

**Prediction of External Corrosion  
for Steel Cylinders—2002 Report**

**Rick Schmoyer  
and  
B. F. Lyon**

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**PREDICTION OF EXTERNAL CORROSION  
FOR STEEL STORAGE CYLINDERS—2002 REPORT**

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and  
B. F. Lyon**

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## EXECUTIVE SUMMARY

The United States Department of Energy (DOE) manages the UF<sub>6</sub> Cylinder Project. The project was formed to maintain and safely manage the depleted uranium hexafluoride (UF<sub>6</sub>) stored in approximately 50,000 carbon steel cylinders. The cylinders are located at three DOE sites: the East Tennessee Technology Park (ETTP) site in Oak Ridge, Tennessee; the Paducah Gaseous Diffusion Plant (PGDP) in Paducah, Kentucky, and the Portsmouth Gaseous Diffusion Plant (PORTS) in Portsmouth, Ohio.

The System Requirements Document (SRD) (LMES 1997a) delineates the requirements of the project, and the actions needed to fulfill these requirements are specified in the System Engineering Management Plan (SEMP) (LMES 1997b). This report documents activities that in whole or part satisfy specific requirements and actions stated in the UF<sub>6</sub> Cylinder Project SRD and SEMP with respect to forecasting cylinder conditions. The results presented here supercede those presented by Lyon (1995, 1996, 1997, 1998, 2000), and Schmoyer and Lyon (2001). Many of the wall thickness projections made in this report are conservative, because they are based on the assumption that corrosion trends will continue, despite activities such as improved monitoring, relocations to better storage, painting, and other improvements in storage conditions relative to the conditions at the times most of the wall thickness measurements were made.

For thin-wall cylinders (design nominal wall thickness 312.5 mils), the critical minimum wall thicknesses criteria used in this report are 0 (breach), 62.5 mils, and 250 mils (1 mil = 0.001 in.). For thick-wall cylinders (design nominal wall thickness 625 mils), the thickness criteria used in this report are 0, 62.5 mils, and 500 mils. The criteria triples are preliminary boundaries identified within the project that indicate (1) loss of material (UF<sub>6</sub>), (2) safe handling and stacking operations, and (3) standards for off-site transport and contents transfer criteria, respectively. In general, these criteria are based on an area of wall thinning. However, the minimum thickness predicted in this report is essentially for a point—an area of about 0.01 square inches—because the thickness measurements on which the predictions are based are essentially for points. For thicknesses criteria greater than zero, conclusions based on minimum point thicknesses are conservative. Because of the interaction of UF<sub>6</sub> with atmospheric moisture and steel, a point breach would deteriorate in a year to one-inch diameter hole (DNFSB 1995), however, and so small-area approximations should be close for the breach criteria.

The most recently collected data, entered into the corrosion model database and not available for the previous report (Schmoyer and Lyon 2001), consists of evaluations of wall loss of 48" thin-wall cylinders: 301 cylinders at Paducah, 101 at ETTP, and 139 at Portsmouth; 14 thick-wall cylinders at Portsmouth; and 99 model 30A cylinders at Paducah. However, because of missing values, repeated measures on the same cylinders, outliers, and other data problems, however, not all of these measurements are necessarily used in the corrosion analysis.

In several cases, difficulty with the data is also due to a mathematical approach to cylinder corrosion modeling that is used in this report, in Schmoyer and Lyon (2001), and in earlier reports by Lyon. Therefore, an alternative approach is also considered in this report. In previous reports, minimum wall thicknesses have been modeled indirectly through separate models of initial thickness and maximum pit depth. In order to estimate minimum wall thicknesses, the initial thickness and maximum pit depth models are combined using mathematics that assumes independence of the statistical distributions of the initial thicknesses and maximum pit depths. Initial thicknesses are modeled from wall thickness maxima measured at relatively uncorroded wall areas of each cylinder. Maximum pit depths for each cylinder are estimated as differences between the initial thickness estimates and measured minimum wall thicknesses. The pit depth maxima estimates are modeled as a function of age and a cylinder grouping based on location.

This indirect modeling approach obviously depends on the maximum wall thickness measurements and how well they emulate original thicknesses. In addition to statistical problems such as high variability typical of statistical maxima, maximum wall thickness can in certain cases be a poor substitute for initial thickness. For example, when corrosion is relatively uniform over a cylinder's surface, the maximum wall thickness tends to be closer to the minimum wall thickness than the initial thickness. These difficulties have lead to results that sometimes seem inconsistent from one cylinder group to the next, and results, such as apparently increasing corrosion rates and even apparently decreasing corrosion, which are inconsistent with theory. High variability of model estimates coupled with conservative approximations have lead to projections that seem too conservative.

Therefore, in addition to the indirect modeling approach, an alternative model is also considered in this report, which is direct in the sense that minimum thickness, which is what is both measured and ultimately estimated, is modeled directly as a function of cylinder age and grouping, and initial thickness estimates, rather than indirectly through separate models of initial thickness and maximum penetration. The direct approach does not depend critically on maximum thickness measurements and does not entail the assumption of statistical independence of maximum pit depths and initial thicknesses (which might fail for example if steel quality and the initial thickness are correlated). The direct approach also admits better incorporation of the information that there is zero corrosion at age zero; the indirect approach does not make good use of this because, as a lognormal model, zero-depth pits are theoretically inadmissible.

Projecting cylinder conditions into the future on the basis of data collected with different goals, sampling schemes, and measurement methods is a difficult task—a task the limitations of which should be understood. Tentatively, the direct model appears to fit the UT cylinder thickness data better than the indirect model, but that conclusion is preliminary and based on judgment. Therefore, both models are used in this report to make forecasts about cylinders. Summaries of projections of numbers of cylinders likely to fail the various minimum thickness criteria are in Table 13 of this report for the indirect approach and in Table 14 for the direct approach.

Both the direct and indirect corrosion models point to bottom rows of C-745-G yard at PGDP and K-1066-K yard at ETPP as the cylinders most at risk. According to the indirect model, the next most at-risk groups are the Portsmouth thin skirted (i.e., at the head/skirt interface) bottom-row cylinders and the Paducah model 30As. The next most at-risk groups according to the direct model are the Portsmouth thin skirted cylinders, both top and bottom. For the direct model, differences among the cylinder groups are not nearly as great as the indirect model suggests. Very few of the thick-wall cylinders show any indication of falling below even the 500 mil thickness criterion. Although corrosion increases over time, these conclusions pertain to both near-term (e.g., FY02) and longer-term projections (e.g., FY2020).

# 1. INTRODUCTION

The United States Department of Energy (DOE) manages the UF<sub>6</sub> Cylinder Project. The project was formed to maintain and safely manage depleted uranium hexafluoride (UF<sub>6</sub>) stored in approximately 50,000 carbon steel cylinders stored at three DOE sites: the East Tennessee Technology Park (ETTP) site in Oak Ridge, Tennessee, the Paducah Gaseous Diffusion Plant (PGDP) in Paducah, Kentucky, and the Portsmouth Gaseous Diffusion Plant (PORTS) in Portsmouth, Ohio.

The System Requirements Document (SRD) (LMES 1997a) delineates the requirements of the project, and the actions needed to fulfill these requirements are specified in the System Engineering Management Plan (SEMP) (LMES 1997b). The report presented herein documents activities that in whole or part satisfy specific requirements and actions stated in the UF<sub>6</sub> Cylinder Project SRD and SEMP with respect to forecasting cylinder conditions. The wall thickness projections made in this report are based on the assumption that the corrosion trends noted will continue. Some planned activities may substantially reduce rates of corrosion, in which case the results presented here are conservative. This report is intended to supercede previous reports by Lyon (1995, 1996, 1997, 1998) and Schmoyer and Lyon (2001).

**System Requirement 1.2.2** states that performance shall be monitored and evaluated to identify potential risks. The related **SEMP Action 2.1.2** is to model corrosion to project cylinder integrity. This report establishes techniques for modeling corrosion rates used in the project to forecast cylinder wall thickness conditions in the future.

**System Requirement 4.1.2** calls for monitoring cylinder conditions. The related **SEMP Action 3.1.2** is to statistically determine the baseline condition of cylinder populations by obtaining quantitative data. This report contains the statistical method used in the project to apply the available quantitative data to cylinder populations. Populations have been established on the basis of historical storage locations (yard and position) and similarity of quantitative data. Wall thickness and corrosion pit depth data have been collected for several subpopulations of cylinders.

**System Requirement 4.2.2** further states that cylinder conditions shall be forecast to direct surveillance and maintenance resources. **Technical Requirement 4.2.2a** is that specific information, as determined by the project, shall be tracked to project the current and future conditions of the system. In addition, **Technical Requirement 4.2.2.b** entails the development of mechanisms to consolidate information for summary-level decision making. **SEMP Action 2.2.1** is to integrate cylinder condition elements to be forecast with cylinder categorization. **SEMP Action 3.1** is to forecast cylinder conditions using parameters identified. Wall thickness, the subject of this report, is one parameter identified in the project to forecast cylinder conditions. The available wall thickness data is used to forecast future cylinder conditions.

**SEMP Action 3.1.1** is to project the number of non-compliant cylinders. The disposition of any particular cylinder for storage, handling, and transfer is based on the condition of the cylinder, where “condition” is ultimately reflected by the minimum wall thickness of a cylinder. The wall thickness criteria used in this report, 0, 62.5, and 250 mils for thin-wall cylinders, and 0, 62.5, and 500 mils for thick-wall cylinders, are preliminary limits identified within the project respectively for (1) loss of material, (2) safe handling and stacking operations, and (3) standard off-site transport and contents transfer. In general, these criteria are based on area of wall thinning, rather than minimum thickness at what is essentially a point—an area of about 0.01 square inches—as used in this report. For thicknesses criteria greater than zero, conclusions based on minimum point thicknesses are, in this respect, conservative. Because of the interaction of UF<sub>6</sub> with atmospheric moisture and steel, a point breach would deteriorate in a year to one-inch diameter hole (DNFSB 1995), however, and so small-area approximations should be close for the breach criteria.

In addition to the modeling approach used in previous versions of this report, an alternative approach is also considered. In the past, minimum wall thickness has been modeled indirectly, through separate models of maximum pit depth and initial thickness. In the alternative approach, minimum wall thicknesses are modeled directly as measured, without converting them to estimates of maximum penetration by subtraction from initial thickness estimates.

The two general approaches to modeling cylinder wall thickness are discussed in Section 2. A history and summary of cylinder wall thickness data collection at Oak Ridge, Paducah, and Portsmouth is presented in Section 3. Section 4 is about model fitting with the models discussed in Section 2 and the data discussed in Section 3. As prescribed in SEMP Action 3.1.1, Section 5 contains projections based on the models fit in Section 4. Separate projections are presented for both the direct and indirect modeling approaches. The two models are evaluated and compared in Section 6. Limitations, conclusions, and recommendations are discussed in Section 7.

## 2. APPROACHES TO MODELING CYLINDER WALL THICKNESS

### 2.1 Direct and Indirect Models

The basic problem addressed in this report is to estimate how many cylinders in various cylinder groups will have minimum thickness below  $z$  by time  $t$ , where  $z$  is a specified thickness criterion. For a cylinder randomly selected from a group, let  $M(t)$  denote the minimum wall thickness at time  $t$ .  $M(t)$  is random because of variations in initial thickness (manufacturing variability), the steel substrate, the corrosion process, and storage conditions.

Let  $P(t) = P(M(t) < z)$  denote the probability that  $M(t)$  is less than  $z$ . For a group of  $N$  cylinders, the expected number of cylinders with minimum thickness below  $z$  at time  $t$  is  $N \times P(t)$ . Because the number of cylinders in a group at risk is affected by maintenance (e.g., painting),  $N$  may change over time. Therefore  $P(t)$  is of interest in its own right, in addition to  $N \times P(t)$ .

One approach to the problem is to measure cylinder wall thicknesses, deliberately trying to locate the actual thickness minima, and to record the observed minima. By doing this for cylinders of various ages and from various groups, the thickness data can be used to model the minima as a function of age, cylinder group, and initial thickness estimates. Initial thickness estimates are based on nominal thickness data (e.g., from design sheets), as well as maximum wall thickness measurements, and judgment. In this report, this approach will be called “direct,” because minimum thicknesses are what is observed, and because the objective is to project minimum thicknesses. An indirect approach is described next.

Let  $C_0(x)$  denote the initial wall thickness at a location  $x$  on a cylinder, and let  $P(t,x)$  denote the amount of corrosion that has occurred at location  $x$  by time  $t$ . Then the minimum thickness is

$$M(t) = \min_x \{C_0(x) - P(t,x)\},$$

where the min is over all points  $x$  on the cylinder. Because the thicknesses of the cylinder walls were not measured when they were first delivered, the joint statistical distribution of  $C_0(x)$  and  $P(t,x)$  is unknown. However,

$$\begin{aligned} M(t) &> \min_x \{C_0(x)\} - \max_x \{P(t,x)\} \\ &= C_0 - \max_x \{P(t,x)\} \\ &= C_0 - P(t) \end{aligned}$$

where  $C_0$  is the initial minimum thickness and  $P(t)$  is the maximum penetration at age  $t$ . In the indirect approach,  $M(t)$  is modeled indirectly through separate models of  $C_0$  and  $P(t)$ . By the above inequality, this is a conservative approach, other bias ignored.

The indirect approach is based on two further assumptions: (1) that  $C_0$  can be estimated using current maximum thickness measurements made at relatively uncorroded cylinder areas, and (2) that  $C_0$  and  $P(t)$  are approximately statistically independent. The first assumption could fail possibly, for example, if

corrosion is uniform over the entire cylinder surface. The second assumption could fail, for example, if steel quality and initial thickness are correlated. The second assumption is needed to estimate the statistical distribution of  $C_0 - P(t)$  from separate estimates of the distributions of  $C_0$  and  $P(t)$ .

If the corrosion process has reached a condition in which, whatever the corrosion history may have been, each cylinder is corroding at some relatively constant (over the year) rate, then modeling future corrosion entails determining the current conditions for each cylinder and estimating the current rate. A feature expected of corrosion, however, is that, in general, its rate decreases with time.

For both the direct and indirect approaches, for a given group of cylinders, the total number of cylinders with a minimum thickness below a given value  $z$  at time  $t$  can be estimated using the relation:

$$\text{Expected number of cylinders with minimum thickness below } z \text{ at time } t = \sum_a (\text{number of cylinders of age } a \text{ at time } t) \times \text{Prob}(M(t) < z), \quad (2.1)$$

where the summation extends over all cylinder ages  $a$ .

## 2.2. The Indirect Approach

### Maximum Pit Depth Models

The indirect approach is based on the power law, which has been used in many previous applications of corrosion modeling (e.g., Felieu et al. 1993a; Felieu et al. 1993b; Legault and Preban 1975; Pourbaix 1982; Mughabghab and Sullivan 1989; Romanoff 1957). The power law is  $P(t)=A \times t^n$ , where  $t$  is age, and  $A$  and  $n$  are constants. For  $n < 1$  the power law allows for “leveling off” in corrosion, which is commonly observed. The model parameters  $A$  and  $n$  can be estimated using the log-linear regression model

$$\log(P)=\log(A) + n \log(t) + \text{random error},$$

which is the estimation approach taken in this report. (All logs in this report are natural logs.) Separate regression models are fit for each of fifteen cylinder groupings. The cylinder groupings are discussed in Sections 3 and 4. To do the regressions, maximum pit depth measurements for each cylinder are estimated from minimum thickness measurements and estimates of initial thickness, which are based on maximum wall thickness measurements made for each cylinder. Regressions with cylinder wall thickness data are discussed in Section 4.

According to Pourbaix (1982), Passano (1934) was the first to use the power law relationship in corrosion prediction. This law is considered to be valid for different types of atmospheres (rural, marine, industrial) and a number of materials. The parameter  $A$  can be interpreted as the corrosion in the first year, and the parameter  $n$  represents the attenuation of the corrosion because of the passivation of the material in the atmosphere (Pourbaix, p.115).

The power law model can be related to the mean (age-averaged) corrosion rate, since the mean corrosion rate is given by  $P/t = A \times t^{n-1}$ . If  $n=1$ , this implies that the age-averaged corrosion rate is constant, while if  $n < 1$  (which is typical), the corrosion rate decreases with time. Mechanistic interpretations of  $n$  have also been made (Horton 1964). If  $n=0.5$ , then the relationship is said to be parabolic, with the corrosion rate controlled by diffusion through the rust layer. If  $n < 0.5$ , this implies that the rust layer is showing protective properties, while if  $n > 0.5$ , the rust layer is not protective because of factors that may be preventing the homogeneous thickening of the rust layer. The power law is used in several Department of

Energy models to predict time to breach due to external corrosion for carbon steel containers in soil.

Because estimates of the “leveling off” ( $n < 1$ ) pattern usually expected for the penetration depth can be sensitive to narrow data ranges, outliers, and other data anomalies, the power law approach should be used with caution. In fact, a failure of either the leveling off ( $n < 1$ ) hypothesis or the increasing corrosion ( $n > 0$ ) hypotheses is observed for nine of the fifteen cylinder groups considered in this report (see Section 4), and an alternative model is then applied. The alternative model is the same, except that  $n$  is constrained to be 1 (Lyon 1995, 1996).<sup>1</sup> This inconsistency between data and assumption—the need for the slope-set-to-one-model—was the main motivation for the direct approach.

In order to address the variability inherent in the corrosion process, it will be assumed that the penetration depths are lognormally distributed at each time. This can also be expressed on the log scale as  $\log(P(t)) \sim N(\log(A) + n \log(t), \sigma_L)$ , where  $N(\mu, \sigma)$  denotes a normal distribution with mean  $\mu$  and standard deviation  $\sigma$ . For this model, on the arithmetic scale, the median is equal to  $At^n$ , the mean is  $At^n \exp[0.5\sigma^2]$ , and the standard deviation is  $At^n \exp[0.5\sigma^2] [\exp(\sigma_L^2) - 1]^{1/2}$ . The coefficient of variation (ratio of the standard deviation to the mean) is constant in time and equal to  $[\exp(\sigma_L^2) - 1]^{1/2}$ .

The lognormal assumption has been checked by goodness of fit tests discussed in previous cylinder reports (Lyon 2000). Given that the data consists of what are considered to be maximum pit depths, it would also be natural to apply extreme-value statistics to this problem. Application of the extreme value distribution (without confidence limits) is discussed in several papers and has also been suggested within this project by Rosen and Glaser (1996). The basic premise underlying the extreme value theory is that the distribution of extremes, under rather general assumptions, should have a specific (parametric) form. Extreme value models are being investigated for the cylinder corrosion. However, for the present indirect-model analyses, lognormal-based methods are used because (1) lognormal-based confidence limits are more straightforward, (2) due to the substantial variability in the data, confidence limits are crucial in the analysis, and (3) the lognormal distribution has many of the same qualitative properties as the extreme value distribution.

### Initial Thickness Models

A stochastic model is used for initial thicknesses, in the indirect modeling approach, because of concerns that variability in initial thickness could be a critical factor (Rosen and Glaser 1995). The initial thickness is approximated using a truncated normal distribution (see Johnson and Kotz, 1970, p 81.) A truncated normal random variable has the distribution of a normal random variable conditional on the normal variable being in the truncated range. The parameters for the truncated normal distribution are estimated from design specifications and regressions with maximum wall thickness data. With the exception of the data at the head/skirt interface (discussed below), this data consists of wall thickness measurements made on the cylinders evaluated in relatively uncorroded regions of the cylinder. These measurements were made using either an automated scanner or a hand-held probe, depending on the particular data set. This initial thickness estimate does not include any general (i.e., uniform) corrosion that may have occurred across the entire cylinder surface. Regressions with maximum wall thickness are discussed in Section 4.

It has been found that the wall thickness is typically much larger in the head/skirt interface than design specifications would indicate. When the first head/skirt thickness data was collected, five manual measurements were made on the cylinder head at a distance of about one inch from the cylinder head/skirt weld (Lykins and Pawel 1997). The initial thickness was set to the measured thickness at the center of the cylinder head plus 10 mils. The extra factor was used because it was found that on several model 48G

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<sup>1</sup>Because,  $n = 0$  is untenable from both a theoretical and practical perspective, the same  $n = 1$  (slope-set-to-one) alternative will be used if the power-law model estimate of  $n$  is either greater than 1 or less than 0.

thin-wall cylinders, the wall thickness was usually 10-20 mils less than that found beneath the plug. This difference was attributed to the forging process to form the contour of the head. That method is not used here, however, because it was found that it does not guarantee that the initial thickness is larger than the measured wall thickness in the head/skirt area. Instead, the maximum of the five measurements plus 10 mils is used as an approximation to the initial thickness.

The design range for the initial thickness of thin-wall cylinders is from 302.5 to 345.5 mils for thin-wall and 615 to 655 mils for thick-wall cylinders. With the exception of cylinders purchased very recently, there is no way to know the distributions of initial thicknesses. The data collected so far suggest that the wall thickness on relatively uncorroded areas of a cylinder is usually larger than the nominal design thickness. In the indirect modeling approach, the lower bound of the range used to model the initial thickness is set to the lower bound of the design specifications: 302.5 mils for thin-wall cylinders, 615 mils for thick-wall cylinders. The upper bound is not taken directly from the design specifications, but is instead set to the largest observed value of the maximum wall thickness.

### 2.3. The Direct Approach

Direct models were investigated as an alternative to the indirect approach, because of anomalous results based on the indirect approach, due in part to the high variability that has been seen in cylinder thickness data. Modeling results for both approaches are discussed in Section 4. There are other reasons, however, besides just data variability, for exploring alternatives to the indirect approach. For example, because of the thermal inertia of the cylinders, literature data for atmospheric corrosion of steel does not necessarily apply to cylinder corrosion modeling.<sup>2</sup> The power law may not apply.

The direct models considered in this report are of the form

$$M(t) = \alpha \times (\text{Initial Thickness Estimate}) + \beta(\text{group}) \times \log(t) + \text{random error}, \quad (\text{for } t > 1)$$

where  $t$  ( $t > 1$ ) is the age in years,  $\alpha$  is a model parameter, and “ $\beta(\text{group})$ ” denotes a model parameter, one for each cylinder group. The cylinder groups are the same for both the indirect and direct approaches, except that thin-wall cylinders are not included in the thick-wall groups (see Sections 3 and 4). According to this model,  $M(1) = \alpha \times (\text{Initial Thickness Estimate})$  is the mean thickness at one year of age, which is essentially, though not exactly, the initial thickness. Thus, initial thickness estimates, which are computed from design specifications, maximum thickness measurements, and judgment, are incorporated into the direct model as predictors, but are further refined by fitting the parameter  $\alpha$ .

Unlike the indirect approach, in which a separate model (with its own intercept and slope) is fit for each cylinder group, in the direct approach considered here, there is only one model. However, the single direct model has separate parameters for each cylinder group. For the indirect model, the total number of parameters (including standard deviations) is three times the number of groups, not including additional estimated parameters in the initial thickness distributions. For the direct model, the total number of parameters is the number of groups plus two (including one for the standard deviation). Having fewer parameters can be either a disadvantage or an advantage. Models with fewer parameters are less flexible, but if they fit, less flexibility reduces the likelihood that anomalous data will lead to anomalous modeling results, which is a difficulty in the indirect approach.

For either the direct or indirect approaches, how random error terms and their variances are modeled can have a critical effect on corrosion projections. Regardless of the mean minimum thickness, if the variance of the (true) error term is high enough, there will always be cylinders whose minimum thicknesses are below

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<sup>2</sup>Steve Pawel, personal communication.



any of the various thickness criteria. This applies to any underlying model, whether direct or indirect. Because, in the direct approach, multiple cylinder groups are handled with one model, high variability in one group affects projections for all groups. Therefore several ways of relating variance to age (and consequential weighted regressions) were considered for the direct model, before deciding on a particular approach. Further, the random error term in the direct model is not assumed to be lognormal or, in fact, to have any particular distribution. A nonparametric method is used instead. This is discussed in Section 4.

In the direct model and in the indirect model with  $0 < n < 1$ , corrosion is assumed to be a concave increasing function of age. In the indirect model the corrosion rate is  $dP(t)/dt = nP(t)/t$  (with a different  $n$  for each cylinder group). Thus the corrosion rate depends on the penetration depth. In the direct model, on the other hand, the corrosion rate is  $-dM(t)/dt = \beta(\text{group})/t$ , which does **not** depend on the penetration depth. This is intentional—to avoid having to estimate the penetration depth, which the maximum thickness data does not seem to support well. However, there are other reasons for this formulation. One reason is the cause of penetration. Penetration depths in the power law model are due to corrosion, not nicks, cuts and handling wear typical of many of the UF<sub>6</sub> cylinders. Another reason is that both  $M(t)$  and  $P(t)$  are extremes (i.e., the **minimum** thickness and the **maximum** penetration). The rate of change of an extreme (e.g., maximum penetration) does not have to satisfy the same relationship as the rate for an individual element, because which particular element is an extreme can change. For all of these reasons, and because it seems to fit, the direct model was considered as a possible alternative to the indirect approach.

## 2.4. Confidence Limits

The method used to calculate confidence limits for the indirect approach involves calculating a convolution of the penetration-depth and initial-thickness statistical distributions, which are assumed to be (statistically) independent. This method is discussed in Appendix B. Confidence limits for the direct approach are based on a nonparametric analog of usual normal-theory confidence limits for individual regression predicted values (Schmoyer 1992). This approach is completely different from the approach to confidence limits for the indirect model. Thus, although the direct and indirect approaches can be compared in general, their confidence limits are not directly comparable. The indirect model confidence limits, which are based on conservative approximations, have in the past seemed too conservative to be useful.

### 3. ULTRASONIC THICKNESS DATA

This section summarizes the ultrasonic thickness (UT) measurement data used in the corrosion models. The previous version of this report (Schmoyer and Lyon 2001) was based on wall thickness data that had been collected through FY2000. This report incorporates additional data collected in FY01.

Two main types of data are used:

- (1) data for predicting overall minimum wall thickness at a point, not including the head/skirt interface
- (2) data for predicting minimum wall thickness at the head/skirt interface

In most cases both minimum and maximum wall thicknesses estimates were measured. The minimum thickness measurements are plotted in Figures 16-30 in Appendix A.

In addition, the data from two breached cylinders discovered in 1992 in K-1066-K yard is used in the corrosion models. External corrosion was considered to be the cause of these breaches. There have been several other breaches discovered (two at ETPP in 1992, two at Portsmouth in 1990, and one at PGDP in 1992), but it was concluded that those breaches were induced by mechanical damage at the time of stacking rather than by external corrosion.<sup>3</sup> Therefore, the only breach data used in the current corrosion modeling is for the two 1992 K-1066-K yard cylinders. Including results for breaches judged due to external corrosion but not sampled randomly slightly biases conclusions toward greater projected likelihoods of failing the various thickness criteria, including breaches.

Tables 1, 2, and 3 summarize the data collected by fiscal year. Whether the various data sets constitute random samples is indicated in the tables. Random sampling is important because compromising it introduces biases into inferences about sampled populations. (An initial sampling plan (Lyon and Lykins 1996) was prepared that included random sampling, and recommended that it be updated to more efficiently fit within the current budget and logistical constraints.) Table 3 summarizes the data collected at the head/skirt interface. Tables 4 through 7 show additional details about the age ranges and yard locations of the cylinders that were measured. In these tables, yard (i.e., location) designations should be regarded as approximations. Cylinders can and are moved during their lifetimes, but only one such location is used to represent each cylinder for the purpose of collecting them into groups for modeling. Complete historical records are usually not available in an electronic form.

#### 3.1. Summary of Measurement Methods

Hand-held UT methods (Lykins and Pawel 1997) were used to collect all wall thickness data except as follows: Several of the data collections used an automated scanner called a P-Scan system (see Schmidt et al 1996 for a description of the equipment). The first effort was performed during 1994 at K-1066-K yard at ETPP. The second was performed during the fall of 1995 at the PGDP, the third was conducted between March and September 1996 at both the Portsmouth and PGDP sites as part of the cylinder relocation efforts. The most recent effort was conducted during FY97, primarily at Portsmouth. The wall thickness data consists of measurements made with the automated scanner for a square region of width and height of about 2.54 mm (0.1 in). The wall thickness data used for the initial thickness consisted of either data collected with the automated scanner near where the maximum pit occurred (with a width and height of approximately the same size as the pit data), or was collected using a hand-held probe for a circular region with a radius of about 2 mm (0.08 in).

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<sup>3</sup> The breaches were caused by a lifting lug of an adjacent cylinder that induced a small crack near a stiffening ring.

**Table 1. Chronological summary of data collected FY92-97—not including the head/skirt interface**

	FY92	FY94	FY95	FY96		FY97	
Thin-Wall Cylinders (nominal wall thickness of 312.5 mils)							
Yard(s)	K1066K	K1066K	C745B/F/K/L	C745F/G/K	X745C	C745G/L	X745C
Method	Visual	P-Scan	P-Scan	P-Scan	P-Scan	P-Scan	P-Scan
Number cylinders	2	138	94	261	473	3	85
Random sampling	No	No	No	No	Yes	No	Yes
Com- ment	Bre- ached Cylin- ders	Intent was that cylinders be selected randomly, but limitations were imposed by scanner	Cylinders selected based on judgement of personnel (Blue, 1995a).	Intent was that 10% of cylinders moved would be evaluated, but space restrictions for selected cylinders prevented this	10% of cylinders moved during FY96 were randomly selected		
Thick-walled Cylinders (nominal wall thickness of 625 mils)							
Yard(s)				C745C	X745C		
Method				P-Scan	P-Scan		
Number cylinders				2	135		
Random sampling				No	Yes		
Com- ment					See above for thin-wall cylinders.		
(No Model 30A Cylinders were sampled during FY92-97.)							

**Table 2. Chronological summary of manual UT data collected FY98-2001—not including the head/skirt interface**

	FY98			FY99			FY2000			FY01		
Thin-Wall Cylinders (nominal wall thickness of 312.5 mils)												
Yard(s)	K1066K	PAD	PORTS	K1066K	PORTS	PAD	K1066K	PAD	PORTS	K1066E, K	PGDP, C, F, K	X745E
Number cyl-inders	40	200	142	30	141	200	58	100	130	101	301	139
Random samp-ling?	Yes	Yes	Yes, but some cylinders were scanned previously	Yes for 29	Yes	Yes	Yes	Yes	Yes, but some cylinders were scanned previously	Yes	Yes	Yes, but some cylinders were scanned pre-viously
Com-ment		Only “good” locations on cylinders were eval-uated, contrary to imple-mentation plan.	Delivery year is unknown for nine cylinders.	One cylinder was chosen by field person-nel due to its known ground contact.	Meth-od-ology used to pick cylin-ders was deve-loped by PORTS person-nel		Top/bot-tom status is not ascertained (but is not used in the current modeling).					

**Table 2 (cont'd). Summary of manual UT data collected FY98-2001—not including the head/skirt interface.**

	FY98			FY99			FY2000			FY01		
Thick-Wall Cylinders (nominal wall thickness of 625 mils)												
Yard(s)			X745E	X745C					X745E			X745E
Number cyl-inders			2	11					25			14
Random samp-ling?				Yes, but some scanned pre-viously					Yes, but some scanned previously			Yes, but some scanned pre-viously
Com-ment												
Model 30A Cylinders												
Yard(s)				—							A, D	
Number cyl-inders				100							99	
Random samp-ling?				No							Yes	
Com-ment				Prelimin-ary list of cylinders to be evaluat-ed was used in place of corrected list.								

**Table 3. Summary of data for estimating wall thickness at head/skirt interface**

	FY97	FY2000	FY01
<b>Thin-Wall Cylinders ( 312.5 mils)*</b>			
Yard(s)	X-745-C	X-745-E	X-745-E
Method	Manual UT	Manual UT	Manual UT
Number of cylinders	233	87	14
Age Range (yr)	38-40	42-44	47-49
Random sampling?	Yes, but some scanned previously	Yes, but some scanned previously	Yes, but some scanned previously
<b>Thick-Wall Cylinders ( 625 mils)*</b>			
Yard(s)	PORTS	PORTS	X-745-E
Method	Manual UT	Manual UT	Manual UT
Number of cylinders	115	23	99
Age Range (yr) when evaluated	36-45	46-48	43-45
Random sampling?	Yes, but some scanned previously	Yes, but some scanned previously	Yes, but some scanned previously

\*Nominal wall thickness

**Table 4. Counts of thin-wall cylinders measured FY92-97 (and age range of cylinders when evaluated)—not including data at the head/skirt interface**

Site	Yard	Row	Fiscal Year				
			'92	'94	'95	'96	'97
ETTP	K1066K	Top	1 (29) <sup>A</sup>	60 (31-36)			
		Bottom	1 (34) <sup>A</sup>	55 (31-38)			
PGDP	C-745-B	Top			4 (39)		
		Bottom			2 (39)		
	C-745-C and D	Top					
		Bottom					
	C-745-F	Top			13 (31-36)		
		Bottom			13 (32-36)	6 (36-37)	
	C-745-G	Top			9 (33-36)	137 (18-37)	
		Bottom			17 (33-36)	98 (5-37)	2 (37)
	C-745-K and L	Top			17 (13-18)		
		Bottom			25 (14-19)	6 (16-37)	1 (38)
PORTS	X-745-C	Top				221 (6-40)	56 (20-36)
		Bottom				252 (6-40)	29 (8-36)
	X-745-E	Top					
		Bottom					

<sup>A</sup>These are the two cylinders that were determined to breach from external corrosion (Barber et al 1994)..

**Table 5. Counts of thin-wall cylinders measured FY98-01 (and age range of cylinders when evaluated)—not including data at the head/skirt interface**

Site	Yard <sup>A</sup>	Row	Fiscal Year			
			'98	'99 <sup>B</sup>	2000	'01
ETTP	K-1066-E					24 (25-43)
		Top	21 (40-42)	13 (36-42)		
	K-1066-K	Bottom	19 (40-42)	17 (36-42)	58 (37-42)	76 (38-43)
PGDP	C-745-B	Top				61 (13-45)
		Bottom				56 (13-45)
	C-745-C and D	Top		1 (25)	1 (12-12)	7 (24-25)
		Bottom		7 (24-30)	1 (18-18)	77 (20-41)
	C-745-F	Top		10 (21-40)	5 (9-41)	31 (19-42)
		Bottom		20 (17-40)	15 (9-41)	68 (19-42)
	C-745-G	Top		21 (22-40)	12 (23-41)	
		Bottom		78 (8-40)	39 (22-41)	
	C-745-K and L	Top		27 (17-22)	8 (19-24)	1 (25)
		Bottom		30 (10-39)	11 ((7-41)	
PORTS	X-745-C	Top	57 (8-36) <sup>C</sup>	56 (12-43) <sup>A</sup>	15 (10-38) <sup>C</sup>	
		Bottom	63 (9-39) <sup>C</sup>	85 (12-43) <sup>A</sup>	10 (11-40) <sup>C</sup>	
	X-745-E	Top	4 (9-11) <sup>C</sup>		60 (11-44) <sup>C</sup>	81 (26-45)
		Bottom	18 (8-9) <sup>C</sup>		45 (23-44) <sup>C</sup>	58 (24-45)

<sup>A</sup>The yard here denotes a best estimate, for each cylinder, of the yard or former yard at which the cylinder was stored the longest. In some cases this could not be determined.

<sup>B</sup>FY99 Paducah cylinders were classified according to present rather than former yard status in the FY01 report.

<sup>C</sup>Top/bottom status estimated from information available; historical information on top/bottom status not available for Portsmouth cylinders.

**Table 6. Counts of thick-wall cylinders measured (and ages ranges of cylinders when evaluated)—not including data at the head/skirt interface**

Site	Yard	Row	Fiscal Year					
			'96	'97	'98	'99	2000	'01
PGDP	C-745-C	Top	1 (42)					
		Bottom	1 (44)					
PORTS	X-745-C	Top	50 (42-45)			11 (45-48)		
		Bottom	65 (42-45)					
	X-745-E	Top			1 (46-46)		16 (46-48)	8 (47-49)
		Bottom			1 (44-44)		9 (46-48)	6 (47-49)

**Table 7. Counts of Model 30A cylinders measured  
(and ages of cylinders when evaluated)**

Site	Yard	Row	Fiscal Year	
			FY99	FY01
PGDP	–	Top	50 (45)	
		Bottom	50 (45)	
	A	Top		61 (47)
		Bottom		36 (47)
	D	Bottom		2 (47)

### 3.2. Data Collection by Fiscal Year

In this section data collections are summarized in order of the fiscal year they were performed.

#### **FY92**

This data consists of two breached cylinders discovered in 1992 in K-1066-K yard, for which it was deemed that external corrosion was the cause of the breach.

#### **FY94**

Between December 1993 and May 1994, wall thickness measurements were made for 136 cylinders in K-1066-K yard (Philpot 1995) using an automated scanner. It was intended that the cylinders selected for measurement be chosen at random, although a random number generator was not used to select them, and there were limitations imposed by the automated scanner (e.g., length of power cord, clearance between adjacent cylinders). For these reasons, the cylinders selected are not a truly random sample from the population, though they may be representative and they may emulate a random sample. For the first 21 cylinders evaluated, only minimum wall thickness data was recorded, while maximum thicknesses were also recorded for the rest of the cylinders. There is concern about the accuracy of the wall thickness data for the first group of cylinders. Further, since maximum thickness data was not recorded for the first 21 cylinders, maximum pit depths could not be used for these cylinders and they are not included in either the direct or indirect-model analyses in this report.

#### **FY95**

During FY95, data was collected for 100 thin-wall cylinders at PGDP using the automated P-scanner (Blue 1995a). The primary purpose of this effort was to assess “the condition of the more vulnerable portion” of the cylinder population at PGDP (Blue 1995a). The cylinders were selected from the C-745-B/F/G/K/L yards on the basis of judgement of the personnel involved, and thus they do not constitute a random sample from any of these yards.

#### **FY96**

During FY96, almost 900 cylinders were evaluated with the automated scanner at the Portsmouth and PGDP sites. Both thin-wall (nominal wall thickness 312.5 mils) and thick-wall cylinders were evaluated (nominal wall thickness 625 mils).

At Portsmouth, 10% of the cylinders that were relocated were selected using a random number generator to evaluate the wall thickness using manual UT measurements. The 10% evaluation criterion was required by a Consent Decree with the Ohio Environmental Protection Agency.



Most of the cylinders evaluated at PGDP were from C-745-G yard, and had been set aside as part of relocation efforts performed during FY95 and FY96. These cylinders were a subset of the approximately 390 cylinders set aside from the first 3900 cylinders moved out of the C-745-G yard. Because of the manner in which these cylinders were selected, these cylinders are a systematic sample only from the first 3900 cylinders moved out of G yard. An additional 6 cylinders from both C-745-F and C-745-K yard were also evaluated. For C-745-F yard, single-stacked cylinders from the north end were selected, while the C-745-K yard cylinders were selected on the basis of ease of accessibility with equipment. Neither of these samples is random.

## **FY97**

During FY97, both head/skirt interface and overall minimum wall thickness data was collected for cylinders at Portsmouth and PGDP. The head/skirt data was collected from cylinders that had been evaluated with the automated scanner in FY96 at Portsmouth, and from two cylinders at PGDP (Lykins and Pawel 1997). The cylinders evaluated at Portsmouth, which had originally been systematically set aside as part of the 10% criterion, were randomly selected from those cylinders moved during the year. Originally, it was suggested that approximately 250 cylinders be evaluated (Lykins 1996). However, budget constraints allowed only 85 evaluations with the P-Scan.

Two cylinders that had been in the bottom row of PGDP C-745-G yard, and one cylinder from C-745-L bottom row, were also evaluated with the P-Scan system in FY97. These were located in the north end of the PGDP C-745-F yard when they were evaluated.

## **FY98**

There were three basic populations sampled from in FY98. The first consisted of 40 thin-wall cylinders randomly selected from K-1066-K yard. These cylinders were chosen from a population of 400 cylinders that were moved to K-1066-E yard during FY98. The second consisted of 200 thin-wall cylinders randomly selected from Paducah yards. The third consisted of 142 thin-wall and 2 thick-wall cylinders in Portsmouth X-745-C and E yards. Some of the Portsmouth cylinders were also measured in FY96. In all cases, the ultrasonic (UT) thickness measurements were done manually. It was confirmed that the Paducah data was representative only of relatively uncorroded locations on each cylinder and therefore cannot be used alone for determining either minimum wall thickness or wall loss. (The Paducah cylinders were re-evaluated in FY99 with the purpose of determining estimates of the thinnest locations on each cylinder.)

## **FY99**

There were four separate sampling efforts in FY99. One consisted of 30 thin-wall cylinders randomly selected from K-1066-K yard. The cylinders were from a subpopulation of 155 cylinders that could be evaluated without moving cylinder movement. A second effort was a re-evaluation of 200 thin-wall cylinders at Paducah that had been evaluated in FY98, in order to estimate the thinnest location on each cylinder. The third effort, which was conducted at Portsmouth, included evaluations of both thin-wall and thick-wall cylinders. In both cases, the UT measurements were done manually. The fourth effort consisted of the evaluation of 100 model 30A cylinders from a population of 1825 30A cylinders at Paducah.

## **FY2000**

Additional data for FY2000 includes manual UT data for 58 48" cylinders from K-1066-K yard in Oak Ridge, 100 48" cylinders at Paducah, and 155 48" cylinders at Portsmouth. Some of the Portsmouth cylinders had been measured before.

## **FY01**

FY01 UT measurements at ETTP were made for 24 cylinders in K-1066-E yard and 76 cylinders in K-1066-K yard. At Paducah, 301 48" cylinders were UT'd from (present or former) B, C, F, and K yards, and 99 30A cylinders from A and D yards were measured. The PGDP cylinders were sampled using a random number generator. At Portsmouth, 139 thin-wall cylinders and 14 thick-wall cylinders, all from X-745-E yard. Some of the Portsmouth cylinders had been measured before.

### **3.3. Summary of Data by Subpopulation**

In this section a description of the UT measurement data is organized by storage-location and cylinder type. The cylinder types are thick-wall, thin-wall, 30A, and "skirted." The designation "skirted" refers to the head/skirt interface location on certain cylinders, both thick and thin-wall. This discussion and the further refinements in Section 4 are the basis for the cylinder groupings used in the corrosion models discussed in Section 4. (Measurement method, P-scan vs manual UT, is the basis for one of the refinements.) Some of the information in this section is a rehash of the data-by-fiscal-year discussion in the last section.

#### **ETTP K-Yard, Thin-Wall Cylinders**

K-1066-K yard, located at the ETTP plant in Oak Ridge, contains 2,942 thin-wall cylinders, ranging in age (in 2002) from about 38 years to 45 years. These cylinders were initially stored at K-1066-G yard at Oak Ridge starting at about 1966, and relocated in 1983 (Barber et al. 1994). During the six month period from 12/93 to 5/94, pit depth and wall thickness measurements were made for 136 cylinders (Philpot 1995) using an automated scanner. It was intended that the cylinders selected for measurement should be chosen at random, although a random number generator was not used to select them, and there were limitations imposed by the automated scanner (e.g., length of power cord, clearance between adjacent cylinders). Thus, strictly speaking, this data is not a random sample, although it may emulate one. For the first 21 cylinders evaluated, only minimum wall thickness data was recorded, while pit depth data was also recorded for the rest of the cylinders. There is concern about the accuracy of the wall thickness data for the first group of cylinders. Further, since no pit depth data was recorded for these cylinders that would allow estimating how much corrosion had occurred, these cylinders are not included in either the indirect or direct-model analyses discussed in this report.

Because of accuracy limits in the equipment used to collect this data, only increments of 5 mils were recorded for pit depth. As a result, there may be several cylinders with the same pit depth measurement, and which, due to data overlaying, appear to be absent in plots of this data (e.g., Figure 1 in Appendix A).

Also included in the data set are the two breached cylinders discovered on K-1066-K yard in 1992 (Barber et al. 1994).

In FY98, 40 cylinders were evaluated out of a subpopulation of 400 cylinders that were being moved to K-1066-E yard. These cylinders were randomly selected for evaluation, and evaluated with manual UT methods. In FY99, 30 more cylinders were evaluated from a subpopulation of 155 cylinders which could be evaluated without requiring any cylinder movements. All but one of these cylinders were chosen randomly, with the additional one selected by field personnel because of its history of ground/water contact. Fifty-eight more cylinders were measured in FY2000, and 100 more were measured in FY01, 24 cylinders from K-1066-E yard and 76 cylinders from K-1066-K yard. In this report, the E-yard cylinders are grouped with the K-yard cylinders for the purpose of corrosion modeling.

## Paducah Thin-Wall Cylinders

For the purpose of corrosion modeling, all PGDP C-745-G yard, bottom-row, thin-wall cylinders are treated as a single population, all other PGDP bottom-row, thin-wall cylinders other are treated as a single population, and all PGDP, top-row, thin-wall cylinders (including C-745-G yard cylinders) are treated as a single population. This decision was based on judgment about conditions of the cylinder yards and data availability. Thus yard designations of other than “G” are not critical, in the Paducah cylinder grouping used for corrosion modeling in this report.

**G-Yard (top and bottom).** The population of cylinders modeled as C-745-G yard cylinders actually consists of those cylinders that were originally in C-745-G yard prior to construction of the new yard and have not been painted. A painting program was initiated for the cylinders moved from C-745-G to C-745-S yard in FY96. All 2,168 cylinders in C-745-S were painted during FY96-97.

Data for C-745-G yard is in five sets, the first three of which were collected using the automated (P-Scan) scanner:

- (1) Data for 26 cylinders that were evaluated in FY95 (Blue 1995a, 1995b).
- (2) Data for measurements made between March and September 1996 on cylinders set aside as part of the relocation efforts performed during 1995 and 1996. A total of 235 cylinders were evaluated (137 from the top row, and 98 from the bottom row). These cylinders are a subset of the approximately 390 cylinders set aside from the first 3,900 cylinders moved out of C-745-G yard. Because of the way these cylinders were selected, they are a systematic sample only from the first 3,900 cylinders moved out of G yard. This weakens statistical statements made about the whole C-745-G yard population on the basis of trends observed for the data collected. There was also concern that the condition of the bottom row cylinders in C-745-G yard affected the accuracy of the equipment for the cylinders evaluated in FY95, as material at the bottom of the pits can result in the equipment underestimating the actual pit depth (Blue 1995b). Checks with hand-held instruments indicated that the pit depths may be underestimated by about 15 mils (Blue 1995b), and for this reason, 15 mils was added to the measured maximum pit depth for these cylinders.
- (3) The third data set consists of two bottom row cylinders that were evaluated in FY97. These cylinders were located in the north end of C-745-F when evaluated. Hand-held measurements using a 2-mm probe were made in 1994 to estimate the minimum wall thickness for eight cylinders in C-745-G yard, but the pit depths were not recorded (Blue 1994). This data is not included in this analysis because it is not possible to reliably estimate the pit depths. This data was used in the analysis discussed in Lyon (1995), because it was the only data available for this yard at that time.
- (4) UT measurements for fifteen top and 44 bottom cylinders in C-745-G yard were evaluated in September FY99 and are used in this report.
- (5) In FY2000, 12 top and 39 bottom G-Yard cylinders were measured. This data is also used in this report.

UT data was also collected for C-745-G yard as a result of two separate efforts performed in FY98 and FY99. Concerns about the quality of this data preclude inclusion in the current report.

**PGDP Yards Other than G-Yard.** PGDP C-745-B Yard contains about 1,500 thin-wall cylinders manufactured between 1954 and 1988. In FY95, six of these cylinders were inspected, all type 48T and manufactured in the period 1956-57. These cylinders had been stored on 10" high concrete piers above the yard surface since April 1967 (Blue 1995a). Four top row and two bottom row cylinders were evaluated,

this particular choice of cylinders being a matter of convenience for the material handlers (Blue 1995a). In FY01, 136 B-yard cylinders were measured, 61 top-row and 56 bottom-row cylinders, with ages ranging from 13 to 45 years.

Manual UT scans for two top and seven bottom row PGDP C-745-C-yard cylinders were added to the database in FY99. Scans for one top and one bottom row PGDP D-yard cylinders were added in FY2000. In FY01, UT measurements were made on 7 top and 77 bottom-row C-yard cylinders.

PGDP C-745-F yard contains approximately 4,500 thin-wall cylinders. The top and bottom rows of this yard were interchanged in 1992 when all bottom row cylinders chocks were replaced, concrete chocks replacing wood. Each row was also relocated south one row. It is likely that some of these bottom row cylinders were in water contact for extended periods of time, although none are now, and conditions in the F-yard are considered to have been better than G-yard conditions. In FY95, 13 top-row and 13 bottom-row F-yard cylinders were evaluated. Both the pit depths and wall thickness were recorded for these cylinders. Hand-held measurements using a 2 mm probe were made in FY94 to estimate the minimum wall thickness for 21 cylinders in C-745-F yard, but the pit depths were not recorded (Blue 1994). This data is not included in the analyses in this report, because it is not possible to reliably estimate the pit depths.<sup>4</sup>

Six cylinders were also evaluated in FY96 from PGDP C-745-F yard. Sixteen top and twenty bottom row cylinder measurements were added to the database in FY99. Five top and fifteen bottom row measurements were added in FY2000, and in FY01, UT measurements were made on 31 top and 68 bottom-row cylinders from this yard.

PGDP C-745-K and C-745-L yards contain a total of about 9,000 Type OM and G cylinders manufactured in the period 1958-1992. These cylinders have been stored on five-inch concrete saddles in gravel yards constructed with an underground drainage system. Data was collected from these yards in FY95-97, and FY99-01. The sampling in FY95 was limited to those cylinders that were manufactured during the period 1976-1982 that had lost large portions of their protective coating (Blue 1995a). A total of 42 cylinders were inspected (39 from K yard, 3 from L yard). Twenty-five cylinders were from the bottom row, and 17 were from the top row. In FY96, 6 cylinders from the bottom row of C-745-K yard were evaluated. In FY97, one cylinder that had been in the bottom row of C-745-L yard was evaluated; it was located in the north end of C-745-F yard when evaluated. One L-yard, bottom cylinders was measured in FY97. In FY99, 27 top and 30 bottom C-745-K and L yard cylinders were measured, with ages ranging from 10 to 39 years. In FY2000, eight top and eleven bottom cylinders were added, with ages ranging from 7 to 41 years. One top-row K-yard cylinder was measured in FY01.

A few cylinders from PGDP yards C-745-M through T have also been measured. Two cylinders were measured in FY99 with ages of eight and nine years. Four top and four bottom row cylinders from M and N yards were measured in FY2000, with ages ranging from ten to fourteen years.

### **Portsmouth, Thin-Wall Cylinders**

There are approximately 14,000 thin-wall cylinders at the Portsmouth site, ranging in age from a few years to over 40 years. Prior to FY96, there were four cylinder storage yards at Portsmouth. These yards were designated X-745-A, X-745-C, X-745-E, and X-745-F. The X-745-A and X-745-C yards were essentially the same yard, but were separated into different sections. The X-745-C yard had six sections, and the X-745-A yard had three sections. The X-745-A and X-745-C yards had a two-tier stacking configuration. The cylinders from the X-745-F yard were single-stacked cylinders. The X-745-E yard was a compacted gravel storage area, but was reconstructed during FY95-96 to a reinforced concrete storage yard. In FY96,

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<sup>4</sup>This data was used in the analysis discussed in Lyon (1995), because it was the only data available for this yard at that time.

a total of 5,708 cylinders were relocated at Portsmouth to meet the new storage requirements.

Cylinders at Portsmouth were moved from single row storage to a two-tiered arrangement around 1976. Prior to this, there were no top row cylinders at Portsmouth. The cylinders had been in their current location until movement activities in FY96. Thus, the “top” row cylinders at Portsmouth discussed here had been in the top row for about 20 years.

In FY96, wall thickness UT measurements were made on 10% of the cylinders that were relocated. The sampled cylinders were selected using a random number generator. The 10% evaluation criterion was required according to the Consent Decree with the Ohio Environmental Protection Agency. These cylinders, as well as other cylinders with handling or storage damage, were evaluated using the automated scanner P-Scan system and hand-held measurements. For the purpose of corrosion modeling, all thin-wall, top-row cylinders at Portsmouth are treated as one group, and all thin-wall, bottom-row cylinders are treated as one group.

During FY96, 473 thin-wall (i.e., nominal thickness 312.5 mils) were measured (221 from the top row, 252 from the bottom row), with an age range of 6-40 years. Eighty five thin-wall cylinders were evaluated during FY97 and about 135 were evaluated during each of FY98-01. Most of these were re-evaluations of the cylinders measured in FY96. Data collected prior to FY98 seems to indicate that cylinder walls are generally thinner than is indicated by the data collected during FY98 and later. Due to this difference, separate analyses are performed using the pre-FY98 and FY98-and-later data sets. Many of the newer data points are duplicates—measurements made on a single cylinder during different FY’s. In these cases, only the most recent measurements were used in the corrosion modeling, which, because of statistical independence requirements, assumes that all UT measurements on any given cylinder were made at (essentially) the same time.

In Lyon (1995), a different data set was used for the Portsmouth site. This data consists of hand-held UT measurements made in 1994 on 125 cylinders. This data is not used in the present analysis because the measurements were not taken at areas known to have accelerated corrosion, such as the saddle interface. Further, the evaluation techniques currently used are more stringent and provide more accurate data than that obtained previously.

### **Thick-Wall Cylinders**

There are approximately 1,860 thick-wall cylinders (nominal wall thickness 625 mils) located at the three sites. During FY96, 135 thick-wall cylinders were evaluated with the P-Scan as part of the relocation efforts at Portsmouth. Two thick-wall cylinders were also evaluated at PGDP. These cylinders were selected because of ease of accessibility (Lykins and Pawel 1997). The ages of these cylinders ranged from 42 to 45 years. The top/bottom status of twenty of these cylinders (when evaluated) is unknown and are not included in the corrosion analysis. In FY97, 70 thick-wall cylinders were evaluated at Portsmouth, half of them from top rows, half from bottom rows, ages from 36 to 45 years. In FY98, two more Portsmouth thick-wall cylinders were evaluated; 11 cylinders were evaluated in FY99; 25 were measured in FY2000; and 14 in FY01.

### **Skirted Cylinders**

About 1,500 thin and thick-wall 48" cylinders at the three sites have skirted ends. Most of these cylinders were manufactured before 1970. Because of a combination of extended time of wetness and differential aeration (Lykins and Pawel 1997), there is a concern that accelerated corrosion in the head/skirt interface crevice is possible. In order to comply with the Director’s Findings and Orders with the Ohio EPA at Portsmouth for cylinder movements performed in FY96, wall thickness measurements were made during FY97 at the head/skirt interface of both thin-wall (233) and thick-wall (115) cylinders. In FY2000, 48 top

and 38 bottom-row thin-wall cylinders (and one with indeterminate top/bottom status) and 15 top and 8 bottom thick-wall cylinders were also evaluated at the head/skirt interface. In FY01, 8 top and 6 bottom-row thick-wall cylinders, and 50 top and 49 bottom-row thin-wall cylinders were measured at the head/skirt interface. This data is used to project the conditions at the head/skirt interface for the entire population of skirted cylinders.

### **Model 30A Cylinders at Paducah**

There are 1,825 model 30A cylinders located at Paducah. Precise historical information is not available on each cylinder, but it is known that all of these cylinders were manufactured around 1954. During FY99, 100 of these cylinders were evaluated via manual UT methods. There were two lists of cylinders to be evaluated. The first was generated in June of 1999. Errors in the list required that it be replaced with an updated list that reflected random sampling. The second list was generated in July 1999, but unfortunately the cylinders evaluated in August 1999 were apparently chosen on the basis of the first list. With this list, the cylinders were chosen from just a few rows of the yard. Nevertheless, this data was used in the analyses for this report. In FY01, UT measurements were made for 99 randomly sampled 30A cylinders.

## 4. DATA ANALYSIS

This section is about regression modeling using the data discussed in Section 3 and the indirect and direct cylinder corrosion models discussed in Section 2. Indirect models are discussed first, in Section 4.1. Section 4.1 is divided into subsections by cylinder group, because separate indirect regression models are run for each group. The basis for the classification of cylinders into fifteen groups is also discussed in this section. The classification is a refinement of the classification discussed in Section 3.3.

Direct-model results, which are based on the same fifteen cylinder groups used for the indirect models, are discussed in Section 4.2. The direct model encompasses all cylinder groups in one regression model. Table 8 summarizes the regression results for the indirect models, and Table 12 summarizes results for the direct model. Projections based on both the direct and indirect fitted regression models are in Section 5, and the two models are compared in Section 6.

Figures 1-15 in Appendix A are scatterplots of maximum pit depth estimates, for each of the fifteen cylinder groups. Figures 16-30 in Appendix A are scatterplots of the minimum thickness measurements for each cylinder group. Figures 1-30 also illustrate the indirect or direct model fitted to the maximum pit depth or minimum thickness data. Figures 16-30 contain charts of the distributions of cylinder ages in underlying group populations from which the data is sampled.

### 4.1 Indirect Model Regressions (by Cylinder Group)

#### K-1066-K and E Yards

This population is treated separately from the other populations, because a large portion of these cylinders were in ground contact for extended periods while they were in a previous yard (K-1066-G yard). Several issues require making assumptions about this data: (1) how to incorporate the two cylinders discovered in 1992 that were deemed to breach due to external corrosion (Barber et al 1994), (2) whether the top and bottom-row populations should be modeled separately, and (3) how to incorporate the data collected in FY98-01 with data collected prior to FY98. In this report, two separate predictions are made for this yard: in the first, the breaches are included with the data collected prior to FY98, and in the second, the data collected in FY98-01 are used.

The data collected via manual UT evaluation in FY98 is significantly different (i.e., medians are different; see Lyon 1998, Appendix D) from the data collected before that with P-Scan equipment. The manually collected data shows in general both a lower amount of wall loss and larger minimum thickness. This is consistent with the results obtained in Schmidt et al (1996), where it was found that the P-Scan measurements under-predicted minimum wall thickness. They found that, generally, the P-Scan method resulted in underestimates of minimum wall thickness by an average of 10-20 mils. However, rather than manipulate the available data (either the P-Scan data or the manual UT data), in this report, the more recent data is used to provide alternate projections about the cylinders. As more years of data become available, the more recent data should supersede the old.

Although there was no a priori intention of comparing the P-Scan and manual UT methods, such comparisons are nevertheless useful. (Also, evaluators would not likely be biased about instrumentation methods, since they were not focused on any such comparisons.) Since six of the cylinders selected at random for evaluation in FY98 and FY99 from K-1066-K yard were previously evaluated with the P-Scan equipment in FY94, additional comparisons were possible (see Table 9 below). In fact, these comparisons were required as part of the process of determining whether and how the different data sets could be combined. The results obtained here are consistent with those obtained in the Schmidt et al analyses:

**Table 8. Summary of Indirect–Model Populations and Modeling Assumptions**

<b>Cylinder Grouping</b>	<b>Population</b>	<b>Model</b>	<b>Sample Size</b>	<b>Intercept</b>	<b>Slope</b>	<b>Std. Dev.</b>	<b>Initial Thickness Sample Size</b>	<b>Initial Mean</b>	<b>Initial Std</b>	<b>Initial Thickness Interval</b>	<b>Total in Population</b>
<b>Thin-Walled</b>	K-1066-K, top and bottom, pre-FY98	Slope Set to 1	117	0.532	1.000	0.456	117	315.1	9.8	[302.5, 340]	2,542
	K-1066-K, evaluated FY98-01	Slope Set to 1	229	-.494	1.000	0.827	229	331.9	24.5	[302.5, 379]	2,542
	C-745-G, bottom	Slope Set to 1	231	-.042	1.000	1.068	231	330.5	13.2	[302.5, 363]	2,064
	PGDP bottom, except G-yard	Fitted Slope	338	1.592	0.330	0.883	338	330.8	12.4	[302.5, 395]	10,299
	PGDP top	Fitted Slope	367	2.137	0.247	0.886	367	331.9	10.3	[302.5, 376]	12,281
	PORTS, top, pre-FY98	Fitted Slope	276	2.422	0.406	0.218	276	331.4	13.2	[302.5, 378]	8,014
	PORTS bottom, pre-FY98	Fitted Slope	281	2.578	0.423	0.280	281	333.9	13.8	[302.5, 375]	8,014
	PORTS Thin, Top, FY98 and later	Slope Set to 1	234	-.045	1.000	0.704	234	356.0	13.8	[302.5, 399]	8,014
	PORTS Thin, Bottom, FY98 and later	Slope Set to 1	248	-.009	1.000	0.779	248	357.2	15.8	[302.5, 430]	8,014



**Table 8 (cont'd). Summary of Indirect-Model Populations and Modeling Assumptions**

<b>Cylinder Grouping</b>	<b>Population</b>	<b>Model</b>	<b>Sample Size</b>	<b>Intercept</b>	<b>Slope</b>	<b>Std. Dev.</b>	<b>Initial Thickness Sample Size</b>	<b>Initial Mean</b>	<b>Initial Std</b>	<b>Initial Thickness Interval</b>	<b>Total in Population</b>
<b>Thick-Walled</b>	PORTS Thin and PORTS and PGDP Thick, top	Fitted Slope	466	3.060	0.174	0.411	63	649.3	30.5	[615, 749]	931
	PORTS Thin and PORTS and PGDP Thick, bottom	Fitted Slope	472	3.348	0.126	0.486	71	647.1	29.6	[615, 761]	931
<b>Skirted</b>	PORTS Thin, skirted, top	Slope Set to 1	153	-.618	1.000	0.697	153	353.7	18.9	[302.5, 435]	3,485
	PORTS Thin, skirted, bottom	Slope Set to 1	167	-.586	1.000	0.874	167	350.5	15.0	[302.5, 388]	3,574
	PORTS Thick, skirted, top and bottom	Slope Set to 1	125	-.001	1.000	0.757	125	770.9	22.6	[615, 846]	1,861
<b>Model 30A</b>	Paducah 30As	Slope Set to 1	199	-.040	1.000	0.817	199	522.0	32.3	[490, 595]	1,825

**Table 9. Comparison of Estimated Minimum Point Wall Thickness Using Different Measurement Methods for Cylinders at K-1066-K Yard**

<b>Cylinder</b>	<b>Estimated Minimum Wall Thickness (mils) Using P-Scan Method (FY94)</b>	<b>Estimated Minimum Wall Thickness (mils) Using Manual Methods (FY98/99)</b>	<b>Difference</b>
5280	230	311	-81
6294	260	304	-44
6622	250	304	-54
7340	140	200	-60
7486	205	220	-15
14375	280	326	-46
<b>Mean (Std. Err.):</b>			<b>49.8 (8.8)</b>

manual evaluation seems to lead to higher estimates of minimum wall thickness. The mean difference here is 50 mils (standard error = 8.8, significance level = .002).

Whether the pre-FY98 or FY98-01 data is used, neither of the best fit estimates of the power-law slope are between 0 and 1. The slope estimates are 1.37 for the pre-FY98 data and -.90 for the FY98-01 data. It is clear from Figures 1 and 2 that the data should indeed lead to these estimates. Therefore, in accordance with the indirect modeling approach described in Section 2, the slope-set-to-one approach is used instead. For the pre-FY98 data, the best fit model with slope set to one is  $\log(P(t)) = .53 + \log(t)$  (or  $P(t)=1.7t$ ), with a standard deviation estimate of .46. For the FY98-01 data, the best fit model is  $\log(P(t)) = -.49 + \log(t)$  (or  $P(t)=.61t$ ), with a standard deviation estimate of .83. Although in either case the estimates of the power-law slope range beyond its assumed bounds, the UT data collected during FY98-01 clearly suggests slower corrosion than would be concluded from the data collected prior to that.

#### **C-745-G Yard, Bottom Rows**

C-745-G yard cylinders represent the worst conditions at the PGDP site. Many of these cylinders were in ground contact for extended periods. Unlike K-1066-K yard at Oak Ridge, there is a wide range of ages for these cylinders. The slope  $n$  estimate for the power law model that best fits the pit depth data for this group turns out to very slightly negative (-.057). Nevertheless, how the data leads to the negative slope estimate is clear from Figure 3. Therefore the slope-set-to-one model is used instead. The best-fit slope-set-to-one model has  $\log(P(t)) = -.04 + \log(t)$  (or  $P(t)=.96 t$ ) with a standard deviation estimate of 1.07.

For the ETTP and Portsmouth thin-wall cylinder groups defined in this report, different groups are used for pre-FY98 and FY98-01 data. The main reason for the dichotomy is the apparent difference between P-Scan and manual UT measurements. The dichotomy was not used for Paducah thin-wall cylinder data, however, or any of the thick-wall data, because, in the past at least, there were not enough manual UT measurements to support it. Although there is more Paducah thin-wall (and more thick-wall) cylinder measurement data available now, the Paducah grouping used in the past will nevertheless be retained here. There are two reasons: (1) The dichotomy leads to different out-year projections and the ultimately unsatisfying suggestion that the projections should somehow depend on the measurement method. (2) With the direct modeling approach, a straightforward adjustment can be made for the measurement methods by

including a term in the model to account for the method. That adjustment is not made for this report, however, as the intent here is to first directly compare the direct and indirect approaches.

### **Paducah Yards Except G, Bottom Rows**

Bottom-row cylinders other than those in G-yard were not in ground contact for extended periods, with the possible exception of some of the F-yard cylinders. Former F yard is considered to be the PGDP yard with the next worst conditions after G yard. However, with additional F-yard cylinders measured in FY01, there appears to be no basis for separating F-yard cylinders from the general non-G yard grouping. Among the non-G-yard groups, F-yard has mean minimum thickness in the middle. Therefore the G/non-G yard division for Paducah cylinders was retained, as in previous versions of this report. The power law model that best fits the pit depth data is  $\log(P(t)) = 1.59 + 0.33 \log(t)$  (or  $P(t) = 4.9 t^{0.33}$ ) with a standard deviation estimate of 0.88. As this estimate of the slope  $n$  is between 0 and 1, the power law is retained as the indirect model for the group.

### **Paducah Yards, Top Rows**

Few, if any, of the cylinders in the top rows of these yards were ever in extended ground contact, and this is assumed to be the case for all of the top row cylinders at PGDP (note that what is modeled here as the C-745-F top row cylinders are currently in the bottom row, and vice versa, due to the relocation that took place in FY92). The power law model that best fits the pit depth data is  $\log(P(t)) = 2.1 + 0.25 \log(t)$  (or  $P(t) = 8.5 t^{0.25}$ ) with a standard deviation estimate of 0.89. As this estimate of the slope  $n$  is between 0 and 1, the power law is retained as the indirect model for the group.

### **Portsmouth thin-wall cylinders, Top Rows**

As with the top row cylinders at PGDP, few of the Portsmouth top row cylinders have ever been in extended ground contact. There are two data sets available, and each is used in a separate analysis to predict the future condition of the population. The first data set consists of evaluations for 337 cylinders, collected using the automated P-Scan equipment prior to FY98. The second consists of data collected for 184 cylinders with manual methods in FY98-01. Many of the Portsmouth cylinders that were measured since FY96 were measured more than once, over the years, and for those, only the most recent measurement is used.

The power law model that best fits (via least squares) the pit depth data collected prior to FY98 is  $\text{Log}(2.73 + 0.31 \log t, 0.30)$ , which has a median predicted pit depth of  $15.4 t^{0.31}$ . For the data collected in FY98 and later, the corresponding power law model is  $\text{Log}(2.17 + 0.32 \log t, 0.45)$ , which has a median predicted pit depth of  $8.8 t^{0.31}$ .

### **Portsmouth thin-wall cylinders, Bottom Rows**

Few of the Portsmouth bottom row cylinders have ever been in extended ground contact. There are two data sets available, and each is used in a separate indirect-model analysis to predict the future condition of the population. The first data set consists of evaluations for 349 cylinders, collected using the automated P-Scan equipment prior to FY98. The second consists of data collected for 185 cylinders with manual methods in FY98-01. Many of the cylinders that were measured since FY97 were measured multiple years, and for those only the most recent measurement is used.

The power law model that best fits the pit depth data collected prior to FY98 is  $\text{Log}(2.70 + 0.37 \log t, 0.41)$ , which has a median predicted pit depth of  $14.8 t^{0.37}$ . For the data collected during FY98-01, the corresponding power law model is  $\text{Log}(2.05 + 0.35 \log t, 0.46)$ , which has a median predicted pit depth of

$7.8 t^{0.35}$ . The model fit with the data collected before FY98 has higher predicted median penetration depths than the model for the FY98 and later data.

### **Portsmouth Thick-Wall cylinders, Top Rows**

There were 63 thick-wall cylinders from top row evaluated in FY96-2000 (62 from Portsmouth, and one from Paducah C-745-C is also included in this group), with an age range of 42-48 years. Due to a concern about using such a narrow age range, and since it is expected that the penetration depth for the thin-wall cylinders will be similar to that for the thick-wall cylinders, the maximum penetration data for thin-wall cylinders in the top row at Portsmouth (403 cylinders) was added to the data set, and a model for penetration depth was then derived.<sup>5</sup> This model is assumed to apply to top-row, thick-wall cylinders at all yards. There are approximately 420 thick-wall cylinders at ETTP in the K-1066-B/E/J yards, 275 at PGDP in the C-745-B/C/D yards, and 1166 thick-wall cylinders at Portsmouth. Using the combined data set, the resulting power law model that fits (via least squares) these pit depth data is  $\text{Log}(3.06 + 0.17 \log t, 0.41)$ , which has a median predicted pit depth of  $21.3 t^{0.17}$ .

### **Portsmouth Thick-Wall cylinders, Bottom Rows**

There were 71 thick-wall cylinders from the bottom row evaluated in FY96-2000 (68 from Portsmouth; and one from Paducah C-745-C), with an age range of 42-48 years. As for the top-row, thick-wall cylinders, maximum pit depth data for the thin-wall cylinders in the bottom row at Portsmouth (401 cylinders) were added to the thick-wall data set, and a model for penetration depth was derived.<sup>5</sup> This model is assumed to apply to bottom-row, thick-wall cylinders at all yards. Using the combined data set, the resulting power law model that fits (via least squares) these pit depth data is  $\text{Log}(3.35 + 0.13 \log t, 0.49)$ , which has a median predicted pit depth of  $28.5 t^{0.13}$ .

### **Thin-Wall Skirted Cylinders, Top**

The wall thickness in the head/skirt interface was evaluated for 153 top-row thin-wall skirted cylinders at Portsmouth during FY96-01. Figure 10 shows that this data suggests a negative power-law slope coefficient  $n$ . The problem appears obviously to be with the data itself, but there does not seem to be a way to decide which of the data should be rejected and which should be kept. If there were a straightforward way to incorporate the constraint  $P(0) = 0$  (i.e.,  $0 = A0^n$ ) into the indirect model, that constraint would counter the data's implied negative slope. Unfortunately, the lognormal power law does not easily admit this constraint (because  $\log(x)$  is undefined at  $x=0$ ). Thus the slope-set-to-one model is used instead. The fitted model for the top row cylinders is  $\text{Log}(-0.61, 0.70) t$ , which has a median predicted pit depth of  $0.54 t$ .

### **Thin-Wall Skirted Cylinders, Bottom**

The wall thickness in the head/skirt interface was evaluated for 167 bottom-row thin-wall skirted cylinders at Portsmouth during FY96-01. Figure 11 shows that the same data problem that occurs for the thin-wall skirted top-row cylinders also occurs for the bottom-row cylinders. Thus the slope-set-to-one model is used instead of the power law. The fitted model for the bottom row cylinders is  $\text{Log}(-0.56, 0.87) t$ , which has a median predicted pit depth of  $0.55 t$ .

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<sup>5</sup>In the direct-model regressions, thin-wall cylinders are not included in the thick-wall groups.

## Thick-Wall Skirted Cylinders

The wall thickness at the head/skirt interface was evaluated for 125 thick-wall skirted cylinders at Portsmouth during FY97-01. For 21 of these cylinders, all evaluated in FY97, the row (top/bottom status) was not available. Again the data suggests a negative power-law slope coefficient  $n$ . That this is almost surely due to measurement bias seems clear, but how to identify which data is biased is unclear. Therefore, the slope-set-to-one model is used, the best fitted version of which is  $\text{Log}(-0.001, 0.76) t$ .

## Model 30A Cylinders at Paducah

There are 1825 model 30A cylinders located at PGDP. Precise information about the age of these cylinders is unknown, but it is known that they were manufactured around 1954. One hundred of these cylinders were evaluated using manual UT techniques in FY99, and 99 more were measured in FY01. In the FY2000 and FY01 versions of this report, these cylinders were modeled as a single group, because the FY99 data showed no significant difference in mean estimated pit depth (or mean minimum thickness), top vs bottom. (The bottom-row cylinders were actually slightly thicker, though not significantly).

For both top and bottom-row cylinders, the sample mean pit depth for the 30As was actually smaller in FY01 than in FY99. This can be seen in Table 10. The difference between the two years (FY99 and FY01) is highly significant for both mean minimum thickness ( $p < .0001$ ) and mean log maximum pit depth ( $p=.002$ ). The reason for the anomalous difference between years is currently unknown and should be investigated, though it may be due only to differences in the sampling algorithm used for the FY99 and FY01 data (see end of Section 3.3). In an analysis of variance, top/bottom status is borderline significant ( $p=.01$ ) for mean log maximum pit depths, though it is not significant for explaining minimum thickness differences, and the difference is minor compared to the difference across the two years. Therefore, for the corrosion modeling, a single combined top-bottom 30A grouping was used, as in the past.

**Table 10. 30A Minimum Thicknesses and Estimated Pit Depths for FY99/01, by Top/Bottom Status**

FY	Top/Bottom Status	N	Mean Minimum Thickness (Standard Error)	Mean Log Maximum Estimated Pit Depth (Standard Error)
'99	Top	54	435.4 (12.4)	3.93 (.10)
	Bottom	46	438.8 (9.01)	4.00 (.10)
'01	Top	61	487.8 (8.8)	3.44 (.11)
	Bottom	38	472.6 (12.2)	3.88 (.14)

As the data suggests, the slope estimate for the indirect power-law model is negative. The slope-set-to-one estimate is used instead and the fitted result is  $\text{Log}(-0.040, 0.82) t$ .

## 4.2. Direct Model Regressions

Without setting the power-law slope to one, the indirect corrosion model would fail for nine of the fifteen cylinder groups. Some of the failures are probably due to the statistically variable and sometimes inconsistent nature of the data, particularly the maximum pit depth estimates, which are computed from maximum thickness measurements used as a proxy for initial thickness. The reason for the failures may also be theoretical, for example because of changed maintenance and storage conditions, or because the power-law model is based on corrosion physics that may apply to small objects such as metal coupons, but

not necessarily to thermally massive storage cylinders. Whether for theoretical reasons or because of practical data limitations, however, it seems like a good idea to try an approach that (1) does not require the estimation of pit-depth maxima, and (2) smooths out data anomalies by imposing more structure than the structure in the indirect approach's fifteen separately-fitted regressions. The direct model does not require pit-depth estimation and provides additional structure.

The direct model requires estimates of the initial cylinder thickness. Design-sheet specifications are a good starting point for initial thicknesses, but, as Table 11 suggests, design-sheet specifications can be improved upon. For each of the fifteen cylinder groups developed in Section 4.1, Table 11 contains 97.5% one-side lower and upper confidence limits (which together compose a 95% confidence interval) for the mean maximum thickness. The confidence limits are computed from wall maximum thickness measurements for each cylinder group. The table also contains the nominal lower and upper design limits, based on design-sheet specifications. In the final column, the table contains an original thickness estimate, which combines the nominal and confidence limits.

The original thickness estimates in Table 11 are computed as follows. As can be seen from the table, except for the Portsmouth thick, skirted, top and bottom group and the PGDP 30As, the confidence limit ranges are not far from the nominal ranges and in most cases overlap them. For the Portsmouth thick, skirted, top and bottom group and the PGDP 30As, the 97.5% LCL was taken as the original thickness estimate, which is the point in the 95% closest to the nominal thickness range. For the other groups, except for the Portsmouth thin, skirted groups, and the Portsmouth thin FY98 and later groups, the confidence intervals and design ranges actually do overlap. For these other groups, when the confidence and nominal ranges overlap, the original thickness estimate was taken as the midpoint of the range of overlap. When the confidence and nominal ranges do not overlap, the nominal range endpoint nearest to the confidence interval was taken as the original thickness estimate.

Thus, except for the Portsmouth Thick, skirted, top and bottom group and the PGDP 30As, the original thickness estimate is defined as follows:

If Nominal Upper < LCL, then Original Estimate = Nominal Upper;  
 Otherwise, if UCL < Nominal Lower, then Original Estimate = Nominal Lower;  
 Otherwise, Original Estimate = [ min(UCL, Nominal Upper) + max(LCL, Nominal Lower) ] / 2.

Because a nominal range endpoint is used when the confidence and nominal ranges do not overlap, this algorithm for estimating the initial thickness favors the nominal specification. The rationale for preferring the nominal specification is that (1) if the original thickness of a cylinder was not uniform, then the maximum thickness (at any time) is likely to be a poor estimate of the original minimum thickness of the cylinder, and (2) the original estimates, so defined, seem to work well in the minimum thickness regression discussed below. For the Portsmouth Thick, skirted, top and bottom group and the PGDP 30As the discrepancy between the confidence limits and the nominal specification was judged to be too big for the nominal specification to be reasonable, and the confidence limit closest to the nominal range was used instead.

A third source of information for estimating the original minimum thickness is the UT **minimum** thickness data itself. Thus the original thickness estimates from Table 11 were used in the direct model regression, as a predictor variable, the effects of which were adjusted in fitting the direct regression model:

$$M(t) = \alpha \times (\text{Original Thickness Estimate}) + \beta(\text{group}) \times \log(t) + \text{random error}, \quad (\text{for } t > 1).$$

Although the original thickness estimate in the direct regression model is assumed only to be an estimate (not the original thickness itself), the  $\alpha \times (\text{Original Thickness Estimate})$  term in the model above actually

**Table 11. Cylinder Wall Thickness Data and Original Thickness Estimate**

<b>Cylinder Group</b>	<b>97.5% LCL</b>	<b>97.5% UCL</b>	<b>Nominal Lower</b>	<b>Nominal Upper</b>	<b>Original Thickness Estimate</b>
ETTP, Thin, top and bottom, pre-FY98	313.3	316.9	302.5	345.5	315.1
PGDP Thin, former G yard, bottom	328.8	332.2	302.5	345.5	330.5
PGDP Thin, bottom, except former G yard	329.5	332.1	302.5	345.5	330.8
PGDP Thin, top	330.9	333.0	302.5	345.5	331.9
PORTS Thin, top, pre-FY98	329.8	332.9	302.5	345.5	331.4
PORTS Thin, bottom, pre-FY98	332.3	335.5	302.5	345.5	333.9
PORTS and PGDP, Thick, top	643.0	653.2	615.0	655.0	648.1
PORTS and PGDP, Thick, bottom	642.0	650.0	615.0	655.0	646.0
PORTS Thin, skirted, top	350.3	356.0	302.5	345.5	345.5
PORTS Thin, skirted, bottom	348.2	352.8	302.5	345.5	345.5
PORTS Thick, skirted, top and bottom*	766.9	774.9	615.0	655.0	766.9
PORTS Thin, Top, FY98 and later	354.2	357.7	302.5	345.5	345.5
PORTS Thin, Bottom, FY98 and later	354.5	358.0	302.5	345.5	345.5
ETTP, Thin, evaluated FY98-01	332.0	334.8	302.5	345.5	333.4
PGDP 30As*	517.4	526.5	343.8	468.8	517.4

\*97.5% LCL used for original thickness estimate for this group (see discussion in main text).

represents the mean thickness at  $t=1$  year of age (i.e., when  $\log(t) = 0$ ). Thus we would expect  $\alpha$  to be close to 1 and smaller than 1, though a departure from this is possible, because of error in the original thickness estimate.

The statistical distribution of the random error term in regression models affects how a regression should be weighted and whether and how the fit of the regression model in one region of the space of predictor variables (i.e., original thickness and age) should be used to make inferences (e.g., predictions) in another region. For example, the variance of the distribution of minimum thickness measurements likely increases with cylinder age. This should be accounted for, because projections about minimum thicknesses at a target age in the future are based on measurements for cylinders at ages less than the target age.

The decision about how a regression should be weighted must in part always be based on judgment. For the direct model regressions, the variance was taken to be proportional to age. Constant-variance (i.e., unweighted) and variance-proportional-to-age-squared weightings were also considered, but the variance-proportional-to-age model was chosen on the basis of residual plots, out-year projections, and judgments about data quality. This basis for the decision about weighting is discussed further below.

The regression weighted by age is easily implemented by dividing the cylinder minimum thicknesses, original thickness estimates and the  $\log(\text{age})$  terms by the square root of age—the ordinary, unweighted regression with the variables so transformed is equivalent to an age-weighted regression of the untransformed variables.

Table 12 shows the results of this regression. The  $R^2$  value for the regression is 97.9%.<sup>6</sup> The  $\alpha$  coefficient for the original thickness estimate, .96, is in the range reasonably close to but less than 1. The  $\beta(\text{group})$  parameters should be negative, because, according to the model  $dM(t)/dt = \beta(\text{group})/t$ . Although the direct model imposes no constraints on the  $\beta(\text{group})$  parameter estimates, all of the estimates do turn out to be negative, and there are no inconsistencies between model and data, at least for the current set of cylinder UT data.

Figures 16-30 show the fitted, direct-model, age-weighted regressions for the fifteen cylinder groups. In addition to plots of regression results, these figures also contain charts of the cylinder age distributions for the populations defined by the cylinder groups. These age distribution are for **all** cylinders in the population, not just for cylinders that were sampled. The age distribution charts show, in particular, the ages and counts for the oldest cylinders in each group, which are the cylinders at greatest risk. Estimates that are averages for entire groups can obscure risks for oldest cylinders, when the oldest cylinders are exceptions relative to the population in general (see Figures 20, 21, 27, 28). The oldest cylinders should in fact be watched most carefully.

In addition to the raw minimum thickness measurement data, the regression plots in Figures 16-30 show the direct-model fitted regression curves and approximate 99% lower confidence limits for minimum thicknesses for individual cylinders over the age ranges in the plots. The lower confidence limit curves are approximations:

$$\text{Probability ( Actual Minimum Thickness at age } t > \text{ Lower Confidence Bound at } t ) \approx .99$$

for any particular age  $t$ . Two different sets of lower confidence limit curves are shown. One set is based on a large-sample approximation (Schmoyer 1992) that does not assume any particular underlying distribution (e.g., normal) for the regression errors. The other lower confidence limits are the usual lower confidence limits for individual predicted values, which are based on the assumption that regression errors have normal distributions. The normal-theory confidence limits are generally (though not necessarily) closer to the regression fitted curve than the large-sample limits. Both the normal-theory and large-sample lower confidence limits suggest that although there are slight declines over time in average minimum wall thicknesses, there is considerable uncertainty about **individual** cylinders, and the uncertainty about individual cylinders increases as projections extend farther ahead in time.

Figures 31 and 32 are plots of the regression residuals, which can be used help decide about the statistical distribution of the regression errors (e.g., whether normal or otherwise), whether the variance-proportional-to-age weighting or some other weighting is appropriate, and whether the regression errors, for the weighting chosen, are approximately uniform (e.g., across ages). Figure 31 shows that for the regression weighted by age, the variance of the residuals is approximately uniform in age. There does appear to be slight tendency for the weighted residuals to fan out with increasing age, but it occurs primarily for the 30A cylinders. As discussed in Section 4.1, however, there are problems with the 30A data. The average minimum thickness is significantly lower in FY02 than in FY99, for example. The 30As could be modeled separately from the other cylinder groups, but one of the goals in the choice of the direct model is to encompass many cylinder groups with one model, so that such data anomalies such as the 30A problem can be smoothed out. Furthermore, no physical theories have yet been offered that would suggest that the 30As, in particular, should be modeled separately from other cylinder groups. Therefore, the 30As were modeled along with the other cylinders, using the variance-proportional-to-age weighting for the regression errors. A more severe,

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<sup>6</sup>Although  $R^2$  statistics for the direct and indirect models are not directly comparable, for reference, the  $R^2$  values for the indirect models were less than .10 for ten of the fifteen indirect models and never exceeded .58 in the other five cases. These are actually the  $R^2$  for the unconstrained two-parameter power law model (even when the slope-set-to-one model is used instead). The  $R^2$  statistic for the slope-set-to-one model, which is an intercept-only model, is by definition always zero.



**Table 12. Direct Model Parameter Estimates and Standard Errors**

<b>Parameter</b>	<b>Estimate</b>	<b>Standard Error</b>
Intercept	0.95	0.01
K-1066-K, top and bottom, pre-FY98	-13.16	1.23
C-745-G, bottom	-8.54	1.15
PGDP bottom, except G-yard	-0.99	1.16
PGDP top	-2.45	1.14
PORTS, top, pre-FY98	-8.43	1.17
PORTS bottom, pre-FY98	-11.72	1.22
PORTS Thin and PORTS and PGDP Thick, top <sup>a</sup>	-4.74	2.11
PORTS Thin and PORTS and PGDP Thick, bottom <sup>a</sup>	-5.69	2.07
PORTS Thin, skirted, top	-0.39	1.21
PORTS Thin, skirted, bottom	-2.27	1.19
PORTS Thick, skirted, top and bottom	-3.15	2.24
PORTS Thin, Top, FY98 and later	-0.79	1.26
PORTS Thin, Bottom, FY98 and later	-0.91	1.25
K-1066-K, evaluated FY98-01	-4.77	1.11
Paducah 30As	-8.04	1.54

<sup>a</sup>Although, for the indirect models, this group included thin cylinders, for the direct approach, it includes only thick-wall cylinders.

variance-proportional-to-age-squared weighting was also tried, but it did not have much effect on either the pattern of residuals or projections based on the rejections.

Figure 32 shows the regression residuals in a normal probability plot. Figure 32 shows that the distribution of residuals (and by extension the distribution of regression errors) is not normal, particularly for the lower (left) side of the distribution, where the residuals are smaller (more negative) than would be expected under normal theory. This suggests that the normal-theory lower confidence limits for individual minimum thickness predictions are likely to be inaccurate, and that the large-sample confidence limits, which are smaller than the normal-based limits, are probably better. Because the use of the normal-theory confidence limits is much more common in regression modeling, they were included in the figures as points of comparison for the large-sample limits.

## 5. WALL THICKNESS PROJECTIONS

Using the fitted, indirect and direct models, projections were made of the number of cylinders with minimum wall thickness less than the preliminary criteria:

1. A wall thickness of zero (a breach), which indicates a possible loss of contained material
2. A wall thickness below 62.5 mils, below which ordinary safe handling and stacking is considered to be impaired
3. A wall thickness representing applicable standards for off-site transport and contents transfer (based on ANSI 14.1 1995): 250 mils for thin-wall cylinders and 500 mils for thick-wall cylinders.

Separate projections were made for the direct and indirect approaches. For model 30A cylinders, there are no published criteria for minimum thicknesses of interest; on the basis of a personal communication with S. J. Pawel, two criteria are reported in the modeling results presented here: 100 mils (the minimum thickness for regular hot feeding), and breach.

These criteria are based on an *area* of wall thinning. However, minimum thickness predicted in this report is for an area of about 0.01 square inches, essentially a point. For thickness criteria greater than zero (breach), using a point thickness may add conservatism to the results in this report. Because of the interaction of UF<sub>6</sub> with atmospheric moisture and the substrate steel, the approximation of a small-area breach with a point breach is probably close (from DNFSB 1995):

A breach in a cylinder allows the external atmosphere to react slowly with the UF<sub>6</sub>. The solid reaction product tends to plug the breach; however, the HF formed releases slowly, attacks the metal cylinder, and enlarges the breach over time. The hole diameter is estimated to increase at a rate of approximately one inch per year.

Table 13 shows numbers of cylinders projected on the basis of the indirect regression model to have minimum wall thickness below various thickness criteria, and Table 14 shows projections based on the direct model. These projections are computed using equation (2.1) and either indirect or direct-model estimates of  $Prob(M(t) < z)$  for the various ages ( $t$ ) and thickness criteria ( $z$ ). The tables reflect only statistical expectations—even if the expectations are exact, there will be random departures from them. Both tables project breaches in 2002 and later years, which is reasonable since breaches have already occurred. Nevertheless, despite random variation, and though breaches have occurred, the numbers of breaches predicted in the tables seem too high. Reasons for these high projections include:

- Many of the cylinders were not sampled randomly (e.g., using a random number generator), but were selected “quasi-randomly” or even with purposive focus on groups thought to be high risk. Two breaches judged to be due to external corrosion are automatically included in the samples, which also biases results slightly toward greater likelihood of breaching.
- The cylinder groupings only roughly approximate the complete storage location history of cylinders. Because cylinders are typically moved from time to time, the “locations” associated with the cylinder groupings would be better represented as combinations of locations. The complete storage histories are not always available, however, in any form (let alone electronic), and the accounting for such an approach would be much more difficult than the direct or indirect approaches used for this report.

- When cylinders are moved, they are usually moved to improved storage locations. Painting programs have protected cylinder surfaces (and cylinders most needing paint have usually been the ones that are painted).

Unraveling the effects of the sampling biases and changing storage locations would be a difficult task. On the other hand, the effect of painting can be accounted for more easily by updating the cylinder population definitions to exclude recently painted cylinders from cylinders at risk. Effects of painting are considered in Schmoyer and Lyon (2001), but the effects of painting are not incorporated directly into the main projections. Cylinders painted within the last ten years, for example, could be excluded from populations considered to be at risk.

Although the projections in Tables 13 and 14 seem conservative, they can still be used on a relative basis, as in prioritizing the cylinder groups or in comparing the direct and indirect models. In addition to projection estimates, Table 13 also contains upper confidence limits for the estimates, based on the indirect model. The numbers in the columns labeled “Estimate” are point estimates computed from the least square estimates of the model parameters. The numbers labeled “95% UCL” are approximate upper 95% confidence limits computed using the method described in Appendix B. The confidence limits take into account variability in the regression parameter estimates and assume that maximum penetration regression errors are lognormally distributed. The point estimates assume lognormal errors but do not account for variability in the regression parameter estimates.

The direct-model projections in Table 14 are based on the same large-sample approximations that are used to derive the lower confidence limits plotted in Figures 16-30 for individual predicted values. Instead of specifying a probability  $p$  (e.g., .99) and determining an approximate bound that a new observation with age  $t$  will exceed with the probability  $p$ , a bound (i.e., thickness criteria)  $z$  is specified, and an estimate of the probability  $Prob(M(t) < z)$  is calculated. For either the direct-model projections or the confidence limits in Figures 16-30, statistical variability in the model parameter estimates is accounted for through an approximation that is essentially exact for large sample sizes. Because variability in the model parameter estimates is accounted for in the direct model approach (just as it is in the usual normal-theory confidence limits for individual values), some conservatism should be expected in the direct model projections, even in the absence of external biases due to sampling, improved storage and maintenance, etc.

For the indirect model, the projection estimates are computed by substituting regression parameter estimates into the lognormal distribution, which is assumed to be the distribution of the regression errors. Variability in parameter estimates is accounted for in the UCL’s, but not in the projection estimates themselves. Thus, the indirect model projection estimates do not incorporate adjustments to account for variability in model parameter estimates and are not conservative because of such adjustments alone.

Because of the quite different assumptions and mathematics used in the direct and indirect approaches, the estimates and upper confidence limits in Table 13 are not directly comparable to the estimates in Table 14. Furthermore, for the indirect model, data for thin-wall cylinders at Portsmouth was included with the data for the thick-wall cylinders in order to derive the model for penetration depth (though the corresponding projections in Table 13 are for thick-wall cylinders only). For the direct model, only data for thick-wall cylinders was included in the thick-wall cylinder groups. Despite these differences, the two tables have the same ultimate purpose in decision-making about cylinders and can be compared from that perspective. Several such comparisons are made in this section, and the direct and indirect approaches are compared more thoroughly in Section 6.

The indirect model points to the Paducah G-yard bottom cylinders as the cylinders most at-risk of either breaches or wall thickness below 62.5 mils. The indirect model projects 64 cylinders below 62.5 mils and 40 breaches, in FY02. Because there are only 2,064 G-yard bottom cylinders, these counts translate to rates of 3.1% and 1.9% for the 62.5 and 0 mil criteria, respectively. The second most at-risk cylinder

group, according to the indirect model, are the K-1066-K cylinders, with 7 and 2 (pre-FY98 data) or 6 and 2 (FY98-01 data) cylinders below 62.5 or 0 mils. As there are 2,542 cylinders in this population, these counts translate to .27% (for 7) or .24% (for 6) and .08% (for 2 breaches). The next most at-risk groups are the Portsmouth thin skirted bottom cylinders and the Paducah 30As. Projections for FY2020 are higher, but comparable. The indirect model projects a total of 61 breaches in FY02.

In terms of rates, the Paducah G-yard bottom and K-1066-K cylinders are also most at risk according to the direct model, though for the direct model, differences among the groups are not nearly as great. The direct model projections for the Paducah G-yard bottom cylinders are 6 (.29%) below 62.5 and 2 (.10%) breaches. The direct model projections for the K-1066-K cylinders are 8 (.31%) below 62.5 and 3 (.12%) breaches. The next most at-risk groups are the Portsmouth thin skirted cylinders (both top and bottom). Projections for FY2020 are higher but comparable, though the increase with time in the projections is slower for the direct model than the indirect model. The direct model projects a total of 39 breaches in FY02, though at least one breach is predicted for every thin-wall group and none for the thick-wall (including skirted) and 30A cylinders.

**Table 13. Summary of Indirect-Model Projections for Target Years and Minimum Thickness Criteria**

Population	Thickness Criterion (mils)	Model	Projected Number of Cylinders Below Thickness Criterion											
			2002		2005		2010		2015		2020		2025	
			Esti- mate	95% UCB	Esti- mate	95% UCB	Esti- mate	95% UCB	Esti- mate	95% UCB	Esti- mate	95% UCB	Esti- mate	95% UCB
K-1066-K, top and bottom, pre-FY98	250	Slope Set to 1	1,414	1,627	1,559	1,780	1,769	1,989	1,939	2,145	2,075	2,260	2,181	2,343
	62.5	Slope Set to 1	7	34	11	47	22	75	37	110	59	154	87	205
	0	Slope Set to 1	2	11	3	16	5	27	10	43	17	63	27	89
K-1066-K, evaluated FY98-01	250	Slope Set to 1	210	324	243	364	301	432	360	500	422	567	484	633
	62.5	Slope Set to 1	6	19	7	23	11	31	15	40	20	51	25	62
	0	Slope Set to 1	2	10	3	12	5	17	7	22	10	29	13	36
C-745-G, bottom	250	Slope Set to 1	470	594	515	640	588	713	658	781	724	844	786	902
	62.5	Slope Set to 1	64	121	74	138	93	165	114	193	135	222	157	250
	0	Slope Set to 1	40	85	48	97	61	118	75	140	91	162	107	185
PGDP bottom, except G-yard	250	Fitted Slope	258	580	286	609	332	659	375	719	415	791	454	873
	62.5	Fitted Slope	4	23	5	25	6	28	8	33	9	38	10	44
	0	Fitted Slope	2	11	2	12	3	14	3	17	4	19	5	23
PGDP top	250	Fitted Slope	619	1,206	662	1,234	728	1,289	787	1,359	842	1,452	893	1,564
	62.5	Fitted Slope	17	69	19	72	22	77	24	85	27	95	30	107
	0	Fitted Slope	8	36	8	38	10	41	11	46	13	51	14	58

**Table 13 (cont'd). Summary of Indirect–Model Projections for Target Years and Minimum Thickness Criteria**

Population	Thickness Criterion (mils)	Model	Projected Number of Cylinders Below Thickness Criterion											
			2002		2005		2010		2015		2020		2025	
			Esti- mate	95% UCB	Esti- mate	95% UCB	Esti- mate	95% UCB	Esti- mate	95% UCB	Esti- mate	95% UCB	Esti- mate	95% UCB
PORTS, top, pre–FY98	250	Fitted Slope	107	203	142	262	217	382	316	531	439	713	585	927
	62.5	Fitted Slope	0	0	0	0	0	0	0	0	0	0	0	0
	0	Fitted Slope	0	0	0	0	0	0	0	0	0	0	0	0
PORTS bottom, pre–FY98	250	Fitted Slope	634	964	785	1,153	1,070	1,494	1,387	1,862	1,721	2,252	2,063	2,651
	62.5	Fitted Slope	0	0	0	0	0	0	0	0	0	0	0	0
	0	Fitted Slope	0	0	0	0	0	0	0	0	0	0	0	0
PORTS Thin, Top, FY98 and later	250	Slope Set to 1	229	395	295	495	431	689	597	911	791	1,154	1,008	1,411
	62.5	Slope Set to 1	5	18	7	24	12	38	19	57	29	81	43	112
	0	Slope Set to 1	2	9	3	12	5	19	8	29	13	42	20	60
PORTS Thin, Bottom, FY98 and later	250	Slope Set to 1	337	544	424	666	594	894	793	1,143	1,014	1,404	1,249	1,671
	62.5	Slope Set to 1	14	40	19	52	29	78	44	111	64	151	89	199
	0	Slope Set to 1	6	21	9	28	14	43	22	63	33	88	46	118

**Table 13 (cont'd). Summary of Indirect–Model Projections for Target Years and Minimum Thickness Criteria**

Population	Thickness Criterion (mils)	Model	Projected Number of Cylinders Below Thickness Criterion											
			2002		2005		2010		2015		2020		2025	
			Esti- mate	95% UCB	Esti- mate	95% UCB	Esti- mate	95% UCB	Esti- mate	95% UCB	Esti- mate	95% UCB	Esti- mate	95% UCB
PORTS Thin and PGDP Thick, top	500	Fitted Slope	1	4	1	4	2	5	2	5	2	6	2	6
	62.5	Fitted Slope	0	0	0	0	0	0	0	0	0	0	0	0
	0	Fitted Slope	0	0	0	0	0	0	0	0	0	0	0	0
PORTS Thin and PGDP Thick, bottom	500	Fitted Slope	9	18	9	19	9	21	10	22	11	24	11	25
	62.5	Fitted Slope	0	0	0	0	0	0	0	0	0	0	0	0
	0	Fitted Slope	0	0	0	0	0	0	0	0	0	0	0	0
PORTS Thin, skirted, top	250	Slope Set to 1	70	162	87	191	119	243	156	300	198	360	244	423
	62.5	Slope Set to 1	1	5	1	6	1	9	2	13	3	17	5	22
	0	Slope Set to 1	0	2	0	3	0	4	1	6	1	8	2	11
PORTS Thin, skirted, bottom	250	Slope Set to 1	190	347	221	390	275	462	333	536	394	611	457	685
	62.5	Slope Set to 1	8	32	10	38	15	50	20	63	26	78	33	93
	0	Slope Set to 1	4	19	5	22	7	30	10	38	13	48	18	58

**Table 13 (cont'd). Summary of Indirect–Model Projections for Target Years and Minimum Thickness Criteria**

Population	Thickness Criterion (mils)	Model	Projected Number of Cylinders Below Thickness Criterion											
			2002		2005		2010		2015		2020		2025	
			Esti- mate	95% UCB	Esti- mate	95% UCB	Esti- mate	95% UCB	Esti- mate	95% UCB	Esti- mate	95% UCB	Esti- mate	95% UCB
PORTS Thick, skirted, top and bottom	500	Slope Set to 1	22	68	27	78	37	97	48	117	60	139	74	162
	62.5	Slope Set to 1	0	4	1	4	1	6	1	8	2	10	2	13
	0	Slope Set to 1	0	3	0	3	1	5	1	6	1	8	2	10
Paducah 30As	100	Slope Set to 1	6	20	7	23	10	30	14	38	18	46	22	56
	62.5	Slope Set to 1	4	15	5	18	8	24	10	30	13	37	17	45
	0	Slope Set to 1	3	10	3	13	5	17	6	21	9	27	11	32



**Table 14. Summary of Direct-Model Projections for Target Years and Thickness Criteria**

Cylinder Grouping	Thickness		Projected Number of Cylinders Below Minimum Thickness Criteria					
	Criterion (mils)	Thick-ness	2002 Esti- mate	2005 Esti- mate	2010 Esti- mate	2015 Esti- mate	2020 Esti- mate	2025 Esti- mate
ETTP, Thin, top and bottom, pre-FY98	250	Thin	1,131	1,167	1,218	1,260	1,295	1,325
	67.5	Thin	12	13	15	16	17	17
	0	Thin	7	7	9	9	10	10
ETTP, Thin, evaluated FY98-01	250	Thin	156	166	184	201	218	235
	67.5	Thin	8	9	9	10	10	11
	0	Thin	3	4	5	6	7	8
PGDP Thin, former G yard, bottom	250	Thin	218	236	263	290	313	334
	67.5	Thin	6	7	8	8	10	11
	0	Thin	2	3	4	5	6	6
PGDP Thin, bottom, except former G yard	250	Thin	209	241	295	350	405	463
	67.5	Thin	10	12	17	22	27	32
	0	Thin	3	4	6	8	10	14
PGDP Thin, top	250	Thin	324	367	443	522	600	671
	67.5	Thin	16	18	25	29	36	41
	0	Thin	5	5	9	12	15	19
PORTS Thin, top, pre-FY98	250	Thin	481	553	678	793	904	1,009
	67.5	Thin	12	14	20	24	28	32
	0	Thin	4	6	7	9	14	16
PORTS Thin, bottom, pre-FY98	250	Thin	720	840	1,020	1,192	1,355	1,501
	67.5	Thin	14	17	22	27	31	34
	0	Thin	5	6	8	12	15	18

**Table 14 (cont'd). Summary of Direct-Model Projections for Target Years and Thickness Criteria**

Cylinder Grouping	Thickness		Projected Number of Cylinders Below Minimum Thickness Criteria					
	Criterion (mils)	Thick-ness	2002 Esti- mate	2005 Esti- mate	2010 Esti- mate	2015 Esti- mate	2020 Esti- mate	2025 Esti- mate
PORTS and PGDP, Thick, top	500	Thick	14	15	17	19	21	24
	67.5	Thick	0	0	0	0	0	0
	0	Thick	0	0	0	0	0	0
PORTS and PGDP, Thick, bottom	500	Thick	16	17	20	22	25	27
	67.5	Thick	0	0	0	0	0	0
	0	Thick	0	0	0	0	0	0
PORTS Thin, skirted, top	250	Thin	78	85	96	108	120	133
	67.5	Thin	8	9	11	11	12	12
	0	Thin	3	3	4	5	7	8
PORTS Thin, skirted, bottom	250	Thin	101	110	126	144	161	176
	67.5	Thin	9	11	11	12	13	14
	0	Thin	3	4	4	6	8	8
PORTS Thin, Top, FY98 and later	250	Thin	101	114	138	165	192	222
	67.5	Thin	6	8	11	15	18	22
	0	Thin	2	2	3	5	7	9
PORTS Thin, Bottom, FY98 and later	250	Thin	102	115	140	167	195	226
	67.5	Thin	6	8	11	15	18	22
	0	Thin	2	2	3	5	7	9

**Table 14 (cont'd). Summary of Direct–Model Projections for Target Years and Thickness Criteria**

Cylinder Grouping	Thickness		Projected Number of Cylinders Below Minimum Thickness Criteria					
	Criterion (mils)	Thick-ness	2002 Esti- mate	2005 Esti- mate	2010 Esti- mate	2015 Esti- mate	2020 Esti- mate	2025 Esti- mate
PORTS Thick, skirted, top and bottom	500	Thick	7	7	8	10	10	11
	67.5	Thick	0	0	0	0	0	0
	0	Thick	0	0	0	0	0	0
PGDP 30As	250	30As	7	8	9	10	11	12
	100	30As	1	1	1	2	2	3
	67.5	30As	0	1	1	1	1	2
	0	30As	0	0	0	0	0	1

## 6. MODEL EVALUATION AND COMPARISON

By fitting the indirect and direct models using only data collected prior to FY01, an assessment of the models can be made by comparing model-based projections for FY01 with actual FY01 sampled results. This approach can be used both to compare the two modeling approaches and to assess the models on an absolute basis.

Several factors complicate such an evaluation, however. For example, the model-based minimum thickness projections are estimates, about which actual measured minimum thicknesses are expected to vary randomly. Another complication is the unlikelihood of a low-probability event such as a thickness below 62.5 mils or a breach. It is the low-probability events that we would most like to predict, yet only higher probability events, such as “thickness < 250 mils,” are typically observed in samples. Adequacy in forecasting numbers of cylinders with thickness below a value in the central part of a thickness distribution does not automatically imply adequacy in forecasting numbers of cylinders with thickness below a value in the lower tail of the distribution.

For the various cylinder groups, Table 15 shows projected and FY01-observed numbers of cylinders with minimum thicknesses falling below 0 and 62.5 mils, and, for thin-wall, thick-wall, and 30A cylinders, below 250, 500, and 100 mils respectively. The projections in Table 15 are for FY01, but they are computed only with data from FY2000 and before. Good conclusions are difficult to draw from Table 15, however, because all of the observed and projected counts for the 0 and 62.5 mil criteria are zeros. Overall, the direct model leads to a prediction that 18 cylinders would have fallen below the 250 mil criterion in FY01; the indirect model leads to a prediction of 32 cylinders. In the sample, 19 thin-wall cylinders were below the 250 mil criterion. There is considerable variability in these outcomes, however. For example, of those 19 thin-wall cylinders, four had minimum thicknesses of 246 or more and could thus just as easily been above the 250 mil criterion.

Comparisons of the direct and indirect models would be based on more data and thus less prone to statistical variation if the all of the data was used both to fit the models and to compare them. Adjustments are necessary, however, when the same data is used both to fit the models and to evaluate their performance. For the fifteen cylinder groups, the direct model has seventeen parameters, including the standard deviation. The indirect model has, for all fifteen groups, 45 parameters, including fifteen standard deviations but **not** including parameters for the initial thickness distribution. Increasing the number of parameters in a model automatically improves model fitting criteria (e.g., the sum of squared residuals), which measure departures between the model-fitted and observed data. However, having more parameters does not automatically imply that a model will provide better model-fitted projections of new, future measurements. (Otherwise arbitrarily high-order polynomials could be used to predict anything.) Increasing the number of parameters can in fact make future projections worse. The same logic applies whether the model fit criteria is based on sums of squared residuals or differences between observed and projected numbers below various thickness criteria. When the same data is used both to fit models and to evaluate their performance, comparisons of the two models should be adjusted to account for differences in numbers of parameters.

Akaike (1974) considered the problem of comparing models with different numbers of parameters and developed a basis for model comparisons that has become known as the Akaike information criterion (AIC). The AIC is defined as

$$\text{AIC} = -2\log\text{-likelihood} + 2(\text{number of free parameters}),$$

where “log-likelihood” denotes the maximized log-likelihood, and the method of maximum likelihood is the

**Table 15. FY2001 Indirect and Direct–Model Projected and Observed Counts for Sampled Cylinders**

<b>Cylinder Grouping</b>	<b>Number in Group Population</b>	<b>Number Sampled from Group</b>	<b>Thickness Spec.</b>	<b>Observed Number Out of Spec.</b>	<b>Indirect Model Projected Number Out of Spec.</b>	<b>Direct Model Projected Number Out of Spec.</b>
ETTP, Thin, evaluated FY98–01	2,542	100	250	8	9	7
			62.5	0	0	0
			0	0	0	0
PGDP Thin, bottom, except former G yard	10,299	201	250	11	7	3
			62.5	0	0	0
			0	0	0	0
PGDP Thin, top	12,281	100	250	0	7	3
			62.5	0	0	0
			0	0	0	0
PORTS and PGDP, Thick, top	931	8	500	0	0	0
			62.5	0	0	0
			0	0	0	0
PORTS and PGDP, Thick, bottom	931	6	500	0	0	0
			62.5	0	0	0
			0	0	0	0
PORTS Thin, skirted, top	3,485	50	250	0	2	1
			62.5	0	0	0
			0	0	0	0
PORTS Thin, skirted, bottom	3,574	49	250	0	4	2
			62.5	0	0	0
			0	0	0	0

**Table 15 (cont'd). FY2001 Indirect and Direct–Model Projected and Observed Counts for Sampled Cylinders**

<b>Cylinder Grouping</b>	<b>Number in Group Population</b>	<b>Number Sampled from Group</b>	<b>Thickness Spec.</b>	<b>Observed Number Out of Spec.</b>	<b>Indirect Model Projected Number Out of Spec.</b>	<b>Direct Model Projected Number Out of Spec.</b>
PORTS Thick, skirted, top and bottom	1,861	14	500	0	0	0
			62.5	0	0	0
			0	0	0	0
PORTS Thin, Top, FY98 and later	8,014	81	250	0	0	1
			62.5	0	0	0
			0	0	0	0
PORTS Thin, Bottom, FY98 and later	8,014	58	250	0	3	1
			62.5	0	0	0
			0	0	0	0
PGDP 30As	1,825	99	100	0	0	0
			62.5	0	0	0
			0	0	0	0

statistical method for estimating the parameters. (The higher the AIC, the worse the model fit.) The second term in the AIC incorporates a penalty proportional to the number of model parameters, because having more parameters reduces the log-likelihood but does not necessarily improve model-based projections of new measurements.

For a given thickness criterion  $C$  and for any minimum thickness measurement  $y$ , let the indicator function  $I$  be defined as

$$I(y) = \begin{cases} 1 & \text{if } y \leq C \\ 0 & \text{if } y > C. \end{cases}$$

For each measured cylinder  $i$  with minimum thickness measurement  $y_i$ ,  $I(y_i)$  is 1 if  $y_i$  is below the thickness criterion  $C$  and 0 otherwise. For each cylinder  $i$ , let  $p_i$  denote the probability that the minimum thickness is below  $C$ . For all measured cylinders, the probability of the observed number with minimum thickness below  $C$  is

$$\prod_{\text{All cylinders } i} p_i^{I(y_i)} (1-p_i)^{1-I(y_i)}.$$

Under either the direct or indirect (or other) model, each  $p_i$  can be estimated using the model's parameter estimates. Let  $\hat{p}_i$  denote such an estimate. For all of the measured cylinders, the likelihood of the observed number with minimum thickness below  $C$  is the probability of the observed number with minimum thickness evaluated at the  $\hat{p}_i$ , and the log-likelihood is thus

$$\sum_{\text{All cylinders } i} I(y_i) \log(\hat{p}_i) + (1-I(y_i)) \log(1-\hat{p}_i).$$

The likelihood function used here is not the same as the squared-error criterion used to estimate either the direct or indirect model parameters, and is therefore not necessarily maximized by the  $\hat{p}_i$ . However, either the direct or indirect models could be fit using this alternative likelihood criterion.

Table 16 shows the direct and indirect-model log-likelihood and AIC criteria for the 0, 62.5, and 250 mil criteria:

**Table 16. Values of the Log-likelihood and Aikake Information Criterion (AIC)**

Thickness Criterion	Indirect Model		Direct Model	
	Log-likelihood	AIC	Log-likelihood	AIC
0	-27.08	144.16	-22.46	78.92
62.5	-22.75	135.50	-22.84	79.68
250	-16,124.66	32,339.32	-583.8	1,201.51

Table 16 shows that even without imposing the AIC penalty for the number of parameters in the model, the log-likelihood for the direct model is essentially as good or better than the log-likelihood for the indirect model, for the 0, 62.5, and 250 mil criteria. Because the likelihood function used here is not the same as the squared-error criterion, the number-of-parameters penalty imposed in the AIC is not necessarily the correct way to adjust the log-likelihood for the number of model parameters. It is clear, however, that a greater penalty should be imposed on the model with more parameters, and so, whatever the adjustment, the direct model seems better than the indirect model for the cylinder thickness data, according to the AIC criteria.



## 7. LIMITATIONS, CONCLUSIONS, AND RECOMMENDATIONS

The UF6 storage cylinder corrosion models developed in this report are intended to be part of a mechanism for decision-making about cylinders, for projecting numbers of non-compliant cylinders, and for forecasting cylinder wall thicknesses in the future. In this section, conclusions made in the previous sections are summarized, caveats and limitations are reiterated, and recommendations are made for future actions.

Two different approaches to corrosion modeling are considered in this report. An indirect model is used to predict minimum wall thickness through separate models of initial thickness and maximum pit depth. The maximum pit depths are not measured directly, but rather are estimated as differences between maximum and minimum measured wall thicknesses. The maximum pit depths are modeled as a function of cylinder age and grouping plus a lognormally distributed error term. In order to estimate minimum wall thicknesses, the initial thickness and maximum pit depth models are combined using mathematics that assumes statistical independence of the distributions of the initial thicknesses and maximum pit depths. The indirect model has been used in previous editions of this report.

The second approach to corrosion modeling relates measured minimum wall thickness directly to cylinder age, grouping, and initial thickness estimates. The initial thickness estimates are incorporated into the minimum thickness model, and the assumption that initial thickness and pit depth are statistically independent is avoided. That assumption could fail, for example, if steel quality and the initial thickness are correlated. This direct-model approach also avoids problems with maximum pit depth estimates, which require good measurements of wall thicknesses maxima measured at relatively uncorroded areas of cylinder surfaces, assumed to be as new. The direct model admits better incorporation of the information that there is zero corrosion at age zero; the indirect model does not make good use of this information, because, in the indirect model, pit depths are lognormally distributed and zero-depth pits are inadmissible.

In the indirect model, maximum pit depths are related to age by a power-law—if the fitted power-law slope is between 0 and 1. If the fitted slope is not between 0 and 1, then the slope is set to 1, and the model in which corrosion increases linearly in time is used instead. Unfortunately, for the data considered in this report, for nine of the fifteen cylinder groups, the power-law does not seem to fit, and the slope-set-to-one model is used instead. The failure of the power-law model is undoubtedly due, at least in part, to limitations of the cylinder thickness data. For various reasons, including a tendency in inspections to focus on deficient rather than good cylinder wall areas, minimum and maximum wall thickness measurements have sometimes been incompatible, and the power-law has not fit the maximum pit depth data very well. These difficulties with the indirect model were the primary reason for considering a direct-model alternative.

The data used for the direct and indirect models is from random or approximately random samples of cylinders collected each fiscal year starting in FY92. With the exception of some of the Portsmouth cylinders, each cylinder in this sample was measured during only one fiscal year. An alternative to this cross-sectional monitoring approach would be longitudinal monitoring, with cylinders measured multiple times over the years. A randomly selected sample of cylinders measured repeatedly over the years could serve as bellwethers for all of the cylinders. Because each cylinder in such a sample could serve as its own control, changes in the sample could be measured more closely than in cross-sectional samples. With each cylinder acting as its own control, models for thickness change could be simpler than models of thickness itself. This approach would not compensate for different measurement methods, such as P-scan vs manual, but biases due to the measurement method might be largely eliminated if great care was taken to use the same measurement techniques each year. Among the Portsmouth cylinders measured FY95-01, 336 have been measured during more than one fiscal year. Although it is unlikely that these cylinders have been measured over a long enough time to detect changes, some or all of these 336 could be incorporated into a longitudinal monitoring program.

Projecting cylinder conditions into the future on the basis of data collected with different goals, sampling schemes, and measurement methods is a difficult task—a task the limitations of which should be understood. Because it is less flexible and data anomalies do not affect it as easily, the direct corrosion model seems to fit the cylinder thickness data better than the indirect model. Yet while less flexibility is an advantage in dealing with noisy or anomalous data, it can be a disadvantage in reflecting the underlying physics of the corrosion process. And although the direct model seems to fit the cylinder data better, projections based on the two models are similar. Thus, there does not yet seem to be ample evidence to support the choice of either corrosion model over the other.

For both the direct and indirect approaches, relative to the variability of the data, corrosion appears to be only weakly related to cylinder age. That cylinder-to-cylinder variability is substantial, even for cylinders of the same age and grouping, is obvious from Figures 1-30. Nevertheless, age has an important and statistically significant effect on the corrosion process, and the oldest cylinders are of greatest concern. Tables 13 and 14 of group-wide numbers of cylinders projected to fall below the various thickness criteria can be misleading if careful attention is not also paid to the oldest and most vulnerable members of each cylinder group. Although **SEMP Action 3.1.1** is to project the number of non-compliant cylinders, attention should also be paid to risks for individual cylinders, particularly for cylinder groups that contain a mixture of new and old cylinders.

Both the direct and indirect corrosion models suggest the bottom rows of C-745-G yard at PGDP and K-1066-K yard at ETPP as the cylinders most at risk. The direct model, however, suggests that differences among the cylinder groups are not nearly as great as the indirect model suggests. According to the indirect model, the next most vulnerable groups are the Portsmouth thin skirted bottom cylinders (i.e., at the head/skirt interface) and the Paducah 30As. The next most vulnerable groups according to the direct model are the Portsmouth thin skirted cylinders, both top and bottom. Very few of the thick-wall cylinders show any likelihood of falling below even the 500 mil thickness criterion. Although corrosion increases over time, these conclusions pertain to both near-term (e.g., FY02) and longer-term projections (e.g., FY2020). More detailed information about the projections is in Tables 13 and 14.

The following caveats and limitations should be kept in mind when considering this report:

- Implicit in either the direct or indirect models is an assumption of age invariance—that newer or older cylinders alike had similar corrosion when they were the same age. The distributions of pit depths or wall thicknesses for 10 year old cylinders in a given population are assumed to be the same no matter when the cylinders were measured.
- Storage (e.g., ground contact) conditions have changed for many cylinders.
- Some cylinders have been painted.
- Environmental changes such as acid rain are not accounted for.
- Cylinder sampling was not always random.
- Significant differences between the data collected at different times for the same yards have been observed. This includes but is not restricted to differences that may be associated with the measurement method (manual or P-scan).
- Literature data for the atmospheric corrosion of steel might not apply to cylinder corrosion modeling, because of the thermal inertia of the cylinders.

- In the indirect model, the maximum pit depths are only estimates, because initial wall thicknesses are estimates from maximum wall thicknesses.
- Group-wide projections for groups with large numbers of cylinders may obscure vulnerabilities in cylinders that are old compared to most of their group.

### **Recommendations:**

1. The values in this report are based on the assumption that the historical trends will continue, and thus represent all baseline projections. Many of the yards are being improved. Future analyses should incorporate these changes, when they can be quantified and accounted for.
2. An inventory of recently painted cylinders should be used to revise the cylinder group population counts (e.g., by excluding from the at-risk cylinders, cylinders painted in the last ten years).
3. The 30A cylinders continue to be something of an anomaly. They seem more variable than the other groups. Potential explanatory variables (besides top/bottom status) should be identified and explored by incorporating them as potential predictors into the current corrosion models.
4. As it appears unlikely that the apparent discrepancy between P-Scan and manual UT results will be resolved, adjustments should be incorporated into the statistical models (at least into the direct model) to attempt to account for the differences between the P-Scan and manual measurements. The pre-FY98 and FY98-and-later cylinder groups should be combined.
5. Extreme value distributions should be investigated for both the indirect and direct models. The extreme value distribution has a physical basis for models of minima or maxima, and might provide an alternative to the lognormal indirect model that does not fail to conform with the power-law in so many cases. The failure of the power-law model could be due to improperly weighting the data in the model fitting, and the weighting is a reflection of the underlying statistical distribution (e.g., lognormal) that is assumed.
6. Particularly for groups that contain a mixture of new and old cylinders, risks for individual cylinders should be calculated and used to prioritize actions for individual cylinders, not just cylinder groups.
7. Although the direct model appears to fit the UT cylinder thickness data better than the indirect model, that conclusion is tentative. Until a clearer picture is established, cylinder thickness data should be analyzed using both the direct and indirect model approaches.
8. A longitudinal design should be considered for future cylinder monitoring at all three cylinder storage sites. The 336 Portsmouth cylinders that have been measured more than once since FY95 should be analyzed to see if any significant changes in these cylinders have occurred during this time period.

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## **APPENDIX A: FIGURES**

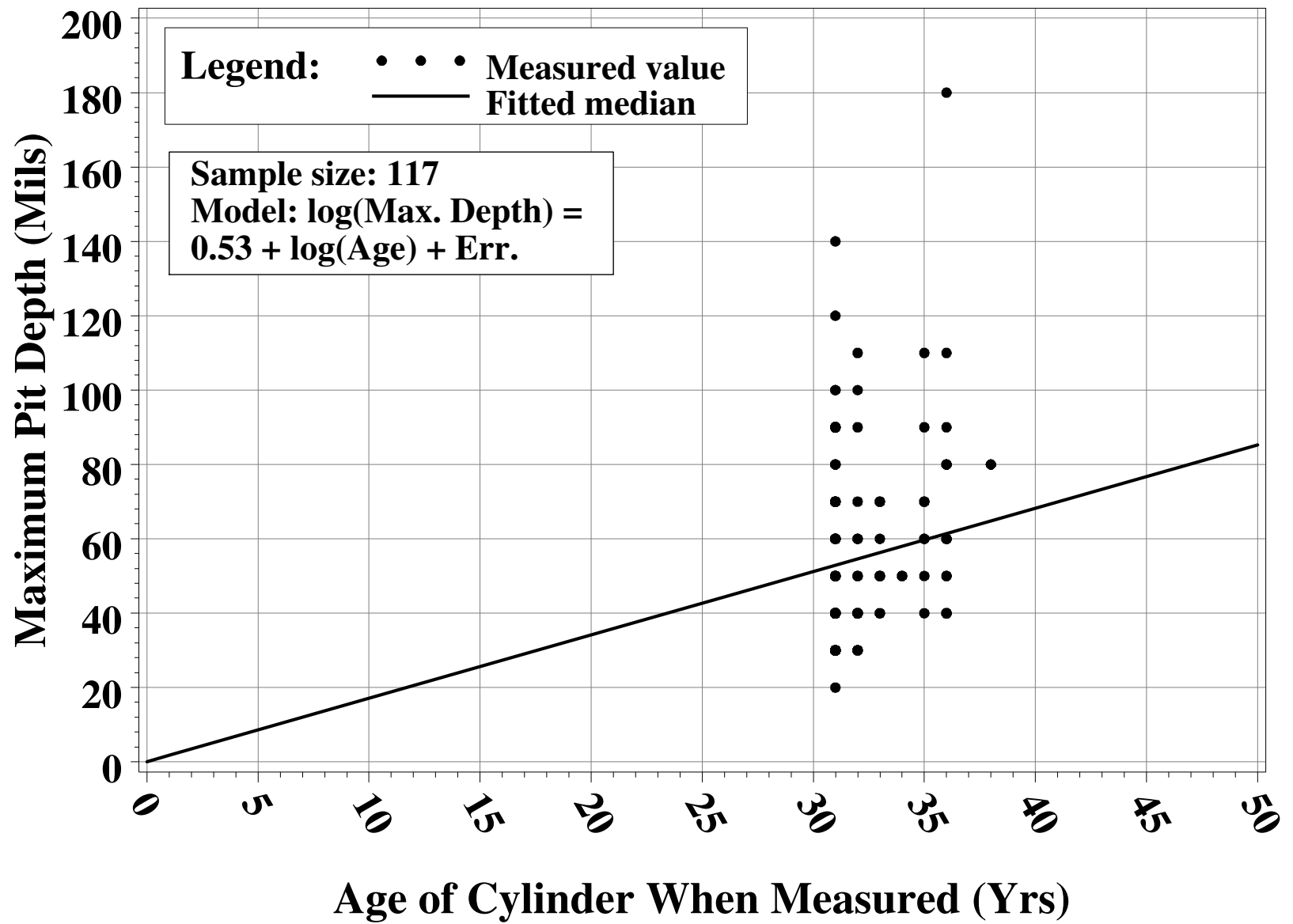


Figure 1. Pit depth estimates for K-1066-K, top and bottom, pre-FY98.

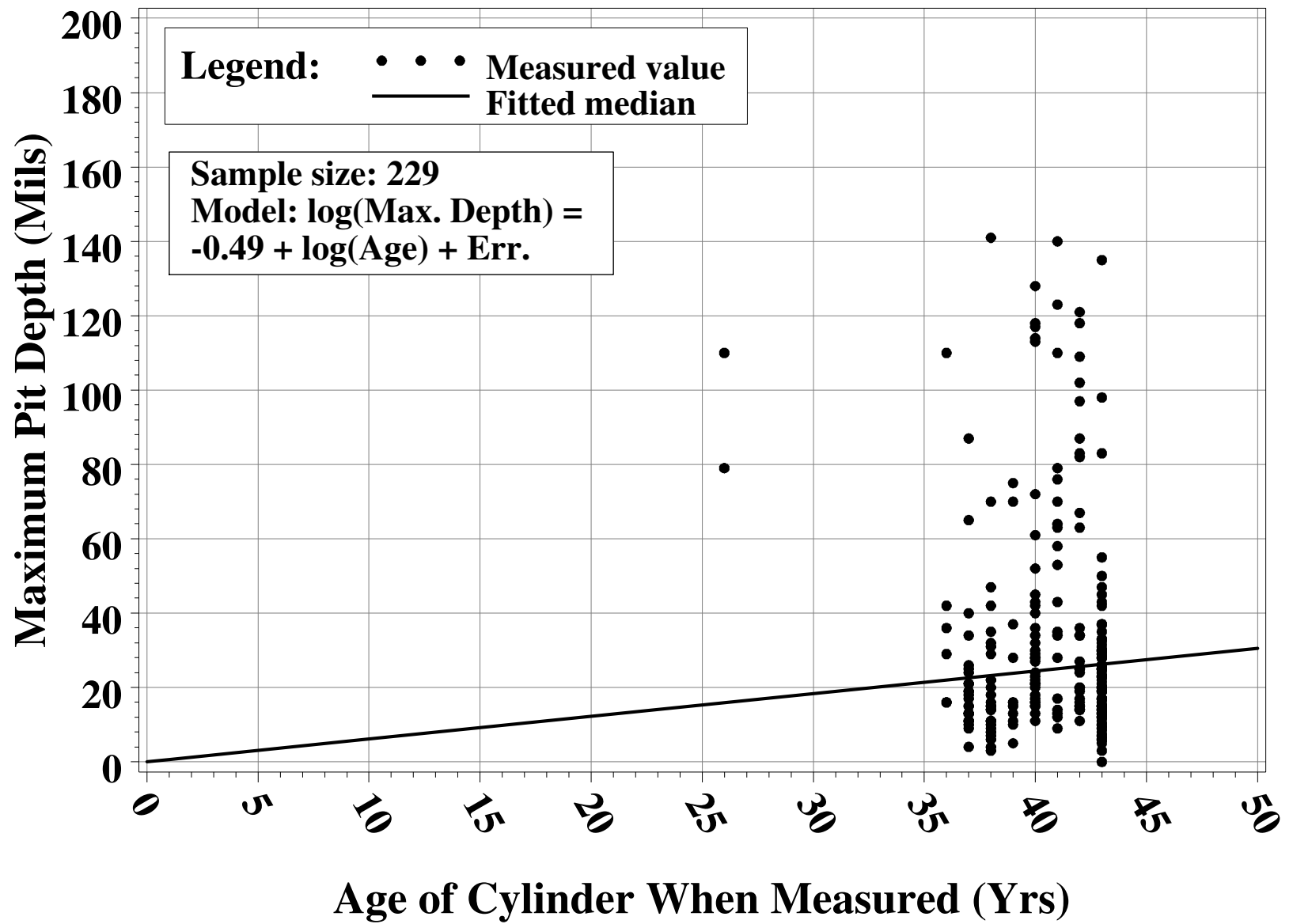


Figure 2. Pit depth estimates for K-1066-K, evaluated FY98-01.



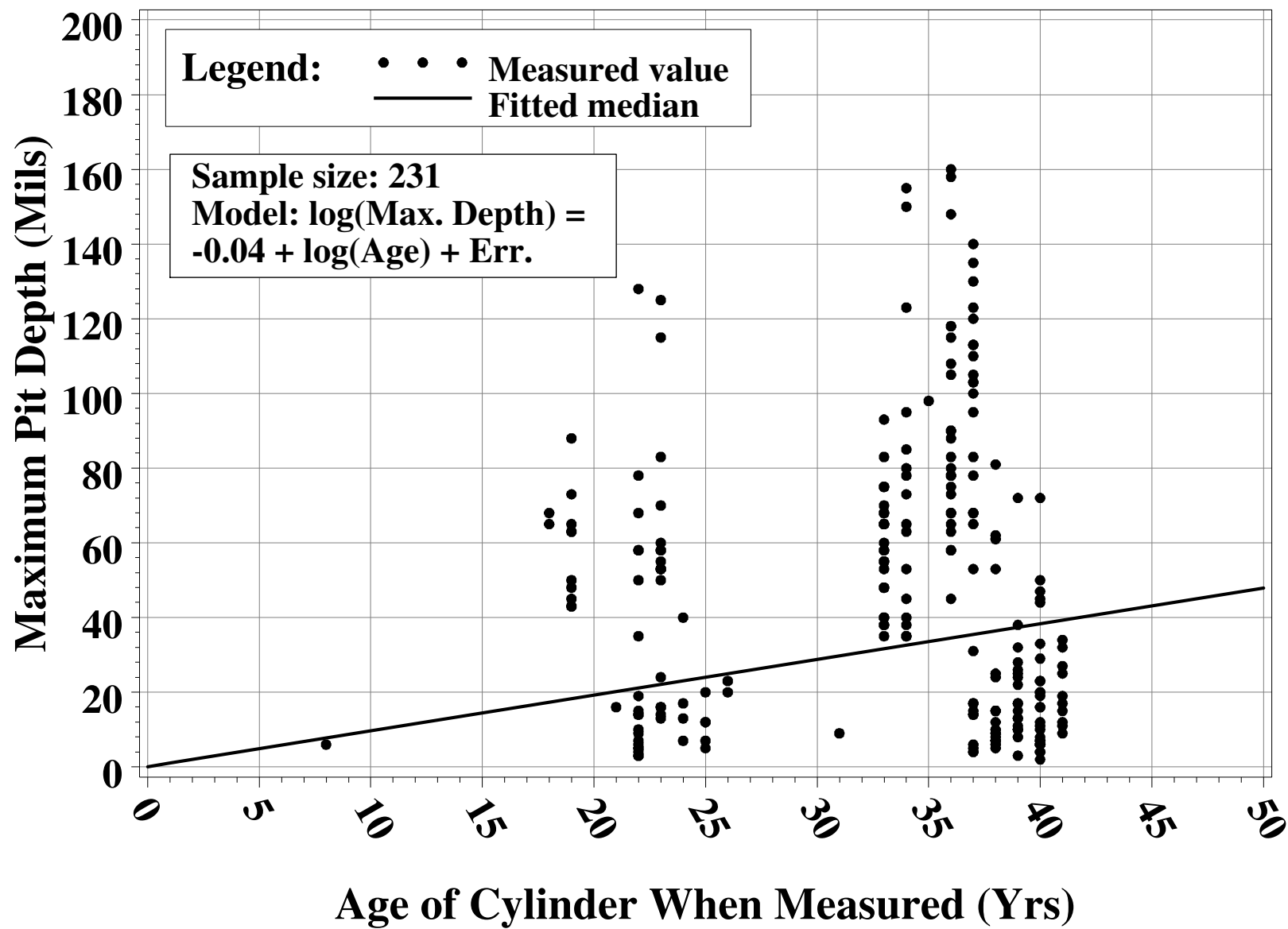


Figure 3. Pit depth estimates for C-745-G, bottom.

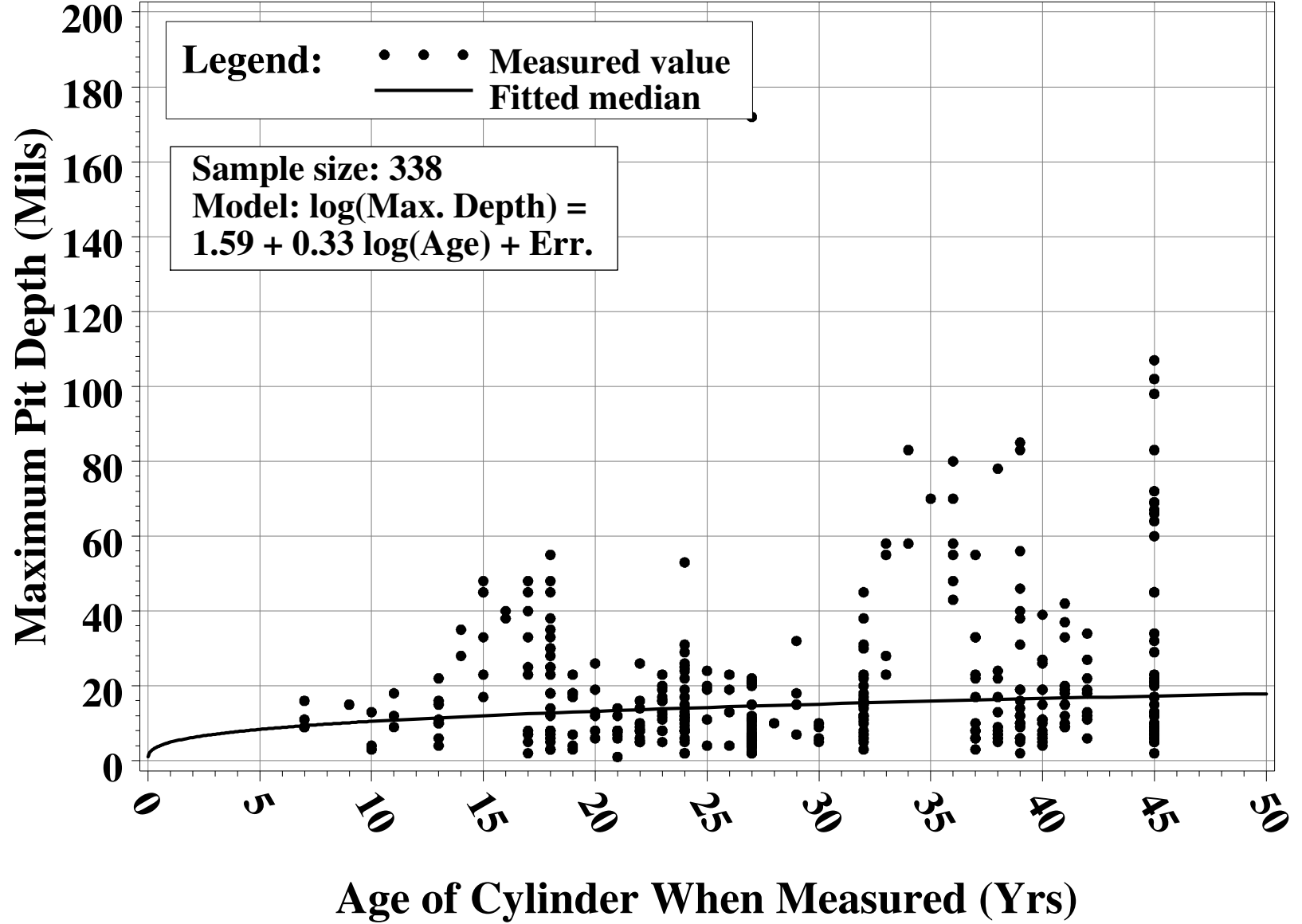


Figure 4. Pit depth estimates for PGDP bottom, except G-yard.

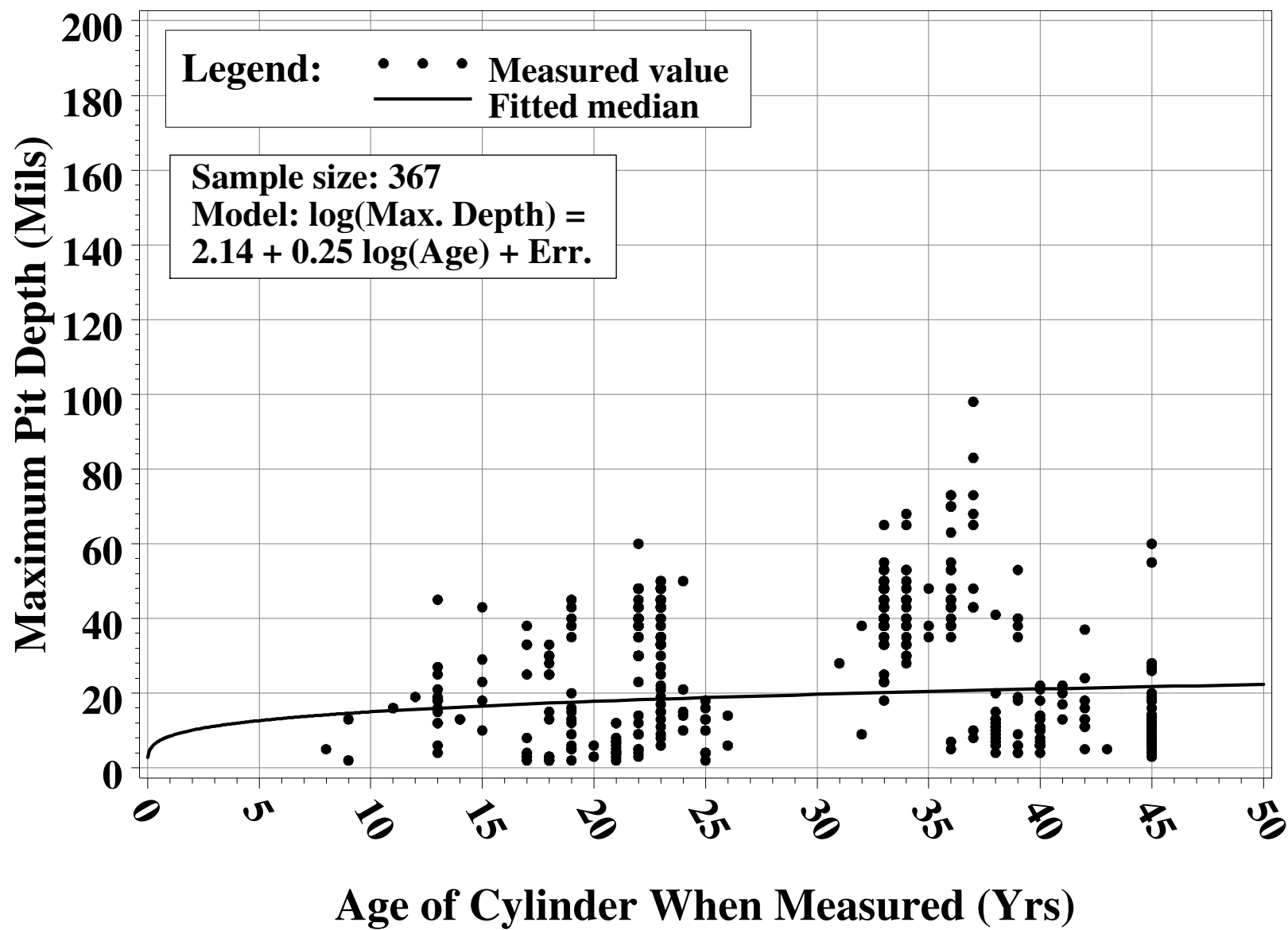


Figure 5. Pit depth estimates for PGDP top.

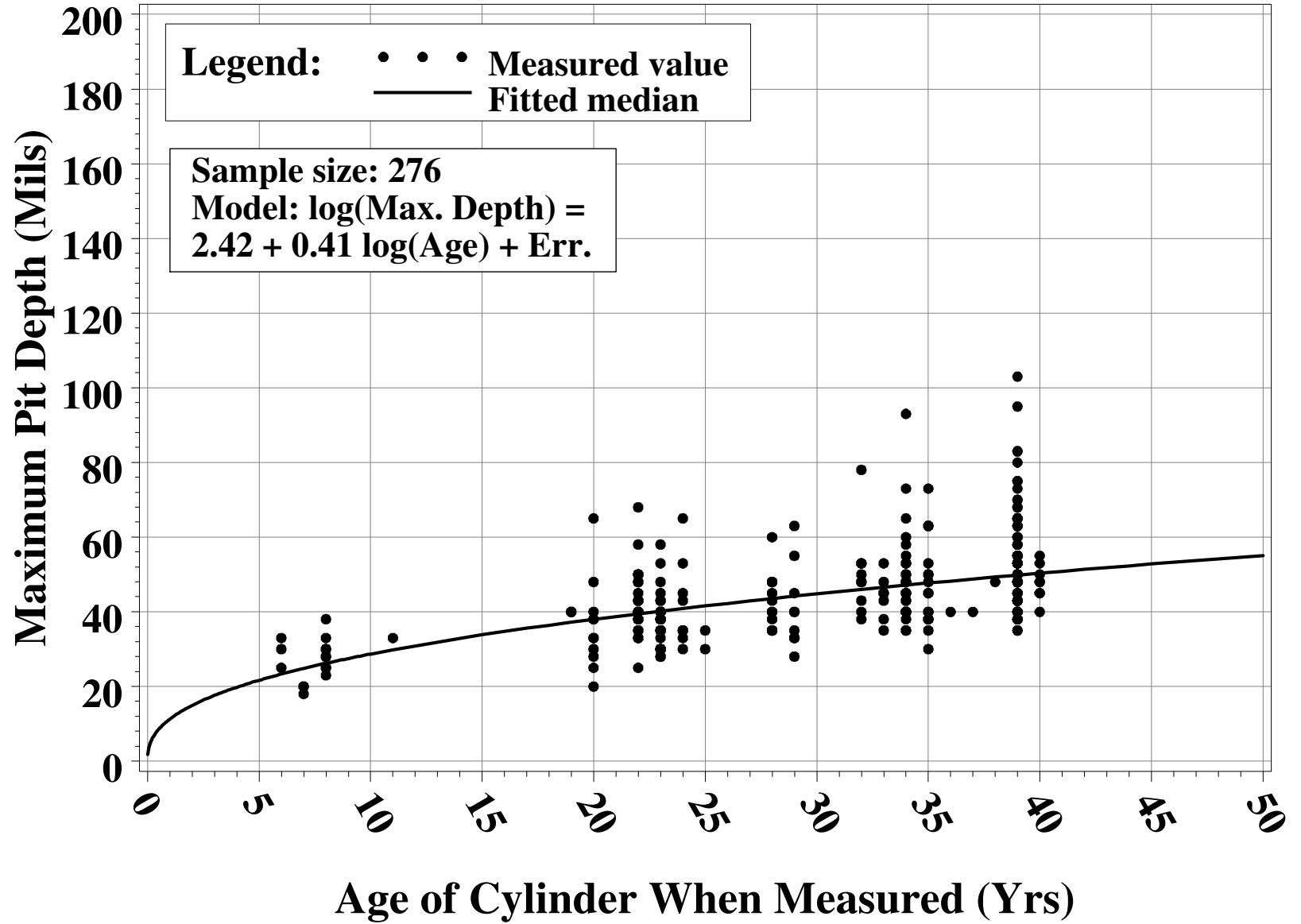


Figure 6. Pit depth estimates for PORTS, top, pre-FY98.

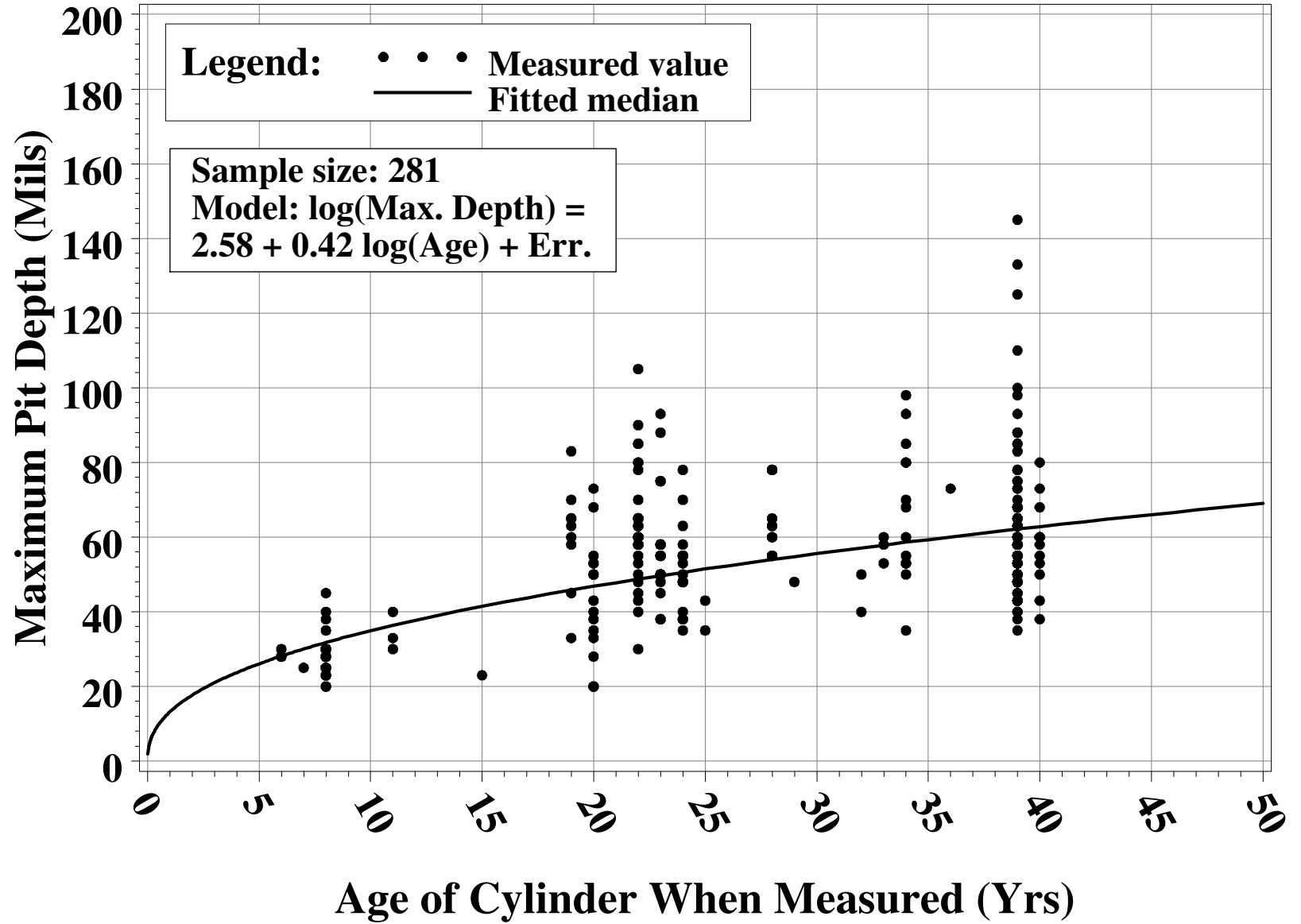


Figure 7. Pit depth estimates for PORTS bottom, pre-FY98.

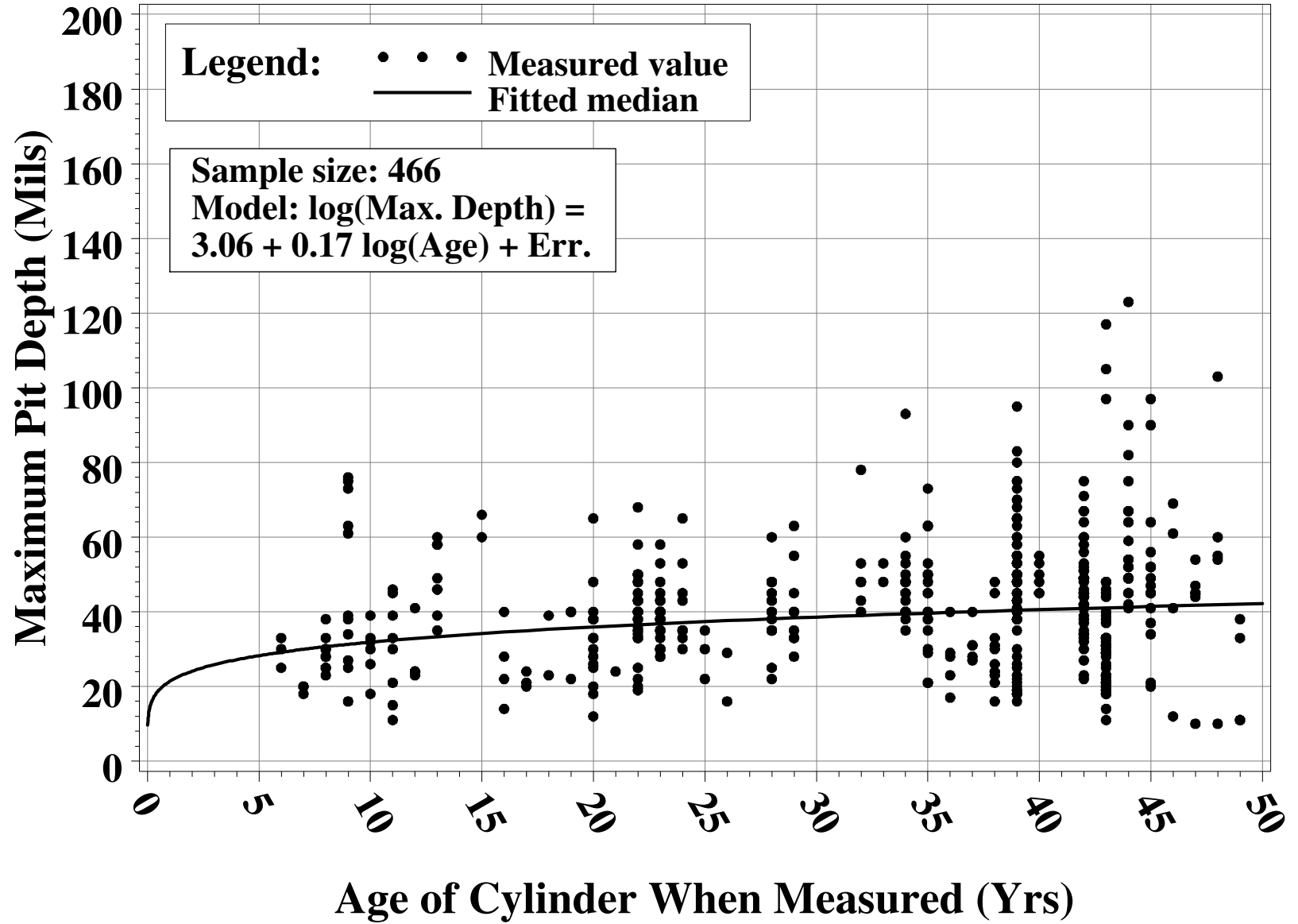


Figure 8. Pit depth estimates for PORTS thin and PORTS and PGDP thick, top.

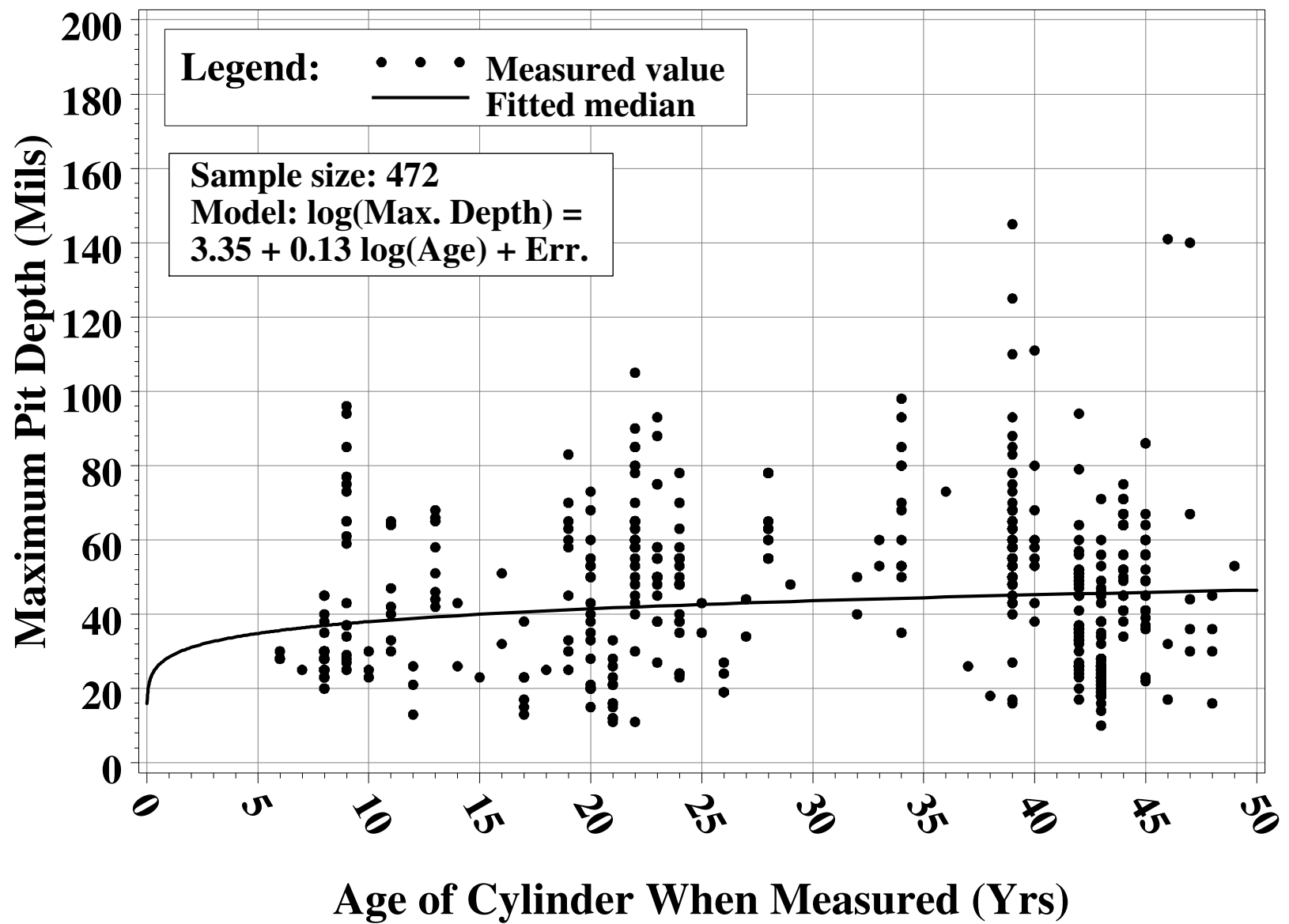


Figure 9. Pit depth estimates for PORTS thin and PORTS and PGDP thick, bottom.

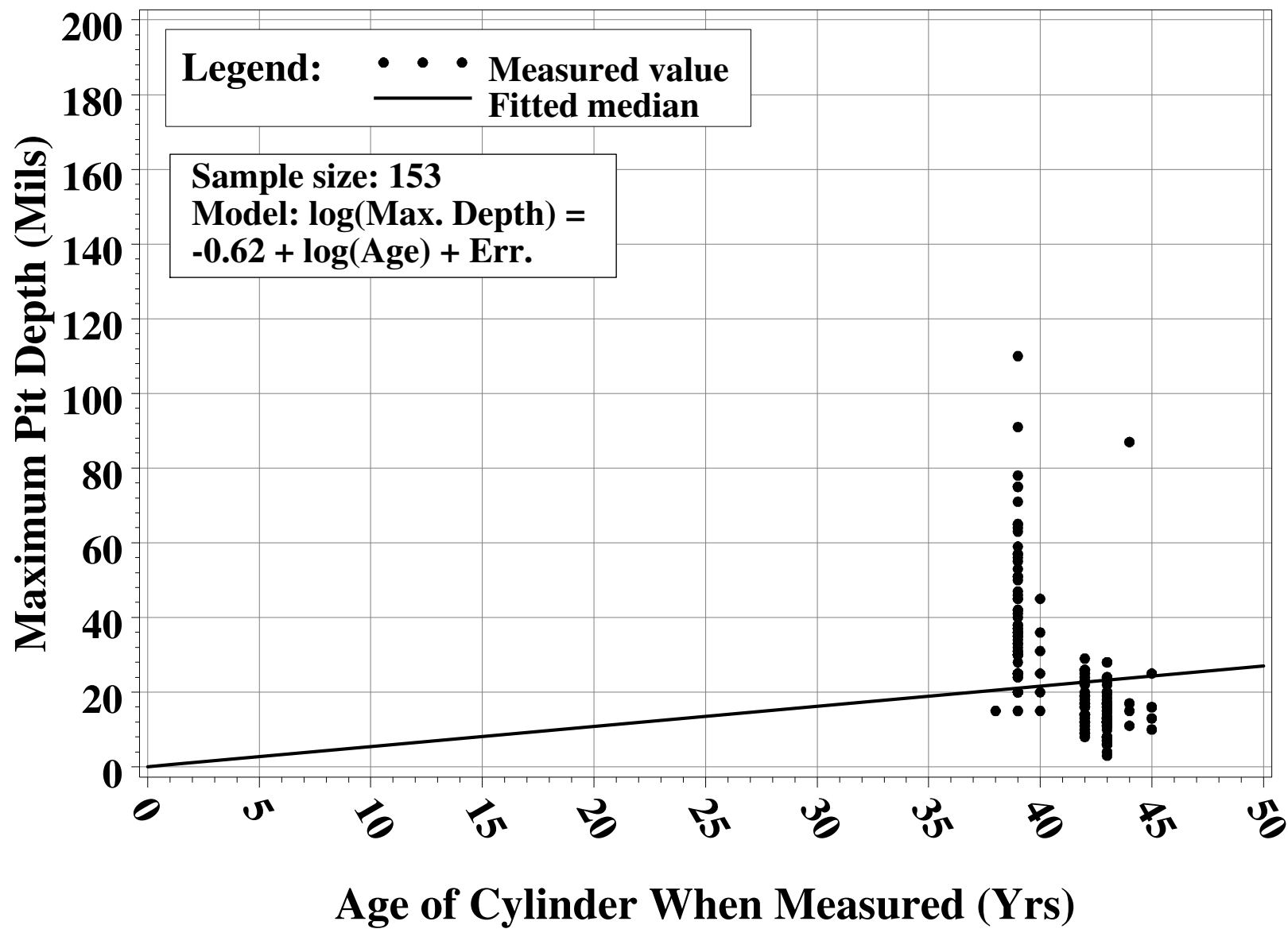


Figure 10. Pit depth estimates for PORTS thin, skirted, top.



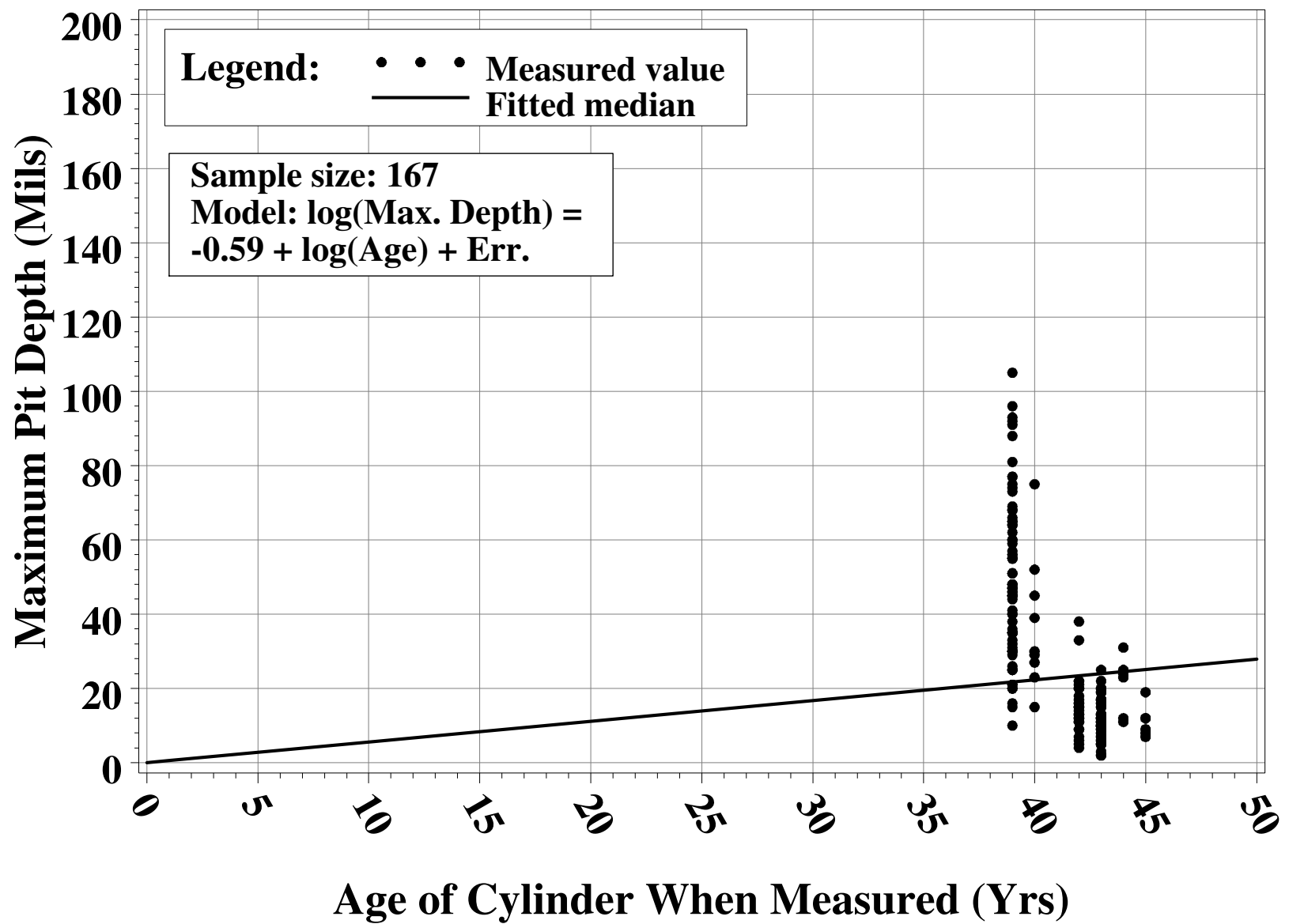


Figure 11. Pit depth estimates for PORTS thin, skirted, bottom.

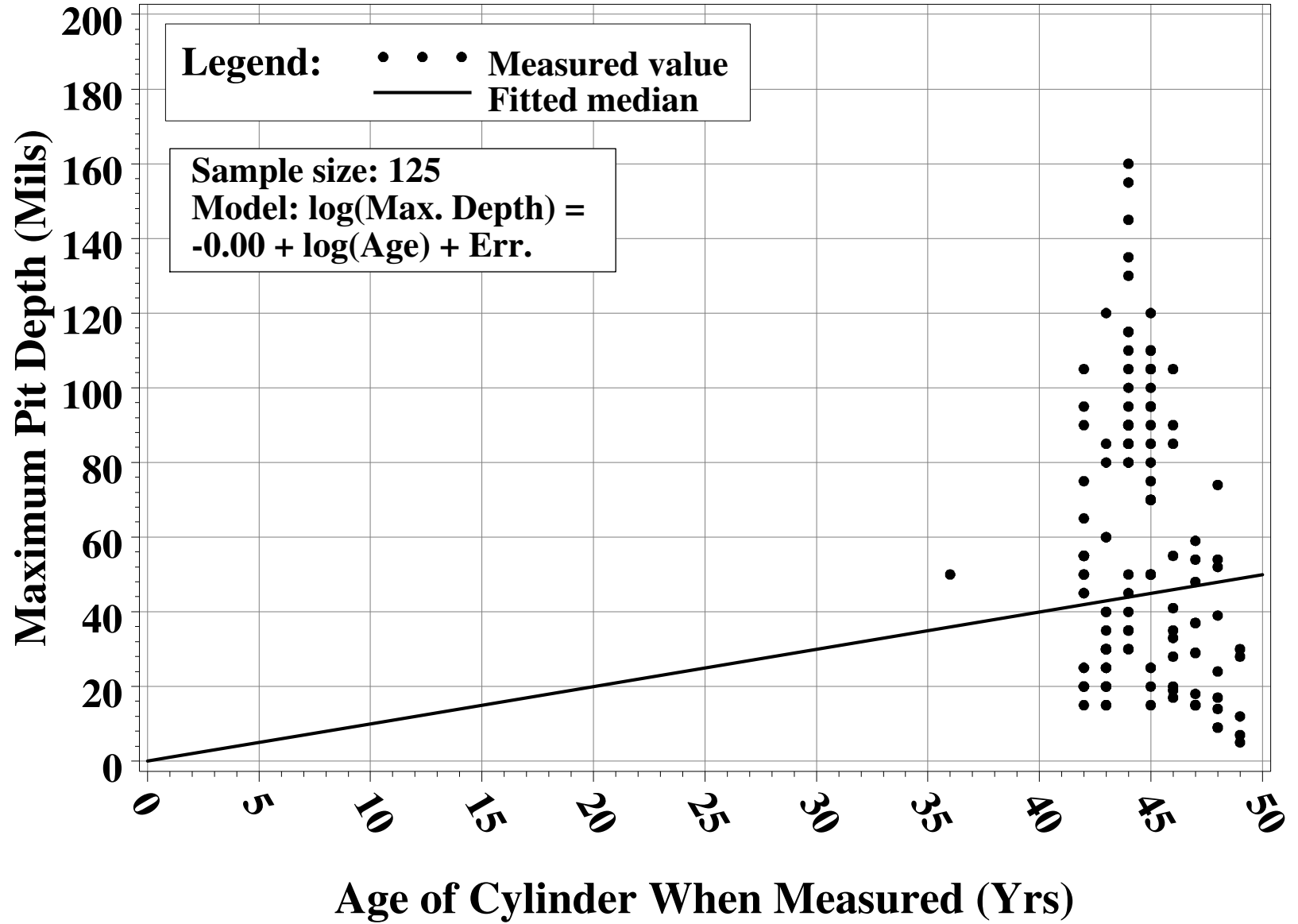


Figure 12. Pit depth estimates for PORTS thick, skirted, top and bottom.

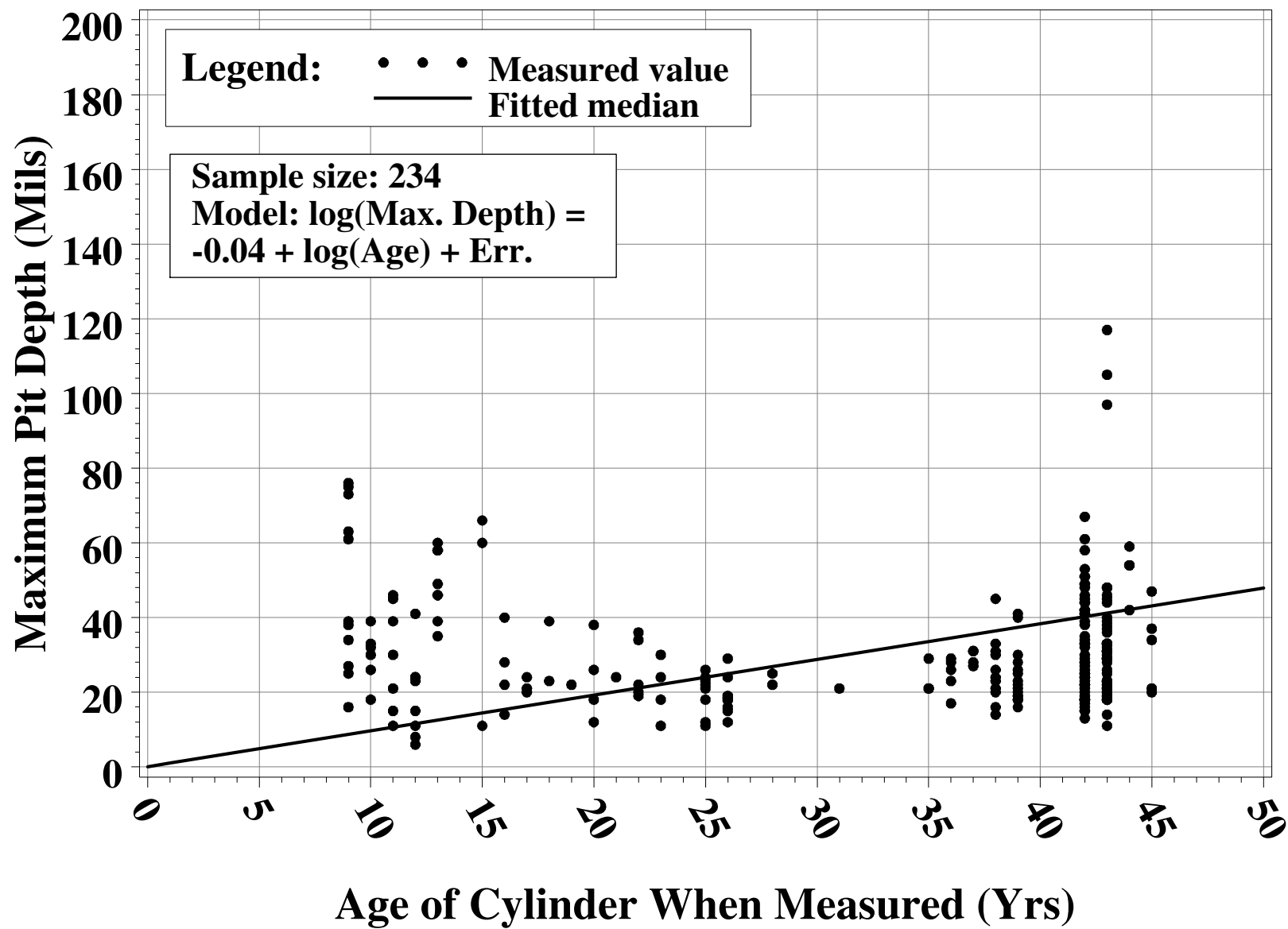


Figure 13. Pit depth estimates for PORTS thin, top, FY98 and later.

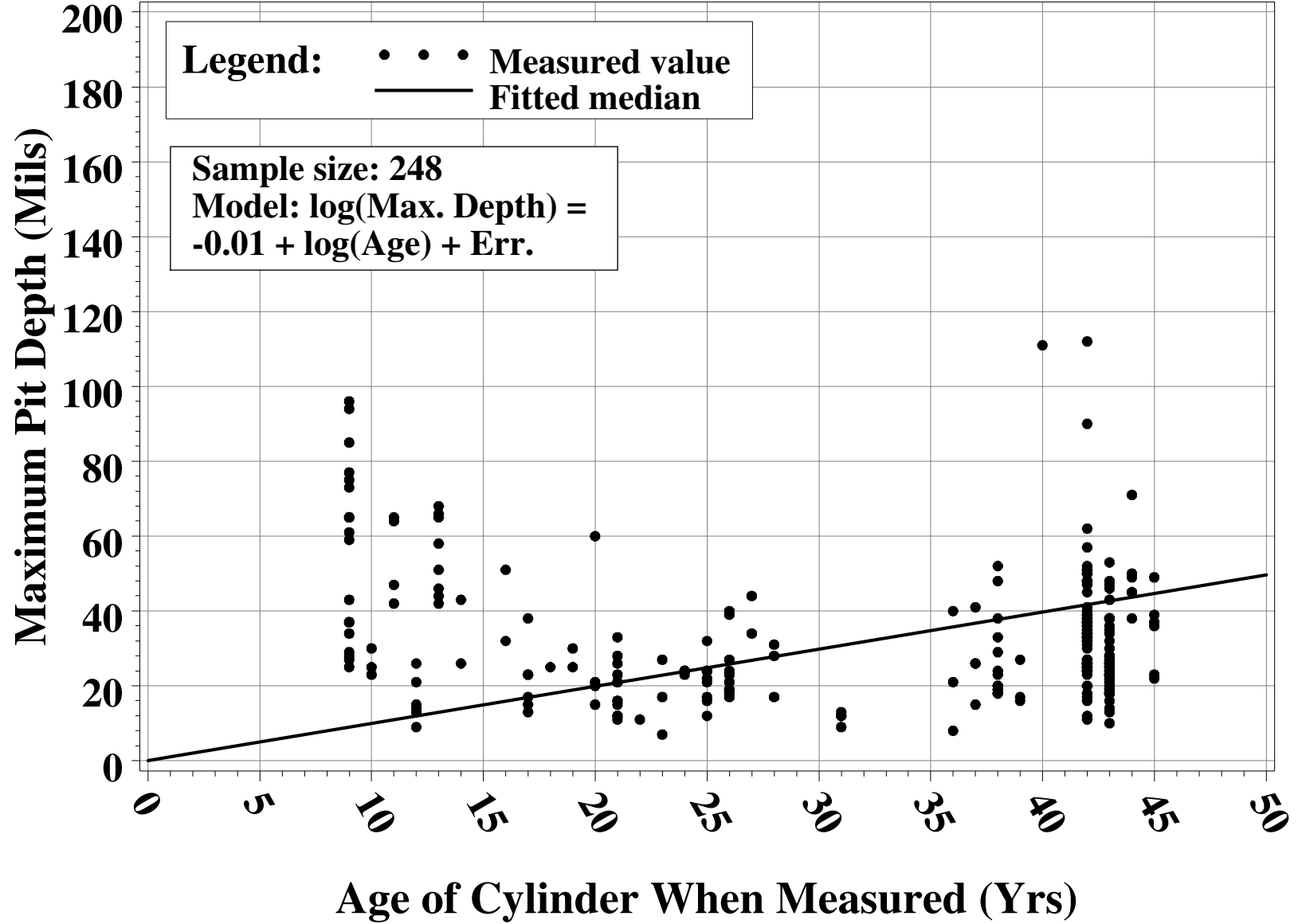


Figure 14. Pit depth estimates for PORTS thin, bottom, FY98 and later.

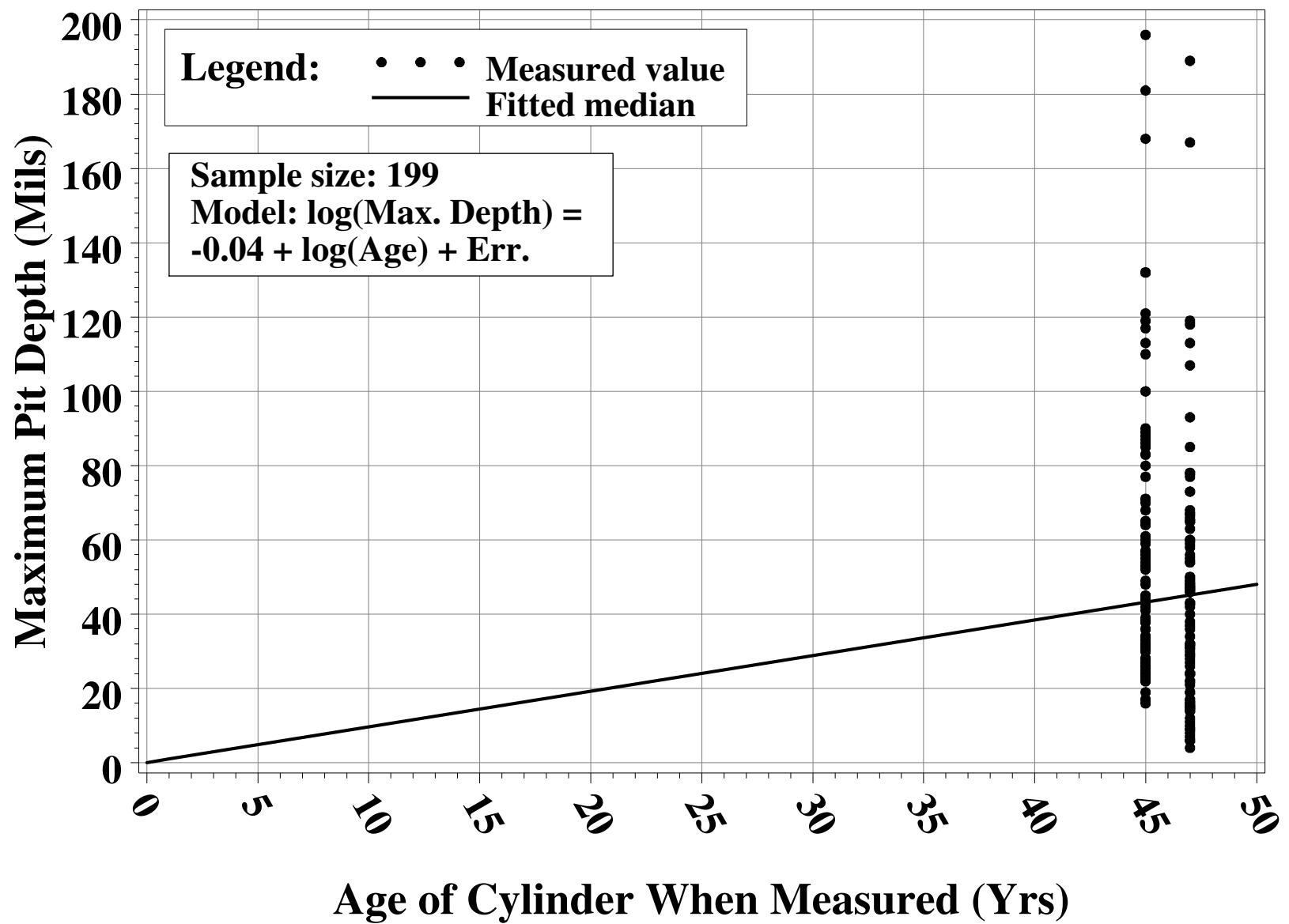
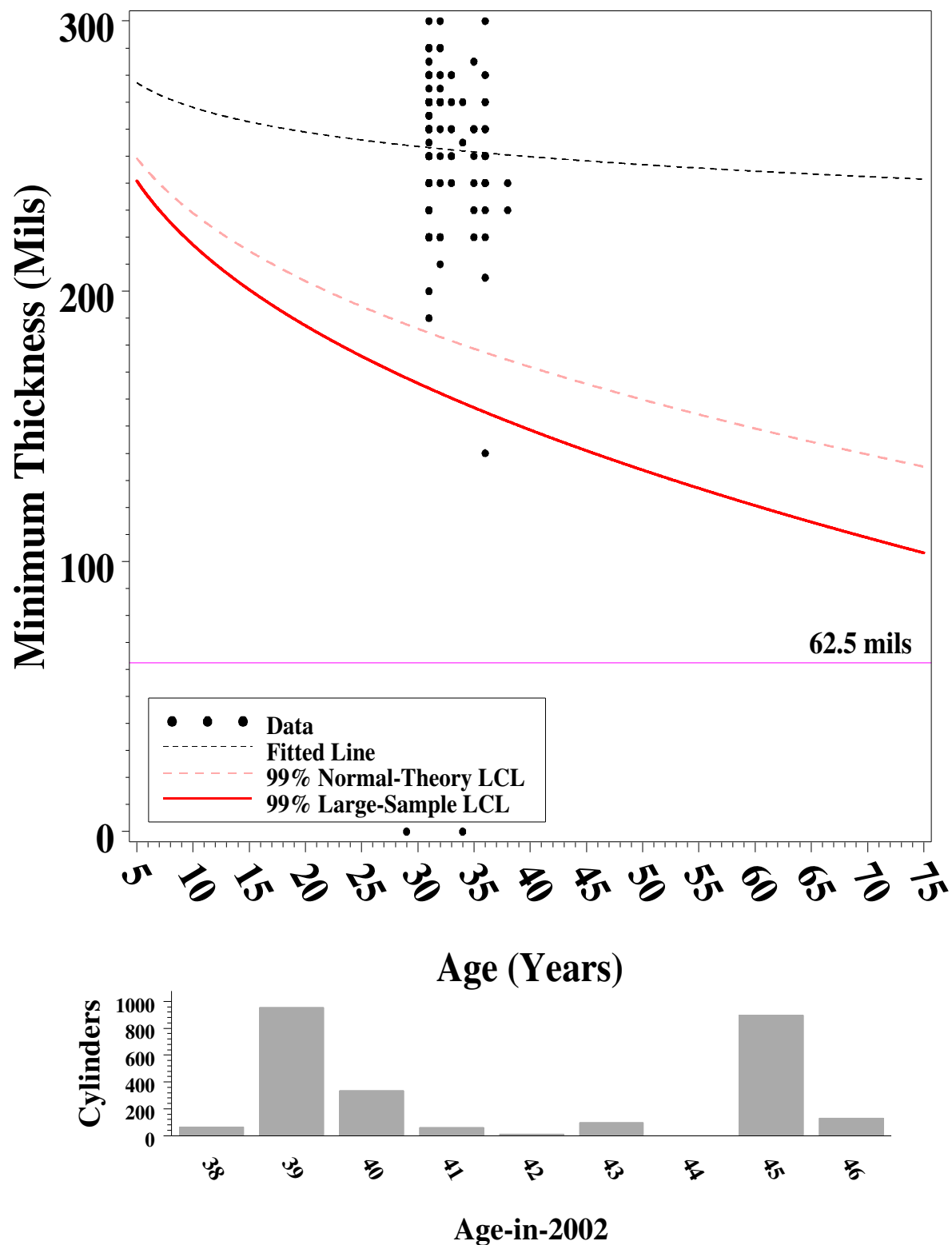


Figure 15. Pit depth estimates for Paducah 30As.



**Figure 16. Minimum thicknesses and cylinder age distribution for ETPP, Thin, top and bottom, pre-FY98.**

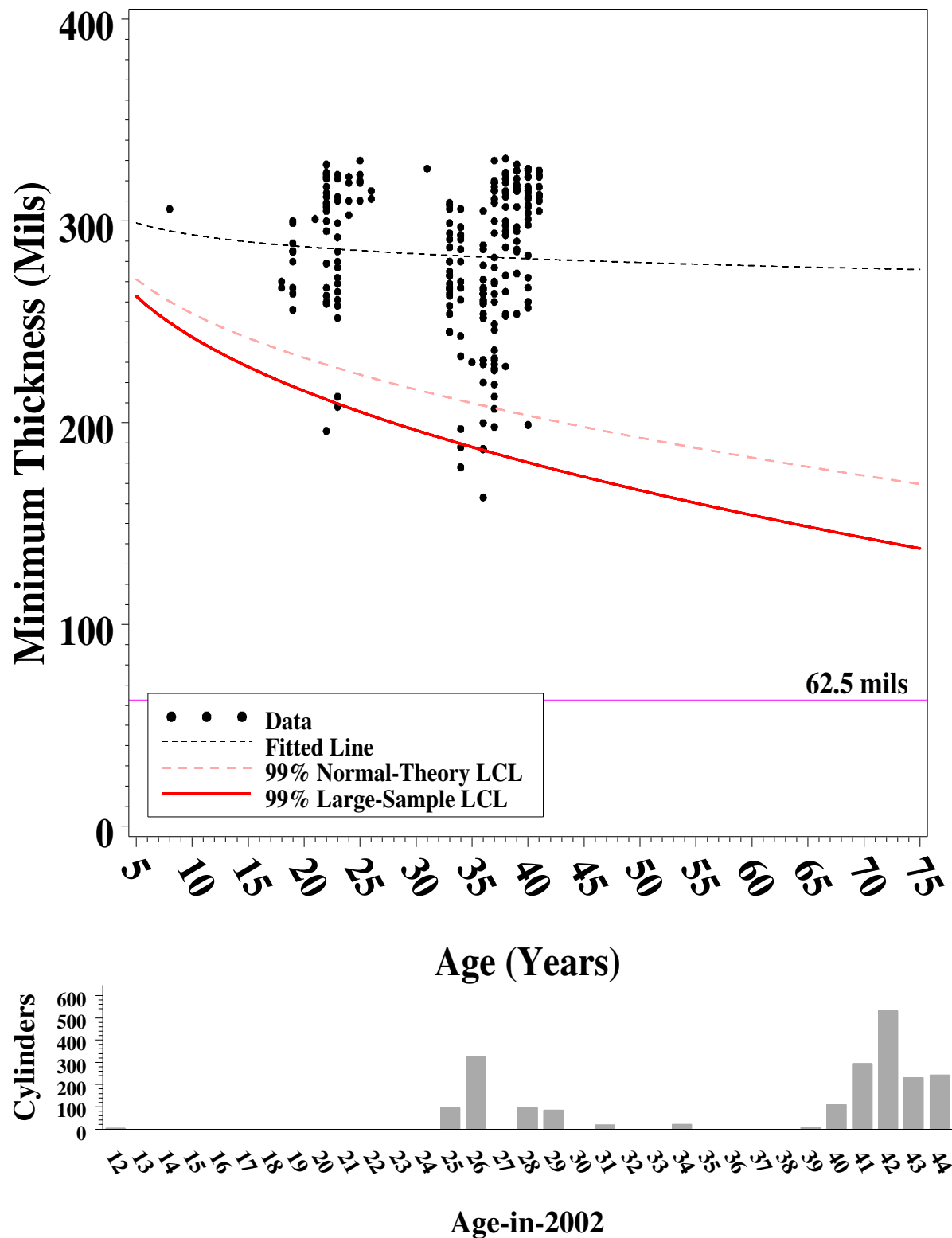
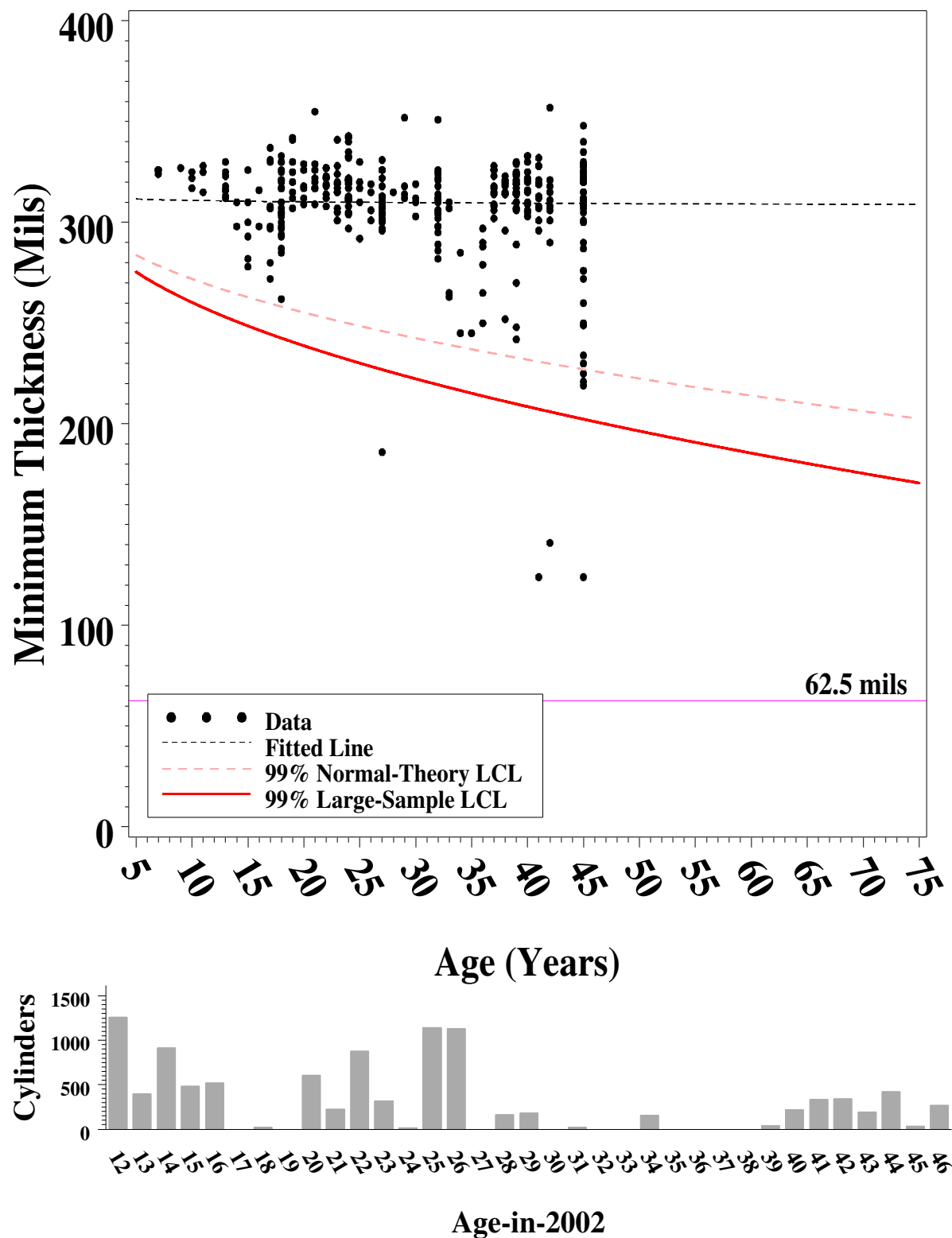
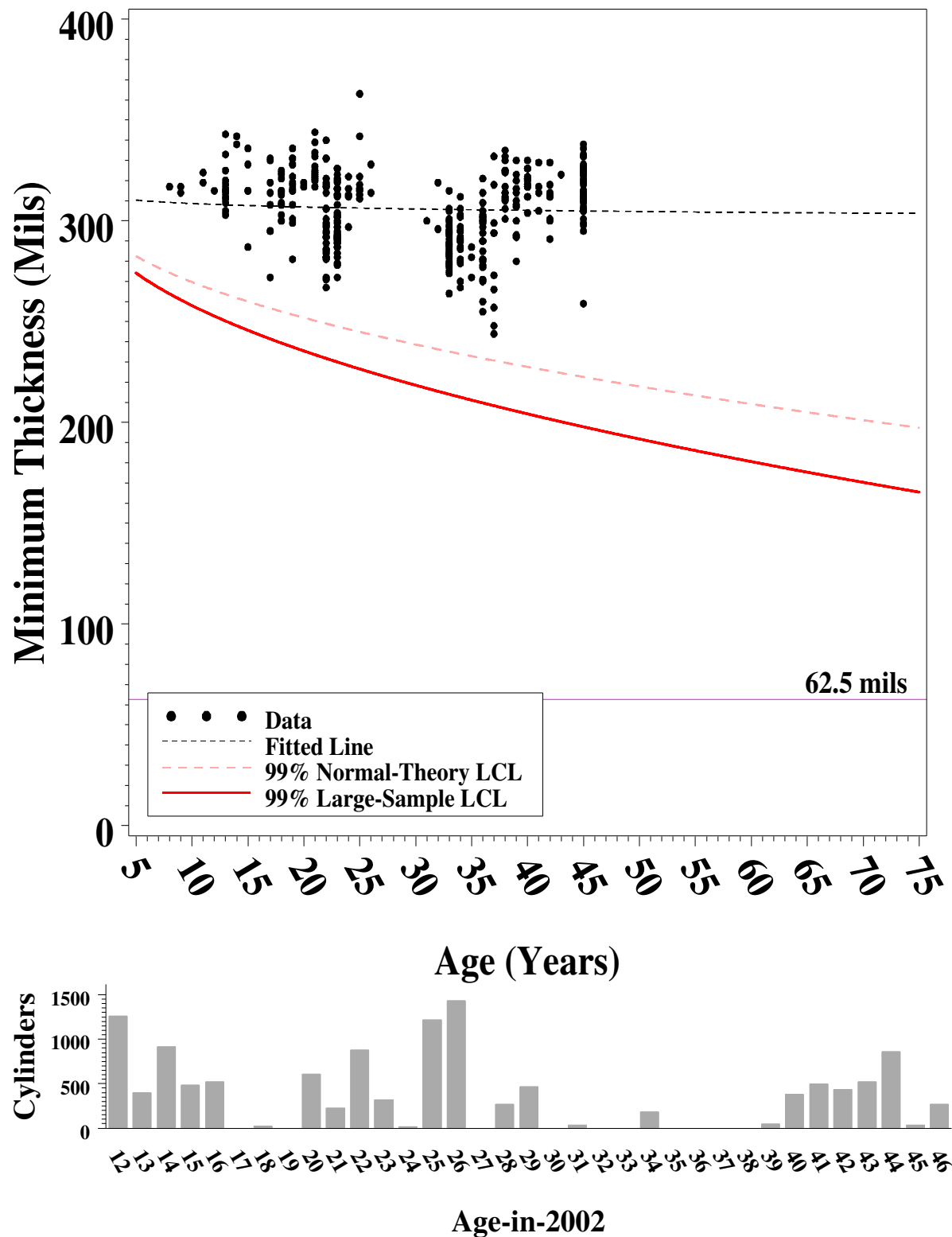


Figure 17. Minimum thicknesses and cylinder age distribution for PGDP Thin, former G yard, bottom.

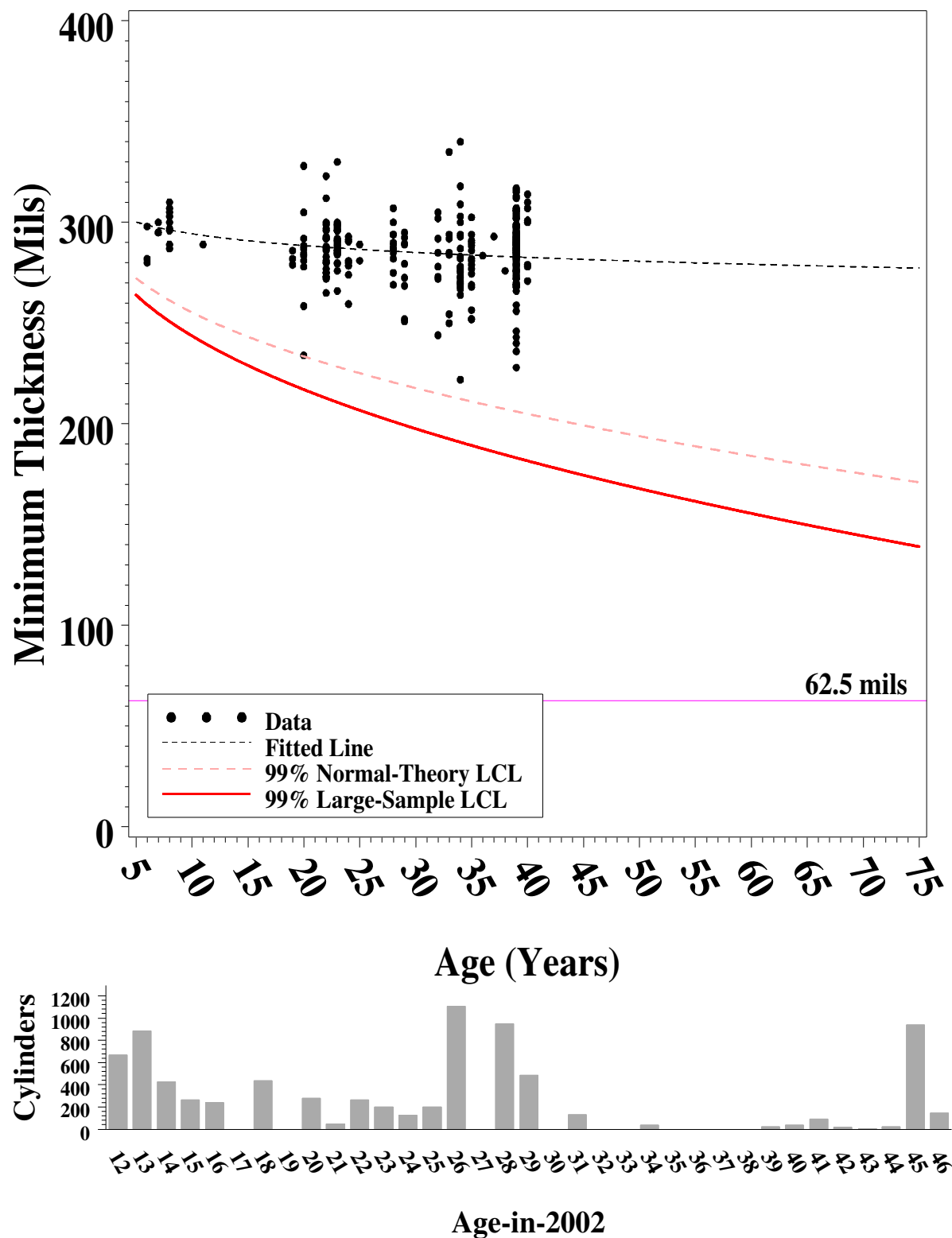


**Figure 18. Minimum thicknesses and cylinder age distribution for PGDP Thin, bottom, except former G yard.**





**Figure 19. Minimum thicknesses and cylinder age distribution for PGDP Thin, top.**



**Figure 20. Minimum thicknesses and cylinder age distribution for PORTS Thin, top, pre-FY98.**

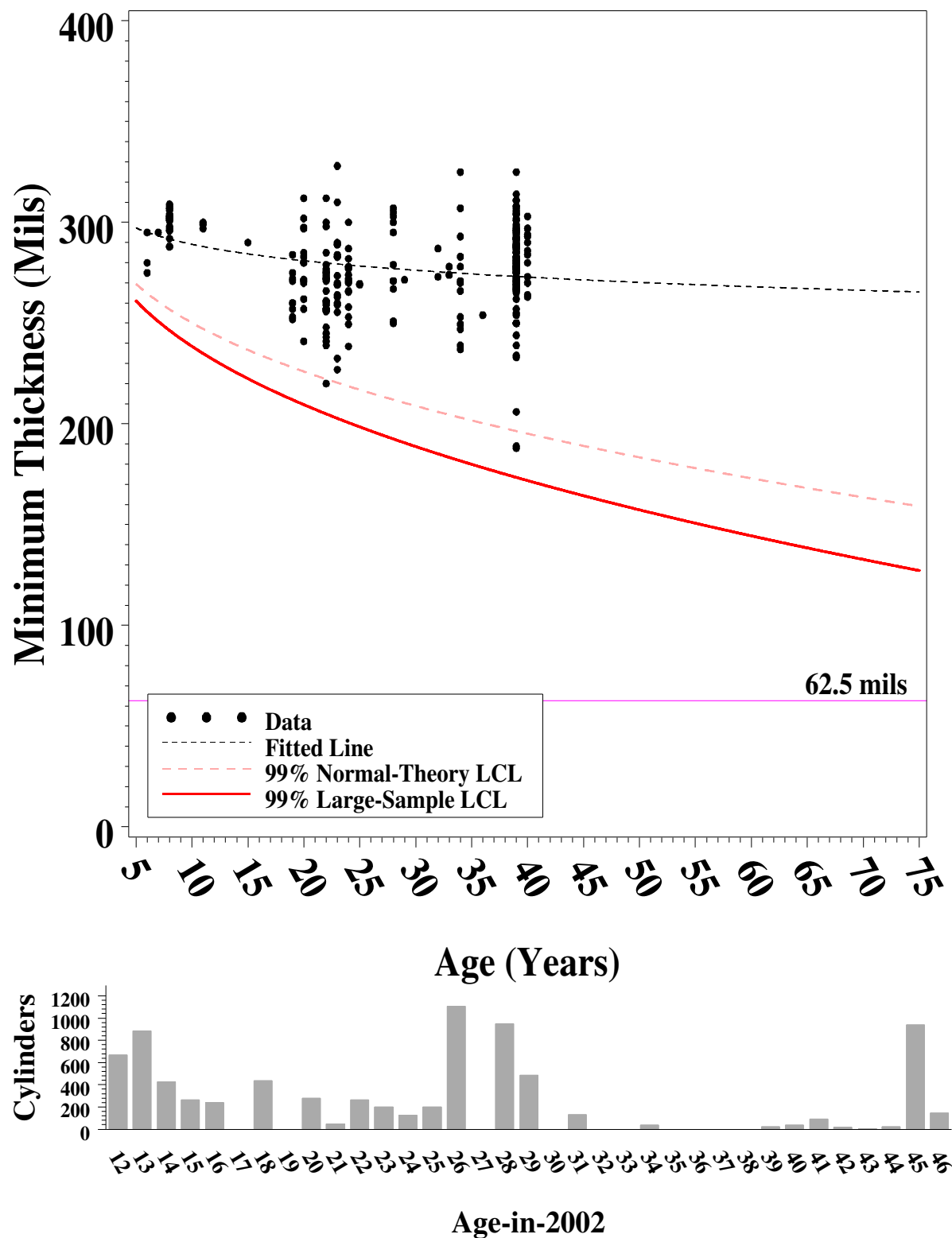
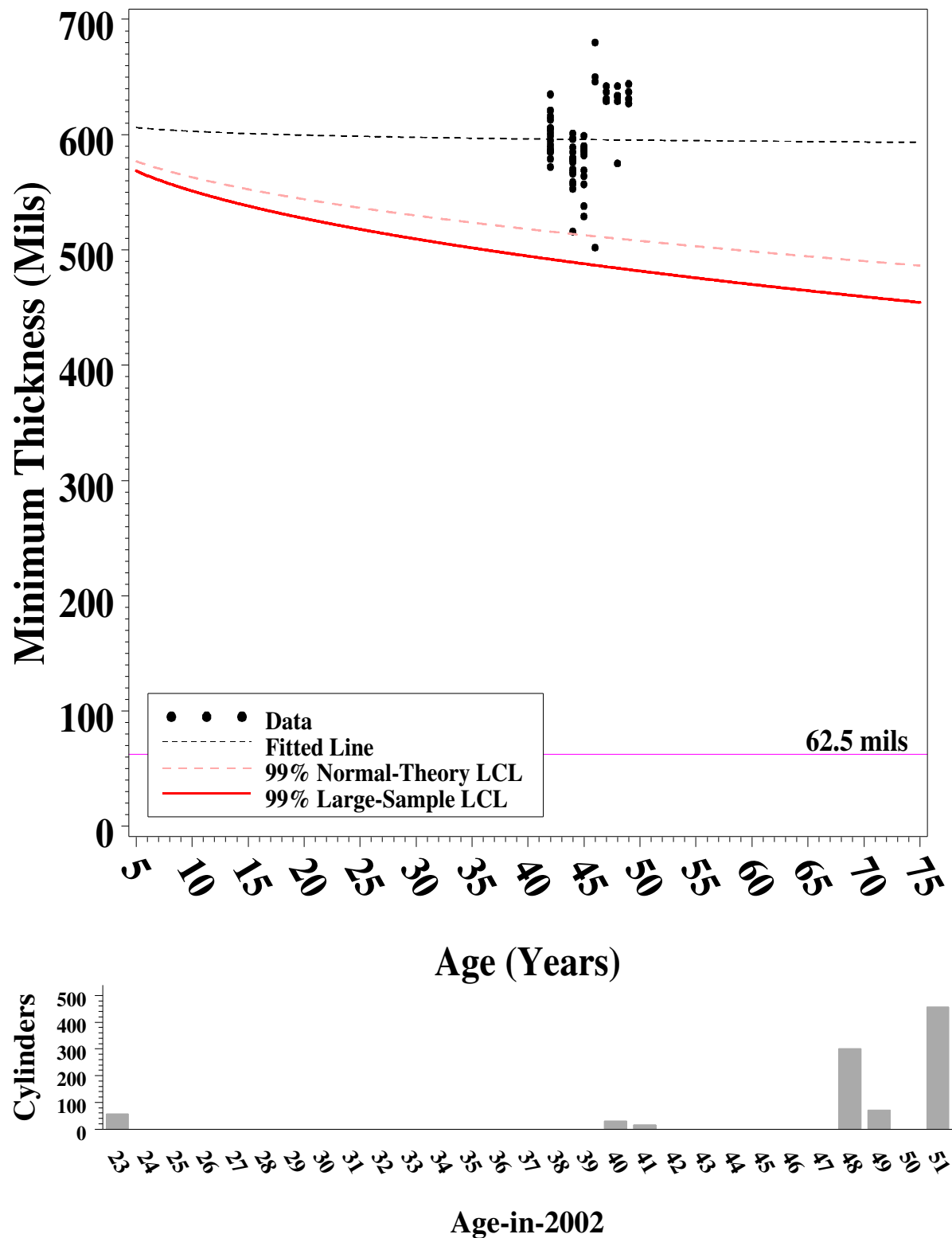
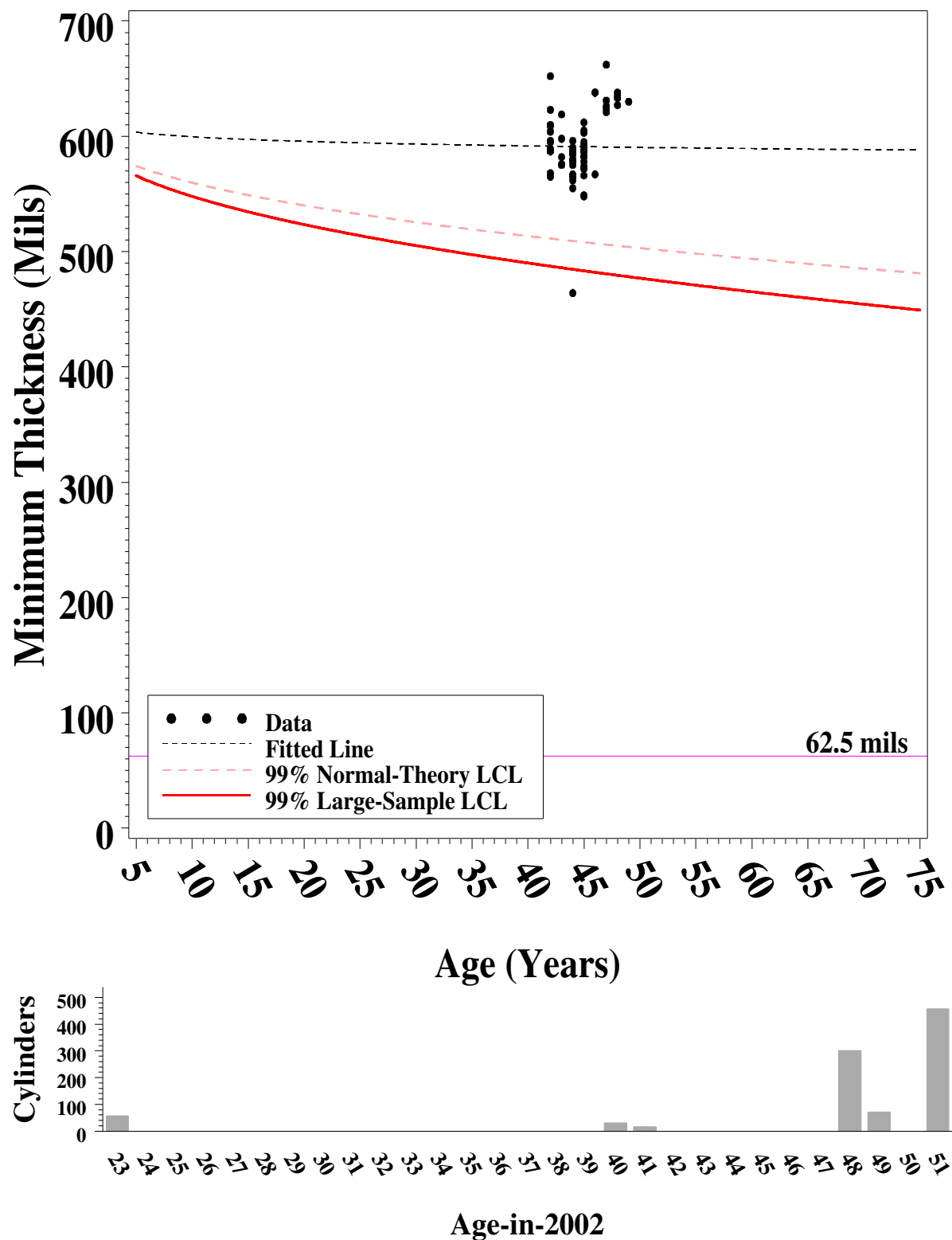


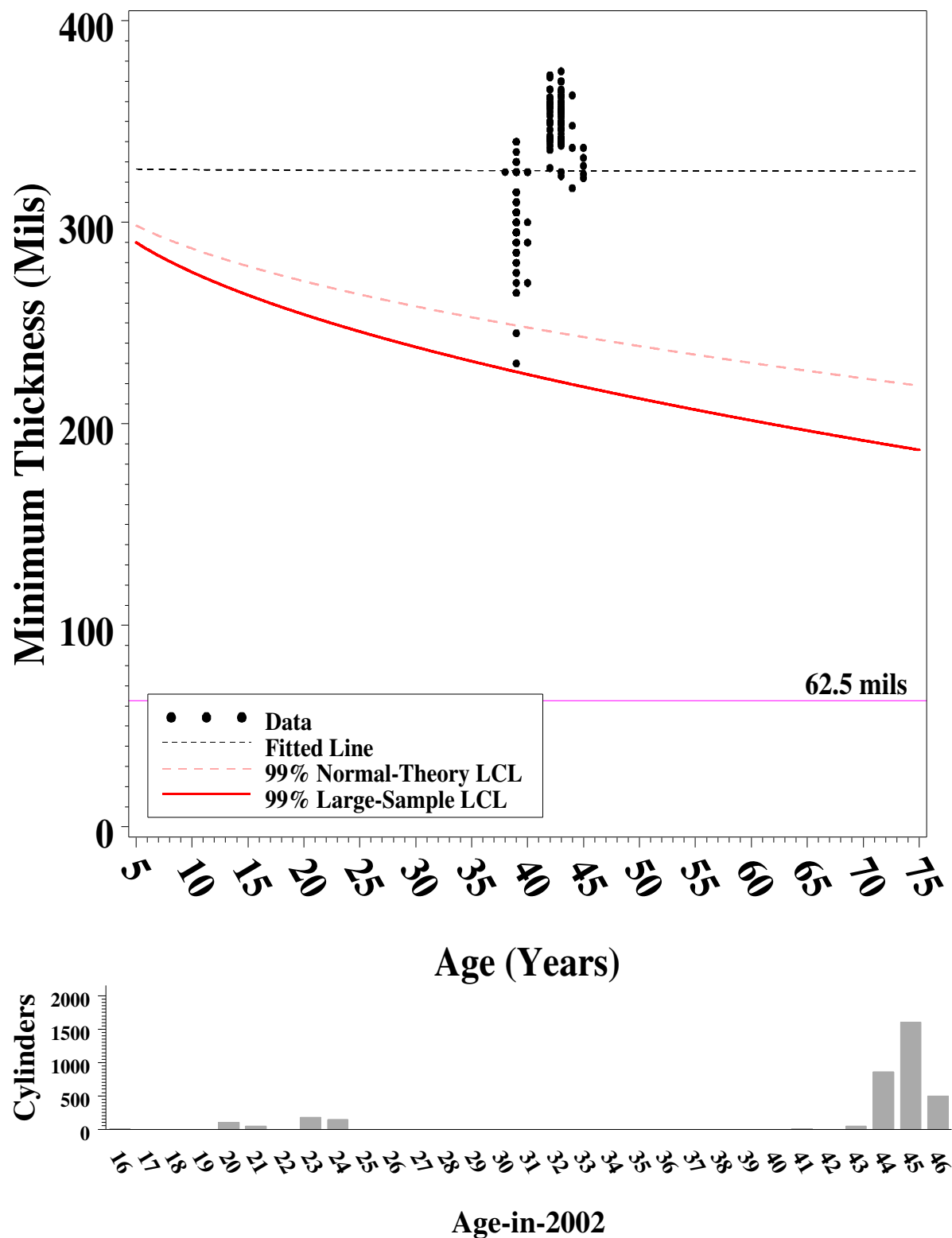
Figure 21. Minimum thicknesses and cylinder age distribution for PORTS Thin, bottom, pre-FY98.



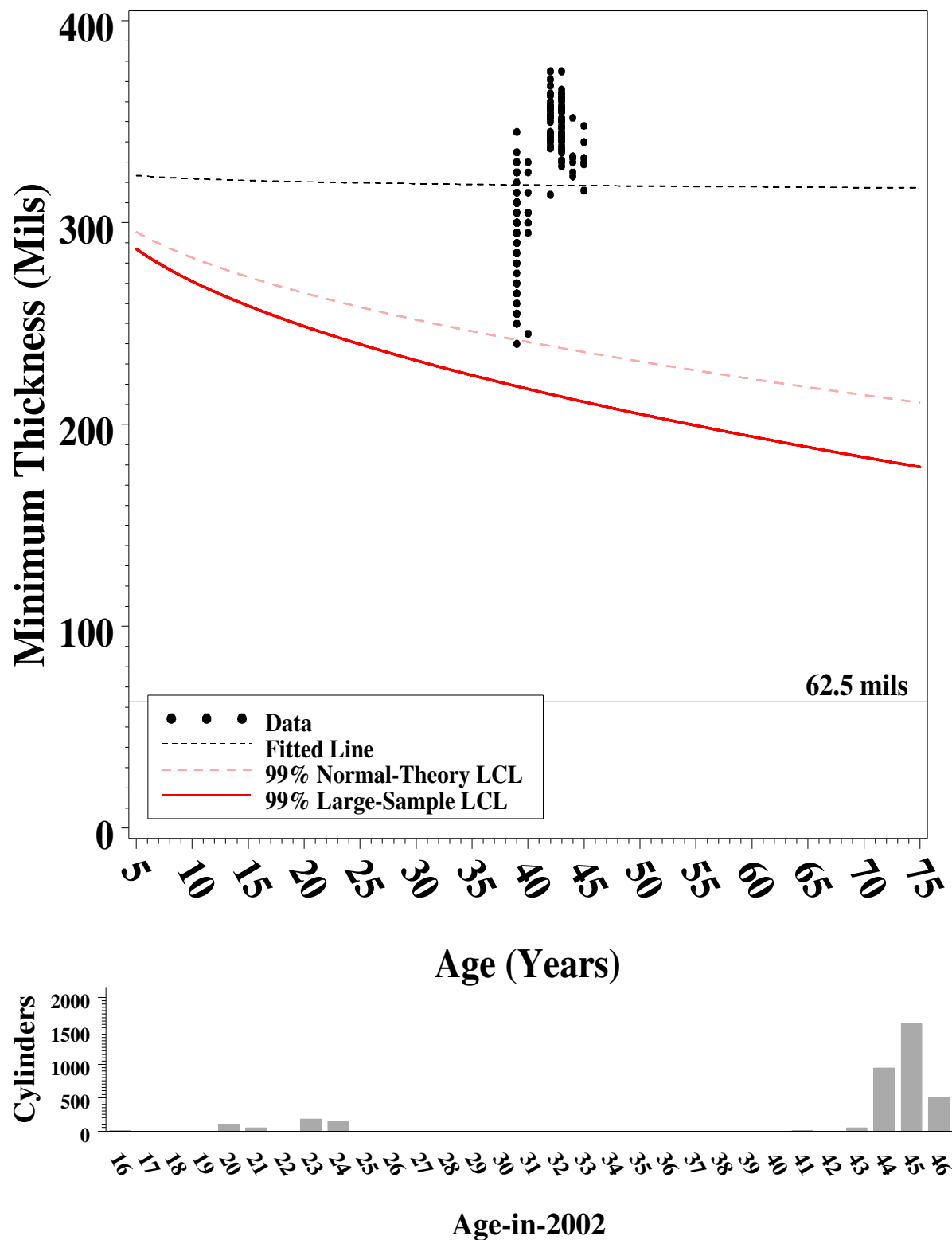
**Figure 22. Minimum thicknesses and cylinder age distribution for PORTS and PGDP, Thick, top.**



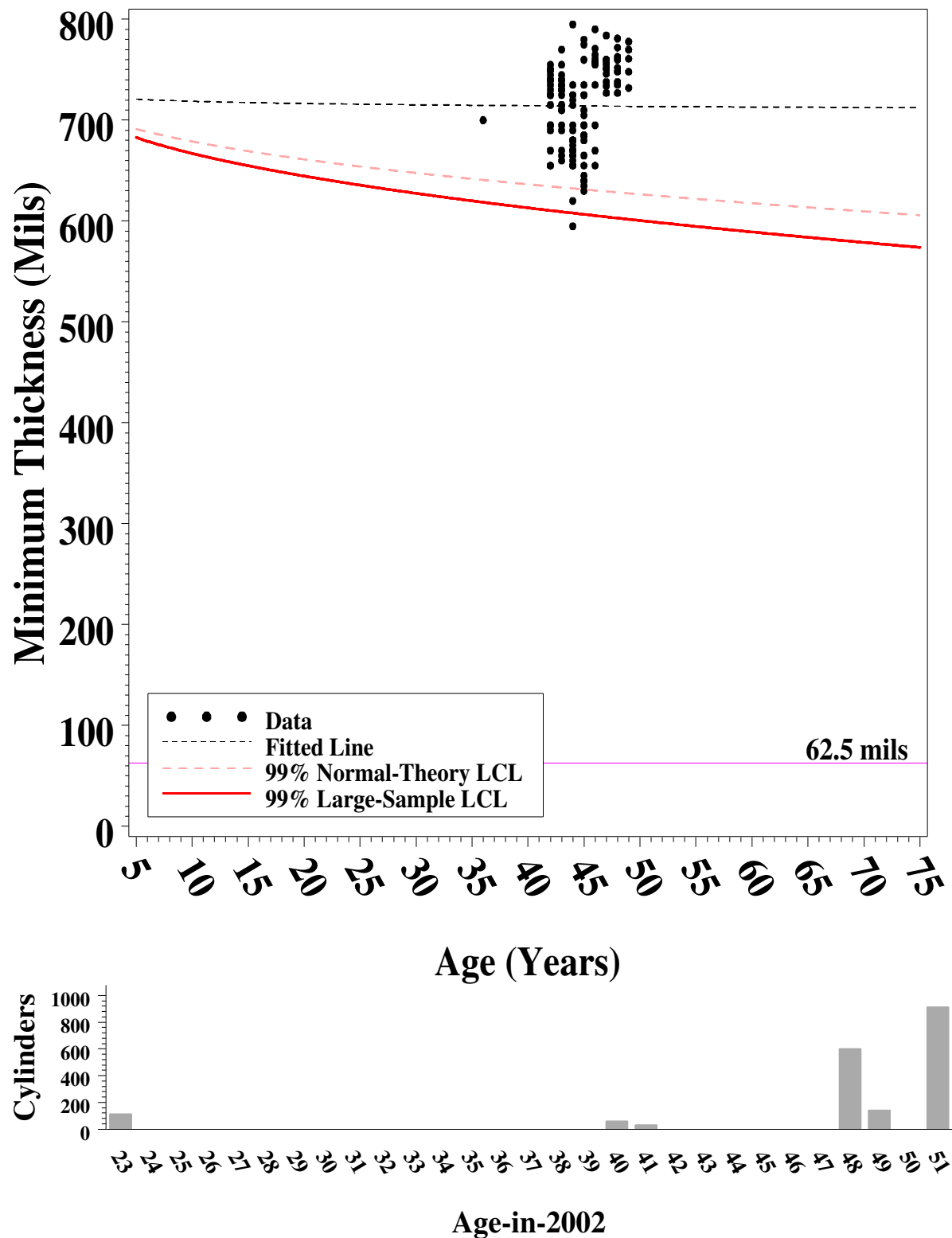
**Figure 23. Minimum thicknesses and cylinder age distribution for PORTS and PGDP, Thick, bottom.**



**Figure 24. Minimum thicknesses and cylinder age distribution for PORTS Thin, skirted, top.**

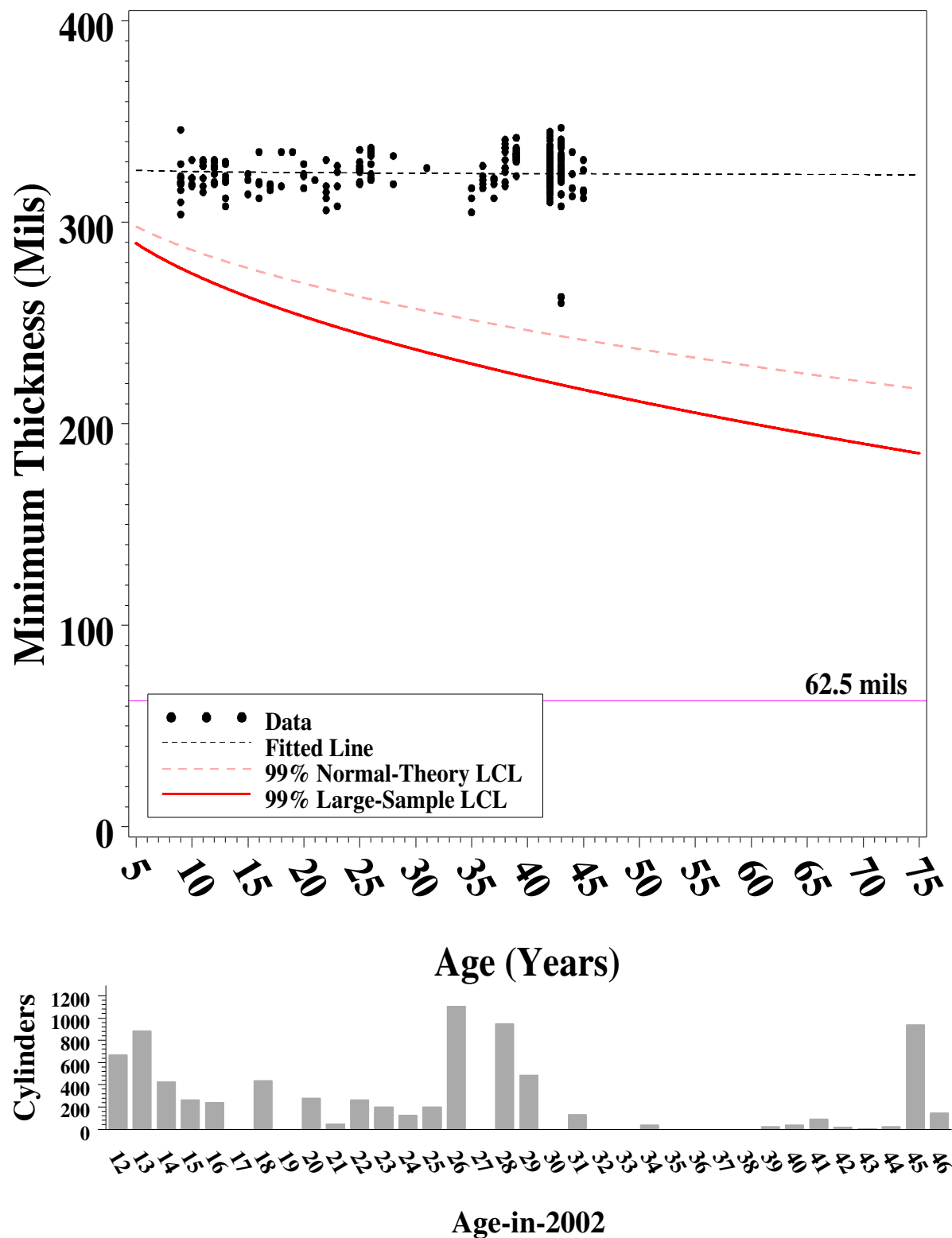


**Figure 25. Minimum thicknesses and cylinder age distribution for PORTS Thin, skirted, bottom.**

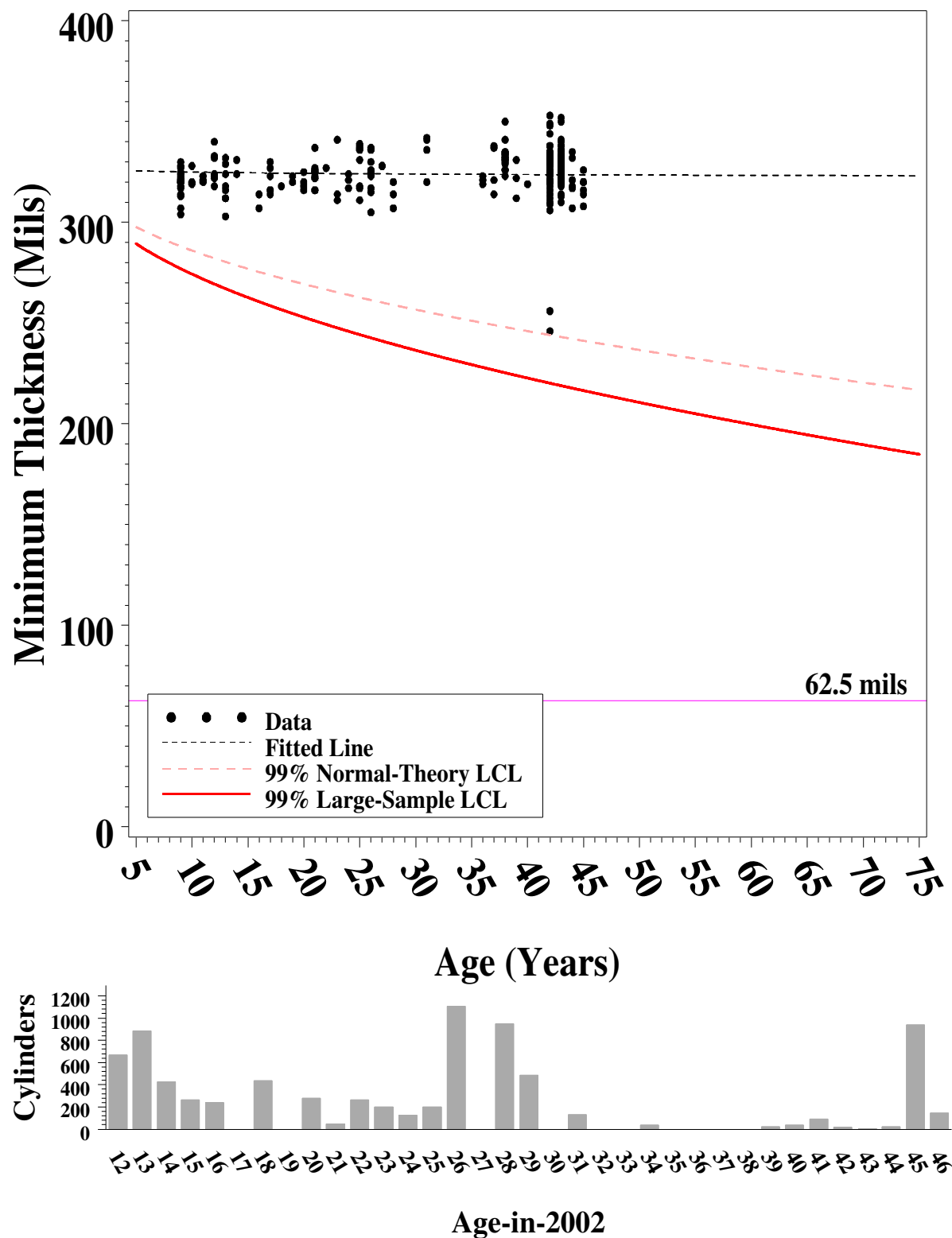


**Figure 26. Minimum thicknesses and cylinder age distribution for PORTS Thick, skirted, top and bottom.**

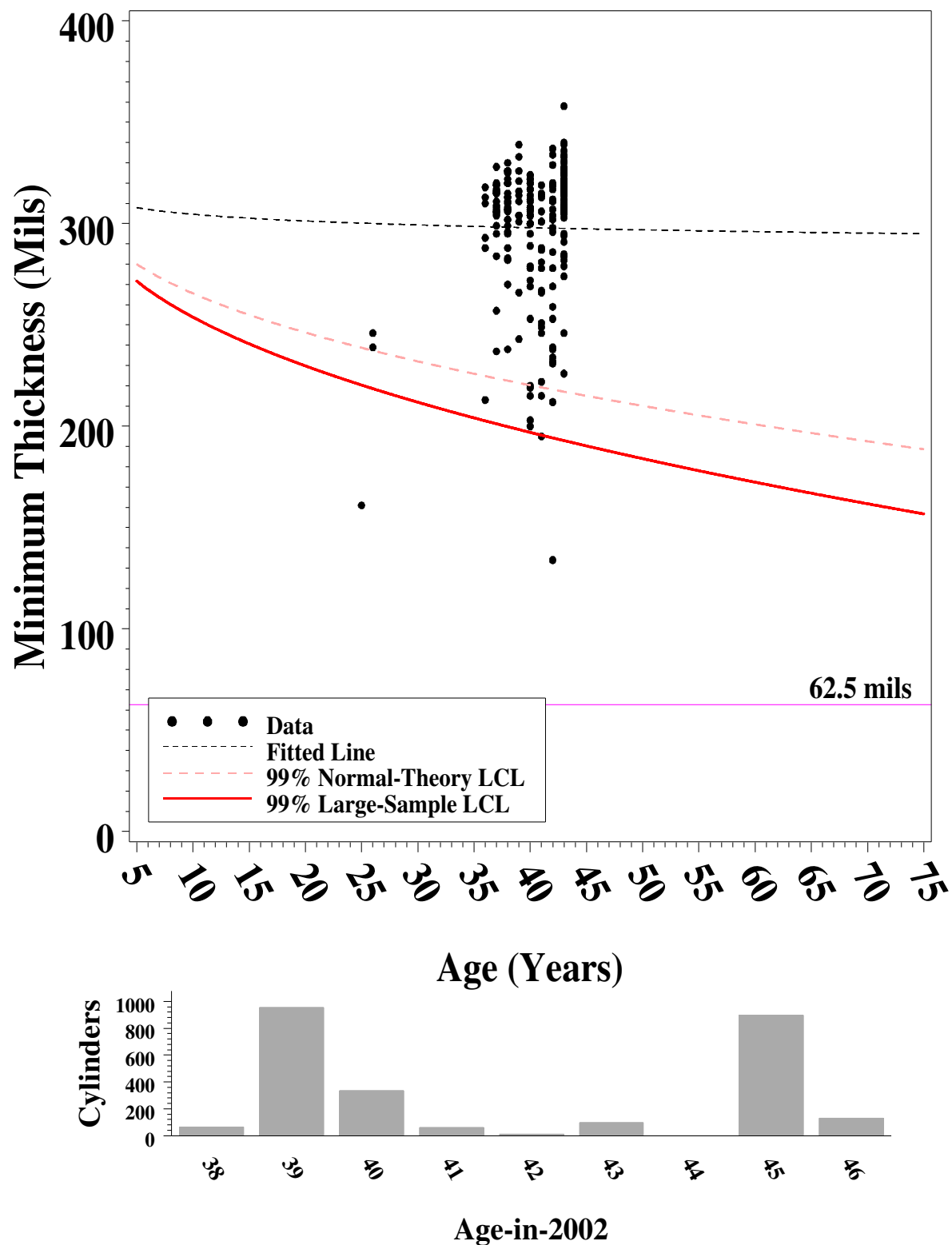




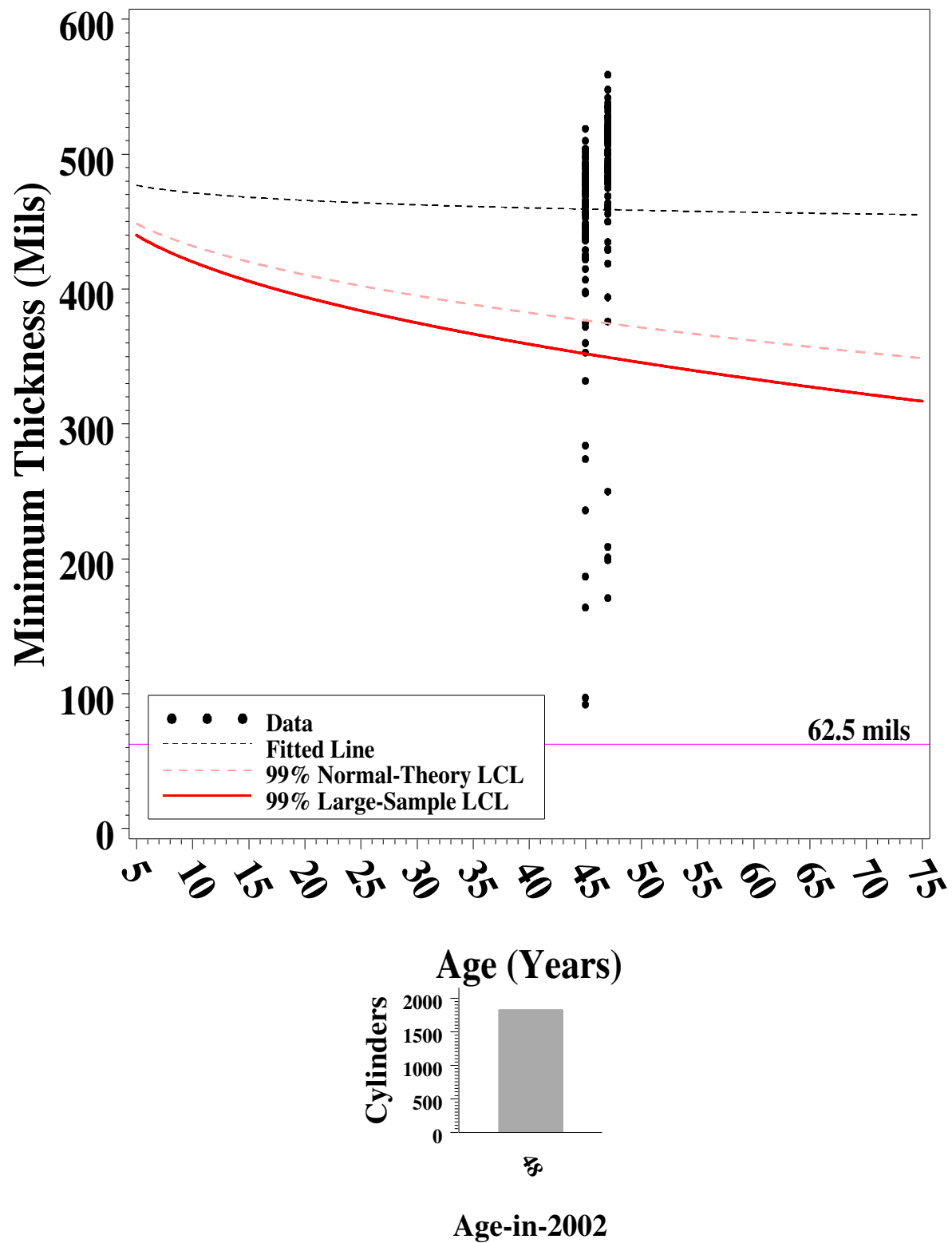
**Figure 27. Minimum thicknesses and cylinder age distribution for PORTS Thin, Top, FY98 and later.**



**Figure 28. Minimum thicknesses and cylinder age distribution for PORTS Thin, Bottom, FY98 and later.**



**Figure 29. Minimum thicknesses and cylinder age distribution for ETTP, Thin, evaluated FY98-01.**



**Figure 30. Minimum thicknesses and cylinder age distribution for PGDP 30As.**

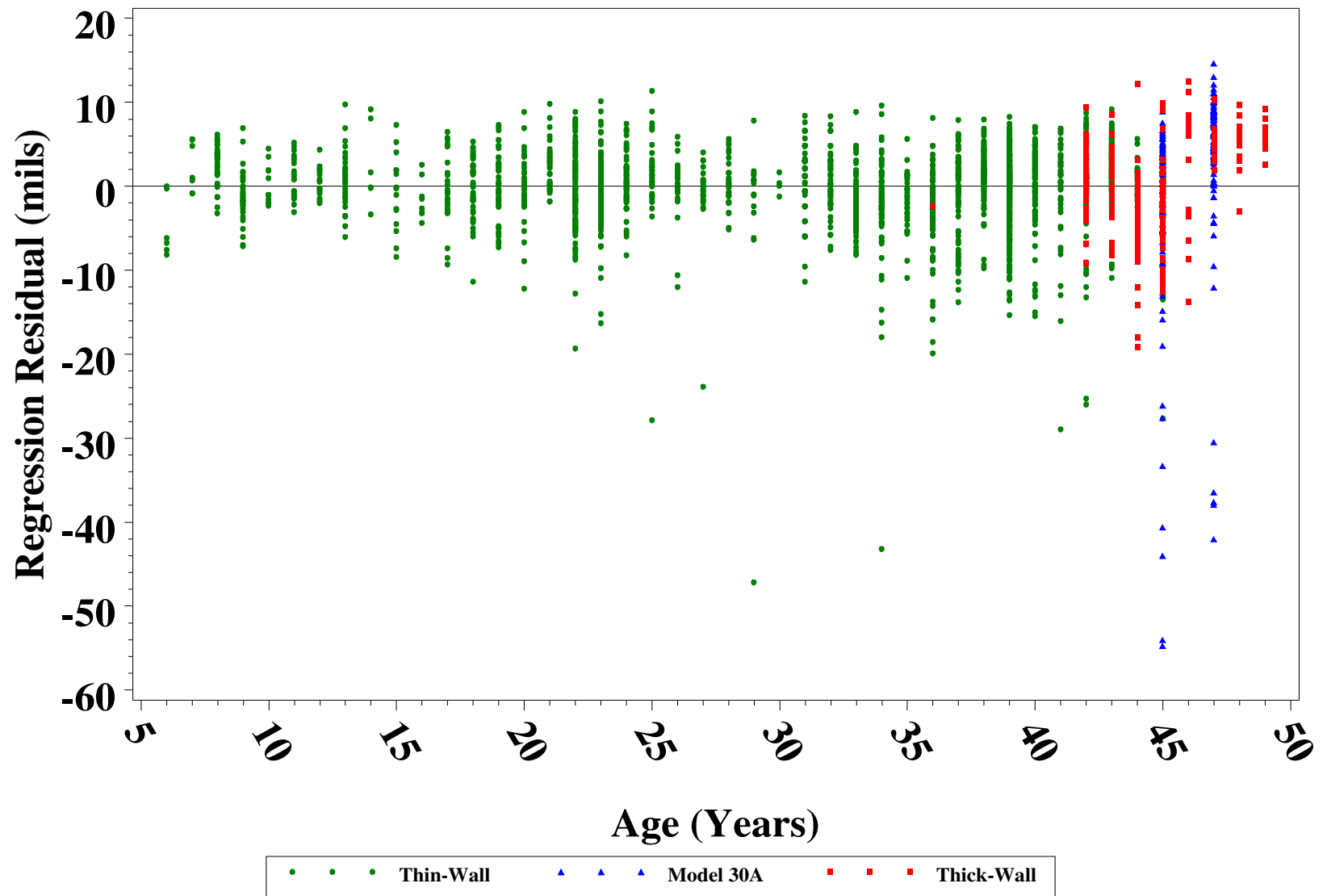
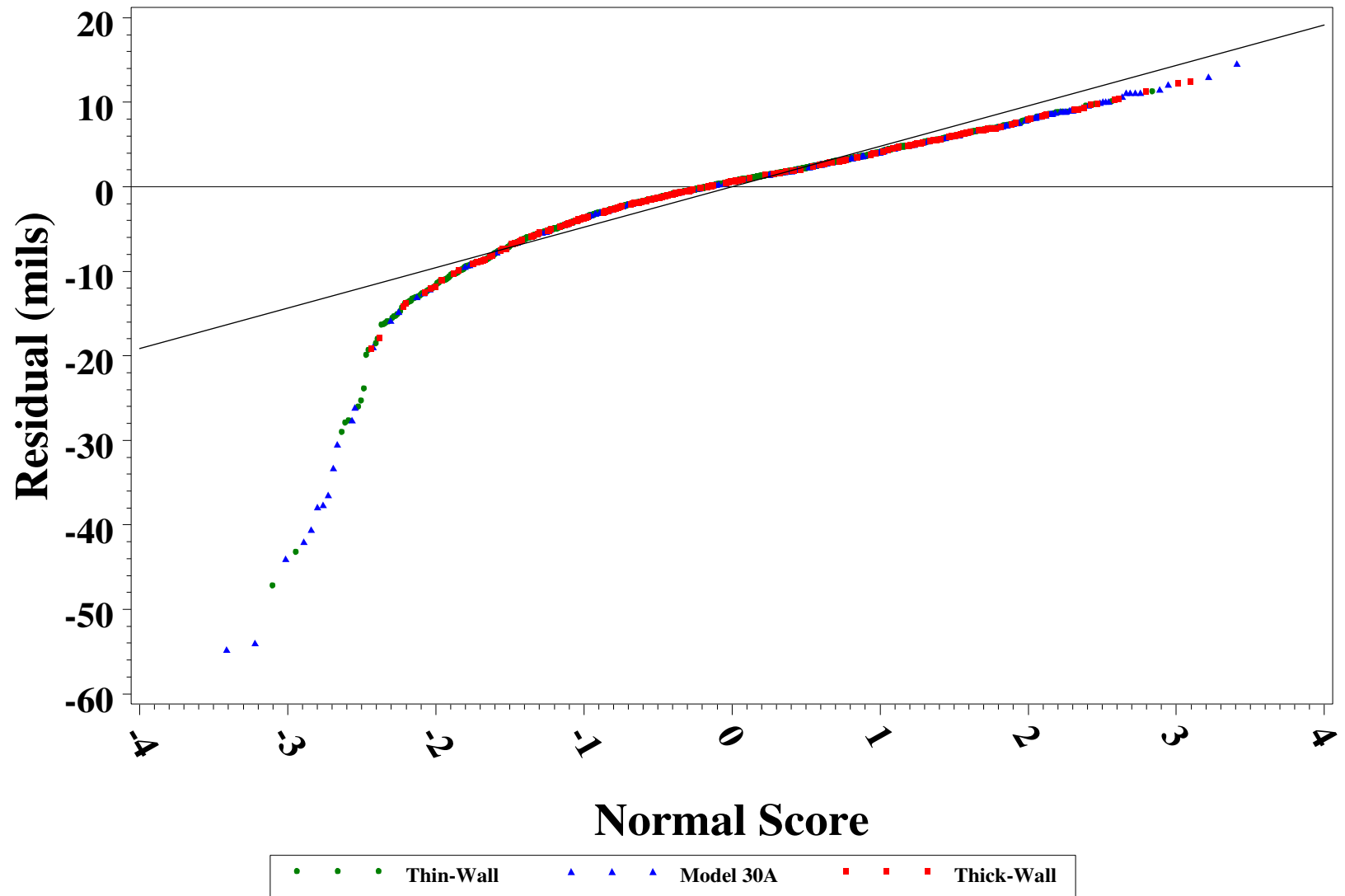


Figure 31. Residuals from minimum thickness regression.



**Figure 32. Normal probability plot for the regression residuals. Systematic departures from the straight reference line suggest non-normal data.**

## APPENDIX B: METHODS FOR THE INDIRECT MODEL

### B.1. Cumulative distribution function for the difference of two distributions

The indirect-model methods discussed in this report are based on the model

$$M(t) = C_0 - P(t) \quad (1)$$

where  $M(t)$  is the minimum wall thickness at cylinder age  $t$ ,  $P(t)$  is the amount of corrosion that results in the minimum wall thickness, and  $C_0$  is the initial thickness where the minimum wall thickness occurs. Both  $P(t)$  and  $C_0$  are taken as random, and calculation of the number of cylinders that have a minimum thickness below a certain thickness  $z$  requires calculating the probability that  $M(t) < z$ . Under model (1), this is equivalent to calculating the probability that  $C_0 - P(t) < z$ . Since  $C_0$  and  $P(t)$  are both random, calculation of this probability is not as straightforward as calculating probabilities for  $C_0$  and  $P(t)$  separately, except for certain special cases (e.g., when  $P(t)$  and  $C_0$  are both normal distributions, in which case the difference is also a normal distribution). In this section, the method of calculating the needed probabilities are developed.

#### General Formula

If the random variable  $W$  is defined by  $W = X - Y$ , where  $X$  and  $Y$  are independent random variables, then any sample  $w$  from  $W$  can be written in the form (not necessarily uniquely)

$$w = F^{-1}(p) - G^{-1}(q)$$

where  $p, q$  are in  $[0, 1]$ , and  $F^{-1}$  and  $G^{-1}$  are the inverse cumulative distribution functions for  $X$  and  $Y$ , respectively. Determination of the probability that  $W < z$  is then equivalent to evaluating

$$\int \int_{A(z)} dp dq \quad (2)$$

where  $A(z)$  is the set defined by  $A(z) = \{(p, q) \mid F^{-1}(p) - G^{-1}(q) < z\}$ . Since  $F^{-1}$  and  $G^{-1}$  are inverse cumulative distribution functions, they are both nondecreasing functions, and so the function  $h(p, q) = F^{-1}(p) - G^{-1}(q)$  is a nondecreasing function of  $p$  and nonincreasing function of  $q$ . This makes evaluation of the integral in Eq. 3 relatively straightforward. First,

$$\begin{aligned} A(z) &= \{(p, q) \mid F^{-1}(p) - G^{-1}(q) < z\} \\ &= \{(p, q) \mid F^{-1}(p) < z + G^{-1}(q)\} \\ &= \{(p, q) \mid p < F(z + G^{-1}(q))\} \end{aligned}$$

and so

$$\begin{aligned}\int \int_{A(z)} dp \, dq &= \int_0^1 \int_0^{F(z+G^{-1}(q))} dp \, dq \\ &= \int_0^1 F(z+G^{-1}(q)) \, dq\end{aligned}$$

Therefore,

$$Prob\{W=X-Y<z\} = \int_0^1 F(z+G^{-1}(q)) \, dq \quad (3)$$

Alternatively, the probability (3) can be written as

$$\int_{-\infty}^{\infty} (1 - G(z-x)) \, dF(x). \quad (4)$$

This follows because

$$Prob(X-Y < z) = Prob(Y > z-X) = \int_{x=-\infty}^{\infty} \int_{\{y > z-x\}} dG(y) \, dF(x) = \int_{x=-\infty}^{\infty} (1 - G(z-x)) \, dF(x).$$

For  $G$  lognormal with log-scale mean and variance  $\mu$  and  $\sigma^2$ , (4) is

$$\int_{-\infty}^{\infty} \left( 1 - N\left( \frac{\log(z-x) - \mu}{\sigma} \right) \right) dF(x) = \int_{-\infty}^{\infty} N\left( \frac{\mu - \log(z-x)}{\sigma} \right) dF(x). \quad (5)$$

where  $N$  denotes the standard normal distribution function.

The integrals (3-5) can be evaluated using the adaptive quadrature method described in Burden and Faires (1989). With this method, subintervals are determined so that the integral is approximated with the desired accuracy using Simpson's rule on each subinterval. This method is generally faster than simpler integration methods to achieve the same accuracy because the ultimate subdivision that is used need not be uniformly spaced over the entire interval of integration; the subintervals can be selected based on the desired accuracy and the variability of the function to be integrated.

## Application

In this report,  $F$  is the cumulative distribution function (cdf) for the initial thickness  $C_0$  which has a truncated normal distribution, and  $G$  is the cdf for the penetration depth  $P(t)$  at a fixed time  $t$  which has a lognormal distribution with mean of the logarithm of the values of  $\mu_L(t)$  and standard deviation of the logarithm of the values of  $\sigma_L$ .



Let  $N(u)$  denote the cdf for the standard normal distribution (this is the normal distribution with mean 0 and standard deviation 1), and denote the  $q$ th quantile of the standard normal distribution by  $n_q$ . Then by the formula above it follows that

$$Prob\{C_0 - P(t) < z\} = \int_0^1 F\left(z + e^{\mu(t) + n_q \sigma(t)}; \mu, \sigma\right) dq$$

where

$$F_{[a,b]}(x; \mu, \sigma) = \begin{cases} 0 & \text{if } x < a \\ \frac{N(x; \mu, \sigma) - N(a; \mu, \sigma)}{N(b; \mu, \sigma) - N(a; \mu, \sigma)} & \text{if } a < x < b \\ 1 & \text{if } x > b \end{cases}$$

where  $N(x; m, s) = N((x-m)/s)$ , where  $N(z)$  is the standard normal distribution.

## B.2. Calculation of Upper Confidence Limits

In the methods used in this report, the maximum penetration depth  $P(t)$  is modeled using a lognormal distribution, with either  $P(t) \sim \text{Log}(\mu_L, \sigma_L) * t$  (slope set to 1) or  $P(t) \sim \text{Log}(\log A + n \text{Log } t, \sigma_L)$ , and the parameters are fit with the data available. The expected number of cylinders with a minimum thickness below a certain thickness  $z$  by a given time  $T$  is calculated by a sum of the form

$$\sum_i Prob(C_0 - P(t_i) < z) \times \{ \text{Number of cylinders of age } t_i \text{ at time } T \} \quad (6)$$

where the sum is over all age classes for the cylinder population of interest.

Given the initial thickness and penetration distributions, the probabilities in (6) can be computed using Simpson's rule, as discussed above. However, the initial thickness and penetration distributions have to be estimated. In this section confidence limits for (6) are developed to account for uncertainty in the estimates of the penetration distribution. The uncertainty in the initial thickness distribution is assumed to be negligible.

The approach taken to calculating a UCL for the sum (6) is based on the Bonferroni inequality, which can be used to determine a value  $\alpha$  such that if an upper  $100\alpha\%$  confidence limit is used for each term in the sum, the final sum will be bounded with at least 95% confidence. However, although expression (6) may have up to 25 terms, the statistical distributions of all of the terms depend on just three parameters—the intercept, slope, and standard deviation from the regression of log-depth on log-age (with uncertainty in the initial thickness distribution assumed negligible.) Therefore, joint confidence limits for the penetration at each age represented in (6) can also be computed from joint confidence limits for the three parameters. This suggests that a more efficient use of the Bonferroni approach would be to use it to derive joint confidence limits for the three parameters, rather than joint confidence limits for all of the terms in (6).

Furthermore, a refinement of the three-parameter Bonferroni approach is possible. Joint confidence limits for the intercept and slope can be used to derive joint UCLs for the penetration log-scale means  $\mu(t_i) = a + b \log(t_i)$  for each age  $t_i$ . But joint confidence limits for the intercept and slope imply joint confidence limits

for **all** points on the curve  $\mu(t) = a + b \log(t)$ , including, for example, points for ages such as  $t = 10,000$  years or  $t = -10,000$  years. In the cylinder modeling, however, the only confidence limits for points on the regression line that are needed are confidence limits for points corresponding to ages of concern—in the range of about 0 to 75 years. As Figure 16 illustrates, the line that interpolates UCLs for the regression line at the endpoints of a range of interest is in fact a joint UCL for all points on the regression line in that range. Because their range is restricted, joint UCLs based on the line restricted to the interval, tend to be tighter than UCLs for the whole line, based on confidence limits for the intercept and slope.

Combining equation (5) and expression (6) gives

$$\sum_i \left\{ \text{Number of cylinders of age } t_i \text{ at time } T \right\} \times \int_{-\infty}^{\infty} N \left( \frac{\mu(t_i) - \log(z - x)}{\sigma} \right) dF(x) \quad (7)$$

for the number of cylinders at time  $T$  for which the thickness criterion  $z$  is violated. It is straightforward to show that expression (7) is increasing in each  $\mu(t_i)$ . Therefore, a UCL for the entire expression can be obtained by substituting UCLs for the individual  $\mu(t_i)$ —if an appropriate limit is also substituted for  $\sigma$ . That limit for  $\sigma$  is discussed next.

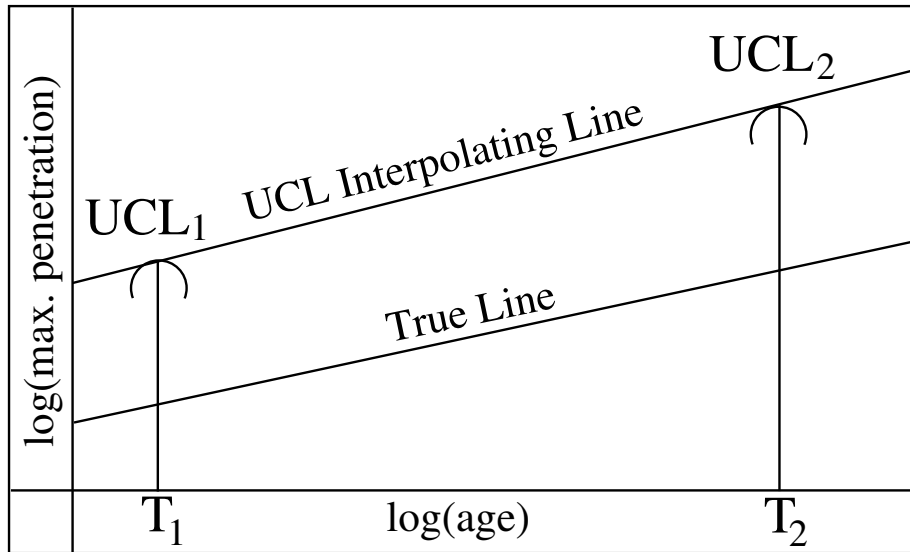


Figure 33. Example of a joint confidence line over an interval, based on two joint UCL's computed at endpoints  $T_1$  and  $T_2$  of the interval.

In a lognormal regression with  $d$  degrees of freedom ( $d = \text{number of observations} - 2$  for a simple line model), the mean squared error (MSE) is an unbiased estimate of  $\sigma^2$ , and  $d \times \text{MSE} / \sigma^2$  has a chi-square distribution with  $d$  degrees of freedom. It follows that  $d \times \text{MSE} / X_{\alpha}^2$  and  $d \times \text{MSE} / X_{1-\alpha}^2$  are upper and lower confidence limits for  $\sigma^2$ , where  $X_{\alpha}^2$  and  $X_{1-\alpha}^2$  denote the  $\alpha$  and  $1-\alpha$  percentiles of the chi-square distribution with  $d$  degrees of freedom. The square roots of the confidence limits are confidence limits for  $\sigma$ .

Because  $\sigma$  is positive, each term in (7) is increasing or flat or decreasing in  $\sigma$ , depending on whether  $\mu(t_i) - \log(z - x)$  is, respectively, negative or zero or positive. If  $\mu(t_i) - \log(z - x) > 0$  for every age  $t_i$ , then a UCL for expression (7) can be obtained by taking  $\sigma$  to be its smallest acceptable value (i.e., its lower confidence limit). On the other hand, if  $\mu(t_i) - \log(z - x) < 0$  for every age  $t_i$ , then the UCL for (7) can be obtained by

taking  $\sigma$  to be its largest acceptable value (UCL). If  $\mu(t_i) - \log(z-x)$  is negative for some of the  $t_i$  and positive for other  $t_i$ , then the UCL for (7) can be determined by a one-dimensional grid search from the lower to upper confidence limits for  $\sigma$ . The adequacy of the grid step size can be guaranteed, because a bound can be determined for the derivative of (7) as a function of  $\sigma$ . (Note that this grid step size is separate from the step size for the numerical integration discussed above. Because, a bound can be determined for the fourth derivative of the of the integrand, the adequacy of the numerical integration step size can also be guaranteed.)

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