical dispersion.

Streamline methods offer an approach that has the potential to satisfy the requirements of high-resolution representation of permeability variation along with representation of the combined effects of phase equilibrium and flow. In this approach, the effects of heterogeneity are captured by calculating the locations of streamlines or streamtubes^{1-6,9}, and the details of the compositional mechanisms are represented as one-dimensional solutions to the flow equations that are mapped along streamlines^{4,5}. In many flows, the positions of streamlines change slowly, and hence the streamlines need not be updated frequently. Streamline calculations for such flows can be orders of magnitude faster than the corresponding finite difference (FD) compositional simulations because the computationally expensive calculation of the pressure field (from which streamlines are determined) is performed relatively infrequently (in contrast with FD methods which do so each time step).

Most compositional streamline calculations performed to date have used numerical simulation for the solutions along individual streamlines. Use of analytical solutions for the 1D compositional flow problem has been limited by the fact that until recently solutions were available only for systems with a limited number of components (usually three or four) and by limitations on streamline updating. Jessen *et al.*⁷ demonstrated an algorithm with which analytical 1D solutions can be obtained for systems with an arbitrary number of components in the gas or the oil, though the systems considered were limited to those in which components do not change volume as they transfer between phases. The restriction on volume change has recently been relaxed allowing for general

application of the analytical 1D solver to streamline simulation as outlined in the following sections.

3. Mathematical Model

In this section we derive the equations that are required to trace streamlines in a 3D heterogeneous porous media. The conservation equations for multicomponent multiphase flow can be written in terms of molar compositions and densities as

$$\sum_{j=1}^{n_p} \frac{\partial}{\partial t} (\phi \rho_j x_{ij} S_j) + \nabla \cdot (\underline{u}_j \rho_j x_{ij}) = q_s \rho_j x_{ij} \quad , \quad i = 1, \dots, nc \quad (3.1)$$

where ϕ is the porosity, ρ_j is the molar density of phase j , x_{ij} is the mole fraction of component i in phase j , S_j is the volume fraction of phase j , \underline{u}_j is the velocity of phase j and q_s represents source/sink terms within the domain. The velocity of phase j can be expressed by Darcy's law

$$\underline{u}_j = -\underline{K} \frac{k_{rj}}{\mu_j} \nabla P_j \quad , \quad \underline{u}_t = \sum_{j=1}^{n_p} \underline{u}_j \quad (3.2)$$

where \underline{u}_t is the total velocity, K is the permeability tensor, k_{rj} is the relative permeability of phase j , μ_j is the viscosity of phase j and P_j is the pressure in phase j . In the examples discussed here we neglect the effects of compressibility and gravity for the purpose of solving for the pressure/velocity field. A summation over all components in Eq. 3.1 combined with the assumption that ρ_j remains constant leads to the governing volume balance equation for incompressible flow

$$\nabla \cdot \underline{u}_t = q_s \quad (3.3)$$

Neglecting capillary effects ($P_i = P_j$) and introducing the total mobility λ_t , the velocity vector can be rewritten as

$$\underline{u}_t = -\underline{K}(\lambda_t \nabla P) \quad (3.4)$$

with λ_t given by

$$\lambda_t = \sum_{j=1}^{n_p} \frac{k_{rj}}{\mu_j} \quad (3.5)$$

Substitution of Eq. 3.4 into Eq. 3.3 leads to the governing pressure equation for incompressible multicomponent multiphase flow in a porous medium

$$\nabla \cdot \underline{\underline{K}}(\lambda_t \nabla P) = -q_s \quad (3.6)$$

Eq. 3.6 can be solved by a standard finite difference approach as outlined by Batycky¹ to obtain the pressure field. Given the pressure field, the face velocities of each gridblock can be evaluated. See Batycky¹ for further details.

3.1 Time of Flight Formulation and Coordinate Transformation

The time of flight^{8,9} is the time required for a particle to travel from a reference location (often taken at an injector) to a given location s along a streamline. The time of flight can be evaluated by the integral

$$\tau(s) = \int_0^s \frac{\phi(\zeta)}{|\underline{u}_t|} d\zeta \quad (3.7)$$

Eq. 3.7 allows the following coordinate transformation¹⁰

$$|\underline{u}_t| \frac{\partial}{\partial s} = \underline{u}_t \cdot \nabla = \phi \frac{\partial}{\partial \tau} \quad (3.8)$$

The key idea behind streamline-based simulation is to decompose the 3D-flow problem into a sequence of 1D displacements along streamlines. To do just that we need to transform the component conservation equation from Cartesian coordinates to the time of flight coordinate by the use of Eq. 3.8. Eq. 3.1 can be rewritten in terms of the overall molar concentration (G_i) and the overall molar flux (H_i) of component i

$$\phi \frac{\partial G_i}{\partial t} + \nabla \cdot (\underline{u}_t H_i) = 0, \quad i = 1, \dots, n_c \quad (3.9)$$

with

$$G_i = \sum_{j=1}^{n_p} x_{ij} \rho_j S_j \quad \text{and} \quad H_i = u_d \sum_{j=1}^{n_p} x_{ij} \rho_j f_j \quad (3.10)$$

where u_d is the dimensionless velocity scaled with respect to the injection velocity and f_j is the fractional flow of phase j . The second term on the left-hand side in Eq. 3.9 can be expanded to

$$\nabla \cdot (\underline{u}_t H_i) = \underline{u}_t \cdot \nabla H_i + H_i \nabla \cdot \underline{u}_t \quad (3.11)$$

Assuming that the effect of compressibility on the velocity field is negligible, the second term on the right-hand side of Eq. 3.11 drops out and Eq. 3.9 can be rewritten as

$$\phi \frac{\partial G_i}{\partial t} + \underline{u}_t \cdot \nabla H_i = 0, \quad i = 1, \dots, n_c \quad (3.12)$$

By applying the coordinate transformation outlined in Eq. 3.8, the final form of the conservation equations along a streamline can be written as

$$\frac{\partial G_i}{\partial t} + \frac{\partial H_i}{\partial \tau} = 0, \quad i = 1, \dots, n_c \quad (3.13)$$

Given constant initial and injection conditions the conservation equations of Eq. 3.13 constitutes a Riemann problem for which the solution is self-similar; the solution can be reported in terms of the dimensionless ratio of independent variables ($\lambda_{sl} = \tau/t$). Consequently, any self-similar 1D-displacement problem that can be mapped along streamlines to describe the flow of multiple components in a 3D heterogeneous porous media.

3.2 Tracing Streamlines

Streamlines are curves in the domain of the porous media along which every point is tangent to the velocity field at a specific time¹¹. Hence, knowing the velocity field allows us to trace the streamlines along which to propagate analytical 1D solutions for any given multicomponent displacement problem. The approach for tracing streamlines in 3DSL use here is based on the work of Pollock⁸ assuming that the total velocity varies linearly within a given gridblock. In 3DSL, streamlines are traced from injectors to producers taking advantage of the properties of an incompressible velocity field. A number of streamlines, specified by the user, are launched of the grid faces of the gridblocks containing an injector. A given streamline is traced forward from the injector by calculating the locations of entry and exit in the next gridblock, assuming linear variation of the velocity with in each gridblock. The increment in the time of flight is recorded by

$$\tau_{sl} = \tau_{sl} + \Delta \tau_i \quad (3.14)$$

where $\Delta \tau_i$ is the incremental time of flight through gridblock i . This procedure is repeated until a producer is reached at which point τ_{sl} is the time it would take a particle to travel from injector to producer along the given streamline, provided that the pressure field remains constant. Each gridblock must be assigned a time of flight for the purpose of mapping the 1D solutions on to the pressure grid as discussed in the following section. As several streamlines may pass through the same gridblock an averaging scheme must be introduced¹:

$$\bar{\tau}_{block_i} = \frac{\sum_{i=1}^{n_{sl}} \Delta \tau_i \bar{\tau}_i}{\sum_{i=1}^{n_{sl}} \Delta \tau_i} \quad (3.15)$$

where

$$\bar{\tau}_i = \frac{\tau_{i,entry} + \tau_{i,exit}}{2} \quad (3.16)$$

4. Mapping 1D Analytical Solutions to Streamlines

In the previous quarterly report, we demonstrated that the mass conservation equations for multicomponent, dispersion-free two-phase flow in one dimension are written as

$$\frac{\partial G_i}{\partial \tau_a} + \frac{\partial H_i}{\partial \xi} = 0, \quad i = 1, \dots, n_c \quad (4.1)$$

with

$$\tau_a = \frac{u_{inj} t}{\phi L} \quad \text{and} \quad \xi = \frac{z}{L} \quad (4.2)$$

where u_{inj} is the injection velocity, t is the time, ϕ is the porosity, L is the overall length of the porous medium and z is the distance from the inlet. Solutions to Eq. 4.1 are self-similar provided constant initial and injection conditions and are constructed by the method of characteristics (MOC). For the 1D problem, the self-similarity variable is given by

$$\lambda_{moc} = \frac{\xi}{\tau_a} = \frac{z\phi}{u_{inj} t} = \frac{\tau_{moc}}{t} \quad (4.3)$$

The analogy between the time of flight τ along a streamline and the time required to reach a specific point in the 1D solution τ_{moc} is evident. Hence, no additional transformations are necessary for mapping analytical 1D solutions along streamlines. The solution to Eq. 4.1 gives the variation of in overall composition and related properties such as saturation and total mobility as a function of the wave velocity λ_{moc} . Knowing the time of flight in a given gridblock allow us to assign an overall composition to that gridblock. In other words, if the time of flight in a gridblock equals 1 and the real time t equals 2, the gridblock is assigned the properties of the analytical solution corresponding to a wave velocity of 0.5.

5. Streamline Simulation Using an Analytical 1D Solver

As in any simulation, 3DSL starts of by assigning a total mobility to all active gridblocks, based on initial conditions of the porous media in question. The next step is to solve for the pressure field and subsequently evaluating the velocity field. Given the velocity field, a specified number of streamlines can be traced and the time of flight for each gridblock can be determined. In the current approach the analytical solution is then propagated along the streamlines from $t = 0$ to $t + \Delta t$ and each gridblock is assigned a new overall composition based on the weighting scheme

$$\overline{G}_j = \frac{\sum_{i=1}^{n_{sl}} \Delta \tau_i q_i G_j(\lambda_i)}{\sum_{i=1}^{n_{sl}} \Delta \tau_i q_i} \quad (5.1)$$

where q_i is the flux associated with streamline i . $G_j(\lambda_i)$ is the overall molar concentration of component j corresponding to the value of the self-similarity variable $\lambda_i(\tau t)$ in the specific gridblock. The flux associated with a given streamline is evaluated by dividing the total flux from the gridblock face, where the streamline is launched, by the total number of streamlines launched from the block face.

As the injected gas invades the porous media, the mobility distribution changes. The change in mobility distribution results in a varying pressure field and consequently causes the streamlines to change locations. Hence, it is necessary to update the pressure field periodically to account properly for mobility contrasts between the injected fluid and the reservoir fluid. However, in displacement problems where the flow is dominated by heterogeneities (preferential flow paths) the streamlines change only slightly over time reducing the number of pressure solves needed to properly predict the sweep. When a pressure solve is required, the updated overall gridblock compositions are used to evaluate the gridblock mobility, and the trace of streamlines and propagation of fluids along the streamlines are repeated for the next time step. An assumption of the current approach is that the time step from t to $t+\Delta t$ can be approximated by a time step from $t = 0$ to $t+\Delta t$. In the following section we demonstrate that this assumption results in good agreement between streamline simulations and conventional FD simulations, at least for the examples considered here.

6. Simulation Examples

To demonstrate the combination of analytical 1D solutions with streamlines to evaluate displacement performance of miscible gas injection processes in a heterogeneous porous media we report two examples:

- 1) 2D 100x10 gridblock areal displacement with horizontal wells.
- 2) 3D 50x50x10 gridblock with vertical wells.

In the two examples, phase equilibrium calculations were performed using the Soave-Redlich-Kwong equation of state, whereas the phase viscosities were calculated by the Lohrenz-Bray-Clark¹² correlation. Relative permeabilities were evaluated by simple quadratic Corey type expressions, with 0.2 residual oil saturation. In both scenarios, the reservoir fluid initially in place was represented by a 15 component mixture. A dry separator gas containing a total of 11 components (81 mole % CH₄) was used to displace the oil. Details of the fluid and relative permeability functions are reported in Jessen *et*

*al.*⁷ At the given reservoir temperature of 368K the minimum miscibility pressure (MMP) is predicted to ~365 atm.⁷

6.1 Two-Dimensional Example

In the first example we displace the oil by injecting gas at a fixed rate over the entire length of an areal slab. The initial pressure in the reservoir is fixed at 365 to ensure that the displacement is near the MMP. Oil is produced at the other end of the slab at a fixed bottom hole pressure of 360 atm. The displacement process was modeled by three different approaches: (1) the finite difference based simulator Eclipse 300, (2) streamlines combined with a dispersion-free analytical 1D solution, and (3) by combining streamlines with a dispersed (100 gridblocks) numerical 1D solution. The predicted recoveries and gas to oil ratios are given in Fig. 1.

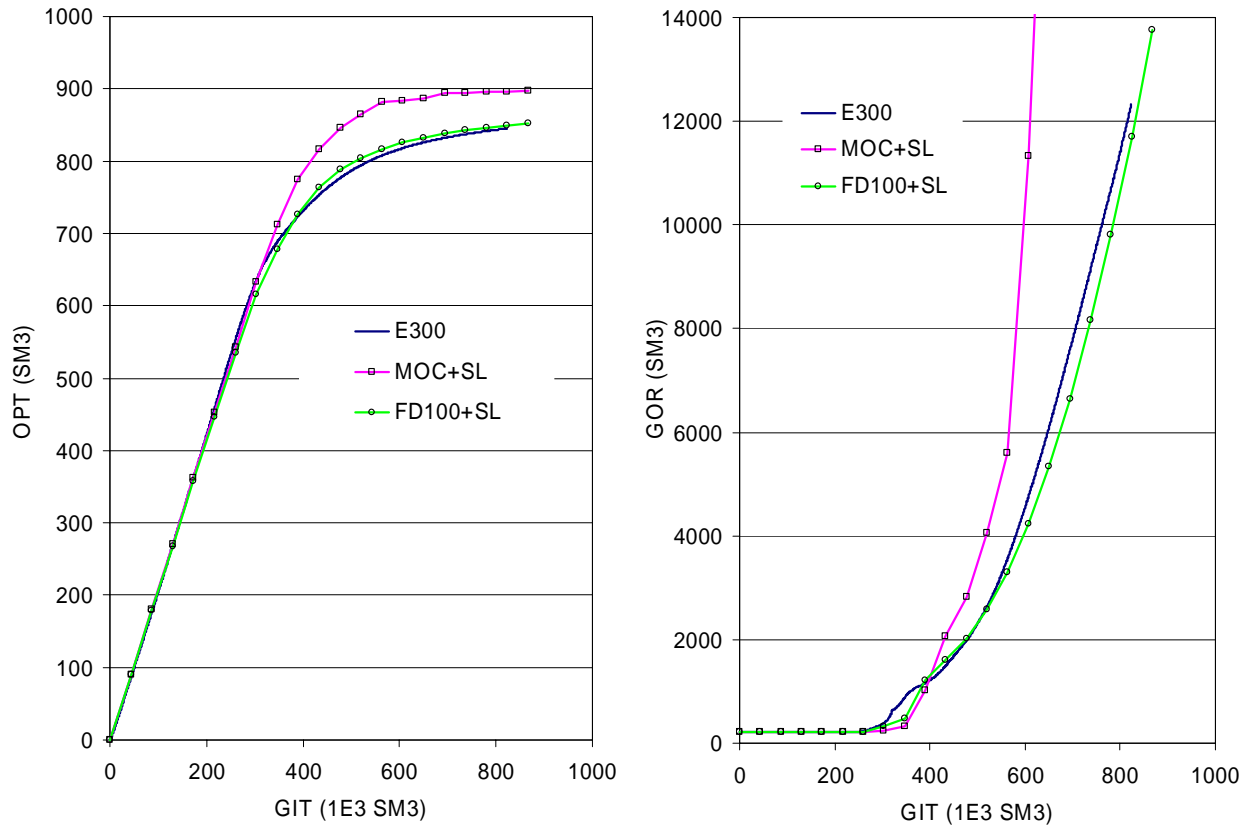


Figure 1: Total oil production (OPT) and gas to oil ratio (GOR) as a function of total gas injected (GIT). (1) E300, (2) analytical 1D solution (MOC) and streamlines (SL), (3) dispersed 1D numerical solution (FD100) + SL.

Fig.1 shows that the recovery predicted by E300 is slightly lower than the equivalent streamline simulation using a dispersion-free 1D solution, whereas the streamline

simulation using a numerical 1D solution (dispersed) is in excellent agreement with E300. This is due to the effects of numerical dispersion in E300¹³. The effects of numerical dispersion are more clearly demonstrated in Fig. 2.

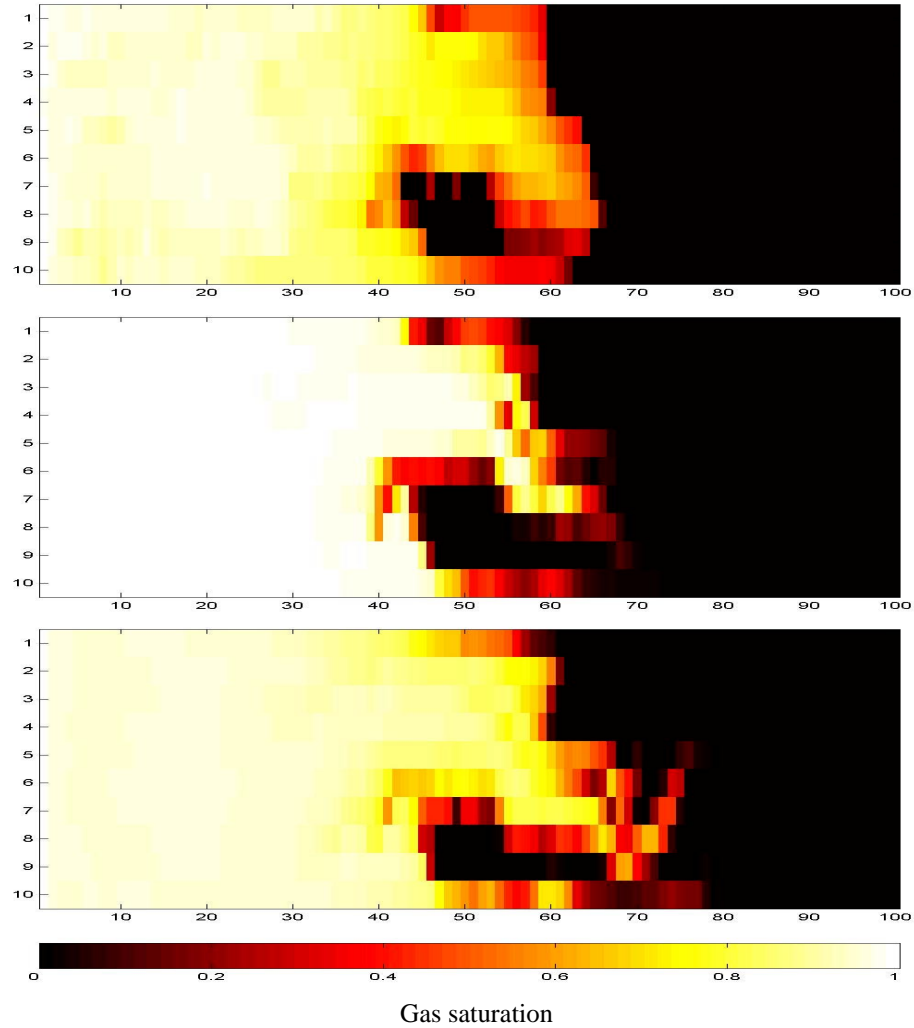


Figure 2: Saturation distribution after 0.4PVI: (1) E300 (top), (2) MOC+SL (middle) and (3) FD100+SL (bottom). CPU requirements: E300 404 sec, 1D+SL: 5 sec.

Fig. 2 shows a snapshot of the displacement process in terms of the saturation distribution at $t = 400$ days. Comparison of the saturation maps from the three simulations indicates that the FD simulation predicts lower local displacement efficiency of the miscible injection process. Numerical dispersion smears out the displacement front resulting in a later breakthrough and a better areal sweep, but with reduced local displacement efficiency in the swept zone. As we add dispersion in the 1D solution to be mapped along streamlines, the saturation distribution approaches more closely that of the E300 simulation. The areal sweep is slightly different, however, due to viscous cross-flow not

fully captured by the streamline approach. However, the CPU requirement for the FD and the SL approaches are very different. Using the approach streamline offers a speed-up of about 80, nearly two orders of magnitude, for this small computational grid.

6.2 Three-Dimensional Example

In the second example we displace the reservoir fluid in a 3D formation corresponding to a quarter of a five spot pattern. The injector and producer are completed over the entire column of the formation. The initial reservoir pressure is fixed at 365 atm and the gas is injected at a fixed rate of 0.001 PV/day. Oil is produced at a fixed bottom hole pressure of 360 atm. The current version of the streamline code does not include gravity. Hence, for the purpose of testing the use of analytical 1D solutions with streamlines and for studying the effects of viscous cross-flow and numerical dispersion, we neglect gravitational forces. Total oil production and GOR predicted by E300 and SL simulation for this displacement process are shown in Fig. 3

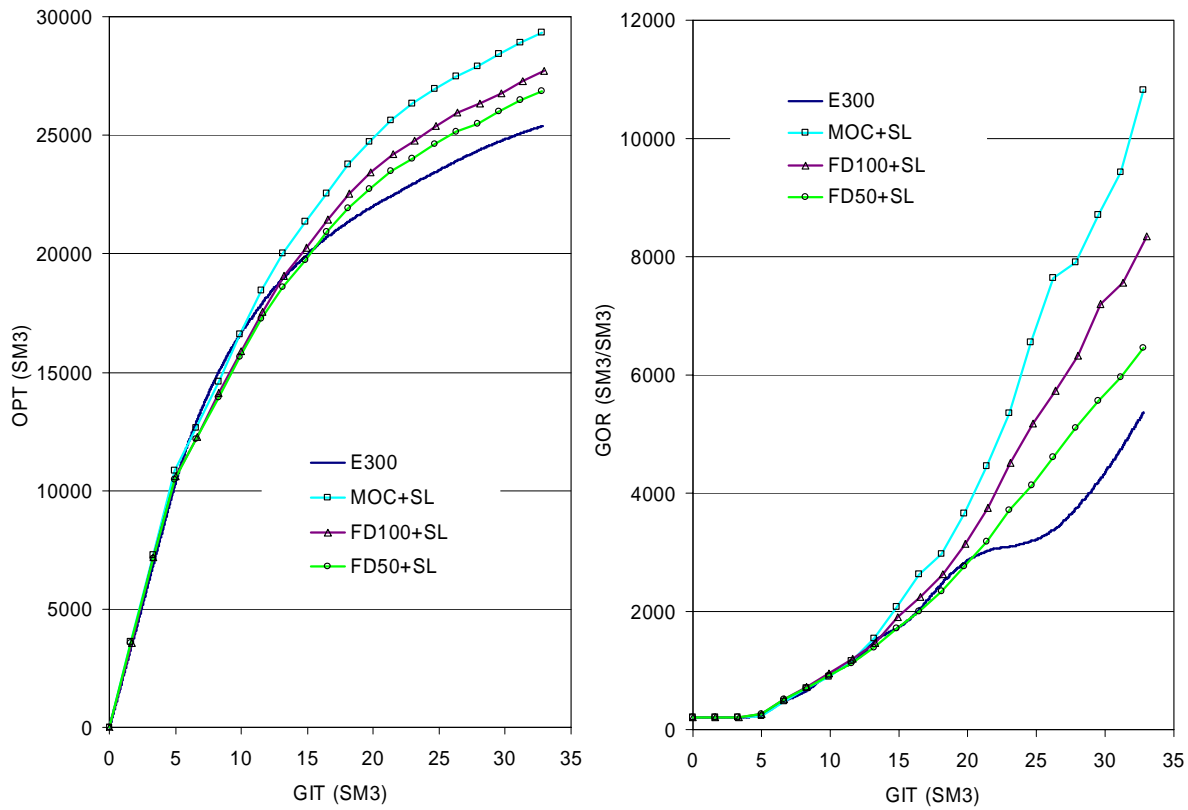


Figure 3: Total oil production (OPT) and gas to oil ratio (GOR) vs. total gas injected (GIT). (1) E300, (2) MOC+SL, (3) FD100+SL and (4) FD50+SL.

The 3D-displacement process summarized in Fig. 3 shows behavior similar to the 2D example. The recovery of oil in place predicted by the E300 simulation is somewhat lower than what is predicted by the MOC+SL method. Again, as we add dispersion to the 1D solutions (FD100 = 100 and FD50 = 50 gridblocks) used in the SL simulations, the predicted oil production and GOR approaches more closely the result of the E300 simulation. Saturation distributions after 200 days of injection are shown in Fig. 4 for (a) Streamline simulation using a dispersed 1D solution (FD50) and (b) E300.

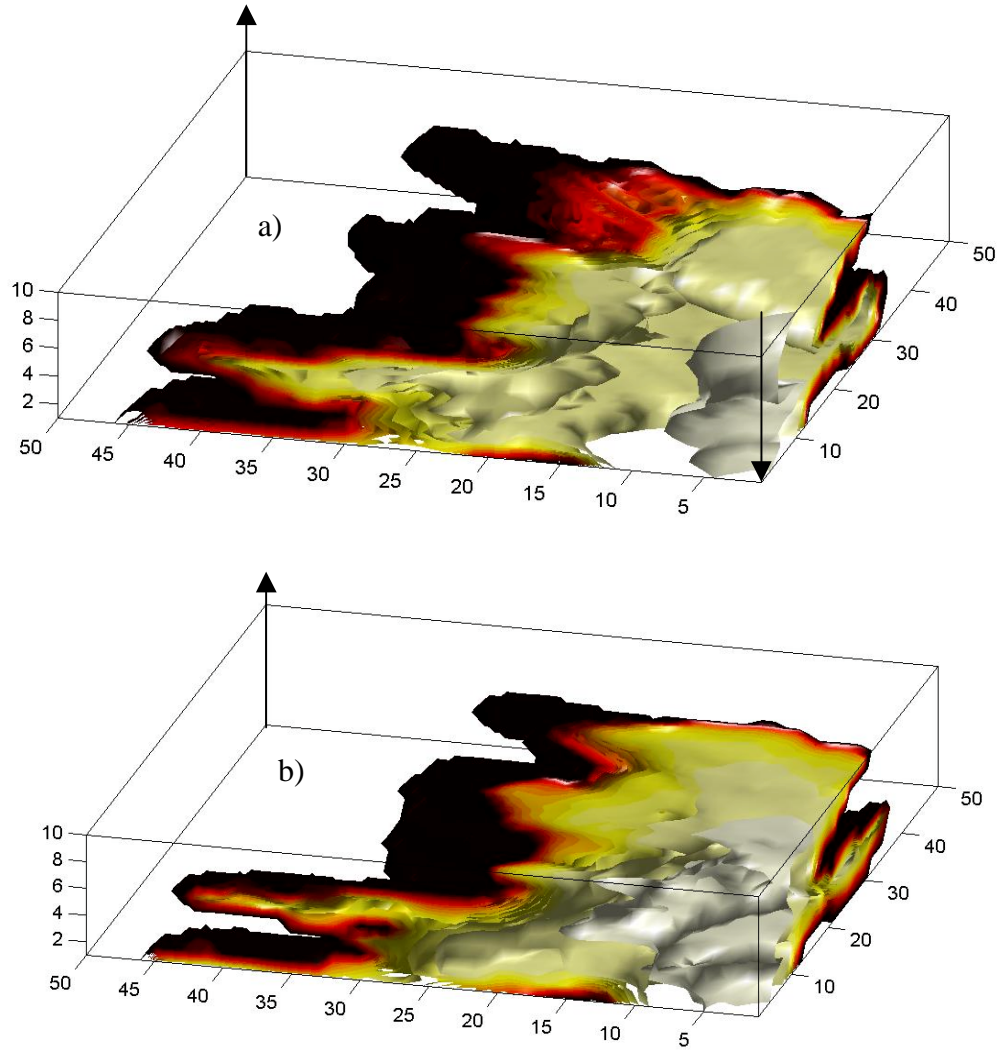


Figure 4: 3D simulation. Saturation distribution after 200 days of injection:
(a) E300 and (b) FD50+SL. CPU (2 PVI): E300 = 2083 min, 1D+SL = 2.55min.

Figs. 3 and 4 indicate that the numerical dispersion in E300 smears out the displacement front, resulting in a later breakthrough and lower GOR but ultimately in a lower overall recovery. The advantage of using streamlines for compositional simulation is more

evident in this example. For this relatively small problem, compared to field scale, the speed-up is close to three orders of magnitude.

7. Discussion and Conclusions

In the previous sections, results from combining analytical dispersion-free 1D solutions with streamline simulation have been presented. The major assumption of this work is that the time step from t to $t+\Delta t$ is approximated accurately by a time step from 0 to $t+\Delta t$. If the flow in a displacement process is not dominated by gravity, this assumption appears to be an excellent approximation. However, for displacements where gravity plays an important role and components move in directions not aligned with the streamlines, the assumption can not be expected to provide accurate results. For gravity dominated flow, the assumption of constant initial and injection conditions used to generate the analytical 1D solutions needs to be relaxed. For gas cycling in condensate fields the effects of gravity are less significant than for oil/gas problems and the suggested approach is expected to produce accurate results at significantly reduced CPU time requirements.

The examples and analysis presented in this report establish that:

1. Dispersion-free 1D solutions to multicomponent gas injection problems generated by the method of characteristics can successfully be combined with streamline methods to predict the performance of a given multicomponent gas injection process.
2. For studies of displacement processes with low impact of gravitational forces the technology is now available for using compositional streamline simulation. Possible speed-ups of 2-3 orders of magnitude relative to conventional FD simulation are available, for small grids of 100 to 25,000 grid blocks. Larger speed-ups will be observed for larger grids.
3. Compositional streamline simulation using dispersion-free 1D solutions offers a limiting case solution to gas displacement processes. Conventional finite difference simulations offer another (dispersed) limiting case.
4. For gas-oil displacement problems where gravity can not be neglected, the compositional streamline simulation using analytical 1D solutions is not yet fully developed but is an area of active research.

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