Design a High-Q Optical Cavity for the Project of Laser Notching H⁻ Beam at 38.5 MHz

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Abstract

Ray matrix formalism is used to represent a two-mirror resonator with a thermal lens in the middle. By tracking a ray vector, which starts from the place where the laser and H⁻ beams intercept, through the optical cavity, the cavity property can be analyzed. The cavity design can be optimized in such a way that at the interception, the spacious jitter of the laser beam caused by the cavity misalignment is the minimum.

Introduction

Since in the project of laser notching H⁻ beam at the Booster injection RF frequency (38.5 MHz), the number of photons used to create an H⁻ beam notch is negligible, the rest of photons in the pulse can be recycled *via* a laser cavity and reused to create as many H⁻ beam notches as possible to cover the entire injection time, usually taking 12 Booster turns (1008 RF periods). At the same time, the energy of the laser pulse must be kept above Ep_{min}=7.25 mJ for the purpose of neutralizing 90% of the H⁻ beam in a double-cross configuration.[1] This can be achieved *via* a high-Q optical cavity in conjunction with a diode-pump system.

Since the Q-factor of an optical cavity is inversely proportional to the round-trip energy loss (δc), any decrease in δc can improve the cavity Q.[2] There are three major causes for δc:1st, power reflection values of two cavity mirrors, R_1 and R_2, are less than 1; 2nd, power transmission values of all the interfaces, T_1, T_2, …., and T_N, inside the cavity, including those from Q-switch, Brewster plate, and the Nd:YAG rod, are less than

1; 3rd, and aperture losses caused by the misalignment of cavity elements, the non-optimized cavity design, or the combination of both. The losses caused by the 1st factor can be minimized by using cavity mirrors with high reflection coatings. As an example, Newport corporation offers Nd:YAG mirrors with the highest power reflection value, which is greater than 99.7%.[3] Similarly, the losses brought by the 2nd factor can be minimized by using optical elements with surface coatings, which have the highest transmission value of 99.9%.[3] Finally, it is important for us to optimize the optical cavity design and keep the aperture loss to a minimum.

Optical Cavity Design

Since the laser and H[-] beams intercept at a place inside the cavity, their intercepting rate (38.5 MHz) determines the optical cavity length of 3.9 m.[1] Also, better the overlap between the laser and H[-] beams at the interception is, higher the neutralizing efficiency of the H[-] beam will be. This requires at the intercepting place:1st, there is a transverse size match between the laser and H[-] beams; 2nd, and compared to the matched beam size of 1 mm,[1] transverse motions of the laser beam are negligible till 1008 round-trips.

All the cavity elements, including drift spaces, are represented by their *ABCD* matrices.[2] Since both Q-switch and Brewster plate are optical mediums with flat surfaces, they can be treated as part of cavity drift spaces. The diode pump causes a thermal lens effect on the Nd:YAG rod, so the Nd:YAG rod is described by a lens with the thermal focal length of *f*. Since without the diode pump, the two surfaces of the Nd:YAG rod are flat, f^{-1} is equal to zero. The schematic of the optical cavity is shown in Fig.1. The left-end cavity mirror has a curvature of r_1, and is represented by matrix *X*1, as shown by eq.1.

$$X1 = \begin{pmatrix} 1 & 0 \\ -2/r_1 & 1 \end{pmatrix}.$$

1

The drift space on the left side of the thermal lens is represented by matrix *X*01, as shown by eq.2.

$$X01 = \begin{pmatrix} 1 & L_1 \\ 0 & 1 \end{pmatrix}.$$

2

The Nd:YAG rod is represented by a thermal lens matrix *F*, as shown by eq.3.

2

$$F = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}.$$

3

The right-end cavity mirror has a curvature of r_2, and is represented by matrix $X2$, as shown by eq.4.

$$X2 = \begin{pmatrix} 1 & 0 \\ -\frac{2}{r_2} & 1 \end{pmatrix}.$$

4

The drift space on the right side of the thermal lens is represented by matrix $X02$, as shown by eq.5.

$$X02 = \begin{pmatrix} 1 & L_2 \\ 0 & 1 \end{pmatrix}.$$

5

Here, $L_2 = L - L_1$, and L is the cavity length (3.9 m). The round-trip matrix M, as shown by eq.6, starts from the place where the laser and H⁻ beams intercept.

$$M = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} = X2 \cdot X02 \cdot F \cdot X01 \cdot X1 \cdot X01 \cdot F \cdot X02.$$

6

The nth round-trip matrix is $M(n)$, as shown by eq.7.

$$M(n) = \begin{pmatrix} A_n & B_n \\ C_n & D_n \end{pmatrix} = M^n.$$

7

We choose the right-end cavity mirror to be the one with the misalignment in the x direction, and it is described by an error vector E, as shown by eq.8.

$$E = \begin{pmatrix} \Delta \\ \Delta' \end{pmatrix}.$$

8

Here, Δ is the position error, and Δ' is the angle error. A ray vector in the x dimension, as measured with respect to the optical axis, right before the 1$^{\text{st}}$ round-trip, in front of the right-end cavity mirror, is denoted by r, as shown by eq.9, x is the position, and x' is the angle.

$$r = \begin{pmatrix} x \\ x' \end{pmatrix}.$$

9

After the ray r propagates through n round-trips in the cavity, it is represented by r_n, as shown by eq.10. The error vector E is kept to the 1$^{\text{st}}$ order.

$$r_n = \begin{pmatrix} x_n \\ x_n' \end{pmatrix} = M^n \cdot r + \sum_{k=1}^{n} M^k \cdot E. \qquad\qquad 10$$

We have all the information and tools needed for designing a stable high-Q optical cavity with the minimum aperture loss. Since x_n and x_n' are obtained using eq.11 and eq.12 respectively,

$$x_n = A_n \cdot x + B_n \cdot x' + \left(\sum_{k=1}^{n} A_k\right) \cdot \Delta + \left(\sum_{k=1}^{n} B_k\right) \cdot \Delta' \qquad\qquad 11$$

$$x_n' = C_n \cdot x + D_n \cdot x' + \left(\sum_{k=1}^{n} C_k\right) \cdot \Delta + \left(\sum_{k=1}^{n} D_k\right) \cdot \Delta' \qquad\qquad 12$$

any divergence in A_n, B_n, C_n, and D_n can cause the ray r to run into the cavity aperture after a certain number of round-trips, as an example, n; or make the error vector E to be magnified until reaching the aperture limit. Since the cavity length is fixed at 3.9 m, the only parameters, which can be varied in the cavity design, are cavity mirror curvatures r_1 and r_2, thermal focal length f, and the drift space length L_1 on the left side of the cavity, as shown in Fig.1.

A_n, B_n, C_n, and D_n can vary either as a *sin* function or as an exponentially growing function. The period and amplitude in the 1st case or the exponential growth rate in the 2nd case are determined by parameters r_1, r_2, f, and L_1. In the choices of r_1, r_2, f, and L_1, one should avoid the exponentially growing situation in order to make an optical cavity stable. In the situation when A_n, B_n, C_n, and D_n are having a *sin*-type variation, additional growths in the position and angle of r, which are caused by the cascade of the error E, are periodically canceled due to the *sin* function property, as shown in eqs.11 and 12. Furthermore, the tolerance to the misalignment of a cavity element can be greatly improved *via* choosing r_1, r_2, f, and L_1 to make the maximum amplitude of A_n, B_n, C_n, and D_n small compared to the cavity aperture.

In the situation that r_1, r_2, f^{-1}, and L_1 are chosen to be 1.0 m, 10.0 m, 1.0602425 m^{-1}, and 1.95 m,[1] A_n, B_n, C_n, and D_n vs. the round-trip number n are shown as the black, red, green, and blue curves in Fig.2(a) respectively. The period of A_n, B_n, C_n, and D_n is 27 round-trips, and their amplitudes are close to 1.0. When the ray vector r and the error vector E are chosen to be $[0.0\ 0.0]^T$ and $[10^{-4}\ 0.0]^T$, position (x_n) and angle (x_n') vs. round-trip number n are shown as the black and blue curves in Fig.2(b) respectively. The

maximum value of x_n is about 0.18 mm, and the period of x_n and x_n' is the same with that of A_n, B_n, C_n, and D_n. Once the aperture of the optical cavity is chosen to be much larger than the maximum of x_n, the cavity condition can be treated to be optimized at the present configuration. Afterwards, we keep r_1, r_2, and L_1 the same with before (1.0 m, 10.0 m, and 1.95 m), and change f^{-1} to half of the value before, 0.53012125 m^{-1}. A_n, B_n, C_n, and D_n vs. the round-trip number n are shown by the black, red, green, and blue curves in Fig.2(c). The amplitude of B_n increases to 7.0, and their period becomes 3 round-trips. Furthermore, the ray vector r and the error vector E are kept to be the same as before, r_n vs. n is shown in Fig.2(d). The maximum value of x_n increases to 0.7 mm, which is worst than the situation in Fig.2(b), since smaller the maximum value of x_n is, better the cavity condition is. It is clear that after r_1, r_2, and L_1 have been fixed to 1.0 m, 10.0 m, and 1.95 m, the maximum value of x_n is extremely sensitive to f^{-1}, and it can be minimized via experimentally adjusting the diode-pump level to make f^{-1} close to 1.0602425 m^{-1}. The diode-pump system can not only provide the optical gain to compensate the round-trip energy loss, but also keep the cavity stable via the adjustable thermal lens effect.

In the case that r_1, r_2, f^{-1}, and L_1 are chosen to be 3.9 m, 3.9 m, 0.0 m^{-1}, and 1.95 m, An, B_n, C_n, and D_n vs. the round-trip number n are shown as the black, red, green, and blue curves in Fig.3(a) respectively. The period of A_n, B_n, C_n, and D_n is 2 round-trips, the amplitude of B_n and C_n is 0.0, and the amplitude of A_n and D_n is 1.0, which satisfy the self-consistent condition of the optical resonator.[2] When the ray vector r and the error vector E are chosen to be the same with before, position (x_n) and angle (x_n') vs. round-trip number n are shown as the black and blue curves in Fig.3(b) respectively, and they are much smaller compared with those in Figs.2(b) and 2(d). When there isn't a need of the diode pump for compensating the round-trip energy loss, the present choice of parameters r_1, r_2, and L_1 is optimal.

Conclusions

The major goal of this work is to find the optimal parameters for cavity elements to reduce the potential aperture-limited loss. Since the transfer matrix for each individual ray in a laser beam is the same, a single ray tracking is sufficient for determining the cavity design, which gives the minimum beam envelop at the interception.

Also, in the situation that the cavity configuration has been fixed, the diode-pump power can be adjusted in the experiment for the purpose of varying the thermal focal length to keep the cavity stable.

References:

[1] X. Yang and C. M. Ankenbrandt, "Design Report of the Laser System for Notching H⁻ Beam at Booster Injection RF Frequency in the 750-keV Linac", FERMILAB-FN-0765-AD.

[2] A. E. Siegman, *Lasers*, 1986, University Science Books, Sausalito, California.

[3] http://www.newport.com.
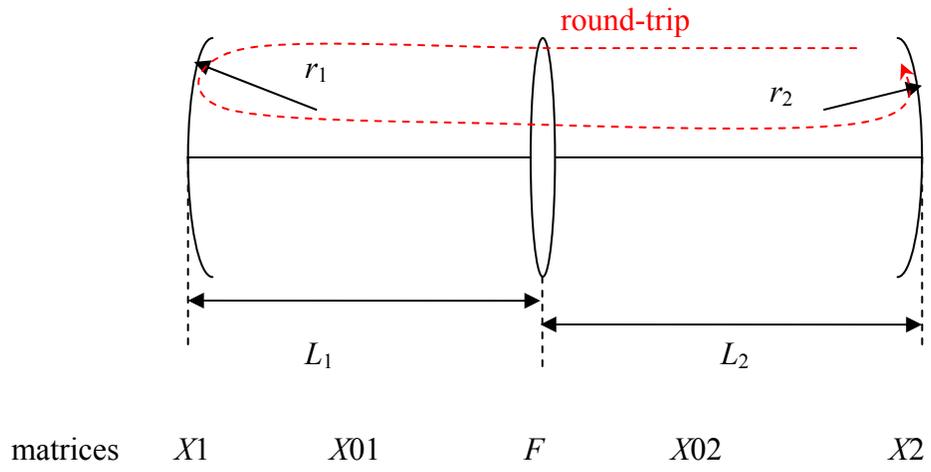
Fig. 1

Fig. 1 schematic of the optical cavity.
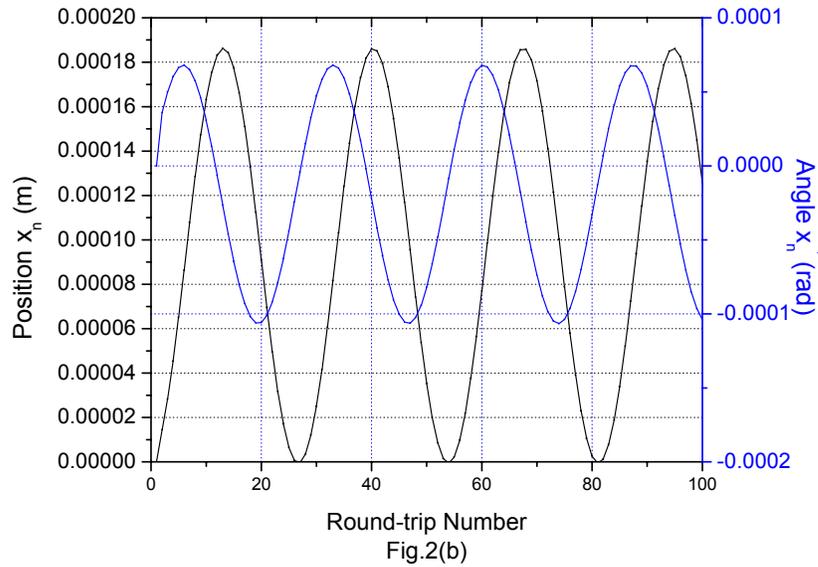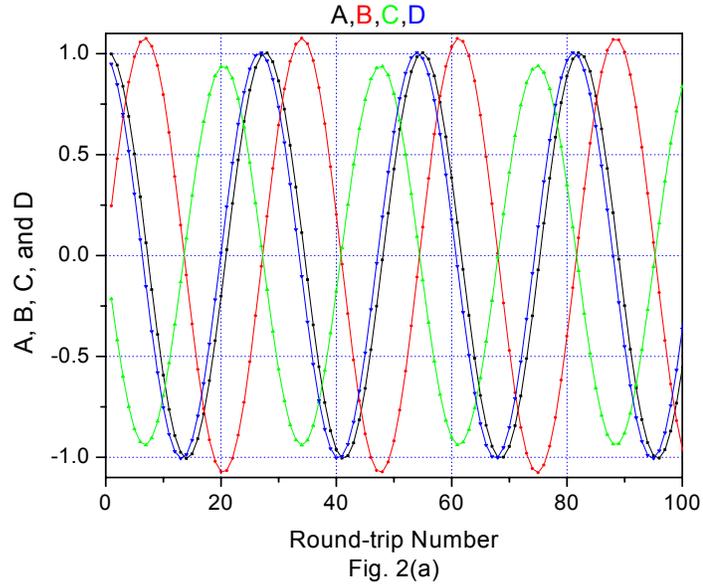
Fig. 2(a)

Fig.2(b)

Fig. 2(a) in the situation that r_1, r_2, f^{-1}, and L_1 are chosen to be 1.0 m, 10.0 m, 1.0602425 m^{-1}, and 1.95 m, A_n, B_n, C_n, and D_n vs. the round-trip number n are shown as the black, red, green, and blue curves respectively.

Fig. 2(b) when the ray vector r and the error vector E are $[0.0\ 0.0]^T$ and $[10^{-4}\ 0.0]^T$, position x_n and angle x_n' vs. the round-trip number n are shown as the black and blue curves respectively.
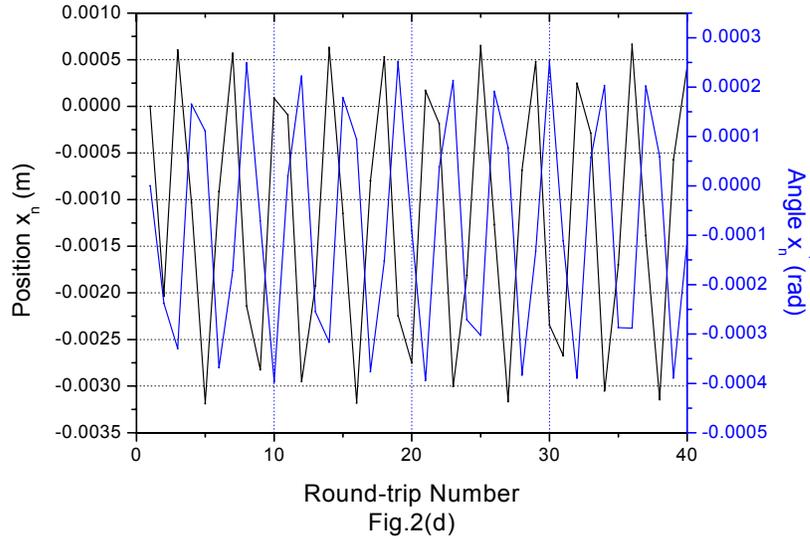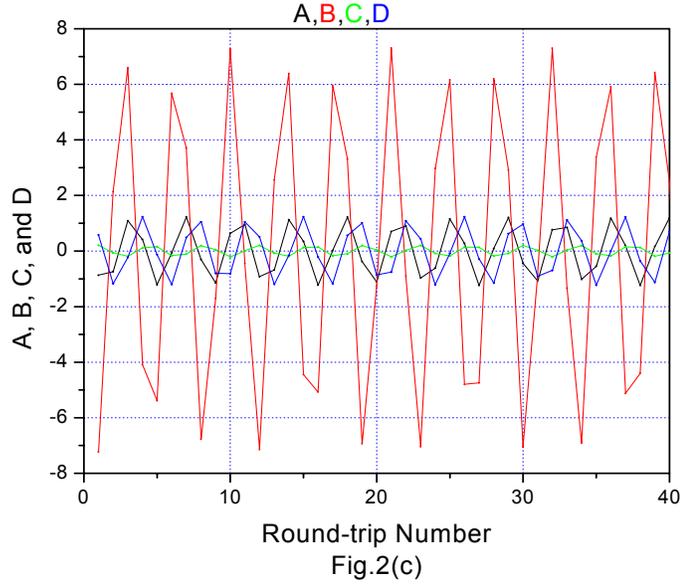
Fig.2(c)

Fig.2(d)

Fig. 2(c) in the situation that r_1, r_2, f^{-1}, and L_1 are chosen to be 1.0 m, 10.0 m, 0.53012125 m^{-1}, and 1.95 m, A_n, B_n, C_n, and D_n vs. the round-trip number n are shown as the black, red, green, and blue curves respectively.

Fig. 2(d) the ray vector r and the error vector E are the same with those in Fig.2(b), position x_n and angle x_n' vs. the round-trip number n are shown as the black and blue curves respectively.
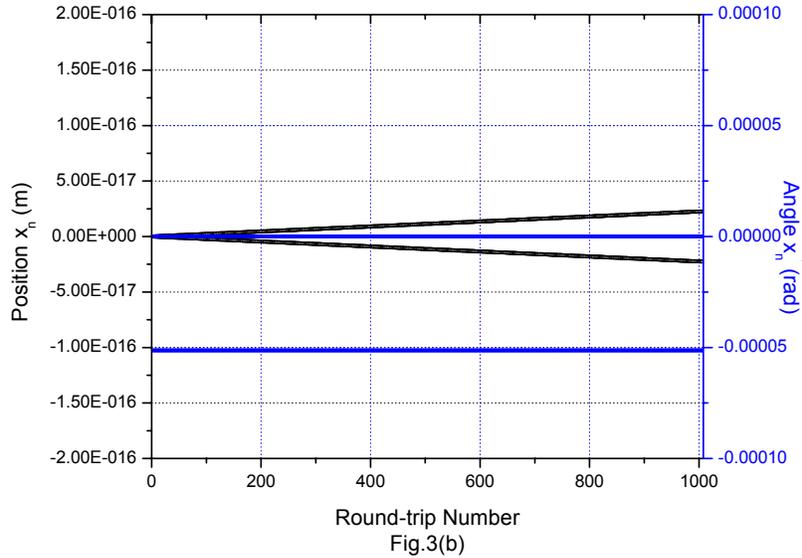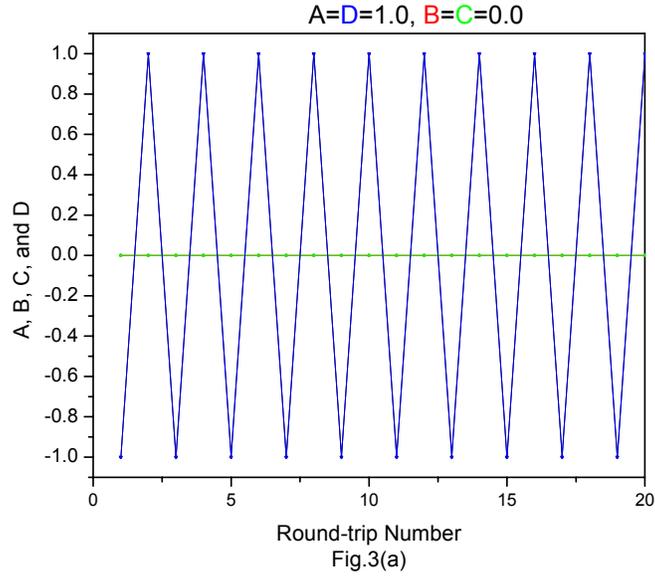
Fig.3(a)

Fig.3(b)

Fig. 3(a) when r_1, r_2, f^{-1}, and L_1 are chosen to be 3.9 m, 3.9 m, 0.0 m^{-1}, and 1.95 m, An, B_n, C_n, and D_n vs. the round-trip number n are shown as the black, red, green, and blue curves respectively.

Fig. 3(b) when the ray vector r and the error vector E are chosen to be the same with those in Fig.2(b), position (x_n) and angle (x_n') vs. round-trip number n are shown as the black and blue curves.