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RON S. LONGACRE

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Physics Department

Brookhaven National Laboratory
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Mass Shifts through Re-scattering

Ron S. Longacre
Brookhaven National Laboratory, Upton, NY 11973, USA[1]

Abstract

In this note we present a model that can produce a mass shift in a resonance due to interference between a scattering amplitude and that amplitude having rescattering through the resonance.

1 Starting point

For the first part of this story we will define what a scattering cross section is. We will only consider in this work elastic scattering of pions. Two pions can scatter at a certain energy which we will call $M_{\pi\pi}$. The differential cross section σ at a given $M_{\pi\pi}$ is

$$\frac{d\sigma}{d\phi d\theta} = \frac{1}{K^2}\left|\sum_{\ell}(2\ell+1)T_\ell P_\ell(cos(\theta))\right|^2 \tag{1}$$

where ϕ and θ are the azimuthal and scattering angles, respectively. T_ℓ is a complex scattering amplitude and ℓ is the angular momentum. P_ℓ is the Legendre polynomial, which is a function of $cos(\theta)$. K is the flux factor equal to the pion momentum in the center of mass. For this note we only consider $\ell = 0$ and $\ell = 1$. The T_0 and T_1 elastic scattering amplitudes are complex amplitudes described by one real number which is in units of angles. The form of the amplitude is

$$T_\ell = e^{i\delta_\ell} sin(\delta_\ell) \tag{2}$$

We note that δ depends on the value of ℓ and $M_{\pi\pi}$. We will use the $\ell = 0$ and $\ell = 1$ δ's given in [1].

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2 Defining A

Let us consider two pions scattering in the final state of the heavy ion collision. The scattering will be either $\ell = 0$ or $\ell = 1$ partial waves. The $M_{\pi\pi}$ of the scattering di-pion system will depend on the probability of the phase space of the overlapping pions. The pions emerge from a close encounter in a defined quantum state with a random phase. We will call this amplitude A and note that the absolute value squared of the amplitude is proportional to the phase space overlap. The emerging pions can re-interact or re-scatter through the quantum state of the pions, which is a partial wave or a phase shift. We have amplitude A plus A times the re-scattering of pions through the phase shift consistent quantum state of A. The correct unitary way to describe this process is given by [2] equation (4.5)

$$T = \frac{V_1 U_1}{D_1} + \frac{\left(V_2 + \frac{D_{12}V_1}{D_1}\right)\left(U_2 + \frac{D_{12}U_1}{D_1}\right)}{D_2 - \frac{D_{12}^2}{D_1}} \tag{3}$$

In the above equation we have two terms, 1 and 2. The first term denoted by 1 is the $\pi\pi$ scattering through p-wave which will become the amplitude A mentioned above, where V is the incoming and U is the outgoing $\pi\pi$ system. The second term denoted by 2 is the direct production of the ρ meson with V being the production, the propagation being D and the decay being U. We see that there are terms D_{12} which involves a loop of pions between scattering pions and the formation of a ρ by the pions.

3 Final equation

The complete derivation is in the appendix. From the appendix we get two terms, one being the direct production of the ρ or $\pi\pi$ p-wave phase shift and the second being the ρ from re-scattering. We now write

$$|T|^2 = \frac{|Dsin(\delta_\ell)|^2}{q^{2\ell+1}} + \frac{|A|^2}{q^{2\ell+1}}\left|\alpha sin(\delta_\ell) + q^{2\ell+1}cos(\delta_\ell)\right|^2 \tag{4}$$

In the above equation $|T|^2$ is the cross section for ρ production, where D is the direct production amplitude and A is the amplitude introduced above for the re-scattering pions into the ρ meson. δ is the $\pi\pi$ phase shift [1]. q is the $\pi\pi$ center of mass. At a given p_T and y bin, D will be a constant as a function of $M_{\pi\pi}$. The α factor is one for rescattering coming from a point source, but goes to zero when rescattering is diffractive. The dependence of A is calculated by the phase space overlap of di-pions added as four vectors and corrected for proper time, with the sum having the correct p_T and y with $M_{\pi\pi}$.

A Appendix

Starting with equation (4.5) from [2]

$$T = \frac{V_1 U_1'}{D_1} + \frac{\left(V_2 + \frac{D_{12} V_1}{D_1}\right)\left(U_2' + \frac{D_{12} U_1'}{D_1}\right)}{D_2 - \frac{D_{12}^2}{D_1}} \tag{5}$$

In order to have the correct threshold kinematics, we define

$$U_1' = U_1 \sqrt{q^{2\ell+1}} \tag{6}$$

$$U_2' = U_2 \sqrt{q^{2\ell+1}} \tag{7}$$

where q is the $\pi\pi$ center of mass momentum and ℓ is the value of the angular momentum. The amplitude A of the text is given by

$$\frac{V_1 U_1}{D_1} = A \tag{8}$$

Thus we have

$$\frac{V_1 U_1'}{D_1} = A\sqrt{q^{2\ell+1}} \tag{9}$$

The phase shift for the ℓ^{th} partial wave will be given by δ_ℓ, where

$$\frac{U_2' U_2'}{D_2} = e^{i\delta_\ell} sin(\delta_\ell) \tag{10}$$

The above equality is true if the D_1 mode plays no role in the $\pi\pi$ scattering in the ℓ^{th} partial wave. But in the initial state there is a large production of D_1. The U's are the basic coupling of the D's to the $\pi\pi$ system. In order to decouple D_1 from the $\pi\pi$ system U_1 must go to zero. We can maintain a finite production of D_1 if we define

$$V_1 = \frac{1}{U_1} \tag{11}$$

Thus the first term in the equation becomes

$$\frac{V_1 U_1'}{D_1} = \frac{\frac{1}{U_1} U_1 \sqrt{q^{2\ell+1}}}{D_1} = \frac{\sqrt{q^{2\ell+1}}}{D_1} = A\sqrt{q^{2\ell+1}} \tag{12}$$

The form of D_{12} is given by

$$D_{12} = \alpha U_1 U_2 + i q^{2\ell+1} U_1 U_2 \tag{13}$$

D_{12} is the real and imaginary part of the two pion loop from state 1 to state 2. The U's are the $\pi\pi$ couplings and the imaginary part goes to zero at the $\pi\pi$ threshold. The α factor is one for rescattering coming from a point source, but goes to zero when rescattering is diffractive. A simple form for α is given by

$$\alpha = (1.0 - \frac{r^2}{r_0^2}) \tag{14}$$

where r is the radius of rescattering in fermis and r_0 is 1.0 fermi or the limiting range of the strong interaction. The second term of the first equation is

$$\frac{\left(V_2 + \frac{D_{12} V_1}{D_1}\right)\left(U_2' + \frac{D_{12} U_1'}{D_1}\right)}{D_2 - \frac{D_{12}^2}{D_1}} \tag{15}$$

Rewriting

$$\frac{\left(V_2 + \frac{\alpha U_1 U_2 V_1}{D_1} + i q^{2\ell+1} \frac{U_1 U_2 V_1}{D_1}\right)\left(U_2' + \frac{D_{12} U_1'}{D_1}\right)}{D_2 - \frac{D_{12}^2}{D_1}} \tag{16}$$

Let us make substitutions

$$V_1 = \frac{1}{U_1}, U_2 = \frac{U_2'}{\sqrt{q^{2\ell+1}}}, \frac{1}{D_1} = A, D_{12} = 0 \tag{17}$$

The second term becomes

$$\frac{\left(V_2 + \frac{A\alpha U_2'}{\sqrt{q^{2\ell+1}}} + i\sqrt{q^{2\ell+1}} A U_2'\right) U_2'}{D_2} \tag{18}$$

The first term is

$$\frac{V_1 U_1'}{D} = A\sqrt{q^{2\ell+1}} \tag{19}$$

Adding the first and the second terms and substituting the phase shift,

$$T = \frac{V_2}{U_2}\frac{e^{i\delta_\ell}sin(\delta_\ell)}{\sqrt{q^{2\ell+1}}} + A\left(\frac{e^{i\delta_\ell}\alpha sin(\delta_\ell)}{\sqrt{q^{2\ell+1}}} + \sqrt{q^{2\ell+1}}e^{i\delta_\ell}cos(\delta_\ell)\right) \tag{20}$$

The term with the factor $\frac{V_2}{U_2}$ is the direct production of the di-pion system. We shall call this amplitude D. The re-scattered amplitude is A and is modified by the di-pion phase shift. These two amplitudes have some random phase and are not coherent. Thus the cross section is

$$|T|^2 = |D|^2\frac{sin^2(\delta_\ell)}{q^{2\ell+1}} + \frac{|A|^2}{q^{2\ell+1}}\left|\alpha sin(\delta_\ell) + q^{2\ell+1}cos(\delta_\ell)\right|^2 \tag{21}$$

References

[1] G. Grayner *et. al.*, Nucl. Phys. B 75 (1974) 189.

[2] R. Aaron and R. S. Longacre, Phys. Rev. D 24 (1981) 1207.