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by

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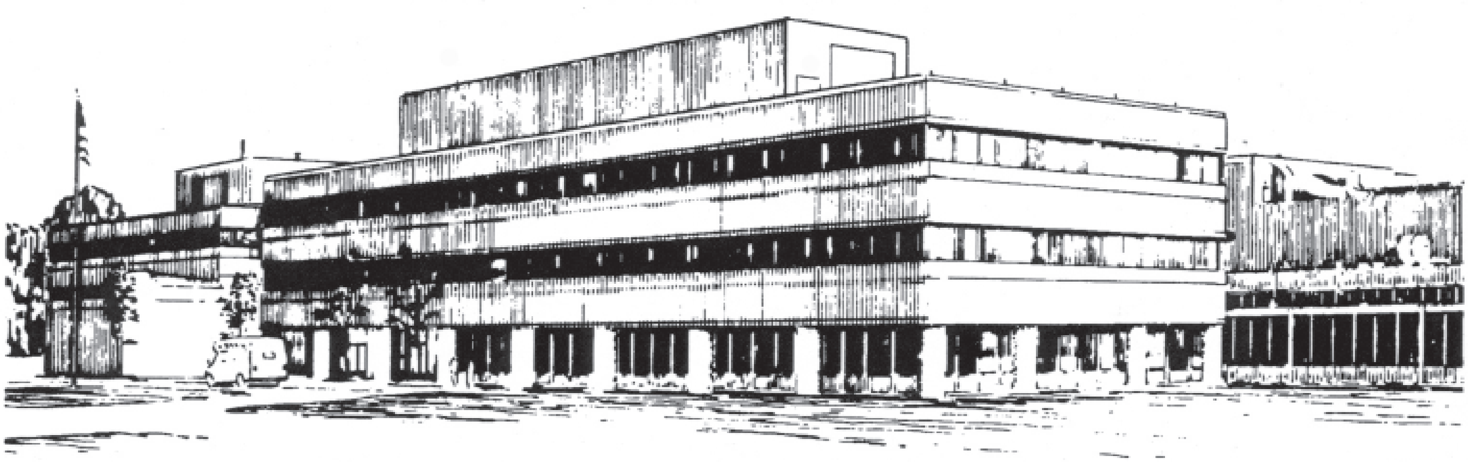

PRINCETON PLASMA PHYSICS LABORATORY
PRINCETON UNIVERSITY, PRINCETON, NEW JERSEY

# On Properties of Compressional Alfvén Eigenmode Instability Driven by Superalfvénic Ions.

N.N. GORELENKOV and C.Z. CHENG

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*Princeton Plasma Physics Laboratory, Princeton University,*
*P.O. Box 451, Princeton, New Jersey 08543 USA*

**Abstract**

Properties of the instability of Compressional Alfvén Eigenmodes (CAE) in tokamak plasmas are studied in the cold plasma approximation with an emphasis on the instability driven by the energetic minority Ion Cyclotron Resonance Heating (ICRH) ions. We apply earlier developed theory [N.N. Gorelenkov and C.Z. Cheng, Nuclear Fusion, **35** (1995) 1743] to compare two cases: Ion Cyclotron Emission (ICE) driven by charged fusion products and ICRH Minority driven ICE (MICE) [J. Cottrell, Phys. Rev. Lett. (2000)] recently observed on JET. Particularly in MICE spectrum, only instabilities with even harmonics of deuterium-cyclotron frequency at the low-field-side plasma edge were reported. Odd deuterium-cyclotron frequency harmonics of ICE spectrum between the cyclotron harmonics of protons can be driven only via the Doppler shifted cyclotron wave-particle resonance of CAEs with fusion products, but are shown to be damped due to the electron Landau damping in experiments on MICE. Excitation of odd harmonics of MICE with high-field-side heating is predicted. Dependencies of the instability on the electron temperature is studied and is shown to be strong. Low electron temperature is required to excite odd harmonics in MICE.

# 1 INTRODUCTION

The theory of thermonuclear cyclotron instability of Compressional Alfvén Eigenmodes (CAE, also called fast Alfvén or magnetosonic eigenmodes), i.e., driven by products of thermonuclear reactions, has significantly progressed in recent years (see Refs. [1, 2, 3, 4, 5, 6] and references therein). It is considered primary in explaining Ion Cyclotron Emission (ICE) in tokamaks. ICE was reported in several experimental papers [7, 8, 9, 10, 11]. In recent experiments on NSTX, CAEs were responsible for sub-cyclotron instabilities driven by Neutral Beam Injection (NBI) ions [12, 13]. Within the concept of thermonuclear CAE instability, two approaches were proposed: perturbative, or weak instability, and nonperturbative (originally in Ref.[14]), or strong instability. In the case of weak instability, the growth rate was considered to be smaller than fast particle bounce period:

$$\gamma_{CAE} \ll \tau_b, \tag{1}$$

and is proportional to fast ("hot") particle density $n_h$. Since hot ion density increases gradually in experimental conditions, one would expect that weak instability would be excited first, at least in those discharges where the density of hot particles is really small. Moreover, in Joint European Torus (JET) experiments [9], ICE amplitude was changing for six orders of magnitude in different discharges and was shown to correlate with the density of fusion products for several orders of magnitude $n_h/n_e = 10^{-9} - 10^{-4}$. The growth rate of strong instability as was presented in Refs.[3, 4, 6] is based on the local homogeneous plasma dielectric properties and was shown to be selfconsistent at the upper limit of this density range. We will show in Appendix A that the strong instability requires a much stronger condition than just reversed Eq.(1) to be selfconsistent. Applied to JET plasma, a strong instability condition reads:

$$\frac{\gamma_{CAE}}{\omega} > \frac{\Delta\tau^{-1}}{\omega} \simeq 0.1\frac{\omega_{cD}}{\omega}, \tag{2}$$

where $\Delta\tau$ is the characteristic time of wave-particle interaction. Since particle interactions with the eigenmode occur only in the vicinity of the Doppler shifted cyclotron resonance, $\Delta\tau$ is much shorter then $\tau_b$.

Recently, a new observations from JET of ICRH hydrogen minority-driven ICE (MICE) [15] showed different properties than ICE driven by fusion protons. Most importantly, in MICE, only instabilities with even harmonics of background deuterium ($D$) cyclotron frequency at the low-field-side (LFS) edge of the plasma were reported $\omega = 2l\omega_{cDedge}$, $l = 1, 2, 3...$, which are equal to integer harmonics of proton ($p$) cyclotron frequency, since $\omega_{cD} = \omega_{cp}/2$. Reconciling ICE and MICE spectra, which have a similar drive source coming from $MeV$ energy protons, presents a challenge to a theory of strong instability. As we will see, the weak instability theory can describe general properties of MICE in JET, which was preliminarily reported in Ref.[16]. In another recent report [17], local theory was applied to describe MICE where the Doppler shift to a local resonance cyclotron condition was assumed to come from the drift velocity only. Such shift may not be sufficient to account for odd $D$-harmonics of ICE spectra if the drift frequency is smaller than Alfvén velocity $v_A$. In addition, proton drift velocity is about the same for MICE and ICE, which challenges the local theory to explain the existence of odd harmonics ICE and their absence in MICE spectra.

As was shown in Ref.[2], odd $D$-harmonics of ICE spectrum can be driven by fusion products via Doppler shifted cyclotron wave-particle resonance, and will be shown to be damped due to the electron Landau damping in experiments on MICE. To excite even harmonics of MICE, high field side heating is proposed. Dependencies of the instability on the electron temperature is studied.

# 2 MODEL OUTLINE

We are applying a perturbation theory described in detail in our previous works [1, 2]. To understand the observable properties of cyclotron instability, one should take into account the poloidally and radially localized structure of CAEs. It is assumed in the perturbation theory that the CAE mode structure is not affected by hot particles, whose population is small near the edge. The localized CAE solutions have the eigenfrequency [18, 19, 2]:

$$\omega^2 = \frac{m^2 v_A^2(r_0)}{r_0^2}(1 + \frac{1+\sigma_i}{\sigma_i}\frac{(2s+1)\Delta^2}{r_0^2}) \tag{3}$$

and the eigenfunction of the perturbed poloidal electric field:

$$E_\theta = \hat{E}_0 \phi_k \left(\frac{\sqrt{2}\theta}{\Theta}\right) \phi_s \left(\frac{\sqrt{2}(r-r_0)}{\Delta}\right) e^{-i\omega t + im(\theta+\epsilon_0 \sin\theta) - in\varphi}. \tag{4}$$

where $\theta$ and $\varphi$ are the poloidal and toroidal angles,

$$\Delta^2/a^2 = \sqrt{2\sigma_i/(1+\sigma_i)}/[m(1+\sigma_i)(1+\epsilon_0)],\ \Theta^2 = \sqrt{4(1+\epsilon_0)/(2s+1)\epsilon_0}\Delta/r_0, \tag{5}$$

$r$ is the minor radius, $\epsilon_0 = r_0/R_0$, $\phi_s(x) = e^{-x^2/2}H_s(x)/\sqrt{n!2^s\sqrt{\pi}}$ and $H_s$ are the $s-th$ order Chebyshev-Hermit functions and polynomials, respectively, $m \gg 1$ is poloidal mode number, $v_A(r_0)$ is the Alfvén velocity evaluated at mode location $r = r_0$, $r_0^2/a^2 = 1/(1+\sigma_i) - (2s+1)\Delta^2/a^2$, $\Delta^2/a^2 = \sqrt{2\sigma_i/(1+\sigma_i)}/[m(1+\sigma_i)]$, and $s$ is an integer representing radial wavenumber, assumed to be 1 in this paper. Here the plasma density profile is chosen in the form:

$$n(r) = n_0(1 - r^2/a^2)^{\sigma_i}, \tag{6}$$

where $n_0$ is the plasma density at the magnetic axis. Rapid variation of the plasma density profile is required near the plasma edge for the eigenmode to be radially localized near the edge, $\sigma_i < 1$.

Fast particles drive the instability of CAEs at those parts of their drift orbits, where the local resonance condition is satisfied:

$$\omega - l\omega_{ch} - k_{||}v_{||h} - k_{\perp}v_{drh} = 0, \tag{7}$$

where $v_{||h}$ and $v_{drh}$ are the parallel and drift particle velocities, $k_{||}$ and $k_{\perp}$ are parallel and perpendicular components of wavevector, respectively. Approximately, the growth rate of such instability due to fast particles is:

$$\gamma_h \sim \frac{n_h}{n_e}\left\langle \frac{\partial \ln f_h}{\partial \ln v_{\perp}} I_{res} \right\rangle,$$

where $f_h$ is the fast particle distribution function, $I_{res}$ is the resonance term responsible for the "effectiveness" of wave-particle interaction at the position of local cyclotron resonance Eq.(7), and $\langle ... \rangle$ means averaging over the phase space (see Ref.[1] for details).

# 3 ICE SPECTRUM

Several factors contributing to the instability of CAE are responsible for the shape of ICE spectra. Since the instability is excited if the damping is smaller than the drive, the most important effect on the spectrum is from the thermal ion cyclotron damping, which damps almost all modes if the resonance layer intersects the mode location [2]. Thus, the instability in a deuterium plasma can be excited with $\omega \sim l\omega_{cDedge}$, $\omega < l\omega_{cDedge}$, i.e., the ion cyclotron resonance layer is required to be outside of the plasma. Hot particles, however, still can be in resonance due to a larger Doppler shift in local cyclotron resonance Eq.(7). In the instability of odd ICE harmonics, the main contribution to the Doppler shift in Eq.(7) comes from the term containing $k_{||}$. However, $k_{||}$ is restricted in absolute value by the requirement of the electron Landau damping to be small.

Electrons contribute to CAE damping through the transit resonance, i.e., condition similar to Eq.(7), but with $l = 0$. In the limit $\omega^2 \gg \omega_{cD}^2$, the electron Landau damping rate is estimated as[2]:

$$\gamma_e \simeq -\omega\beta_e\zeta_e e^{-\zeta_e^2}, \tag{8}$$

where $\zeta_e = \omega/|k_{||}|v_{Te} \simeq (k_\perp/k_{||})\sqrt{m_e/m_i\beta_e}$, $\beta_e = 8\pi n_e T_e/B^2$, $k_{||} \simeq m(1+\epsilon_0)/qR_0 - n/R_0$, and $q$ is the safety factor. Typically, $n \sim m$, which provides enough Doppler shift at the edge for the fusion protons with birth velocity $v_{f0} = 2.4 \times 10^9 cm/sec$ to excite the instability at odd deuterium edge cyclotron harmonics. It requires that $k_{||} \geq \omega_{cDedge}/v_{p0} \simeq 3.5m^{-1}$ for JET [2]. Effects of finite $k_{||}$ on the mode structure and eigenfrequency were considered in Ref.[20] and should not change properties of CAE instabilities presented here, since $k_{||} \ll k_\perp$.

First, consider an example of ICE spectrum in JET plasma [15]. We will use the following plasma parameters: major radius $R_0 = 2.95m$, minor radius $a = 0.95m$, magnetic field $B_0 = 2.5T$, ellipticity $\kappa = 1.8$, density profile given by Eq.(6) with $n_0 = 2.5 \times 10^3 cm^{-3}$, and $\sigma_i = 0.5$, ion and electron temperatures $T_i = T_e = (1 - r/a)keV$, safety factor $q = 0.8\left(1 + 4\left(r/a\right)^3\right)$, hot fusion proton particle density $n_h = 2. \times 10^{-2} n_0 \left(1 + \left(r/a\right)^2\right)^3$. Fusion products are assumed to have a slowing down velocity distribution function with energy cutoff at $\mathcal{E}_{hc} = 3MeV$, and single pitch angle distribution represented by barely trapped particles.

The numerical calculations outlined above reproduce major features of experimentally observed ICE [9] as shown in example Fig.1 (see also [2]). Plotted are the most unstable modes at each frequency. One can see that odd, $3rd$ deuterium cyclotron harmonic has a smaller growth rate and, hence, is expected to be excited with lower amplitude. The instability is restricted to the frequency domain without thermal ion damping, i.e., $\omega \sim l\omega_{cDedge}$, $\omega < l\omega_{cDedge}$.

# 4 MICE SPECTRUM

To avoid electron damping in the presence of a rather small drive from hot particles, one should look for the instability with $\zeta_e \gg 1$, i.e., at $\omega/v_{Te} \gg> |k_{||}|$. It is clear from Eq.(7), that low $v_{||h}$ would require large $k_{||}$ to excite CAE instability at odd harmonics of deuterium cyclotron frequency: $v_{||h}k_{||} = \omega_{cDedge}$. In the case of ICRH minority driven instability, $v_{||h}$ is fixed and small since the resonant layer is typically located at the plasma center and all particles are well trapped $v_{||h} = \sqrt{\mathcal{E} - \mu B} \simeq v_h\sqrt{\epsilon} \Rightarrow k_{||} = \omega_{cDedge}/v_h\sqrt{\epsilon}$. Thus, it is harder to excite odd harmonics of MICE. This is in contrast with ICE odd harmonics driven by fusion products represented mostly by barely trapped particles with larger $v_{||h} \simeq v_h\sqrt{2\epsilon}$, i.e., it requires smaller $k_{||} = \omega_{cDedge}/v_{||h}\sqrt{2\epsilon}$. Eq.(8) shows that electron damping should be stronger (exponentially in $k_{||}^2$) for CAE instability to be responsible for MICE odd harmonics. Here we will assume the same characteristic velocity for hot particles in both cases, MICE and ICE. This is justified for MICE by the requirements that hot particles are present at the mode location, which requires large particle radial excursion from the center consistent with particle energy $\mathcal{E} = 3MeV$ [15].

To study CAE properties, we perform calculations using the same perturbative formulation used above and reported previously [2] for typical JET plasma parameters. The velocity distribution of hot particles is Maxwellian in energy with temperature $T_h = 1MeV$, having a single pitch angle corresponding to the resonance layer at $R = R_c$, $\lambda \equiv \mu B_0/\mathcal{E} = R_c/R_0$. Note that there is a natural energy cutoff when the particle orbit becomes wide enough and reaches plasma edge. The cutoff energy is a function of the plasma minor radius at the particle bounce point and is on order of $3MeV$ [15]. In our simulations, it is calculated based on particle drift orbit trajectory.

Unlike our previous study, the spectrum is derived by summing up the growth rates of unstable CAE modes for a given frequency. Each frequency determines the poloidal mode number Eq.(3) so that a few modes with different toroidal mode numbers can be unstable. This presents a stronger dependence on the electron temperature and is probably similar to the experimentally observed conditions of multi mode excitation. In the experiment, one would have a small frequency splitting due to finite $k_{||}$, which not always can be seen since nonlinear mode evolution can easily broaden the eigenfrequency of CAEs.

For simplicity, we look for the odd $l = 3$ harmonic instability. It is unstable if $v_{||}$ becomes large enough, i.e., at pitch angle $\lambda < 1$, which corresponds to the case of high field side heating. Fig. 2 gives a dependence of odd $l = 3$ MICE and even $l = 4$ peaks on the minority "thermal" velocity at $\lambda = 0.95$, at which the major radius of the resonance layer is $R_c = R_0\lambda = 2.8$. Both peaks have threshold in $v_{h0}$ velocity, which agrees with observations of the even peaks [15]. This is required for protons to reach the edge from the center.

Since electron damping plays a crucial role in forming MICE, we changed the electron temperature to simulate this dependence in Fig.3. It can be seen that the growth rate of the odd harmonic peak is very sensitive to both electron temperature and hot particle velocity. Thus conditions for the unstable CAEs are low electron temperature and low $\lambda$, which means high field side ICRH.

# 5 SUMMARY

Weak instability theory is shown to be selfconsistent in describing both ICE and MICE spectra, driven by fusion products and ICRH hydrogen minority ions, respectively. Strong dependence of MICE spectra on the electron temperature is demonstrated to explain the disappearance of odd deuterium cyclotron harmonics. Low electron temperature and high field side ICRH are required to excite odd harmonics. The excitation of MICE seems to have a threshold in fast particle velocity. Large particle velocity is required for particles to reach the low field side of the plasma, where CAEs are localized. This is in agreement with JET observations.

# 6 ACKNOWLEDGMENTS

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# A STRONG VERSUS WEAK INSTABILITY

Weak instability, which contrasts with the strong instability (basis for the local theory) approach adopted in Refs.[3, 4] (and references therein) is applicable if

$$\gamma_{CAE} < \Delta\tau^{-1}, \tag{9}$$

where $\Delta\tau$ is the characteristic time of wave-particle interaction, $\Delta\tau = \min\left(\tau_{res}, \Theta qR/v_{||}\right)$. Here $\tau_{res}$ is the time of particle interaction with the perturbation near the local resonance and $\Theta qR/v_{||}$ is the time a particle spends in the mode localization region. Typically, the resonance interaction is happening in the vicinity of the local resonance determined by the condition Eq.(7). The upper estimate for $\Delta\tau$ (and the lower estimate for $\gamma_{CAE}$) in JET is $\Delta\tau = \Theta qR/v_{||} = 0.5 \times 10^{-6} sec$ for $\Theta = 1$, $R = 3m$, $v_H = 10^9 cm/sec$, $q = 2$, $\chi = 0.5$. This gives the validity condition for the weak instability theory:

$$\frac{\gamma_{CAE}}{\omega} < 0.1\frac{\omega_{cD}}{\omega}, \tag{10}$$

which is typically never violated within the weak instability theory.

# References


[1] Gorelenkov, N.N., Cheng, C.Z., Phys. Plasmas **2** (1995) 1961.

[2] Gorelenkov, N.N., Cheng, C.Z., Nucl. Fusion **35** (1995) 1743.

[3] Fulop, T., Lisak, M., Nucl. Fusion **37** (1997) 1281.

[4] Fulop, T., Lisak, M., Nucl. Fusion **38** (1998) 761.

[5] Belikov, V.S., *et.al.,* Nucl.Fusion, **35** (1995) 1603.

[6] Kolesnichenko, Ya.I., Lisak, M., Anderson, D., Nucl. Fusion **40** (2000) 1419.

[7] Greene, G. J., *et.al.*, in *Proceedings of 17th European Conference on Controlled Fusion and Plasma Heating*, Amsterdam, Netherlands, 1990, edited by G. Briffod, Adri Nijsen-Vis, and F. C. Shüller (European Physical Society, Petit-Lancy, Switzerland, 1990), Part IV, Vol. **14**B, p.1540.

[8] Cottrell, G.A., Dendy, R.O., Phys. Rev. Lett. **60** (1988) 33 .

[9] Cottrell, G.A., et.al., Nucl. Fusion **33** (1993) 1365.

[10] Cauffman, S., Majeski, R., Rev. Sci. Instrum **66** (1995) 817.

[11] Cauffman, S., Majeski, R., McClements, K.G., *et.al.* Nucl.Fusion, **35** (1995) 1597.

[12] Fredrickson, E., Gorelenkov, N.N., C.Z.Cheng, *et.al.,* Phys. Rev. Lett. **87** (2001) 145001.

[13] Gorelenkov, N.N., *et.al.,* submitted to Nuclear Fusion.

[14] Mikhailovskii, A.B., in *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, New York, 1986), Vol.9, p.103.

[15] Cottrell, G.A., Phys. Rev. Lett. **84** (2000) 2397.

[16] Gorelenkov, N.N., Cheng, C.Z., Nazikian, R., *Proc. of International Sherwood Fusion Theory Meeting* (2000) 1D35.

[17] Mcclements, K.G., Dendy, R..O., Cottrell, G.A., in *Proceedings of 27th European Conference on Controlled Fusion and Plasma Physics*. Budapest, 12-16 June 2000 ECA Vol. **24**B (2000) 1517 -1520.

[18] Mahajan S.M., Ross, D.W., Phys. Fluids **26** (1983) 2561.

[19] Coppi, B., et.al., Phys. Fluids **29** (1986) 4060.

[20] Fulop, T., Lisak, M., Kolesnichenko, Ya. I., *et.al.,* Phys. Plasmas **7** (2000) 1479.

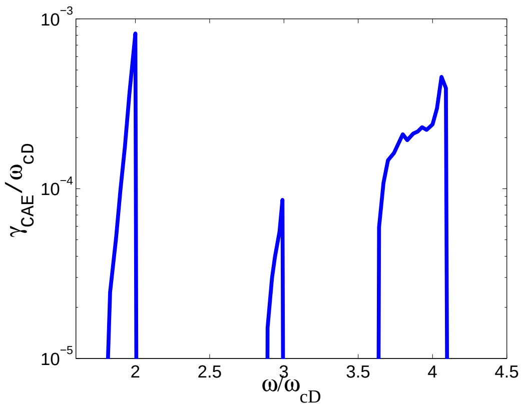


Figure 1: ICE spectrum driven by fusion protons.

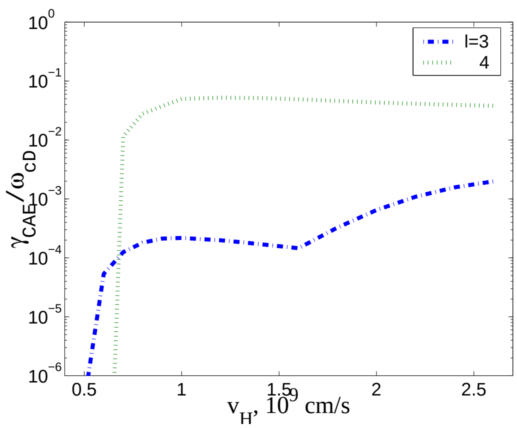


Figure 2: MICE odd $l = 3$ and even $l = 4$ deuterium cyclotron frequency harmonic growth rate as a function of minority ion velocity $v_{h0} = \sqrt{2T_h/m_h}$. The cutoff energy is fixed at $\mathcal{E}_{hc} = 3MeV$.

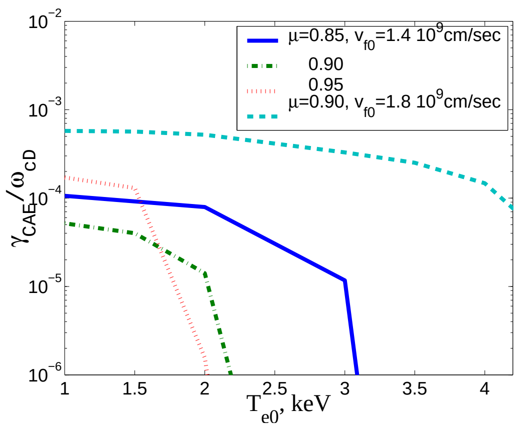


Figure 3: MICE odd $l = 3$ harmonic growth rate dependence on the electron temperature, where $T_{e0}$ is the electron temperature at the center, for two values of hot minority velocity $v_{h0} = 1.4 \times 10^9 cm/sec$ ($\mu = 0.85$, 0.90, 0.95) and $v_{h0} = 1.8 \times 10^9 cm/sec$ ($\mu = 0.90$) .

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Information Services
Princeton Plasma Physics Laboratory
P.O. Box 451
Princeton, NJ 08543

Phone: 609-243-2750
Fax: 609-243-2751
e-mail: pppl_info@pppl.gov
Internet Address: http://www.pppl.gov