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A Theoretical Description of Inhomogeneous Turbulence

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Abstract

This is the final report of a three-year, Laboratory-Directed Research and Development (LDRD) project at the Los Alamos National Laboratory (LANL). In this LDRD, we have developed a highly compact and descriptive formalism that allows us to broach the theoretically formidable morass of inhomogeneous turbulence. Our formalism has two novel aspects: (a) an adaptation of helicity basis functions to represent an arbitrary incompressible channel flow and (b) the invocation of a hypothesis of random phase. A result of this compact formalism is that the mathematical description of inhomogeneous turbulence looks much like that of homogeneous turbulence -- at the moment, the most rigorously explored terrain in turbulence research. As a result, we can explore the effect of boundaries on such important quantities as the gradients of mean flow, mean pressure, triple-velocity correlations and pressure velocity correlations, all of which vanish under the conventional, but artificial, assumption that the turbulence is statistically spatially uniform. Under suitable conditions, we have predicted that a mean flow gradient can develop even when none is initially present.

Background and Research Objectives

Turbulence is pervasive on all scales in the natural world from the tempest in a pot of tea, to storms on Jupiter, to cataclysmic events in galaxies. Feynman termed fluid turbulence the last and greatest problem of classical physics. Nevertheless, since the close of the 19th century, the elucidation of turbulence has remained among the most challenging of physics problems because, for the most part, it has been relegated to stagnation with only phenomenological patching. Instead, physicists turned their attention to the new and exciting physics of the very small and the very large, setting aside the old and severe difficulties of classical mesoscopic matter. Most of the existing theoretical research on fluid turbulence starts out with the simplifying assumptions of statistical homogeneity and isotropy, and even of spatial periodicity, although they provide only a naïve semblance of physical reality. The justification for this emphasis has been falling back on the old lamppost saw: "In search of a solution, one might as well start looking where there is some light, even if the relevant

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solution lies elsewhere." Homogeneity has lit up the theoretical realm where simple diagonal Fourier representations hold sway.

A whole collection of calculational techniques, each with its own strengths and weaknesses have been developed, principally by Kraichnan, but also by many others such as Edwards[1] and Orszag[2]. Promising initial steps were first presented by Kraichnan who formulated the Direct Interaction Approximation (DIA) for bounded buoyancy-driven turbulence[3] and again by Kraichnan who formulated the Test-Field Model (TFM) for arbitrary orthogonal basis vectors in somewhat more general geometries[4]. The consequent complexities of the Test-Field Model led Kraichnan[5] to a truncation of the expression for the evolution of the eddy-damping (time-scale), thus leading to a final, simplified model that resembled Orszag's eddy-damped quasinormal Markovian (EDQNM) model[2].

Our own interest in turbulence was triggered by the intriguing configuration of a mean reversed-toroidal magnetic field that was turbulently generated in a pinch fusion device. The reversed nature persisted on time scales much greater than what would be expected on the basis of classical resistive diffusion. Analogous persistent magnetic field structures occur also in astrophysical environments. The explanation for such persistent mean field structures requires the incorporation of the effect of a geometry's boundedness, certainly a hallmark of any earth-based fusion device. A statistical theory can account for many of the mean properties of an ideal "reversed-field pinch." But in any purely statistical treatment of an ideal continuous medium, lurks the ultraviolet catastrophe of Rayleigh and Jeans; resistive and viscous dissipation are vital to avoiding this pathology. Aside from two seminal papers of Robert Kraichnan[5,6] providing formalism that does not lead to tractable numerical computations with even contemporary computers, there has been virtually no theoretical attempt to treat inhomogeneous turbulence from the bottom up.

Importance to LANL's Science and Technology Base and National R&D Need

Historically, theoretical effort to understand fluid turbulence has been focused on regimes of greatest aesthetic and mathematical simplicity, regimes having a high symmetry; such as the regime of statistically homogeneous and isotropic turbulence. The advantage of such investigations is, of course, the utility of Fourier analysis. The disadvantage is their obvious lack of physical realism; i.e., their inapplicability to inhomogeneous turbulence whose mean parameters have spatial gradients, the hallmark of all relevant problems. These problems extend from drag forces on transportation vehicles, combustion in automotive engines, and flow through turbines, to meteorological and astrophysical phenomena, eruptions of volcanoes and forest fires, and weapons physics. All of these examples are germane to LANL and relevant to a science and technology base able to provide solutions to critical national problems: such as environmental pollution, natural disasters, climate, energy efficiency, weapons and weapons monitoring, biological and chemical warfare.

With the Laboratory's current research mandate regarding *science-based* stockpile stewardship and the concomitant concern to maintain technical leadership in the associated areas of science, the results of our inhomogeneous research provides a tool with much potential to solve and understand heretofore intractable problems.

The applications of this turbulence research to magnetohydrodynamics has direct relevance to the reversed-field pinch and spheromaks, ingredients of this nation's innovative confinement concepts in magnetic fusion energy that receives theoretical support from Los Alamos.

Scientific Approach and Accomplishments

During the past 3 years, we developed a treatment of bounded turbulence[1]. We utilized a theoretical closure that has been used extensively by others to describe homogeneous fluid turbulence to obtain a quantitative first principles' treatment of a viscous channel flow bounded by two parallel, free-slip planar boundaries. We implemented this closure in the context of a helicity-based representation of a channel flow of an incompressible fluid.

The basis used for representation of the bounded incompressible flow is a simple linear combination of the clockwise and counter-clockwise polarization vectors well known in optics for the representation of a (homogeneous) beam of light. These polarization vectors have been generalized in a seminal paper of Chandrasekhar and Kendall to a spherical geometry[7]. With Montgomery, Vahala, we used the Chandrasekhar-Kendall technique to analyze the statistics of the magnetohydrodynamic turbulence in a cylinder[8]. We introduced these mathematical techniques to the statistical treatment of inhomogeneous turbulence employing the well-known closures of DIA, TFM, and EDQNM[3,4]. (In the realm of homogeneous turbulence, we had already demonstrated that EDQNM could specify the evolution of an arbitrary, anisotropic, and mirror-symmetry-violating turbulence with only four coupled equations involving four scalar functions[2]. Heretofore, the only literature on the subject utilized 9 equations with 9 functions[9]. Clearly there were 5 linear dependencies that had not been removed!)

We next introduced a hypothesis of random phase[3] in order to obtain the same simplification provided by the naïve, but often-used, assumption of spatial uniformity of the statistics of the turbulence. The non-linearity of the equations motivates the idea that the phases of different solenoidal components will be random. If no approximation like this were to be valid, there would be scant hope of developing a theory of spatially non-uniform turbulence based on our existing concepts. By means of a hypothesis of random-phase, we demonstrated calculations of the evolution of macroscopic mean fluid structures of inhomogeneous turbulence, pressure gradients, for example, and quantities that either had been previously assumed to be negligible (e.g., gradient of the pressure-velocity correlation) or had been previously modeled *ad hoc*, such as the gradient of the triple-velocity correlation, which we find to be negligible (see Figs. 1 and 2)[5]. Another significant new result is the prediction that mean flow gradients can emerge out of the interaction of turbulent eddies even when no such gradients are initially present[5]. In direct numerical simulations of a Navier-Stokes fluid in a free-slip channel, we have demonstrated the excellence of the random-phase hypothesis for fully developed turbulence in the absence of mean flows (see Fig. 3)[6]. This work received further clarification through reference 7.

We have started to extend this analysis to the two-fluid-like case of MHD turbulence. In this case, more demands are placed on the closure than are placed on the closure for ordinary fluid turbulence. The turbulent fluid energy spectrum must remain positive at all times; the turbulent magnetic energy spectrum must remain positive at all times; and the cross-helicity spectrum (whose integral yields the spatial integral of the scalar product of the velocity field with the magnetic field) critical to the turbulent MHD dynamo must satisfy a Schwarz inequality. Thus, one must show that the three evolution equations of the three spectra obtained from the closure yield evolving spectra that always satisfy these three "realizability" conditions. In the absence of mean fields, we have done so using the Elsasser field variables, $\mathbf{v}\pm\mathbf{B}$.

This research has been published in Physical Review Letters, Physics of Fluids, Physical Review E, and has been accepted for publishing in the Journal of Fluid Mechanics with supporting documentation to be archived in their Editorial Office. This research has formed the basis for numerous invited talks over the past several years. Most recently, this research will be a principal focus of an international turbulence workshop to take place in Lyon, France in May of 2000.

Publications

1. Turner, L., "Using Helicity to Characterize Homogeneous and Inhomogeneous Turbulent Dynamics," accepted for publication in J. Fluid Mech.

2. Turner, L., "Helicity Decomposition of Evolution of Incompressible Turbulence. I. Homogeneous, Anisotropic, Helical Case," archived in Editorial Office of *J. Fluid Mech.*, LA-UR 96-618.

3. Turner, L., "Helicity Decomposition of Evolution of Incompressible Turbulence. II. Inhomogeneous Case - Free-Slip Channel," archived in Editorial Office of *J. Fluid Mech.,* LA-UR 96-3257.

4. Turner, L., "Helicity Decomposition of Evolution of Incompressible Turbulence. III. Direct-Interaction Approximation and Test-Field Models of Two-and Three-dimensional Homogeneous and Free-Slip Channel Cases," archived in Editorial Office of *J. Fluid Mech.,* LA-UR 97-339.

5. Turner, L., "Macroscopic Structures of Inhomogeneous, Navier-Stokes Turbulence," Phys. Fluids **11**, 2367-2380 (1999).

6. Ulitsky, M., Clark, T., and Turner, L., "Testing a Random Phase Approximation for Bounded Turbulent Flow," Physical Review E **59**, 5511-5522 (1999).

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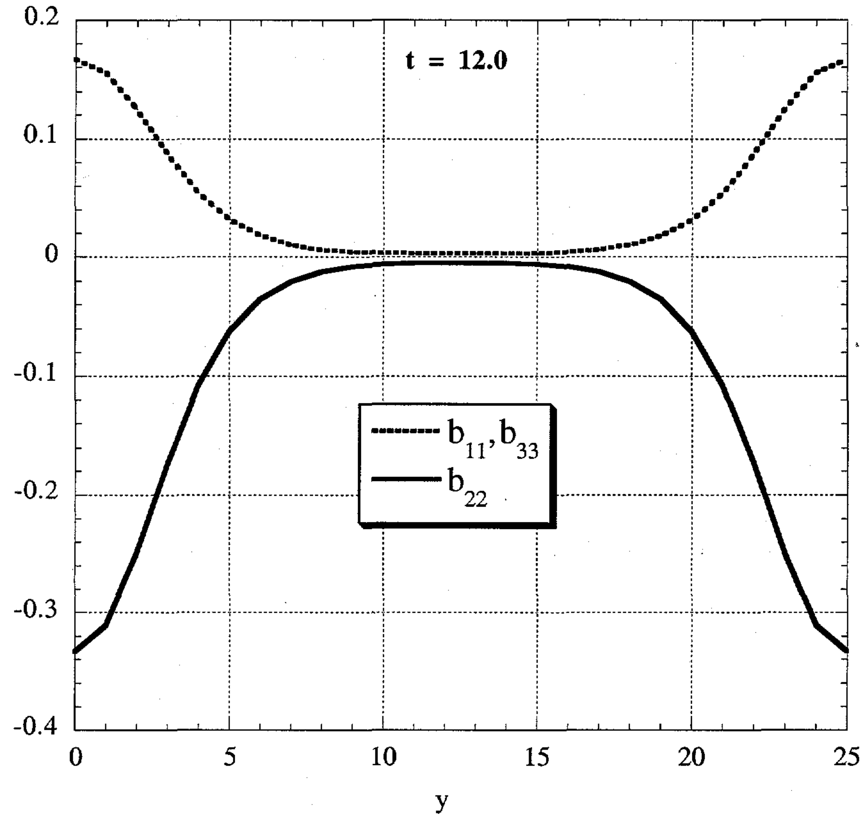
Figures

Figure 1 Example of non-vanishing elements of the normalized anisotropy tensor, b_{ij}, as a function of y. The free-slip channel boundaries are located at $y=0$ and at $y=25$.

Figure 2 Example of the rate of change of mean turbulent kinetic energy and its contributory quantities: the gradient of the mean pressure-velocity correlation, the mean viscous damping term, and the gradient of the mean triple velocity correlation.

Figure 3 Comparison of two probability density functions of the correlation between two spectral coefficients: one calculation based on random-phase, the other based on an ensemble of direct numerical simulations of the Navier-Stokes equation of a freely decaying turbulence. The slight discrepancy is due to the presence of a finite viscosity.



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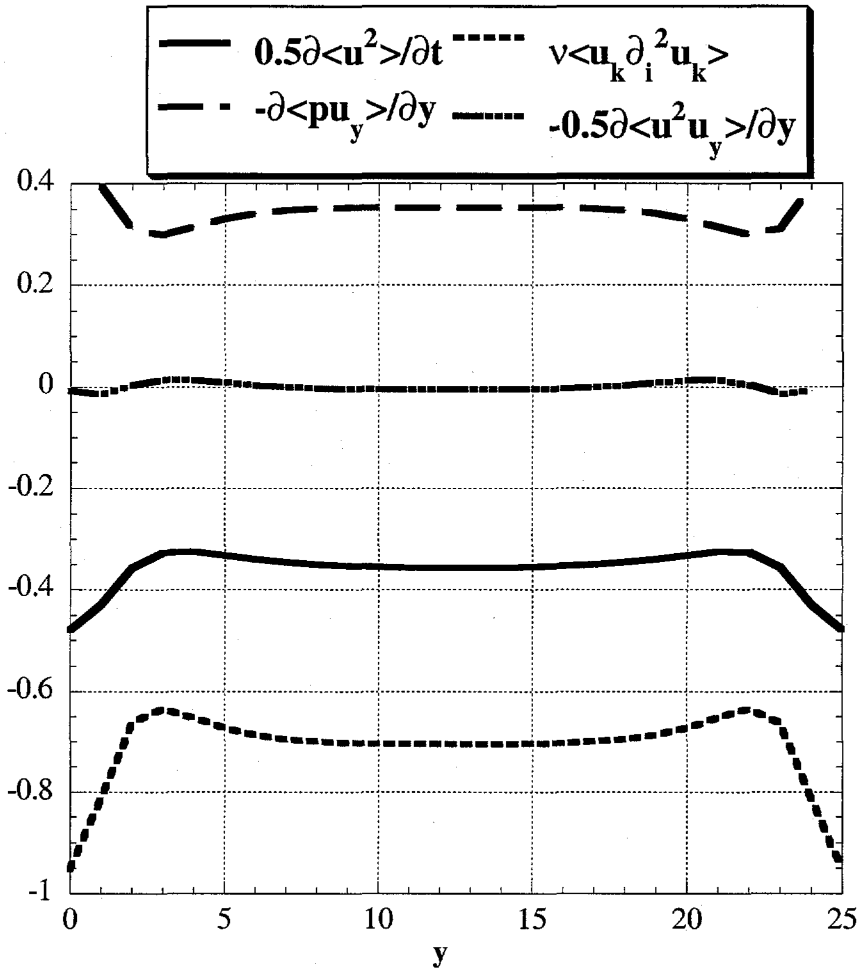

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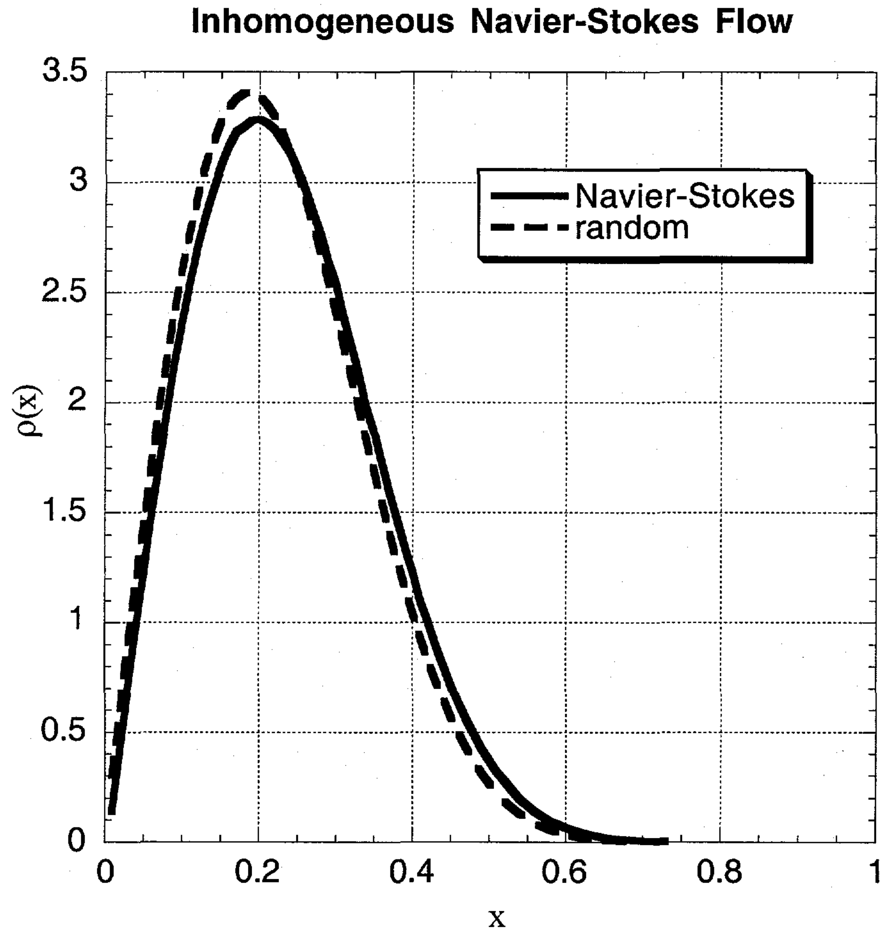

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