

Final Technical Report
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Title: "Mathematical and Numerical Studies of Nonstandard
Difference Equation Models of Differential Equations"

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INTRODUCTION

This Final Technical Report on DOE Grant DE-FG02-97ER25342 provides a concise summary of the major research and professional activities done under funding from this grant. The complete details and explanations for each aspect of the completed results can be obtained by consulting my Annual Progress Reports and the publications listed in Appendices A, B, and C of this document.

RESEARCH PROGRAM

Much of modern science and technology depends upon the ability to obtain solutions to differential equations that arise in the modeling of various systems [1, 2, 3]. In most instances the complexities of these equations do not permit the existence of analytic solutions that can be represented in terms of a finite number of the elementary functions [4]. Consequently, numerical integration methods must be used. An important technique to obtain numerical solutions is the application of finite-differences. However, an issue of great importance is the relationship between the numerical solutions and the actual (not known, *a priori*) analytical solutions to the differential equations. This leads the researcher to various questions related to local truncation errors, consistency, convergence, and stability [5, 6, 7].

The major thrust of our research was to continue our investigations of so-called non-standard finite-difference schemes as formulated by the PI [8]. These schemes do not follow the standard rules used to model continuous differential equations by discrete difference equations. The two major aspects of this procedure consist of generalizing the definition of the discrete derivative and using a nonlocal model (on the computational grid or lattice) for nonlinear terms that may occur in the differential equations.

Our aim was to investigate the construction of nonstandard finite-difference schemes for several classes of ordinary and partial differential equations. These equations are simple enough to be tractable, yet, have enough complexity to be both mathematically and scientifically interesting. It should be noted that all of the equations studied model some physical phenomena under the appropriate set of experimental conditions.

The major goal of this project was to better understand the process of constructing

finite-difference models for differential equations. In particular, we demonstrated the value of using nonstandard finite-difference procedures. A secondary goal was to construct and study a variety of analytical techniques that can be used to investigate the mathematical properties of the obtained difference equations. These mathematical procedures are of interest in their own right [9] and are a valuable contribution to the mathematics research literature on difference equations.

CENTRAL PRINCIPLES

The following is a concise summary of the central principles for constructing nonstandard finite difference schemes. The details for why a particular construction is done is given in my book [8]. Also, see references [13, 17, 23] given in Appendix B.

* In general, derivatives must be modeled by discrete analogues that take the form

$$\frac{dx}{dt} \rightarrow \frac{x_{k+1} - \psi(h)x_k}{\phi(h)}, \quad (1)$$

where $h = \Delta t$; $t_k = hk$, $k = \text{integer}$; x_k is an approximation for $x(t_k)$; and the functions $\psi(h)$ and $\phi(h)$ have the properties

$$\psi(h) = 1 + O(h), \quad \phi(h) = h + O(h^2). \quad (2)$$

The functions $\psi(h)$ and $\phi(h)$ are called, respectively, the “numerator” and “denominator” functions.

* Nonlinear terms appearing in a differential equation generally are modeled by non-local discrete representations. For example, in the ODE

$$\frac{dx}{dt} = x - x^2, \quad (3)$$

the x^2 term is to be replaced by

$$x \rightarrow x_{k+1}x_k. \quad (4)$$

Likewise in the Fisher PDE

$$u_t = u_{xx} + u(1 - u), \quad u = u(x, t), \quad (5)$$

the u^2 term has the discrete representation [8]

$$u^2 \rightarrow \left(\frac{u_{m+1}^k + u_m^k + u_{m-1}^k}{3} \right) u_m^k, \quad (6)$$

where $x_m = m(\Delta x)$, $t_k = k(\Delta t)$, and u_m^k is an approximation for $u(x_m, t_k)$.

* The finite difference schemes and their solutions should satisfy the same special conditions as the corresponding differential equations and their solutions. The following properties often occur in the ODE's and PDE's that model important dynamical systems in the natural and engineering sciences:

- (i) positivity of solutions,
- (ii) monotonicity of solutions,
- (iii) boundedness of solutions,
- (iv) the existence of special solutions and their associated stability behaviors.

If the discrete schemes do not have the required properties of the associated differential equations, then numerical instabilities are certain to appear.

PHILOSOPHY OF RESEARCH PROGRAM

The major task of our research activities was not to construct the "best" possible finite difference scheme for a given system of differential equations, where "best" might be defined in terms of their accuracy or convergence properties [5, 6, 7]. (Note that all of the nonstandard schemes are "stable" and consistent with the original equations.) We have constructed proof-of-principal nonstandard schemes which demonstrate that discretization methods exist such that the elementary types of numerical instabilities do not show up in these schemes. However, methods to increase the accuracy of nonstandard schemes do exist and have been implemented for several types of both ordinary and partial differential equations. The paper of Price et al. [10] illustrates one possibility for achieving this goal.

SUMMARY OF MAJOR RESULTS

The PI and his collaborators have explored a wide variety of issues related to the construction, analysis, understanding, and application of nonstandard finite difference schemes. Major publications representing this work are given in Appendix B. Below we

provide a brief summary organized into six topics. (All references refer to those listed in Appendix B.)

1. Review Articles

Three review articles have been written by the PI to bring to a wider audience the general features of how to construct nonstandard schemes and to illustrate their application. These reviews occur in references [13, 17, 23, 34].

2. Exact Finite Difference Schemes

For a limited number of ODE's and PDE's, exact finite difference schemes exist and can be found. In references [2, 12, 17, 23] we show how such exact schemes can be constructed and use them to understand the nature of nonstandard schemes. Of particular interest are the results which come from both linear and nonlinear PDE's. A general summary of many of these results is given in R. E. Mickens, *Nonstandard Finite Difference Models of Differential Equations* (World Scientific; River Edge, NJ; 1994).

3. "Best" Nonstandard Schemes

A *best finite difference scheme* is a discrete model of a differential equation that is constructed according to the modeling rules of the nonstandard method. In general, best schemes are not *exact schemes*; however, they offer the opportunity of constructing discrete models for differential equations that do not possess the usual numerical instabilities. The current situation does not *a priori* necessarily lead to a unique discrete model for a given differential equation. But all do have the property that the usual numerical instabilities do not occur.

"Best" finite differences schemes have been constructed and studied for a broad range of differential equations:

- * reaction-convection-diffusion PDE's [9, 14, 21, 22, 24]
- * the dissipative complex equation [11]
- * conservative oscillators [25, 28, 32]
- * the van der Pol oscillator [31, 33]

4. PDE's, Positivity, and Step-Size Relations

Many of the PDE's modeling dynamical systems in the natural and engineering sciences have dependent variables which cannot take on negative values, i.e., they correspond to particle numbers and/or densities, absolute temperatures, etc. Consequently, this feature of positivity must be built into the corresponding finite difference model. We have showed that this can be done for a large class of both single and coupled nonlinear PDE's which model actual systems of practical interest. An important additional aspect of these nonstandard schemes is that they are "effectively" explicit and functional relations can be derived between the time and space step-sizes. The relevant publications are given in references [3, 8, 16, 26, 29, 30].

5. General Discrete Modeling: ODE's

In several publications, we have focused on issues related specifically to the construction of nonstandard finite difference schemes for ordinary differential equations. In particular, a detailed analysis was done on a system composed of coupled nonlinear oscillators [5]. The important elements incorporated into this work were the construction of a "discrete" energy function having the property of being invariant under a discrete time-reversal operation! Two other papers investigated the suppression of numerically induced chaos by use of nonstandard finite difference schemes [18, 19]. Two other relevant publications discussing related issues are references [4, 20]. In particular, [20] showed the major impact and difference between using "forward" and "backward" shifted discrete derivatives of the same order. In all cases, the central feature was to construct nonstandard schemes such that the usual numerical instabilities do not occur.

6. Asymptotic Methods for Second-Order Equations

Linear second-order ODE's give rise to linear second-order difference equations, i.e., ΔE 's. The analysis of a finite difference scheme, whether standard or nonstandard, may require the determination of the asymptotic behavior of its solutions. We have investigated the asymptotic behavior of second-order ΔE 's for which no existing mathematical theory exists. (See for example, J. Wimp, *Computation with Recurrence Relations* (Pitman,

Boston 1984); Appendix B.) In particular, we have considered a nonstandard scheme for the general class of ODE's

$$\frac{d^2 y}{dx^2} + f(x)y = 0, \quad (1)$$

which is

$$y_{m+1} + y_{m-1} = \left[2 \cos \left(h \sqrt{f_m} \right) \right] y_m, \quad (2)$$

where $f_m = f(\Delta x m)$. Note that $f(x) = x$ corresponds to the Airy equation, while

$$f(x) = 1 + \frac{a}{x} + \frac{b}{x^2}, \quad (3)$$

is the radial equation for the Coulomb problem in nuclear physics. We have constructed the required asymptotic expansions and showed how to calculate the various parameters in this representation. In the near future, we will write a general review article on this new contribution to the subject of asymptotic solutions of second-order difference equations. (Again, note that Eq. (2) is not of the standard form and thus none of the known theorems apply to it, especially for the case where f_m is a polynomial function of m .) The references [1, 6, 7, 15, 27] contain our contributions to this topic.

IMPACT OF RESEARCH

To date, the research performed under this DOE Grant has led to the following outcomes:

- * Achieved a better understanding of the construction process for finite-difference schemes for determining (some) of the conditions necessary for the genesis of numerical instabilities ...
- * Allowed the ability to better model differential equations by finite-difference schemes having the property that the elementary types of numerical instabilities will not occur ...
- * Provided an understanding of both the "physical" and "mathematical" conditions necessary for the determination of functional relations between the various step-sizes for partial differential equations ...
- * Provided a list/catalog of both "exact" and "best" finite-difference schemes for several important classes of ordinary and partial differential equations ...

- * Provided a partial theoretical understanding of the rules used in the modeling procedure for constructing nonstandard schemes ... in particular, the functional forms for the denominator functions that occur in discrete derivatives ...
- * Constructed new mathematical procedures for calculating the asymptotic properties of the solutions to certain classes of second-order, linear difference equations ...
- * Constructed new techniques to calculate accurate analytic approximations to the solutions of difference equations ...

A measure of the success of this research project is that its methods are beginning to be routinely incorporated into the general activities of the numerical analysis community involved with the numerical integration of differential equations. A number of researchers have applied my nonstandard finite difference schemes to problems arising in technology and mathematics. A selected list of references is given below. Of interest is the fact that Cole claims savings, in terms of computational time, by factors of 10^4 for certain scattering problems involving the numerical solutions of Maxwell's equations.

Selected References to Applications of Nonstandard Schemes

- * H. V. Kojouharov and B. M. Chen, "Nonstandard methods for the convective transport equation with nonlinear reactions," *Numerical Methods for Partial Differential Equations* 14, 467 (1998). (This is an application appearing in atmospheric pollution control, enhanced oil recovery, etc.)
- * R. Meyer-Spasche, "Difference schemes of optimum degree of implicitness for a family of simple ODE's with blow-up solutions," *Journal of Computational and Applied Mathematics* 97, 137 (1998).
- * J. B. Cole, "A high-accuracy realization of the Yee algorithm using non-standard finite differences," *IEEE Transactions on Microwave Theory and Techniques* 45, 991 (1997).
- * S. N. Elaydi, "Discrete competitive and cooperative models of Lotka-Volterra type," *Journal of Computational and Applied Analysis* (accepted for publication).
- * R. Anguelov and J.M.-S. Lubuma, "Contributions to the mathematics of the non-standard finite difference method and applications," *Numerical Methods for Partial Differential Equations* 17, 518 (2001).
- * R. E. Mickens (editor), *Applications of Nonstandard Finite Difference Schemes* (World Scientific, Singapore, 2000).

The last reference is an edited volume giving expanded versions of invited talks presented in a minisymposium at the 1999 SIAM Annual Meeting held in Atlanta, Georgia. The

minisymposium, "Nonstandard Finite Difference Schemes: Theory and Applications," introduced the concept of nonstandard schemes and demonstrated their power and usefulness by applying these techniques to various problems.

References

1. D. Potter, *Computational Physics* (Wiley, New York, 1973).
2. W. M. Washington and C. L. Parkinson, *An Introduction to Three-Dimensional Climate Modeling* (University Science Books; Mill Valley, CA; 1986).
3. M. Holt, *Numerical Methods in Fluid Dynamics* (Springer-Verlag, Berlin, 1984, 2nd edition).
4. D. Zwillinger, *Handbook of Differential Equations* (Academic Press, Boston, 1989).
5. F. B. Hildebrand, *Finite-Difference Equations and Simulations* (Prentice-Hall; Englewood Cliffs, NJ; 1968).
6. D. Greenspan and V. Casulli, *Numerical Analysis for Applied Mathematics, Science and Engineering* (Addison-Wesley; Redwood City, CA; 1988).
7. K. W. Morton and D. F. Mayers, *Numerical Solution of Partial Differential Equations* (Cambridge University Press, Cambridge, 1994).
8. R. E. Mickens, *Nonstandard Finite Difference Models of Differential Equations* (World Scientific; River Edge, NJ; 1994).
9. R. E. Mickens, *Difference Equations: Theory and Applications* (Van Nostrand Reinhold, New York, 1990). The PI has published more than thirty papers on nonstandard schemes. A complete list of early works is given in reference [8], pp. 243–246.
10. W. G. Price, Y. Wang, and E. H. Twizell, "A second-order, chaos-free, explicit method for the numerical solution of a cubic reaction problem in neurophysiology," *Numerical Methods for Partial Differential Equations* 9, 213–229 (1993).

APPENDIX A

Abstracts

The following is a listing of the major published abstracts related to work carried out under funds provided by the DOE Grant.

1. R. E. Mickens, "Discrete Modelling of Continuous Systems: The Details Matter," (with K. Oyedele) *Bulletin of the American Physical Society* 41, 1155 (1996).
2. R. E. Mickens, "Frontier Applications in Computer Methods for Dynamic Systems," *AMSIE '96* (The 1996 AAAS Annual Meeting and Science Innovation Exposition; Baltimore, MD; February 8-13, 1996); pp. S-42 and pp. A-47.
3. R. E. Mickens, "Asymptotic Solutions to Discrete Coulomb Equations," *Program of the 911th AMS Meeting: Louisiana State University; Baton Rouge, Louisiana* (American Mathematical Society, 1996). Abstract 911-39-59, pp. 421.
4. K. Oyedele and R. E. Mickens, "Numerical Integration Schemes for the Dissipative Complex Equation," *Bulletin of the American Physical Society* 41, 1667 (1996).
5. R. E. Mickens, "Existence of Periodic Solutions for a Generalized Conservative Oscillator," *Abstracts: The Georgia Tech-UAB International Conference on Differential Equations and Mathematical Physics*, (Atlanta, GA; 23-29 March 1997); pp. 24.
6. R. E. Mickens, "Investigation of Nonstandard Finite-Difference Schemes," *HBCU's Research Conference: Agenda and Abstracts* (April 9-10, 1997; Ohio Aerospace Institute; Cleveland, OH); pp. 18.
7. R. E. Mickens, "Finite-Difference Schemes for a Scalar Reaction-Convection PDE," *Bulletin of the American Physical Society* 42, 1110 (1997).
8. R. E. Mickens, "Relations Between Time and Space Step-Sizes in Numerical Schemes for PDE's that follow from a Positivity Condition," *Program of the 922nd AMS Meeting: Wayne State University; Detroit, MI* (American Mathematical Society, 1997). Abstracts 922-35-169, pp. 392.
9. R. E. Mickens, "A Mathematical Model of Dieting," *Abstracts: Conference on Mathematical Models in Medical and Health Sciences* (Vanderbilt University; Nashville, TN; May 28-31, 1997); pp. 23.
10. R. E. Mickens, "Nonstandard Finite Difference Schemes: A Status Report," *Book of Abstracts: Third International Conference on Theoretical and Computational Acoustics* (Naval Undersea Warfare Center Division; Newport, RI; 1997); Section 2.1 Computational Methods I.
11. R. E. Mickens, "Numerical Integration of Reaction-Diffusion PDE's," *Abstracts: Third Midwestern-Southeastern Atlantic Joint Regional Conference on Differential Equations* (Vanderbilt University; Nashville, TN; November 7-9, 1997); pp. 26.
12. R. E. Mickens, "Difference Equation Models of Reaction-Diffusion PDE's," *Abstracts of Papers Presented to the American Mathematical Society* 19, 137 (1998). Abstract #930-65-268.
13. R. E. Mickens, "The Construction of Finite Difference Schemes Having Special Properties," *HBCU's Research Conference: Agenda and Abstracts* (NASA-Lewis Research Center/Ohio Aerospace Institute; Cleveland, OH; April 8-9, 1998); pp. 14.

14. Kale Oyedeji and Ronald E. Mickens, "Nonstandard Explicit Discretizations of First-Order ODE's," *Abstracts, 18th Annual Southeastern-Atlantic Regional Conference on Differential Equations* (Auburn University; Auburn, AL; October 16-17, 1998); pp. 15.
15. Ronald E. Mickens, "Explicit Discretizations of First-Order ODE's," *Abstracts of Papers Presented to the American Mathematical Society* 20, 88 (1999); Abstract #939-39-193.
16. R. E. Mickens, "Influence of Spatial Discretizations on Nonstandard Finite Difference Schemes for Nonlinear PDE's," *Book of Abstracts. The 1999 IMACS International Conference on Nonlinear Evolution Equations and Wave Phenomena: Computation and Theory* (Georgia Center for Continuing Education, University of Georgia; Athens, GA), pp. 58.
17. R. E. Mickens, "Review of Nonstandard Finite Difference Methods," 1999 *SIAM Annual Meeting: Final Program and Abstracts* (SIAM; Philadelphia, PA; 1999) pp. 184.
18. R. E. Mickens, "Discrete Models of Dynamical Systems," *Abstracts: Fourth Mississippi State Conference on Differential Equations and Computational Simulations* (Mississippi State University, MS; May 21-22, 1999), pp. 28.
19. R. E. Mickens, "A Nonstandard Finite Difference Scheme for Strongly Nonlinear Duffing Equations," *Abstracts of Talks Presented at the 19th Southeastern-Atlantic Regional Conference on Differential Equations* (University of Richmond; Richmond, VA; October 22-23, 1999).
20. R. E. Mickens, "Pulse Vaccination in a Discrete Epidemic Model," *Abstracts of Papers Presented to the American Mathematical Society* 21, 101 (2000).
21. R. E. Mickens, "Discrete Derivatives," *Bulletin of the American Physical Society* 45, 22 (2000).
22. K. Oyedeji and R. E. Mickens, "Discrete Hamiltonian Models for the Harmonic Oscillator," *Bulletin of the American Physical Society* 45, 28 (2000).
23. R. E. Mickens, "Asymptotic Solutions to a Discrete Airy Equation," *Abstracts of Papers Presented to the American Mathematical Society* 22, 95 (2001). Abstract 962-39-1430.
24. R. E. Mickens, "Discrete Models of Periodic Diseases," *Abstracts of the 25th Annual Meeting of the Society for Industrial and Applied Mathematics, Southeastern Atlantic Section* (Coastal Carolina University; Conway, SC; March 16-17, 2001); pp. 5.
25. K. O. Oyedeji and R. E. Mickens, "A Nonstandard Finite Difference Scheme for the Lotka-Volterra Predator-Prey Model," *Georgia Journal of Science* 59, 67 (2001).

The following two abstracts appear in *Book of Abstracts, The Second IMACS International Conference on Nonlinear Evolution Equations and Wave Phenomena: Computation and Theory* (Georgia Center for Continuing Education; Athens, GA; April 9-12, 2001):

26. R. E. Mickens, "A Nonstandard Finite Difference Scheme for the Burgers Equation with a Combustion Reaction Term," pp. 78.
27. R. E. Mickens, "A Nonstandard Finite Difference Scheme for the Brusselator Reaction-Diffusion System," pp. 115.

APPENDIX B

Major Peer-reviewed Publications

1. R. E. Mickens, "WKB Procedure for Schrödinger Type Difference Equations," with I. Rāmadhani, in V. Lakshmikantham, editor, *World Congress of Nonlinear Analysts '92* (Walter de Gruyter, Berlin, 1996); Volume 4, pps. 3907–3912.
2. R. E. Mickens, "Exact Finite Difference Schemes for the Wave Equation with Spherical Symmetry," *Journal of Difference Equations and Applications* 2, 263–269 (1996).
3. R. E. Mickens, "Relation Between the Time and Space Step-Sizes for the Fisher Partial Differential Equation," *International Journal of Applied Sciences and Applications* 2, 423–424 (1996).
4. R. E. Mickens, "Discrete Models of Continuous Systems: The Details Matter," with O. Oyediji, in S. K. Dey and J. Ziebarth, editors, *Proceedings of Advances in Scientific Computing and Modeling* (Eastern Illinois University; Charleston, IL; October 12–14, 1995); pps. 91–95.
5. R. E. Mickens, "Construction of Finite Difference Schemes for Coupled Nonlinear Oscillators Derived From a Discrete Energy Function," *Journal of Difference Equations and Applications* 2, 185–193 (1996).
6. R. E. Mickens, "Asymptotic Properties of Solutions to Discrete Coulomb Equations," in Special Issue: Advanced in Difference Equations II, *Computers and Mathematics with Applications* 36, 285–289 (1998).
7. R. E. Mickens, "Asymptotic Properties of Solutions to Two Discrete Airy Equations," *Journal of Difference Equations and Applications* 3, 231–239 (1998).
8. R. E. Mickens, "Relation Between the Time and Space Step-Sizes in Nonstandard Finite-Difference Schemes for the Fisher Equation," *Numerical Methods for Partial Differential Equations* 13, 51–55 (1997).
9. R. E. Mickens, "Nonstandard Finite Difference Scheme for a Scalar Reaction-Convection PDE," *Journal of Difference Equations and Applications* 3, 359–367 (1998).
10. R. E. Mickens, "A Model of Dieting," with D. N. Brewley and M. L. Russell, *Society for Industrial and Applied Mathematics Review* 40, 667–672 (1998).
11. R. E. Mickens, "Numerical Integration of the Dissipative Complex Equation," with K. O. Oyediji, *International Journal of Applied Science and Computations* 4, 99–103 (1997).
12. R. E. Mickens, "Exact Finite-Difference Schemes for Two-Dimensional Advection Equations," *Journal of Sound and Vibration* 207, 426–428 (1997).
13. R. E. Mickens, "Nonstandard Finite Difference Schemes: A Status Report," in Yu-Chiung Teng et al., editors *Theoretical and Computational Acoustics '97* (World Scientific Publishing Company, Singapore), pps. 419–428.
14. R. E. Mickens, "A Finite Difference Scheme for Traveling Wave Solutions to Burgers Equation," *Numerical Methods for Partial Differential Equations* 14, 815–820 (1998).
15. R. E. Mickens, "On the Asymptotic Behavior of a Second-Order Linear Difference Equation," *Journal of Difference Equations and Applications* 5, 111–112 (1999).

16. R. E. Mickens, "Nonstandard Finite Difference Schemes for Reaction-Diffusion Equations," *Numerical Methods for Partial Differential Equations* 15, 201–214 (1999).
17. R. E. Mickens, "An Introduction to Nonstandard Finite Difference Schemes," *Journal of Computational Acoustics* 7, 39–58 (1999).
18. R. E. Mickens, "Suppression of Numerically Induced Chaos with Nonstandard Finite Difference Schemes," with Alicia Serfaty de Markus, *Journal of Computational and Applied Mathematics* 106, 317–324 (1999).
19. R. E. Mickens, "Discretizations of Nonlinear Differential Equations Using Explicit Nonstandard Methods," *Journal of Computational and Applied Mathematics* 110, 181–185 (1999).
20. R. E. Mickens, "Influence of Spatial Discretizations on Nonstandard Finite Difference Schemes for Nonlinear PDE's," *International Journal of Applied Science and Computations* 6, 89–95 (1999).
21. R. E. Mickens, "Analysis of a Finite-Difference Scheme for a Linear Advection-Diffusion-Reaction Equation," *Journal of Sound and Vibration* 236, 901–903 (2000).
22. R. E. Mickens, "Nonstandard Finite Difference Schemes for Reaction-Diffusion Equations Having Linear Advection," *Numerical Methods for Partial Differential Equations* 16, 361–364 (2000).
23. R. E. Mickens, "Nonstandard Finite Difference Schemes," Chapter 1, pps. 1–54, in R. E. Mickens (Editor), *Applications of Nonstandard Finite Difference Schemes* (World Scientific, Singapore, 2000).
24. R. E. Mickens, "A Nonstandard Finite Difference Scheme for a Nonlinear PDE Having Diffusive Shock Wave Solutions," *Journal of Mathematics and Computers in Simulation* 55, 549–555 (2001).
25. R. E. Mickens, "A Nonstandard Finite-Difference Scheme for Conservative Oscillators," *Journal of Sound and Vibration* 240, 587–591 (2001).
26. R. E. Mickens, "The Role of Positivity in the Construction of Nonstandard Finite Difference Schemes for PDE's," pps. 294–307, in David Schultz, Bruce Wade, Jesus Vigo-Aguiar, and Suhrit K. Dey (editors), *Proceedings of International Conference on Scientific Computing and Mathematical Modeling* (University of Wisconsin-Milwaukee; Milwaukee, WI; May 25–27, 2000).
27. R. E. Mickens, "Asymptotic Solutions to a Discrete Airy Equation," *Journal of Difference Equations and Applications* (reviewed and accepted for publication).
28. R. E. Mickens, "Mathematical and Numerical Study of the Duffing-Harmonic Oscillator," *Journal of Sound and Vibration* 244, 563–567 (2001).
29. R. E. Mickens, "Mathematical Analysis of a Nonstandard Finite Difference Scheme for the Fisher Equation with Nonlinear Diffusion," *Journal of Computational Methods in Applied Sciences and Engineering* 1, 135–142 (2001).
30. R. E. Mickens, "A Nonstandard Finite-Difference Scheme for a Fisher PDE Having Nonlinear Diffusion," *Computers and Mathematics with Applications* (reviewed and accepted for publication).

31. R. E. Mickens, "Analytical and Numerical Study of a Nonstandard Finite Difference Scheme for the Unplugged van der Pol Equation," *Journal of Sound and Vibration* 245, 757-761 (2001).
32. R. E. Mickens, "Generalized Harmonic Balance/Numerical Method for Determining Analytical Approximations to the Periodic Solutions of the $x^{4/3}$ Potential," *Journal of Sound and Vibration* (reviewed and accepted for publication) with Karega Cooper.
33. R. E. Mickens, "Numerical Study of a Nonstandard Finite-Difference Scheme for the van der Pol Equation," *Journal of Sound and Vibration* (reviewed and accepted for publication) with A. B. Gumel.
34. R. E. Mickens, "Nonstandard Finite Difference Schemes for Differential Equations," *Journal of Difference Equations and Applications* (reviewed and accepted for publication).

APPENDIX C

Book

- * Ronald E. Mickens, editor, *Application of Nonstandard Finite Difference Schemes* (World Scientific, Singapore, 2000).

The above indicated book provides expanded versions of invited presentations that were given in a minisymposium held at the 1999 SIAM Annual Meeting held in Atlanta, Georgia. The title of the minisymposium was "Nonstandard Finite Difference Schemes: Theory and Applications." It consisted of four lectures by individuals actively working in the area. The main purpose of the minisymposium was to introduce the concepts of non-standard finite difference schemes and demonstrate their power and usefulness by applying these techniques to problems involving nonlinear convection-diffusion-reactions, electromagnetic scattering, and Lotka-Volterra type interactions in population dynamics.

APPENDIX D

Major Conference/Seminar Presentations of Dr. Ronald E. Mickens

1. "The Qualitative Theory of ODE's," Department of Mathematics and Physics, Tennessee State University; Nashville, TN; October 30, 1996
2. "Existence of Periodic Solutions for a Generalized Conservative Oscillator," Georgia Tech-UAB International Conference on Differential Equations and Mathematical Physics; Atlanta, GA; March 24, 1997
3. "Nonstandard Finite Difference Schemes," Department of Mathematics, Howard University; Washington, DC; April 18, 1997
4. "Relations Between Time and Space Step-Sizes in Numerical Schemes for PDE's that follow from a Positivity Condition," American Mathematical Society Detroit Meeting; Wayne State University; Detroit, MI; May 3, 1997
5. "Numerical Tests of a New Finite-Difference Scheme for the TDSE," 26th Southeastern Theoretical Chemistry Association Conference, University of Alabama at Birmingham; Birmingham, AL; May 16, 1997
6. "Nonstandard Finite Difference Schemes: A Status Report," Third International Conference on Theoretical and Computational Acoustics, Holiday Inn North; Newark, NJ; July 14, 1997

The following two presentations were given at Trinity University; San Antonio, TX; October 9, 1997:

7. "Qualitative Theory of Differential Equations," Department of Mathematics Major's Seminar
8. "Numerical Integration of Reaction-Diffusion PDE's," Mathematics Department Colloquium
9. "Regulation of Singular ODE's Modeling Oscillating Systems," 926th American Mathematical Society Meeting, Georgia Institute of Technology; Atlanta, GA; October 18, 1997
10. "Numerical Integration of Reaction-Diffusion PDE's," Third Midwest-Southeastern Atlantic Joint Regional Conference on Differential Equations, Vanderbilt University; Nashville, TN; November 9, 1997
11. "Difference Equation Models of Reaction-Diffusion PDE's," Special Session on "Difference Equations and Applications," Joint Mathematics Meeting of the AMS and MAA, Baltimore Convention Center; Baltimore, MD; January 10, 1998
12. "Regulation of Singular ODE's Modeling Oscillating Mechanical Systems," Special Session on "Dynamical System-1," 13th U.S. National Congress on Applied Mechanics, University of Florida; Gainesville, FL; June 26, 1998
13. "Application of Mathematics to the Biomedical Sciences," UNCF Premedical Summer Institute at Fisk and Vanderbilt Universities; Nashville, TN; July 13, 1998

14. "Explicit Discretizations of First-Order ODE's," Special Session on Discrete Models and Difference Equations; Joint Mathematics Meetings (AMS-MAA); San Antonio, TX; January 14, 1999
15. "Analytic Approximation to the Periodic Solution of a Modified van der Pol Equation," The 1999 International Conference on Differential Equations and Mathematical Physics, University of Alabama-Birmingham; March 17, 1999
16. "The Third Revolution in Science: Nonlinear Dynamics," Scientist Symposium: Launching Young Minds into a Universe of Success, sponsored by the Endeavor Space Club, Morehouse College; Atlanta, GA; April 1, 1999
17. "The Influence of spatial Discretizations on Nonstandard Finite Difference Schemes for Nonlinear PDE's," Session on Singular Perturbations, Dynamics, and Nonstandard Difference Schemes at The 1999 IMACS International Conference on Nonlinear Evolution Equations and Wave Equations, University of Georgia; Athens, GA; April 12, 1999
18. "Numerical Integration of Chemical Reaction-Diffusion Partial Differential Equations," 28th Southeast Theoretical Chemistry Association Meeting, University of Memphis; Memphis, TN; April 14, 1999
19. "Review of Nonstandard Finite Difference Methods," in Minisymposium – Nonstandard Finite Difference Methods: Theory and Applications; 1999 SIAM Annual Meeting; Atlanta, GA; May 14, 1999
20. "Discrete Models of Dynamical Systems," Fourth Mississippi State Conference on Differential Equations and Computational Simulations, Wise Center, Mississippi State University, MS; May 22, 1999
21. "The z -Transform," Fourier Transforms and Wavelet Theory Workshop, Physics Program, Spelman College; Atlanta, GA; June 2, 1999
22. "Qualitative Methods in Differential Equations," Mathematics Department, Xavier University; New Orleans, LA; September 29, 1999
23. "A Nonstandard Finite Difference Scheme for a Strongly Nonlinear Duffing Equation," 19th Annual Southeastern-Atlantic Regional Conference on Differential Equations, University of Richmond; Richmond, VA; October 23, 1999
24. "Pulse Vaccination in a Discrete Epidemic Model," Joint Mathematics Meetings of AMS, MAA and SIAM; Special Session of the AMS on Difference Equations and Their Applications in Social and Natural Sciences, II, Washington, DC; January 22, 2000
25. "Application of Nonlinear Dynamics and Fractals to the Biosciences," Department of Physics, Mississippi State University; Starkville, MS; April 24, 2000
26. "A New Predator-Prey Model," Session on Mathematical Biology and Other Models I, Year 2000 International Conference on Dynamical Systems and Differential Equations; Kennesaw State University; Kennesaw, GA; May 18, 2000
27. "The Role of Positivity in the Construction of Nonstandard Finite Difference Schemes for PDE's," IMACS 2000-International Conference on Scientific Computing and Mathematical Modeling; University of Wisconsin-Milwaukee; Milwaukee, WI; May 25, 2000

28. "Discrete Derivatives," Session KA: Computational and Theoretical Physics I, at the 67th Annual Southeastern Section Meeting of the American Physical Society; Mississippi State University; Starkville, MS; November 4, 2000
29. Asymptotic Solutions to a Discrete Airy Equation," AMS Special Session on Asymptotic Behavior of Difference Equations with Applications, II; Joint Mathematics Meetings; New Orleans Marriott Hotel; New Orleans, LA; January 10, 2001
30. "Nonstandard Finite Difference Schemes," University of New Orleans Mathematics Colloquium; New Orleans, LA; January 11, 2001
31. "The Qualitative Theory of Differential Equations," Mathematics Seminar; Morgan State University; Baltimore, MD; January 26, 2001
32. "Discrete Models of Periodic Diseases," SIAM-Southeastern Atlantic Section Meeting; Coastal Carolina University; Conway, SC; March 16, 2001

The following two lectures were given at the Second IMACS International Conference on Nonlinear Evolution Equations and Wave Phenomena: Computation and Theory held during April 9–12, 2001 at the Georgia Center for Continuing Education; Athens, GA:

33. "A Nonstandard Finite Difference Scheme for the Burgers Equation with a Combustion Reaction Term," Special Session on "Mathematical and Computational Approaches to Applied Propagation Problems," April 9, 2001
34. "A Nonstandard Finite Difference Scheme for the Brusselator Reaction-Diffusion System"
35. "Mathematical and Numerical Study of a Dieting Model," Mathematics Department, Stamford University, Birmingham, AL; May 15, 2001

APPENDIX E

Collaborators

The following individuals have been involved with me on various aspects of constructing and analyzing nonstandard finite difference schemes for ODE's and PDE's.

Matthias Ehrhardt
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APPENDIX F

Major Professional Activities

1. I serve on the editorial boards of the following peer-reviewed scientific research journals:
 - * *The Journal of Difference Equations and Applications*
 - * *International Journal on Applied Science and Computation*
 - * *Journal of Computational Methods in Sciences and Engineering*

2. During the course of the DOE Grant (1996–2001), I reviewed sixty-one (61) scientific manuscripts for the following research publications:
 - * *Journal of Sound and Vibration*
 - * *Numerical Methods for Partial Differential Equations*
 - * *Journal of Difference Equations and Applications*
 - * *CAARMS-6 Proceedings*
 - * *Proceeding A, The Royal Society (United Kingdom)*
 - * *Dynamic Systems and Applications*
 - * *Computers and Mathematics with Applications*
 - * *Journal of Applied Mathematics*
 - * *Journal of Computational and Applied Mathematics*
 - * *Proceedings, 18th Biennial ASME Conference on Mechanical Vibration and Noise*
 - * *The Mathematical Intelligencer*
 - * *Applied Mathematics Letters*
 - * *Journal of Mathematics and Computers in Simulation*

3. Again, during the course of the DOE Grant (1996–2001), I provided mail reviewed of research proposals to the following research funding agencies (the numbers represent the number of proposals):
 - * Army Research Office (1)
 - * Department of Energy (4)
 - * National Institutes of Health (3)
 - * NASA (2)
 - * National Science Foundation (3)

All of the proposals were in the area of computational methods applied to problems in the natural, engineering, and biomedical sciences.