SANDIA REPORT

SAND201-0406
Unlimited Release
Printed February 2001

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Merle E. Riley

Prepared by
Sandia National Laboratories
Albuquerque, New Mexico 87185 and Livermore, California 94550

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Dust in the Ion Wind: A Model for Plasma Dust Particle Dynamics

Merle E. Riley
Plasma Sciences Department
P. O. Box 5800
Sandia National Laboratories
Albuquerque, NM 87185-1423

Abstract

A model is developed for the forces acting on a micrometer-size particle (dust) suspended within a plasma sheath. The significant forces acting on a single particle are gravity, neutral gas drag, electric field, and the ion wind due to ion flow to the electrode. It is shown that an instability in the small-amplitude dust oscillation might exist if the conditions are appropriate. In such a case the forcing term due to the ion wind exceeds the damping of the gas drag. The basic physical cause for the instability is that the ion wind force can be a decreasing function of the relative ion-particle velocity. However it seems very unlikely the appropriate conditions for instability are present in typical dusty plasmas.

1. Introduction

There are recent observations of self-oscillation of dust particles suspended within the near-sheath or sheath regions of dusty plasmas.[1] The oscillations are perpendicular to the plane of the sheath (vertical) are most noticeable at lower background pressure. These observations and subsequent analyses prompted the present investigation of the forces acting on dusty plasma particles.

The rf (radio frequency, typically 13.56 MHz) fields used to create the plasma are much too high in frequency to couple directly to the motion of the dust particles. In many cases these fields do not even perturb the ion motion, especially for heavier ions. The rf field affects the particles only indirectly via the electron response. In this work it will be assumed that all the forces acting on the dust particles have been averaged over the rf cycle.

We will show in this work that there is a possibility of unstable motion when the ion wind force on the particle is sufficiently strong. Since the ion wind force is predominately a decreasing function of the relative velocity of the ions and the particle, this can lead to unstable oscillatory motion. Potentially, this would appear to be an almost universal phenomenon for plasma dust particles.

It should be noted here that there is a tremendous amount of literature appearing on the problem of dusty plasmas at the current time. Journals devoted to plasma physics have several articles per month appearing on plasma dust, especially in regard to dust collective wave phenomena. Unfortunately, we cannot afford to discuss all these studies in this work.

2. Equations of Motion of Plasma Dust Particle

We will consider only plasmas formed in electropositive gases. A plasma with a considerable fraction of negative ions would complicate the analysis beyond what we wish to explore. Typical plasma dust particles are suspended at the bottom of the plasma in a nearly planar region, suggesting that gravity has a dominant role. This is not universally true, but the relevant point is that we can approximate the forces and motion of the particle as a one-dimensional problem with gravity defining the coordinate axis. Consider a system with the z-axis pointed upwards. Newton's equation for the dust particle at z is

$$m_d \ddot{z} = -m_d\, g + q_d E_{sh} - m_d \gamma_{gas} \dot{z} + f_w, \tag{1}$$

where we use "dot notation" to denote time derivatives. z is the dust particle vertical coordinate, m_d is the mass of the dust particle, g is the acceleration of gravity, q_d is the charge on the particle, E_{sh} is the plasma sheath electric field along z at the position of the particle, γ_{gas} is the damping rate (1/s) due to collisions with the neutral gas background, and f_w is the "wind force" on the particle due to ion flow within the non-uniform plasma sheath. The wind force includes all forces on the particle due to its presence in a non-uniform plasma *except* those isolated in the $q_d\, E_{sh}$ term. We will use SI units unless otherwise noted. It is seen that g and γ_{gas} are positive constants, q_d and E_{sh} are both negative and possibly functions of z and $\dot{z}$, and f_w is also negative and a function of z and $\dot{z}$. We immediately scale the dust mass out of Eq.(1) to give the equation of motion (EOM):

$$\ddot{z} = -g + Q_d E_{sh} - \gamma_{gas}\dot{z} + F_w \tag{2}$$

where $Q_d = q_d / m_d$ and $F_w = f_w / m_d$.

The gas damping is estimated from a simple kinetic argument based on the velocity of the particle relative to the gas fluid velocity.[2] The result for the force acting on a spherical non-accommodating dust particle is

$$\vec{f}_{gas} \approx -\tfrac{4}{3}\pi a^2 m_{gas} n_{gas} v_{Tgas} \vec{v}_d \tag{3}$$

where $\vec{v}_d$ is the particle velocity and $v_{Tgas} = (8kT/\pi m_{gas})^{1/2}$ is the thermal velocity of the gas molecules. We have assumed the gas to be stationary. The particle radius is denoted as a. Evaluating the damping coefficient determines that

$$\gamma_{gas} = \tfrac{4}{3}\pi a^2 m_{gas} n_{gas} v_{Tgas} / m_d\,. \tag{4}$$

This seems appropriate considering the small size of the dust particles and the large mean free path of the gas molecules. We note that the case of complete accommodation of the gas on the particle gives the identical coefficient.[2]

2.1. Electric field within the plasma sheath

In order to calculate the electric field we have to develop a reasonably accurate model for the sheath and pre-sheath region. This could even be time dependent for some of the situations of interest, but that is difficult to accomplish. We will use an electrostatic model that incorporates linearization of the charge densities in terms of the potential field appropriate for the region of the plasma where the particles tend to accumulate. For the basic charging mechanism believed to be operative for the dust particles, we know that the dust can acquire a negative charge only where electrons are present. This means that the particles are not present, or at least not at rest, in the Child-Langmuir region of the sheath which is nearly devoid of electrons. The most likely location is the inner sheath or pre-sheath region where the positive ions and electrons are nearly equal in density. Thus we will be making approximations that take advantage of the smallness of the electric field.

All of the analysis in this work can be developed from any basic plasma physics text.[3] The sheath electric field, E_{sh}, is the macroscopic field due to the statistically averaged motion of the ions and electrons within the plasma in the absence of the particle.[3] The sheath field is a function of only the spatial coordinate z. The distortion of the local space charge density due to the presence of the charge on the dust particle is a separate effect. This Debye shielding distortion is mostly *spherically symmetric* about the particle and does not contribute greatly to the electric field there, especially insofar as the electrons are concerned. If the particle lies in a homogeneous isotropic plasma, there are *no* forces on it due to the plasma except the stochastic ones leading to Brownian motion. Non-spherical corrections are included in the "ion wind" term in the

EOM. These have the form of momentum transfer from the scattering ions to the particle. Another point is that the flow of charged particles through the sheath consists of nearly equal fluxes of ions and electrons. Consequently the momentum transfer to the particle is dominated by the ions by three orders of magnitude due to the ion-to-electron mass ratio.

It is fair to ask why the charge on the macroscopic dust particle is not shielded from the plasma sheath field. In other words, why do we single out the $q_d\, E_{sh}$ term in the EOM? Such shielding exists, but it is a second order effect. The plasma sheath has a non-zero electric field because the electrons cannot shield the electrodes (the applied field) from the plasma bulk in that region. There is another sheath about the particle itself. It has been proven elsewhere[4] that the linearization of the Poisson equation with Boltzmann distributions for the charged particles leads to linear superposition of potentials from the particle sources. This means that we should treat these fields as additive. We separately account for the weaker field at the position of the particle due to the surrounding, slightly non-isotropic, particle sheath by separately solving for ion scattering.

Typical sheath solutions utilize the Poisson equation, a Boltzmann distribution assumption for the electrons, and an approximation for the ion motion across the sheath. We will assume that the plasma bulk is a region of uniform electric potential, which is referenced at zero volts. A canonical Boltzmann distribution of electrons at temperature T_e would give:

$$n_e(\vec{r}) = n_B \exp(e\phi(\vec{r})/kT_e)\,, \tag{5}$$

where ϕ denotes the potential relative to the Bohm point where the plasma density is n_B. The values at the Bohm point are assumed to be close to the *bulk* properties; in fact, we treat the Bohm point values as being the limiting values that we would find in the bulk region of the plasma. This furnishes a determination of the electron density *throughout* the sheath, including the vicinity of the particle when present. However to solve for the sheath field we will replace this by the electron distribution in the absence of the particle, in which case we have:

$$n_e(z)=n_B \exp(e\phi(z)/kT_e) \tag{6}$$

with only a dependence on the vertical coordinate.

The Poisson equation is

$$\phi''(z)=-\frac{e}{\varepsilon_o}(n_i-n_e) \tag{7}$$

where e is the absolute value of the charge on the electron, and n_i and n_e are the ion and electron number densities. We already have Eq.(6) for the electron density. What we need now is an approximation for the ion density in the sheath.

A fluid description of the ions solves the constant-flux continuity equation as long as there are no significant sources or sinks of ions within the sheath region,

$$\Phi_i \equiv n_i(z)v_i(z)=n_B v_B \ , \tag{8}$$

with v_i being the ion fluid or drift velocity. At the bulk boundary region this is typically set to the Bohm velocity, v_B:

$$v_B=(kT_e/m_i)^{1/2} \quad . \tag{9}$$

One should remember that the Bohm velocity is not so much a physical constraint or condition, but a definition of the spatial point where the ions have that velocity. We assume that the ion energy is approximately conserved within this region as the ions are in free fall across the main part of the sheath. This is given by the energy-conservation equation,

$$\begin{aligned} K_i(z) + e\phi(z) &= \tfrac{1}{2} m_i {v_B}^2 \\ &= \tfrac{1}{2} kT_e \; (Bohm) \,, \\ K_i(z) &= \tfrac{1}{2} m_i v_i(z)^2 \quad . \end{aligned} \tag{10}$$

K_i is the ion kinetic energy. We can combine Eqs.(6) -(10) to give:

$$\phi'' = -\frac{e n_B}{\varepsilon_o}\left((1 - \frac{2e}{kT_e}\phi)^{-1/2} - \exp(e\phi / kT_e) \right. \quad . \tag{11}$$

One can do a first integration of this equation, to give an algebraic relation of E_{sh} and ϕ, but not a complete integral. In any case we do not need the first integral in the following analysis.

At the beginning of this section we discussed a reason for expecting that the electric field would be small in the location of the dust particles. This is also the same as expecting that the change in potential from the bulk will not be very large compared to the electron temperature in the region of dust localization. We can make immediate use of this argument for small $e\phi / kT_e$ in finding an approximate solution of the Poisson equation. To do so, we linearize the Poisson equation by expanding in powers of $e\phi / kT_e$ on the right hand side of Eq. (11). With the ion velocity in the bulk set to the Bohm value, the leading term is quadratic in ϕ and the equation may be solved explicitly. This is:

$$\phi'' = -\frac{e}{\lambda_e^2 kT_e}\phi^2 \quad , \tag{12}$$

where the electron Debye radius λ_e is defined by

$$\lambda_e^2 = \frac{\varepsilon_o \, kT_e}{n_B \, e^2} \quad . \tag{13}$$

The solution of Eq.(12) with boundary conditions (b.c.) in the bulk (large z) region set for zero potential and zero electric field is given in terms of one integration constant, z_1:

$$\begin{aligned} \phi(z) &= -\frac{6kT_e}{e}\frac{\lambda_e^2}{(z-z_1)^2} \; , \\ E_{sh}(z) &= -\frac{12kT_e}{e}\frac{\lambda_e^2}{(z-z_1)^3} \quad . \end{aligned} \tag{14}$$

z_1 is the point of singularity of the approximate solution. This point should lie far enough outside the region of interest that it is unimportant. In general we are restricted to $z - z_1 > O(\lambda_e)$. Another consideration is that the particle can only accumulate negative charge where the electron density is non-zero. Thus we do not find particles trapped in the high-field regions of the wall sheath. If we specify potential b.c. at some point, say at $z = 0$, which is taken to be the wall at potential ϕ_w, we can eliminate z_1 in terms of ϕ_w and have the solutions:

$$\begin{aligned} \phi(z) &= \phi_w \frac{1}{(z/z_1 - 1)^2} \; , \\ E_{sh}(z) &= \phi_w \frac{2/z_1}{(z/z_1 - 1)^3} \; , \\ z_1 &= -\lambda_e \sqrt{-6\,kT_e / e\phi_w} \; . \end{aligned} \tag{15}$$

When written in this form, it is important to remember that ϕ_w is applied at $z=0$. The singular point lies beyond the wall, outside the domain of the solution. However one should appreciate that the approximate solution given by Eq.(14) or Eq.(15) is not valid over a sufficient range in distance to reach the actual wall position. Thus we should have an alternative method of fixing the location of the presheath-to-sheath transition region. Until we have a way of connecting the approximate solution given by Eq.(15) to the electrode wall, z_1 is just a free parameter. In other words our approximate solution and its properties are translationally indeterminate.

We note the following character of the field that we have constructed. First of all the potential and electric field both approach zero as we move into the plasma bulk. The electron density becomes constant there, as does the ion density. The ion fluid velocity approaches the Bohm velocity, corresponding to the ions having half the electron thermal energy. This is not really consistent since one needs an electric field to drive the ion drift motion. This electric field should be the ambipolar field set up within the plasma by the small charge separation.

2.2. Ion Momentum Transfer to Dust Particles

The scattering of ions in the screened field about the particle contains the non-spherical correction to the electric field at the position of the particle. Although a treatment by particle scattering seems somewhat inconsistent with the canonical electron distribution and monoenergetic ion distribution assumed elsewhere, it is really just a calculation of the correction to the isotropic assumption contained in the isolated Debye-screened particle field. There is a constant flow of ions and electrons past the suspended dust particles. The near equality of the electron and ion fluxes, the similarity of the interactions (shielded Coulomb), and the large mass ratio says that ions dominate the plasma forces applied to the particle. In this section we obtain an approximation to the rate of momentum transfer to the particle (ion wind force) based on scattering from a Debye interaction.

Consider classical scattering of a directed beam of ions of flux $\Phi_i = n_i v_i$ from a stationary particle. Φ_i is a negative quantity in the geometry we have set up for the sheath. After scattering, the ions are deflected to an angle $\Theta(v_i, b)$, leaving with nearly same energy as before because of the large dust-to-ion mass ratio. The quantity b is the impact parameter for an individual trajectory. Each ion changes the forward momentum of the particle by a small increment

$$\Delta(m_d v_d) = m_i v_i (1 - \cos(\Theta(v_i, b))) \quad . \tag{16}$$

Summing over all impact parameters and multiplying by the ion flux, we get an expression for the force of the ions on the particle:

$$\begin{gathered} f_w = |\Phi_i| m_i v_i \sigma_{mt}(v_i) \\ \sigma_{mt}(v_i) = 2\pi \int_0^\infty b db (1 - \cos(\Theta(v_i, b))) \end{gathered} \quad . \tag{17}$$

σ_{mt} is the momentum transfer cross section. f_w is a function of the ion velocity, which in turn is a function of position within the sheath.

The evaluation of $\sigma_{mt}(v_i)$ is a problem in scattering theory. We will use a numerical fit from the literature for scattering from the shielded potential.[5] The result is:

$$\sigma_{mt}(v_i) \approx B^2 c_1 \ln(1 + c_2 \lambda_e^2 / B^2) \quad ,$$
$$c_1 = 0.9369 \quad ,$$
$$c_2 = 61.32 \quad , \qquad (18)$$
$$B = A / K_i \quad ,$$
$$A = -e q_d / 4\pi\varepsilon_o \quad ,$$

where K_i is the ion kinetic energy. In this representation of the cross section, λ_e could be the combined electron and ion shielding length.[5]

A fully self-consistent study of particle charging and ion momentum transfer by Choi and Kushner[6] predicts cross sections and charging results which are not too different than used here. Choi and Kushner note the decrease of ion wind force with relative ion-particle velocity, but do not speculate on the possibility of oscillation instability.

At this point all that is needed to solve for the trajectory of the dust particle in the sheath is known, or at least approximated. These numerical solutions will be done after the next section.

2.3. Charging Rate of the Particle

Although we can estimate the charge on the particle due to the local plasma environment, it is better to calculate the charge from estimates of the rate of electron and ion impact. We will assume that the collisions with the particle result in unit sticking or accommodation on the particle surface. This is not necessarily true, and corrections can easily be made if more information is known about the process. The charging rate of a particle is written in terms of the incident ion and electron currents to the surface:

$$\dot{q}_d = I_i + I_e . \qquad (19)$$

The electron current is easily evaluated in terms of the Boltzmann assumption about the electron distribution function in the sheath and plasma. The local density at position z is related to the bulk (or Bohm point) density by means of Eq.(6) in order to simplify the result:

$$\begin{aligned} I_e &= -e4\pi a^2 \Phi_{ed} \\ &= -e4\pi a^2 \tfrac{1}{4} n_e(z) v_{Te} \exp\left(e(V_d - \phi(z)/kT_e\right). \\ &= -e\pi a^2 n_B v_{Te} \exp\left(e(V_d - V_B)/kT_e\right) \end{aligned} \tag{20}$$

Φ_{ed} is the electron flux to the surface, v_{Te} is the electron thermal velocity, $v_{Te} = (8kT_e/\pi m_e)^{1/2}$, and the potential at the reference point V_B is defined to be zero. Note that the only dynamic dependence remaining in I_e is in the surface potential. The rise in potential at the dust surface above the local sheath potential is estimated from the capacity relation of a charged sphere:

$$V_d - \phi(z) \approx \frac{1}{4\pi\varepsilon_o} q_d / a \tag{21}$$

One could treat either V_d or q_d as the unknown property of the particle for the purposes of numerical solution.

The ion current is not so easily approximated due to the ion orbiting. There are many studies of ion capture and scattering from small charged bodies because the physical situation is the same as that of plasma probe analysis. Many of these have been discussed in the context of dust particles.[7] We use the microcanonical distribution function for ions ignoring multiple turning points and absorptive corrections.[8] This limiting case of the complex general theory gives:

$$
\begin{aligned}
I_i &= e\pi a^2 n_i(z) v_i(z)\left(1 - \frac{e\Delta V}{K_i(z)}\right.\\
&= e\pi a^2 \Phi_i \left(1 - \frac{e(V_d - \phi(z))}{K_i(z)}\right)
\end{aligned}
\qquad (22)
$$

ΔV is the fall in potential of an ion as it encounters the particle surface at the local position in the sheath. The ion flux, Φ_i, is constant through the sheath. K_i is the kinetic energy of the ions as given in Eq.(10).

If we sum the electron and ion currents to zero, we obtain a value for the steady-state charge and potential carried by the particle.

3. Equilibrium Position and Oscillation of the Particle

Once we have the forces acting on the dust particle, we can solve for the equilibrium position and small displacements about that point. One complication is the variation of the charge on the particle with position and velocity. This is possible, but the variation is estimated to have a smaller effect than the coupling introduced by the variations in the ion wind. We do not explore that effect in this section, but will include it in the numerical solutions to be given later. Rewrite the EOM for the particle, Eq.(2), assuming that the dust charge is constant:

$$
\ddot{z} = -g + Q_d^o E_{sh}(z) - \gamma_{gas}\dot{z} + F_w(v_i(z) - \dot{z}). \qquad (23)
$$

Eq.(23) is now expanded through first order in the particle variables:

$$
\begin{aligned}
\ddot{z} \approx &-g + Q_d^o\left(E_{sh}(z_o) + (z - z_o)E_o'(z_o)\right) - \gamma_{gas}\dot{z}\\
&+ F_w(v_i(z_o)) + \left.\frac{dF_w}{dv_i}\right|_{z=z_o}\left(\left.\frac{dv_i}{dz}\right|_{z=z_o}(z - z_o) - \dot{z}\right.
\end{aligned}
\quad . \qquad (24)
$$

We will find the equilibrium point and oscillation frequency analytically from this fully linearized Eq.(24).

We make use of an order-of-magnitude approximation to the momentum transfer cross section in order to find a simple estimate of the wind effect at the equilibrium position. This is, where A is given in Eq.(18),

$$\sigma_{mt} \sim A^2 / K_i^2 . \tag{25}$$

From this one can evaluate the derivative in the Eq.(24) for γ_w directly. The charge on the particle is given approximately by the capacity combined with an estimate of the floating potential. We use these,

$$\begin{aligned} q_d^o &\approx 4\pi\varepsilon_o a V_f \\ V_f &= -L kT_e / e \end{aligned} , \tag{26}$$

to eliminate the unknown charge. L is a factor of order unity that would account for the deviation of V_f from the electron temperature in eV, which could be found from Eqs.(19)-(22) by setting $\dot{q}_d$ to zero. For a typical plasma sheath, L is about 4.

The equilibrium point is the root, z_o, of the equation found by setting the sum of the first, second, and fifth terms on the RHS of Eq.(24) to zero:

$$g = Q_d^o E_{sh}(z_o) + F_w(v_i(z_o)) . \tag{27}$$

The general solution (root of a quadratic) is not written out here because of its detail. However the solution when the ion wind is negligible compared to gravity is not so difficult. This gives z_o:

$$z_o \approx z_1 + (kT_e / e)\left(36L\varepsilon_o^2 / n_B e g \rho a^2\right)^{1/3}. \quad (28)$$

ρ is the mass density of the particle. We give this relation to show all the direct scaling of the particle location on particle and plasma properties. The scaling in z_1 as defined in Eq.(15) is given in terms of ϕ_w as

$$z_1 = -(kT_e / e)(-6\varepsilon_o / \phi_w n_B e)^{1/2}. \quad (29)$$

Because of the limitations of our solution for the electric field, this relation is *not* generally useful for predicting the absolute position of the particle within the plasma sheath.

The formula for the oscillation frequency is likewise complicated as determined by the third and sixth terms on the RHS of Eq.(24). We again write down only the limiting form that applies when the ion wind is small compared to gravity:

$$\nu_o \approx \frac{\sqrt{3}}{2\pi}\left((a^2 \rho n_B g^4 / L)(e / kT_e)^3 (e / 36\varepsilon_o^2)\right)^{1/6}. \quad (30)$$

As expected, ν_o does not depend on z_1 or ϕ_w.The seventh term on the RHS of Eq.(24) in $\dot{z}$ is a non-conservative force, which damps (when positive) at a rate we call γ_w. The expression for γ_w is:

$$\gamma_w = \left.\frac{dF_w}{dv_i}\right|_{z=z_o} = \frac{m_i}{m_d}\left|\Phi_i\right| \frac{d}{dv_i}\left(v_i \sigma(v_i)\right)\Big|_{z=z_o}. \quad (31)$$

The ion flux Φ_i is constant. The definition is such that a positive γ_w creates damping of the motion. There are two terms in the above derivative. The first is simple and always leaves a positive contribution to γ_w. The second term requires us to find the velocity derivative of $\sigma(v_i)$. Since $\sigma(v_i)$ is a decreasing function of $|v_i|$, it is seen that this term makes a negative contribution to γ_w. This introduces the possibility of driven oscillations – an instability in the motion. We now examine this term using the above approximations to obtain an analytic result. This gives a negative answer indicating that the ion wind always pushes towards instability:

$$\gamma_w = -\frac{m_i}{m_d}|\Phi_i| a^2 \left(\frac{kT_e}{K_i(z_o)} \right)^2 3L^2 \quad . \tag{32}$$

If we set $K_i = kT_e$, which represents a reasonable value of the ion kinetic energy if we are near the Bohm point, we greatly simplify the expression for the wind-damping coefficient. We can take the ratio of Eq.(32) to the gas damping coefficient given in Eq.(4) to develop a condition for absolute instability or growth:

$$\frac{|\gamma_w|}{\gamma_{gas}} \approx \frac{|\Phi_i|}{\Phi_{gas}} \frac{9L^2}{4\pi} > 1 \tag{33}$$

Φ_{gas} is the thermal flux of the background gas. We see that only a highly ionized gas with large particle charge can be unstable to particle oscillation. This seems unlikely.

4. Numerical Solutions for Particle Motion

The above analytic estimates of the damping and instability are not completely general for examining the particle motion. In this section we give numerical solutions of the particle trajectories while trapped within a plasma sheath. Basically we are just solving Eqs.(1) or (2) for the particle trajectory, including the electric field developed in Section 2.1, the wind force developed in Section 2.2, and the additional EOM for the charging rate developed in Section 2.3.

The first reported case of particle self-oscillation[1] or instability estimated the following dust and plasma conditions:

$$
\begin{aligned}
& n_B \approx 0.5e8 / cm^3 \\
& a = 2.5 \mu m \\
& kT_e / e \approx 1.0 \, eV \\
& \rho = 1.0 \; gm / cm^3 \\
& m_i = 40 \, amu \\
& Pgas = 5 \, mTorr
\end{aligned}
$$

We have decreased the plasma density by a factor of two as an estimate of the sheath density at the Bohm point compared to the measured bulk value. The above give the analytical estimates:

$$
\begin{aligned}
& \lambda_e = 1050 \, \mu m \\
& \nu_o = 16.2 Hz \\
& \phi(z_o) = -0.82 V \\
& K_i(z_o) = 1.32 \, eV \\
& q_d / e = 6940
\end{aligned}
$$

The electron Debye length is larger than their reported mean interparticle separation in the crystalline structure, but this may be due to the influence of the ion Debye length to shorten the shielding length in the crystal. More relevant is the oscillation frequency, which is the right order. The potential and ion kinetic energy at the equilibrium point are reasonably larger than the Bohm values at the start of the ion fall through the sheath. A trajectory for a perturbed oscillation with these conditions is shown in Fig. 1. For these circumstances, the wind acceleration is $-0.15\, m/s^2$, small compared to gravity. The wind damping coefficient is likewise small, five orders of magnitude smaller than the gas damping. In Figure 2 we show the variation of the particle charge during the oscillation. The result is converging to a value close to the simple estimate given in the above table.

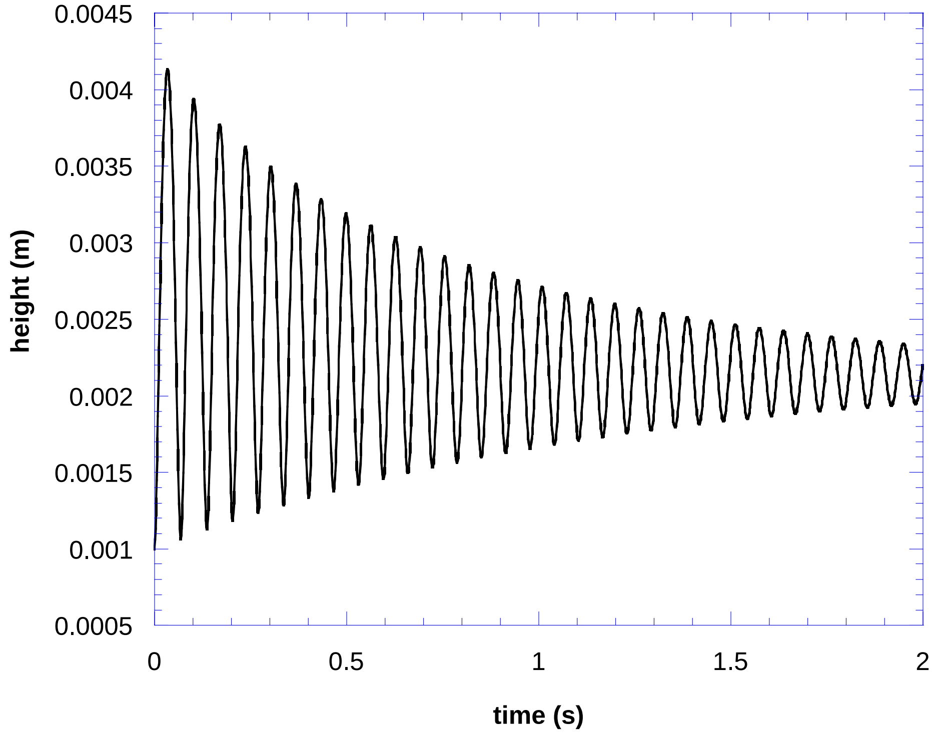

Figure 1. Oscillation of a perturbed particle in conditions similar to those reported in the literature.[1] The damping is due to gas collisions. The frequency is close to the value predicted by Eq. (33), namely 16.2 Hz.

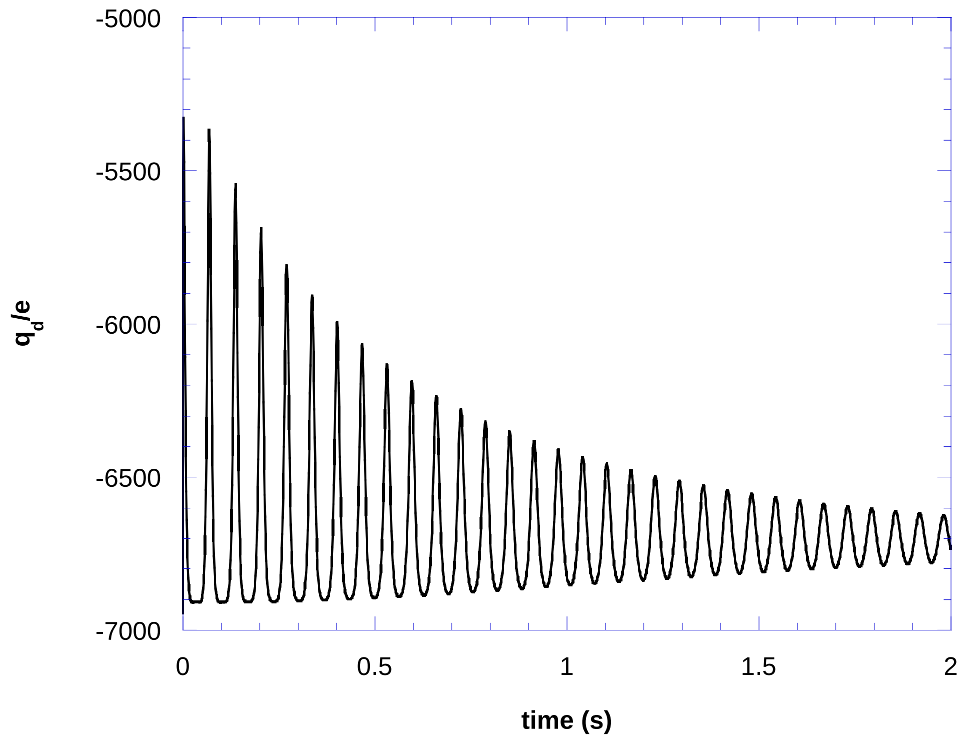

Figure 2. Time dependence of the particle charge in electron units for the trajectory shown in Figure 1. To be noted by comparison with Figure 1 is that the variation is proportional to the position and not the velocity; the latter would be required for this variation to create an instability in the motion.

In other experiments,[9] we have the estimates of the relevant particle and plasma conditions with the density again reduced by a factor of two:

$$
\begin{aligned}
&n_B \approx 1.0e10/cm^3 \\
&a = 7.5\mu m \\
&kT_e/e \approx 2.0eV \\
&\rho = 1.0\ gm/cm^3 \\
&m_i = 40\,amu \\
&Pgas = 60mTorr
\end{aligned}
$$

The above gave estimates for the dust properties:

$$
\begin{aligned}
&\lambda_e = 105\,\mu m \\
&\nu_o = 104Hz \\
&\phi(z_o) = -0.60V \\
&K_i(z_o) = 1.6eV \\
&q_d = -20800
\end{aligned}
$$

with a wind acceleration of $-46m/s^2$, and γ_w three orders of magnitude smaller than the gas damping. The oscillation is shown in Figure 3. We can modify the conditions to make the frequency more resonant with 60 Hz line frequency, which is observed in the experiments. For example, reducing the plasma density by a factor of two gives the trajectory shown in Figure 4. The wind acceleration is $-26m/s^2$ in this case and the γ_w/γ_{gas} ratio is order 10^{-4}. However these experiments were conducted with a capacitively-coupled rf drive, which we suspect does not resemble the theoretical model for the electric field developed here.

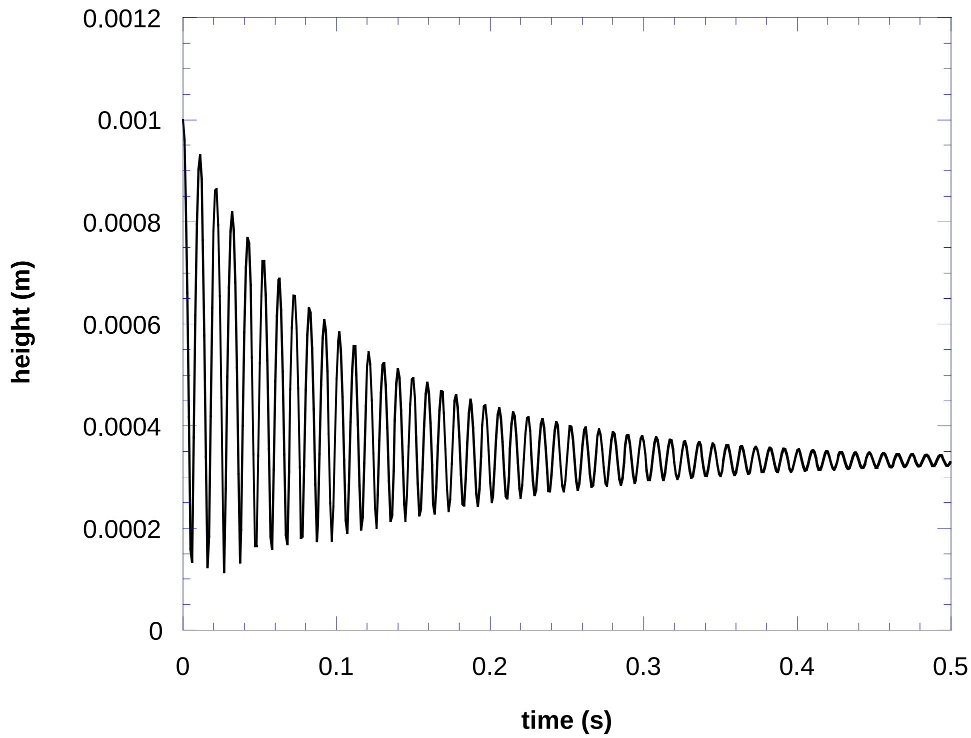

Figure 3. Oscillation of a perturbed particle in conditions of an experiment.[9] The damping is due to gas collisions. The frequency is too large compared to the observed near-resonance with 60 Hz line frequency. The vertical displacement is not predictive.

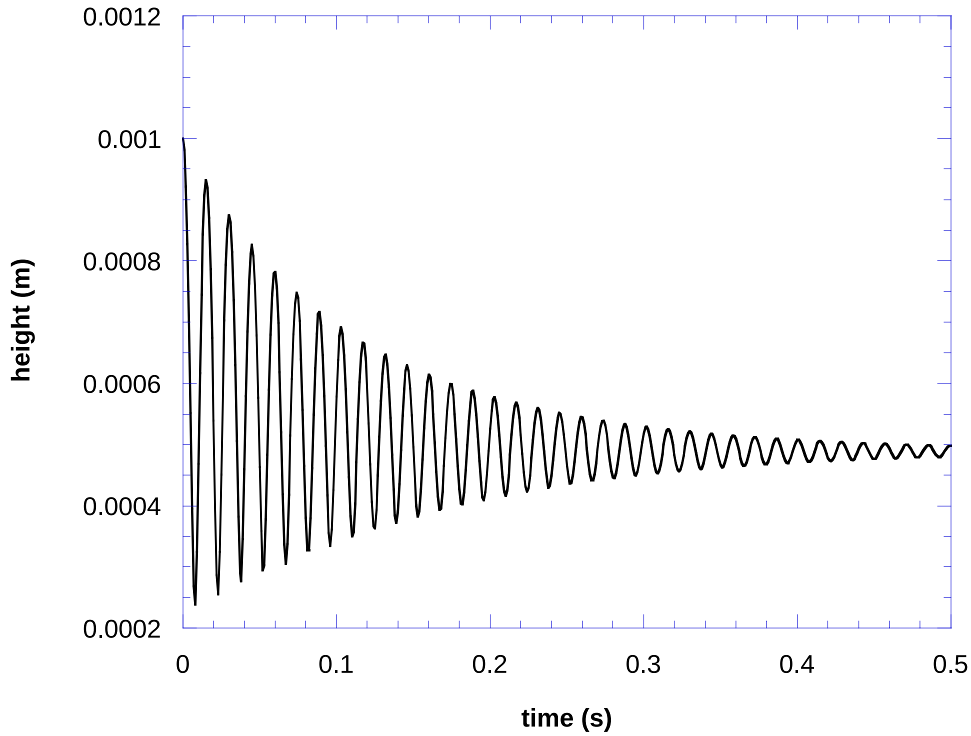

Figure 4. Oscillation of a perturbed particle under conditions shown in Fig. 4 except that the plasma density is lowered by a factor of two. The damping is due to gas collisions. The frequency is closer to the observed near-resonance with 60 Hz line frequency. The vertical displacement is not predictive.

5. Conclusions

It is apparent that the model developed here does not predict spontaneous oscillations, or instability, of the dust particles. We do make fairly good predictions of the natural oscillation frequency and gas damping of the motion under circumstances where the plasma properties are reasonably well known and potentially in harmony with the model assumptions.

Acknowledgment

I thank Rick Buss, Greg Hebner, Paul Miller, and John Torczynski for helpful discussions.

This work was supported by the U. S. Department of Energy under Contract DE-AC04-94AL85000 at Sandia National Labs. Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy.

Appendix

An alternative solution for the electric field in the sheath-bulk region can determined as follows. Instead of assuming that the ion density in the Poisson equation is controlled by the flux (Eq.(8)) and energy conservation (Eq.(10)), we assume that the ion density is constant within this region. This is not a baseless assumption. The region of the Bohm point may in fact be collisional, in which case the conservation of ion energy equation will not be valid. A collisional, canonical, ion distribution would predict a density *increase* where the potential begins to drop in the sheath. The free-fall limit predicts a density *decrease* in the same case. The intermediate constant density assumption leads to a linearized Poisson equation:

$$\phi'' = -\frac{e n_B}{\varepsilon_o}\left(1 - \exp(e\phi / kT_e)\right) \approx \frac{1}{\lambda_e^2}\phi \quad . \tag{1}$$

This has the immediate solutions defined in terms of the wall potential b.c.:

$$\begin{aligned} \phi(z) &= \phi_w \exp(-z/\lambda_e) \\ E_{sh}(z) &= \phi_w / \lambda_e \exp(-z/\lambda_e) \end{aligned}, \tag{2}$$

with the integration constants chosen such that the large-z limits are vanishing in the plasma bulk. The particle equilibrium point (*without* the wind term) and oscillation frequency are just:

$$\begin{aligned} z_o &= \lambda_e \log(q_d^o \phi_w / g m_d \lambda_e) \\ \nu_o &= \frac{1}{2\pi}\sqrt{g/\lambda_e} \end{aligned} \tag{3}$$

The EOM for the particle motion require much the same analysis as in the text. However the simpler nature of the above formula (compare Eq.(30)) suggests that it may be of use in analyzing the oscillation frequency. Of course the absolute position of the particle is not necessarily determined accurately with the above. For the two experimental cases discussed in the text, we get 15 and 49 Hz, not too far from the observed oscillation frequencies.

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