PFP Generalized Geometry Holdup Calculations and Total Measurement Uncertainty

Prepared for the U.S. Department of Energy
Assistant Secretary for Environmental Management

Project Hanford Management Contractor for the
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Fluor Hanford

P.O. Box 1000
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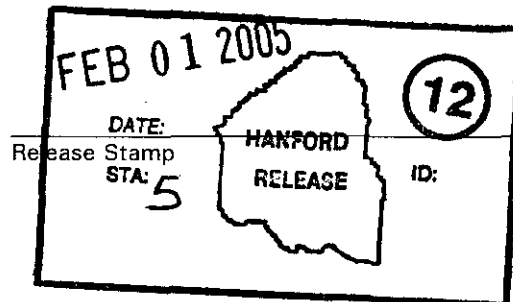Assistant Secretary for Environmental Management

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PLUTONIUM FINISHING PLANT ANALYTICAL LABORATORY

PFP Generalized Geometry Holdup Calculations and Total Measurement Uncertainty

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1.0 INTRODUCTION

A collimated portable gamma-ray detector will be used to quantify the plutonium content of items that can be approximated as a point, line, or area geometry with respect to the detector. These items can include ducts, piping, gloveboxes, isolated equipment inside of gloveboxes, and HEPA filters. The Generalized Geometry Holdup (GGH) model is used for the reduction of counting data.

This document specifies the calculations to reduce counting data into contained plutonium and the associated total measurement uncertainty.

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APPENDIX A   CALCULATIONS

A.1.   Mass/Concentration at Each Measurement Location.

A.1.1.   Area Source.

The surface concentration of plutonium will be calculated for each measurement location as follows:

$$S_{Pu} = \frac{\left[(tot - R \times pkbg)_{sample} - (tot - R \times pkbg)_{background}\right] \times CF(AT) \times Ka}{t \times f \times w}$$

where:

S_{Pu} is the surface concentration of plutonium present in units of grams per square inch.

tot is the total spectrum counts in the Primary Region of Interest (ROI),

R is the ratio of the number of channels comprising the primary ROI to the number of channels comprising the Compton background ROI

pkbg is the total counts in the Compton background ROI,

$(\ldots)_{sample}$ is the spectrum taken in the glovebox assay shot

$(\ldots)_{background}$ is the spectrum taken in the background shot

CF(AT) is the attenuation correction factors,

Ka is the area source calibration constant, defined with equation 17 in ZA-948-395, *Mass Based Calibration Data Package for Portable Non-Destructive Assay (NDA) Equipment*. The calibration constant is included in the calibration report and has units of
grams Pu – 239 × min/count/inch²,

t is the count time in minutes,

f is the mass fraction of Pu-239, and

w is the weighting factor due to end effects described in section A.1.1.1 below.

Note: The term R for ratio is equal to 1 for high and moderate resolution detectors in which the software automatically calculates the background to the photopeak.

A.1.1.1.   End Effects and Weighting Factors

When the detector is located outside of the item being assayed (as opposed to inside of a glove port), the detector is often centered on the outside edge or corner of the item being assayed. In such cases, the assay item fills ½ or

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¼ of the field of view of the detector, respectively. In effect, the combination of 2 edge measurements or 4 corner measurements approximates a single interior measurement. Correspondingly, a weighting factor, w, is assigned 0.5 or 0.25, edge and corner assays, respectively.

The weighting factor for interior locations is assigned 1.0. By similar logic, the weighting factor for internal corners where ¾ of the item is in the field of view is 0.75.

Additional weighting factors may be developed for geometries that partially fill the detection field of view.

A.1.2. Line Source

The linear concentration of plutonium in grams per inch will be calculated for each measurement location as follows:

$$L_{Pu} = \frac{\left[\left(tot - R \times pkbg\right)_{sample} - \left(tot - R \times pkbg\right)_{background}\right] \times CF(AT) \times CF(ICF) \times d \times K_L}{t \times f \times w}$$

where:

L_{Pu} is the linear concentration of plutonium present in units of grams plutonium per inch.

d is the most probable distance from the detector to the midpoint of deposit in inches,

CF(ICF) is the item correction factor (finite width correction factor) described in A.3, below,

K_L is the line source calibration constant, defined with equation 7 in ZA-948-395, *Mass Based Calibration Data Package for Portable Non-Destructive Assay (NDA) Equipment.* The calibration constant is included in the calibration report and has units of
grams Pu − 239 × min/count/inch² ,

w is the weighting factor described in A.1.2.1 below, and

A.1.2.1. End Effects and Weighting Factors

When the detector is centered at the end of a line source, the assay item only fills ½ of the detector's field of view. In effect, the combination of two end measurements approximates a single interior measurement. A weighting factor, w, of 0.5 will be assigned to assays positioned at the end of a line source.

A.1.3. Point Source

The contained mass of plutonium in grams will be calculated for each measurement location as follows:

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$$P_{Pu} = \frac{\left[(tot - R \times pkbg)_{sample} - (tot - R \times pkbg)_{background}\right] \times CF(AT) \times CF(ICF) \times d^2}{t \times f \times E_K}$$

where:

P_{Pu} is the mass of plutonium in units of grams,

$E_K{}^1$ is the point source efficiency, defined with equation 1 in ZA-948-395, *Mass Based Calibration Data Package for Portable Non-Destructive Assay (NDA) Equipment*. The calibration constant is included in the calibration report and has units of *counts·inch²/gram Pu-239/min.*

A.2.   Attenuation Correction Factors

A.2.1.   The attenuation correction factor for slab shields is calculated from tabulated attenuation coefficients:

A.2.1.1.   The mass attenuation coefficient is the NIST value published at http://physics.nist.gov/PhysRefData/Xcom/Text/XCOM.html

A.2.1.2.   Calculate an Attenuation Correction Factor following the general example:

$$CF(AT) = e^{\left(u/\rho\right) \times \rho \times X}$$

where:

u/ρ is the mass attenuation coefficient (converted from tabulated values in cm²/g into units of in²/g)

ρ is the density (converted from tabulated values of lb/ft³ or g/cm³ into units of g/in³), and

X is the path length through the shield (in inches).

A.2.1.3.   Attenuation correction factor for slab shields may also be calculated from measured transmission data:

$$CF(AT) = \frac{1}{T^N}$$

where:

T is the measured transmission through a single thickness of the shield, and

N is the number of shields.

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[1] The notation E_K is for Cecil Kindle who published an early version of the NDA techniques employed at PFP: ARH-SA-248, In Situ Measurement of Residual Plutonium, June 30, 1976.

A.2.2. Attenuation correction factor for matrix (distributed) shields is calculated as follows for non-self-attenuating matrices:

$$CF(AT) = \frac{\left(\frac{u'}{\rho}\right) \times \rho \times X}{1 - e^{-\left(\frac{u}{\rho}\right) \times \rho \times X}}$$

Alternately, the correction factor for matrix shields may be calculated from measured transmission factors:

$$CF(AT) = \frac{-\ln T}{1 - T}$$

Matrix shields are not typically used for simple line sources such as pipes or ducts unless significantly thick deposits are present in which the non plutonium portion has greater attenuation properties than from the self-attenuation portion of the contained plutonium.

A.2.3. If multiple shields are present, the overall attenuation correction factor for the measurement location is the product of the correction factor for each shield, s:

$$CF(AT)_{overall} = \prod_s CF(AT)_s$$

A.3. Item Correction Factor[2] (Applicable to line and point sources)

The correction factor for "wide" lines and "fat" points is estimated by applying the general methodology specified in LA-13600-MS, *Achieving Higher Accuracy in the Gamma-Ray Spectroscopic Assay of Holdup*, P.A. Russo, Los Alamos National Laboratory, September 2000.

The detector response decreases as a source is moved away from the normal axis of the collimator by a function that is approximated by a Gaussian function. A source located at the edge of a line or at the edge of a point source with width, w, has a reduced detector response relative to the detector response to a source located at the center. The response at the edge of the line or point is determined as follows:

$$r_{w/2} = \exp\left[-0.5\left(\frac{2.354 \times w/2}{FWHM \times d}\right)^2\right]$$

---

[2] The item correction factor is the historical name at PFP for what is also called the finite width correction by the wider NDA community. It is also called the geometry correction. The historical ICF calculation was based on either a direct summation of the radial response curve or of the integration to a polynomial fit of the radial response curve.

where:

$r_{w/2}$ is the detector response at the edge of the line source relative to the response at the central location,

w is the most plausible width of the line in inches,

d is the most plausible distance from the detector face to the line, in inches, and

$FWHM$ is the width of the radial response function at one half the maximum value. The value is unitless in terms of width/distance and is determined during calibration from counting sources at multiple locations.

Correspondingly, for point source items, the detector response from a source located at the edge of an object with height, h, relative to the detector response from a source located at the center of the item is determined as follows:

$$r_{h/2} = \exp\left[-0.5\left(\frac{2.354 \times h/2}{FWHM \times d}\right)^2\right]$$

where:

$r_{h/2}$ is the detector response at the top and bottom of the item relative to the response at the central location,

h is the most plausible height of the item in inches,

The detector response integrated over the line or point width, R_w, is approximated by one half of the difference in the response at the edge and at the center of the item as follows:

$$R_w = \frac{1 + r_{w/2}}{2}$$

For point sources, the detector response integrated over the item height , R_h, is correspondingly calculated:

$$R_h = \frac{1 + r_{h/2}}{2}$$

The item correction factor, CF(ICF), for a line source is determined at each counting location as the inverse of the response as follows:

$$CF(ICF)_{line} = \frac{1}{R_w} = \frac{2}{1 + r_{w/2}}$$

The item correction factor, CF(ICF), for a point source is determined at each counting location as an inverse of the product of responses as follows:

$$CF(ICF)_{point} = \frac{1}{R_w \times R_h} = \frac{4}{(1+r_{w/2}) \times (1+r_{h/2})}$$

A.4.   Self-attenuation Correction

A.4.1.   Self attenuation effects is generally not calculated for area sources. When necessary, self attenuation effects may be calculated in accordance with LA-13600-MS, *Achieving Higher Accuracy in the Gamma-Ray Spectroscopic Assay of Holdup*, P.A. Russo, Los Alamos National Laboratory, September 2000. In cases where self attenuation is considered, the uncertainty will be propagated into the total measurement uncertainty.

A.4.2.   Line and Point Sources

The self-attenuation correction is non-linear and is applied to each individual measurement after all other corrections have been made. The following steps outline the calculation of the self-attenuation correction factor.

A.4.2.1.   Measured Areal Density

The measured areal density, $(\rho x)_{meas}$, in grams per square inch, for an individual measurement is calculated using the formula below. The measured areal density is the measured surface concentration.

For line sources:

$$(\rho x)_{meas} = \frac{L_{Pu}}{w}(1+f_U)$$

where:

L_{Pu} is the linear concentration of plutonium for a given measurement in grams/inch,

w is the width of the "wide" line in inches, and

f_U is the weight percent of uranium in the system relative to plutonium, when present.

For point sources:

$$(\rho x)_{meas} = \frac{P_{Pu}}{w \times h}(1+f_U)$$

where:

P_{Pu} is the plutonium mass for a given measurement in grams,

w is the width of the "fat" point in inches, and

h is the height of the "fat" point in inches

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A.4.2.2.   Infinite Thickness Test

A test is then performed to determine if the deposit has detection characteristics such that it cannot be differentiated from an infinitely thick deposit. Deposits that approach or exceed infinite thick detection characteristics should be evaluated in further detail.

Line sources:

$$\mu/\rho(\rho x)_{meas} + 3\mu/\rho\,\hat{\sigma}_{L_{Pu}}(\rho x)_{meas} < 1$$

where:

μ/ρ is the mass attenuation coefficient with the value of 0.0388 in^2/g for PuO2,

$(\rho x)_{meas}$ is the measured areal density, calculated above, and

$\hat{\sigma}_{L_{Pu}}$ is the relative uncertainty for the measurement location defined in section A.5.3, below.

Point Sources:

$$\mu/\rho(\rho x)_{meas} + 3\mu/\rho\,\hat{\sigma}_{P_{Pu}}(\rho x)_{meas} < 1$$

where:

$\hat{\sigma}_{P_{Pu}}$ is the relative uncertainty for the measurement location defined in section A.5.3, below.

A.4.2.2.1.   IF the test result is less than 1, THEN continue to section A.4.2.3.

A.4.2.2.2.   IF the test result is greater than 1, STOP and evaluate.

A.4.2.3.   Corrected Areal Density

The areal density corrected for self attenuation effects, $(\rho x)_{corr}$, is calculated by the equation below. The self attenuation correction below is derived as a solution to the equation in A.2.2 for a self-attenuating actinide deposit where $(\rho x)_{corr}$ is the actual areal density of the deposit and the ratio of actual areal density, $(\rho x)_{corr}$, to the measured areal density, $(\rho x)_{meas}$, is equivalent to the attenuation correction factor, CF(AT).

$$(\rho x)_{corr} = \frac{-1}{\mu/\rho}\ln[1 - \mu/\rho(\rho x)_{meas}]$$

The adjusted linear concentration of plutonium, corrected for self-attenuation is:

Line Sources:

$$L_{SAcorrPu} = \frac{(\rho x)_{corr} \times w}{1 + f_U}$$

Point Sources:

$$P_{SAcorrPu} = \frac{(\rho x)_{corr} \times w \times h}{1 + f_U}$$

A.5.   Total Measurement Uncertainty

The total measurement uncertainty (TMU) includes all identified sources of uncertainty that affect the quality of a final measured value. These sources of uncertainty are usually referred to as either random or systematic (bias) uncertainties.

A.5.1.   Systematic Uncertainty

Short-term systematic uncertainty is an uncertainty that affects some, but not all, members of a data set. Long-term systematic uncertainty (or bias) is an uncertainty that affects all members of a data set. The attenuation correction factors, the area source calibration constant, and the mass fraction of Pu-239 are considered to be systematic uncertainty terms.

A.5.1.1.   Uncertainty in the Calibration Constant

The uncertainty for the calibration constant (σ_{Ka}, σ_{Kl}, σ_{Ek},) is considered a long-term systematic component. It is defined in ZA-948-395, *Mass Based Calibration Data Package for Portable Non-Destructive Assay (NDA) Equipment.*

The origin of a significant portion of the uncertainty in the calibration constant is a common to all detectors and to all geometries. Therefore, when multiple detectors or geometries are included in the assay, a typical uncertainty in the calibration constant should be chosen and applied to all geometries and detectors. The calibration uncertainty is given the notation σ_K for all geometries and detectors.

A.5.1.2.   Mass Fraction Uncertainty

The uncertainty for the mass fraction of Pu-239 is considered a long-term systematic component. It is determined either through isotopic measurements or is estimated from an expected range of isotopics provided by the Cognizant Engineer. The difference between the high and low end of the range is taken to represent 4 standard deviations of a normal distribution. The relative uncertainty in the mass fraction, σ_f, varies with location and measurement circumstances.

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A.5.1.3. Attenuation Uncertainties

A.5.1.3.1. Uncertainty in Each Shield

The uncertainty in the correction factor for each shield is estimated from correction factors associated with the range of possible shield thicknesses for the assay situation. The range (largest plausible correction factor − smallest plausible correction factor) is assumed to represent four standard deviations of a normal distribution. The relative standard deviation for the attenuation correction factor is estimated as follows:

$$\hat{\sigma}_{CF(AT)} = \frac{largest\ plausible\ correction\ factor - smallest\ plausible\ correction\ factor}{4 \times most\ probable\ correction\ factor}$$

A.5.1.3.2. General Attenuation Uncertainty

Shielding calculations that estimate field conditions typically include additional sources of uncertainty compared to those normally considered when assessing largest and smallest plausible correction factors used in the determination of attenuation uncertainty. These additional sources of uncertainty include the following:

- Measurements are sometimes made at angles through materials.

- Use of a tape measure, not a high precision micrometer or caliper to measure items.

- Use of a mass attenuation coefficient for a tabulated single energy photon, whereas the measurement ROI includes photons of multiple energies.

- Use of tabulated mass attenuation coefficients that are a function of atomic number and gamma ray energy. The mass attenuation coefficient is multiplied by the material density to arrive at the linear attenuation coefficient. There are many cases when the density of a shielding material is not well known and hence is a source of uncertainty. (Applies to calculation of external shields only, not self-attenuation effects).

- Use of empirically determined coefficients that are subject to measurement uncertainties associated with mocking up a shield and performing a measurement.

- Measurements through glove ports that may have a variable thickness of material through which the assay is performed.

The general attenuation uncertainty, $\sigma_{general\text{-}AT}$, is judged to be approximately 10% and is included in all measurements, except when

all these additional sources are specifically addressed in sections
A.5.1.3.1, A.5.1.3.3, or A.5.1.3.4.

A.5.1.3.3.   Long-Term Systematic Attenuation Uncertainties

Since both short-term and long-term systematic attenuation
uncertainties are possible, it is necessary to identify attenuation shields
common to all measurement locations (long-term) and those which are
common to only some measurement locations (short-term). The long-
term relative systematic attenuation uncertainty, determined from the
relative uncertainty in the common shields is:

$$\hat{\sigma}_{CF(AT)_{long-term}} = \sqrt{\sum_{j} \sigma^2_{CF(AT)_j} + \sigma^2_{general-(AT)}}$$

where:

j is each shield common to all measurements.

$\sigma_{general-AT}$ is the general attenuation uncertainty applied in cases
described in section A.5.1.3.2.

Note: Line and point sources will generally be comprised entirely
of common shields that can be related to long–term systematic
uncertainties. However, when assaying a line or point source that is
included as a portion of a larger system, in order to facilitate
combining overall uncertainties, it is recommended to consider
common shields to the larger system as long-term systematic
uncertainties and shields specific to the individual source as short-
term systematic uncertainties.

A.5.1.3.4.   Short-Term Systematic Attenuation Uncertainties

The short-term systematic uncertainty is calculated similar to the long-
term systematic uncertainty. Consider the measurement uncertainty for
all shields not considered in the long-term systematic uncertainty of
the preceding paragraph.

A.5.1.3.4.1.   Segregate individual measurements into distinct populations with
similar attenuation characteristics. Each distinct population group
should be assigned a relative attenuation uncertainty that represents
each member in the group. The assigned attenuation uncertainty
for the population is notated as $\hat{\sigma}_{CF(AT)_p}$.

A.5.1.3.4.2.   For area and line sources, calculate an effective measurement area
(*ea*), effective measurement length (*el*) for each measurement
location as follows:

Area Sources:

$$ea = area_{surface} \times \frac{w}{\sum_i w_i}$$

where:

ea is the effective area,

area$_{surface}$ is the area of the surface,

w is the weighting factor for the measurement location,

i is each measurement location.

Line sources:

$$el = length \times \frac{w}{\sum_i w_i}$$

where:

el is the effective length,

length is the length of the item.

A.5.1.3.4.3. Calculate effective mass (*em*) for each measurement location as follows:

Area Sources:

$$em = ea \times S_{Pu}$$

Line Sources:

$$em = el \times L_{SAcorrPu}$$

Point Sources:

$$em = \frac{P_{SAcorrPu}}{N}$$

where:

N is the number of measurements taken of the point source.

A.5.1.3.4.4. Calculate effective mass for each population group (em_p).

$$em_p = \sum_i em_i$$

where:

i is each measurement in the population group, and

p identifies the population group.

A.5.1.3.5.   Overall relative Attenuation Uncertainty

A.5.1.3.5.1. Calculate the total effective mass (em_{tot}) for the entire system (all area, line and point measurements reported as a combined total value).

$$em_{tot} = \sum_p em_p$$

A.5.1.3.5.2. Calculate the overall relative attenuation uncertainty for the entire system as a combination of the short-term and long-term uncertainties as follows:

$$\sigma_{CF(AT)_{overall}} = \frac{\sqrt{\sum_p (em_p \times \sigma_{CF(AT)_p})^2 + (em_{tot} \times \sigma_{CF(AT)long-term})^2}}{em_{tot.}}$$

A.5.1.4.   Method Systematic Uncertainty

There is an overall method uncertainty that is inherent to portable NDA. The method uncertainty is taken to account for each of the following sources of uncertainty, as applicable to the assay being performed. Selection of method uncertainty values shall be documented in the calculation and reviewed by a second scientist. Documentation should include the selection of material distribution uncertainty.

A.5.1.4.1.   Contaminated Glove Uncertainty (Applicable to area, line and point sources)

Assays which are made through glove ports assume that there is no activity associated with the glove, i.e. it is clean. This is not always the case. The effect of assuming a glove to be clean when it is not is an overestimation of activity. It is judged that the overestimation could be as high as a factor of 1.5, or 50% high. It is not realistic to be higher

under normal circumstances because measurements are generally made through the upper arm portion of the glove. Materials are not handled in the upper arm portion of the glove therefore, it is judged that the upper arm portion of the glove would likely not contain more than ½ the surface concentration as the average surface activity in the glovebox.

Adjustments for contaminated gloves are not made to the assay value, instead they are factored into the uncertainty. The probability distribution is assumed to be a one-sided distribution where a 50% overestimation of the actual contamination represents the approximate upper boundary at 3 standard deviations. The contaminated glove contribution to the method uncertainty, is thus estimated to be no more than 17% of the actual contamination. The contaminated glove uncertainty, $\sigma_{contaminated\ glove}$, is included in the method uncertainty when gloves are visibly contaminated. A default value of one-half the maximum value, or 9%, should be considered for normal assay situations when gloves have moderate levels of visible contamination. Other values ranging from 0 to 17% may be used for $\sigma_{contaminated\ glove}$ based on technical judgment.

The same arguments and uncertainty can be assigned for measurements through any surface that is assumed clean such as bagout ports and glass windows.

A.5.1.4.2.   Intervening Equipment Uncertainty (Applicable to area sources)

Assays made through glove ports, bagout ports and glass windows assume all the activity is on the opposite surface. When there is intervening process equipment, it may be included in assay measurements from both sides. In effect the plutonium source would be assayed twice resulting in an overestimation of activity. It is judged that the overestimation could be as high as a factor of 1.5, or 50% high. This effect is realized whether or not it is a one-sided measurement of a glovebox or a 2-sided measurement.

Adjustments for intervening equipment are not made to the assay value, instead they are factored into the uncertainty. The probability distribution is assumed to be a one-sided distribution where a 50% overestimation represents the approximate upper boundary at 3 standard deviations. The contribution to the method uncertainty is estimated to be typically 17% for intervening equipment that is effectively measured twice. A default intervening equipment uncertainty, $\sigma_{intervening\ equipment}$, of 17% is included in the method uncertainty when measurements are made through glove ports and there is intervening process equipment that is effectively assayed

twice. Other values may be used for $\sigma_{intervening\ equipment}$ based on technical judgment.

This analysis does not apply to items that may contain large deposits in which the holdup is considered separately.

A.5.1.4.3. Ledges Uncertainty (Applicable to area sources)

When a measurement is made through the flat portion of a glovebox wall or window, the detector viewing angle sometimes would not include ledges between panels and around windows edges on the near surface, but would on the far surface. Material may preferentially accumulate on the ledges and around the windows edges. In this case, results would be calculated as if material were distributed on both surfaces, when in actuality, the accumulation points on the near surface may not be well represented. The mass could be underestimated by as much as ½, or 50% low.

Adjustment for ledges is not made to the assay value; instead it is factored into the uncertainty. The probability distribution is assumed to be a one-sided distribution where a 50% underestimation is to represent the approximate upper boundary at 3 standard deviations. The ledges contribution to the method uncertainty, is thus estimated at 17%. The ledges uncertainty, σ_{ledges}, is included in the method uncertainty when measurements are made through flat portions of the surface and significant portions of the ledges and window edges are not well represented on the near surface of the measurement. Other values may be used based on technical judgment.

A.5.1.4.4. Material Distribution Uncertainty (Applicable to area and line sources)

Measurements spaced at the effective L have some sensitivity to the location of discrete point material distributions. In addition, there may be a diminished response at the edge of a surface or line. It is not generally practical to space adjacent measurement shots uniformly and to also be spaced at the effective L.

Figures 1-3 have been developed to show the effects of the position of a single point source along a line in relation to measurement positions. Figure 1 shows the effect from a normally spaced measurement system at the effective L. Figure 2 shows the effect of a somewhat un-even spacing of measurement locations. Figure 3 shows the effect of a highly un-even spacing of measurement locations.

In each figure, the detectors are in a fixed location at the center of the shown radial response curves function for individual measurements where the effective length has a width/distance of 1.15 (the small light bell-shaped curves). The x-axis represents the location of a single point source in a generic glovebox (normalized to units of effective L).

The Y-axis represents the individual detector responses to a point
source located accordingly on the X-axis. The thick black line
represents the average response of the overall system to a point source
located on the X-axis. In these figures it is assumed that uneven
spacing only occurs in a single dimension. For area sources, it is
assumed that the spacing in the opposite dimension is at the effective
L.

Figure 1 illustrates the variations that are expected from normally
spaced measurements. In addition Figure 2 shows the effects of the
third measurement position (from the left) being spaced too close
(0.75L) to the second measurement location. Figure 2 also shows the
effects of the fourth measurement position (from the left) being spaced
too far (1.25L) from the third measurement location. Figure 3 is
similar, except the spacing is closer at 0.5 L and further at 1.5L.

Figure 1. The response of a series of measurements that would result from a point
source located where the spacing between adjacent shots is normal spacing.

The overall response of the
measurement system as a
function of source position.
Max = 1.084
Min = 0.913
Average = 0.995

Response of each
individual
measurement due
to position of the
of the source

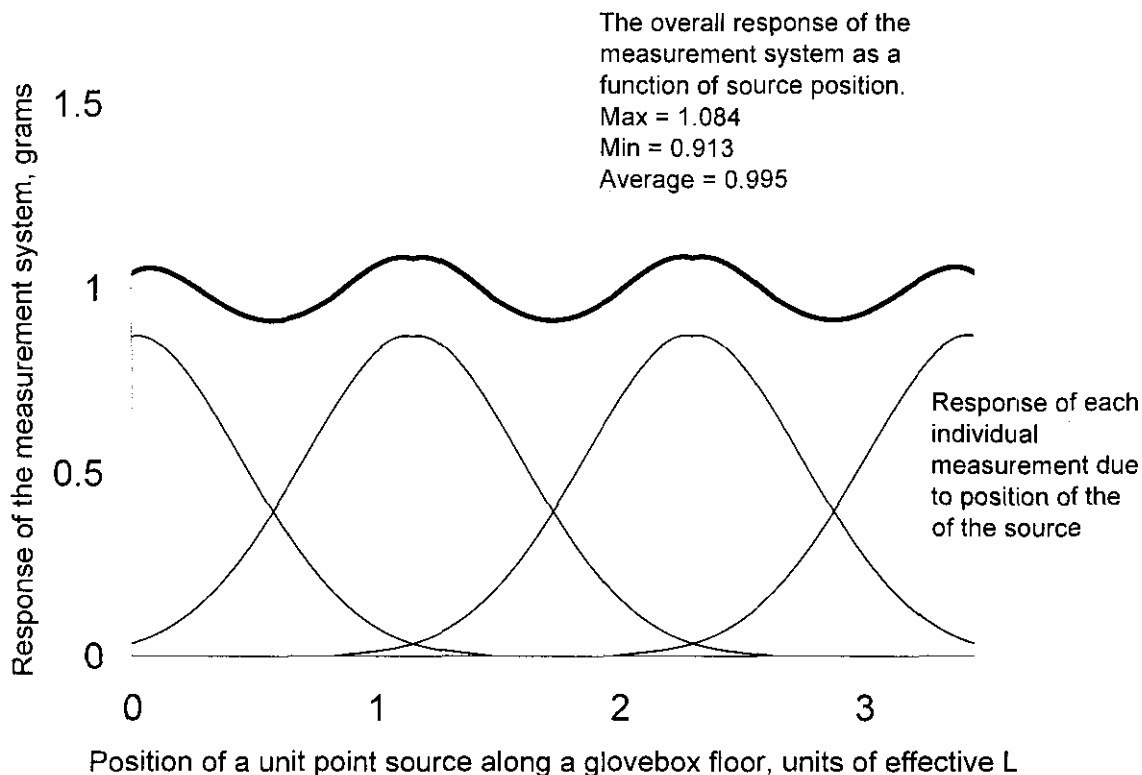Position of a unit point source along a glovebox floor, units of effective L

Figure 1 is taken to represent the extreme case of positioning effects
due to assaying a localized deposit when the detector is normally
spaced at the effective L. In reality, material distributions tend to be
spread out. Thus, the uncertainty due to material distribution is taken
to be represented by a normal distribution where the minimum and

maximum values of the uncertainty due to material distribution are at the lower and upper ends of the probability distribution of attenuation correction values at the approximate 99% confidence level. The difference between the maximum and minimum values from the figure is taken to be six (6) standard deviations, or 3% in each dimension. The 2-dimensional material distribution contribution, $\sigma_{\text{material distribution}}$, to the method uncertainty is estimated at 4% by summing each dimension in quadrature for cases where the measurement shots are ideally spaced at or near the effective L in both dimensions.

Figure 2. The response of a series of measurements that would result where the spacing between shots 2 and 3 is moderately closer than normal spacing and the spacing between shots 3 and 4 are moderately further than normal spacing.

The overall response of the measurement system as a function of source position.
Max = 1.29
Min = 0.58
Average = 0.997

Response of each individual measurement due to position of the of the source

Position of a unit point source along a glovebox floor, units of effective L
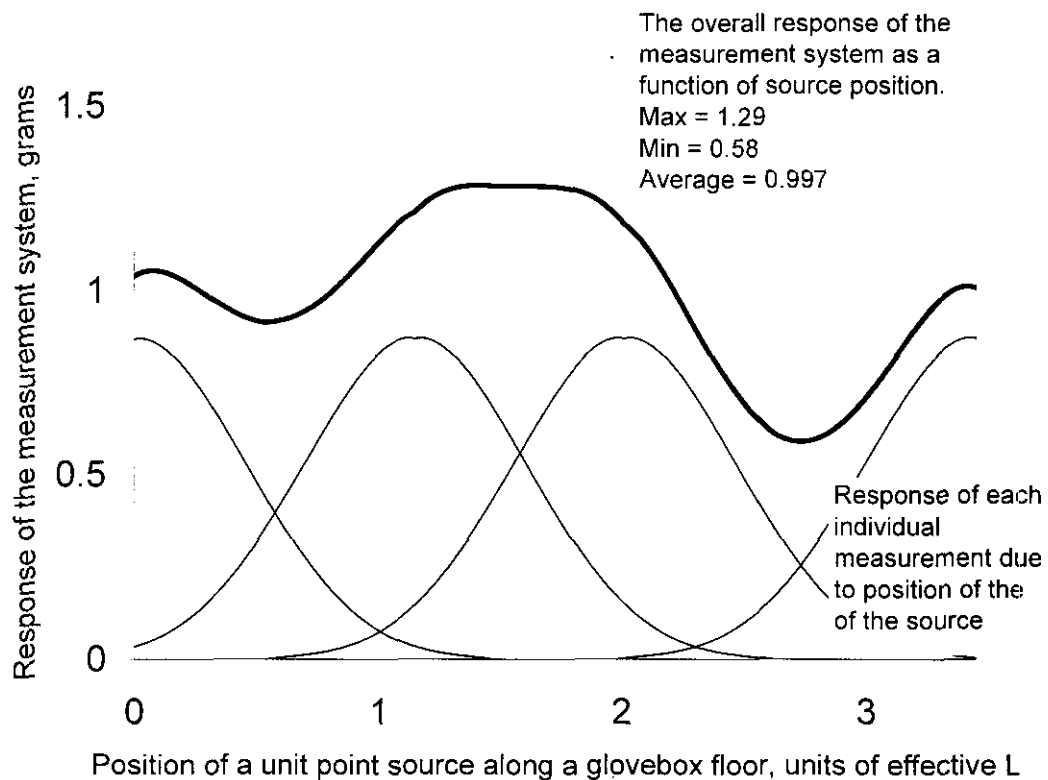
Figure 2 is taken to represent the extreme case of positioning effects due to assaying a localized deposit when the detector placement deviates from normal spacing (either too close or too far). In reality, material distributions tend to be spread out. Thus, the uncertainty due to material distribution is taken to be represented by a normal distribution where the minimum and maximum values of the uncertainty due to material distribution are at the lower and upper ends of the probability distribution of attenuation correction values at the approximate 99% confidence level. The difference between the

maximum and minimum values from the figure is taken to be six (6) standard deviations, or 12% in each dimension. The material distribution contribution, $\sigma_{material\ distribution}$, to the method uncertainty is thus estimated at 12% for cases where the measurement shot overlap is within +/- 25% of the effective L in one dimension. In two dimensions, the material distribution contribution, $\sigma_{material\ distribution}$, to the method uncertainty is estimated at 16% by summing each dimension in quadrature for cases where the measurement shot overlap is within +/- 25% of the effective L in both dimensions. Other values may be used based on consideration of uniformity in material distribution and end effects.

Figure 3. The response of a series of measurements that would result where the spacing between shots 2 and 3 is much closer than normal spacing and the spacing between shots 3 and 4 are much further than normal spacing.

Response of the measurement system, grams

The overall response of the measurement system as a function of source position.
Max = 1.64
Min = 0.35
Average = 0.997

Response of each individual measurement due to position of the of the source

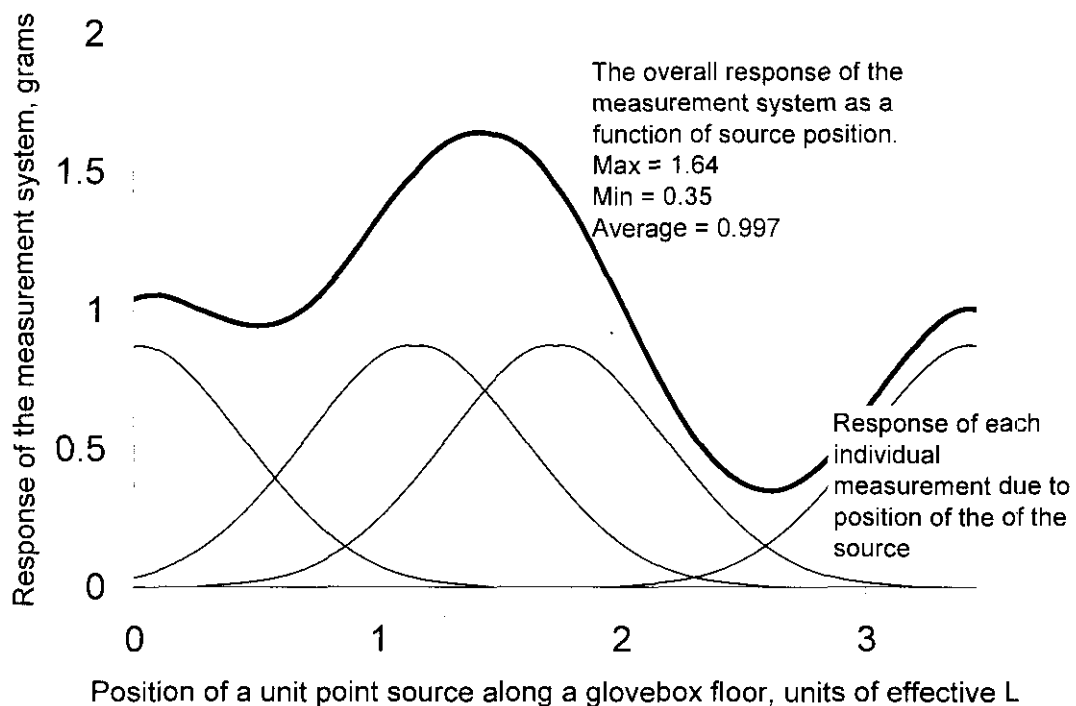Position of a unit point source along a glovebox floor, units of effective L

Figure 3 is taken to represent the extreme case of positioning effects when the detector is not spaced consistently across a glovebox. The exaggerated and diminished response from overshooting or undershooting a localized deposit dominates the overall response. In reality, material distributions tend to be spread out. Thus, the uncertainty due to material distribution is taken to be represented by a normal distribution where the minimum and maximum values of the

uncertainty due to material distribution are at the lower and upper ends of the probability distribution of attenuation correction values at the approximate 99% confidence level. The difference between the maximum and minimum values from the figure is taken to be six (6) standard deviations. The material distribution contribution, $\sigma_{material\ distribution}$, to the method uncertainty is thus estimated at 22% for cases where the measurement shot overlap is greater than +/- 25% of the effective L in one dimension. The two dimensional measurement uncertainty uncertainty is estimated at 31% by summing each dimension in quadrature for cases where the measurement shot overlap is greater than +/- 25% of the effective L in both dimensions. A default uncertainty of +/-31% is assigned for angled shots of an area source surface. Other values may be used based on technical judgment. Higher uncertainties may be assigned for difficult assay situations.

A.5.1.4.5.   Forward Background (Applicable to area, line and point sources)

In some instances there is background interference from plutonium deposits forward of the detector that are not part of the assay but are within the viewing angle of the detector. A forward background uncertainty, $\sigma_{forward\ bkg}$, shall be assigned based on technical judgment when the assay has significant forward background. The choice of uncertainty values should be based on several forward background measurements.

A.5.1.4.6.   Sorenson Uncertainty (Applicable to line and point sources)

Measurement assays assume the detector is aiming directly at the object. However, this is not always the case and results in an underestimation of the activity. It is judged that the angle could be as much as $15°$ off center in an extreme case. The effect, the Sorenson factor, of being $15°$ off center is equivalent to underestimating the sample activity by an amount equal to 11% with a typical collimated NaI detector. The probability distribution is assumed to be a one-sided distribution where an 11% underestimation of the actual contamination represents the approximate upper boundary at 3 standard deviations. The Sorenson contribution to the method uncertainty, $\sigma_{Sorenson}$, is thus estimated to be 4% and will be applied to all line and point measurement assays.

A.5.1.4.7.   Overall Method Uncertainty

A.5.1.4.7.1.   The overall method uncertainty is estimated from population groupings in a manner similar to section A.5.1.3.4. The overall method uncertainty is notated as $\hat{\sigma}_{method}$.

A.5.1.4.7.2. For each type of identified uncertainty, assign a method uncertainty to the individual measurements. Segregate individual measurements into distinct populations with similar method characteristics. Each distinct population should be assigned the appropriate uncertainty that represents each member of the group.

A.5.1.4.7.3. Calculate effective mass for each population group using the formula and method of sectionA.5.1.3.4.4.

A.5.1.4.7.4. Calculate the overall relative method uncertainty for the entire system as a combination of the short-term and long-term uncertainties, as applicable to the assay being performed:

$$\hat{\sigma}_{method} = \frac{\sqrt{\begin{array}{l} \sum_p \left(em_p \times \hat{\sigma}_{contam\ glove}\right)^2 + \sum_p \left(em_p \times \hat{\sigma}_{micrv\ eqip.}\right)^2 + \sum_p \left(em_p \times \hat{\sigma}_{ledges}\right)^2 + \\ \sum_p \left(em_p \times \hat{\sigma}_{material\ distrib}\right)^2 + \sum_p \left(em_p \times \hat{\sigma}_{forward\ bkg}\right)^2 + \sum_p \left(em_p \times \hat{\sigma}_{Sorenson}\right)^2 \end{array}}}{em_{tot}}$$

A.5.1.5. Distance Uncertainty (Applicable to line and point sources)

The distance from the detector to the deposit has an uncertainty because the deposit often has physical thickness and because the exact distance to the deposit is not known. Distance uncertainty is considered to be a short-term systematic uncertainty component in relation to line sources, as the detector is typically held a consistent distance from the near surface of a duct (or other line source) for multiple measurements along the line. Thus, the distance bias to the actual deposit would be relatively consistent for all measurements. Distance uncertainty is generally considered a random uncertainty in relation to point sources, as a point source is typically counted on multiple sides. Thus, the distance bias to the actual deposit would be variable depending on the rotation of the object. Distance uncertainty is not applicable to area sources.

The uncertainty in the distance for each measurement is estimated from the range of distances for the assay situation. The range (largest plausible distance – smallest plausible distance) is assumed to represent four standard deviations of a normal distribution. The relative standard deviation for the distance for an individual measurement is estimated as follows:

$$\hat{\sigma}_{d_i} = \frac{largest\ plausible\ distance - smallest\ plausible\ distance}{4 \times most\ probable\ distance}$$

A.5.1.5.1. Line Sources: Assign a distance uncertainty to the individual measurements. Segregate individual line source measurements into distinct populations with similar distance characteristics. Each distinct population should be assigned the appropriate uncertainty that represents each member of the group.

For example: If an 8 inch straight duct were measured from both the top and the bottom, then the set of measurements from the top would be assigned to a distinct population and the set of measurements from the bottom would be assigned a separate distinct population grouping. This is because the material may actually be distributed on the bottom of the duct, and therefore a measurement from the top would have a different systematic uncertainty than a measurement from the bottom. The separate population groupings will exist regardless of whether or not the distances and assigned uncertainty values are the same for each population.

A.5.1.5.2. Calculate effective mass for each line source population group using the formula and method of section A.5.1.3.4.

A.5.1.5.3. Calculate the overall relative distance uncertainty for the entire line source as a combination of the short-term uncertainty as follows:

$$\hat{\sigma}_d = \frac{\sqrt{\sum_p \left(em_p \times \hat{\sigma}_{d_p}\right)^2}}{em_{tot}}$$

A.5.1.6. Uncertainty in the Item Correction Factor (ICF) (Applicable to line and point sources)

The uncertainty associated with the ICF is due to both the uncertainty of the position of the deposit within the line width or size of the point source and also due to the uncertainty in the width of a line or size of the point source. The ICF uncertainty is considered a short-term systematic uncertainty.

A.5.1.6.1. Uncertainty in the position of the deposit.

The uncertainty in the ICF due to the variability in the position of the deposit is estimated from the spread of a correction factor calculated for a source located at the edge of the item and calculated at the center of the item. The spread is assumed to represent 4 standard deviations of a normal distribution.

In one dimension, the width of the line and width of a point source, the relative positional distribution uncertainty in the ICF, $\sigma_{ICF\text{-}pos\text{-}w}$, of the source width is estimated as follows:

$$\hat{\sigma}_{ICF-pos-w} = \frac{CF_{edge} - CF_{center}}{4 \times ICF} = \frac{\frac{1}{r_{w/2}} - 1}{4 \times \frac{2}{1 + r_{w/2}}} = \frac{1}{8}\left(\frac{1}{r_{w/2}} - r_{w/2}\right)$$

In addition, for point sources, the uncertainty with respect to the second dimension, or the item height is similar:

$$\hat{\sigma}_{ICF-pos-h} = \frac{CF_{top,bottom} - CF_{center}}{4 \times ICF} = \frac{1}{8}\left(\frac{1}{r_{h/2}} - r_{h/2}\right)$$

A.5.1.6.2. **ICF Uncertainty in Relation to the Width and Height Uncertainty.**

In one dimension, the width of the line and width of a point source, the uncertainty in the ICF due to an uncertainty in the width of the line or point source is estimated from the range of ICFs calculated for the assay situation due to the range of corresponding item widths. The range (the ICF calculated from the largest plausible width– the ICF calculated from the smallest plausible width) is assumed to represent four standard deviations of a normal distribution. The relative width uncertainty in the ICF, σ_{ICF-w}, is estimated as follows:

$$\hat{\sigma}_{ICF-w} = \frac{ICF_{largest\ width} - ICF_{smallest\ width}}{4 \times ICF}$$

In addition for point sources, the uncertainty with respect to the second dimension, the item height is similar:

$$\hat{\sigma}_{ICF-h} = \frac{ICF_{largest\ height} - ICF_{smallest\ height}}{4 \times ICF}$$

A.5.1.6.3. **ICF Uncertainty for an Individual measurement.**

The relative ICF uncertainty, σ_{ICF}, for each individual measurement is a combination of the positional and width uncertainty:

$$\hat{\sigma}_{ICF} = \sqrt{\hat{\sigma}_{ICF-w}^2 + \hat{\sigma}_{ICF-h}^2 + \hat{\sigma}_{ICF-pos-w}^2 + \hat{\sigma}_{ICF-pos-h}^2}$$

A.5.1.6.4. **Calculation of Overall ICF Uncertainty**

Assign an ICF uncertainty to the individual measurements. Segregate individual measurements into distinct populations with similar ICF characteristics. Each distinct population should be assigned the appropriate uncertainty that represents each member of the group.

For example: If an 8 inch straight duct were to taper down to a 6 inch straight duct and be measured along the entire length, then the set of measurements for the 8 inch section would be assigned to a distinct population and the set of measurements from the 6 inch section would be assigned a separate distinct population grouping.

A.5.1.6.4.1. Calculate effective mass for each population group using the formula and method of section A.5.1.3.4.

A.5.1.6.4.2. Calculate the overall ICF uncertainty for the entire item as a combination of the short-term uncertainties as follows:

$$\hat{\sigma}_{ICF} = \frac{\sqrt{\sum_{p}\left(em_p \times \hat{\sigma}_{ICF_p}\right)^2}}{em_{tot}}$$

A.5.1.7.   Overall Systematic Uncertainty in the Plutonium Mass

The overall relative systematic uncertainty of the measurement is calculated as follows:

$$\hat{\sigma}(relative\ systematic)_{m_{Pu}} = \sqrt{\hat{\sigma}_f^2 + \hat{\sigma}_K^2 + \hat{\sigma}_{CF(AT)_{overall}}^2 + \hat{\sigma}_{ICF}^2 + \hat{\sigma}_{method}^2 + \hat{\sigma}_{length}^2 + \hat{\sigma}_{area}^2 + \hat{\sigma}_d^2}$$

where:

area is the systematic component associated with the area of the item.
length is the systematic component associated with the length of the item.

d is the distance uncertainty in relation to lines sources.

The systematic component associated with the length and area measurement is the uncertainty associated with the measuring device. It is generally negligible. It is listed in the above equation for completeness, but will not necessarily be included in actual calculations.

The overall absolute systematic uncertainty of the measurement is then:

$$\hat{\sigma}(systematic)_{m_{Pu}} = m_{Pu} \times \hat{\sigma}(relative\ systematic)_{m_{Pu}}$$

where:

m_{Pu} is the overall plutonium mass, defined in section A.7, below.

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A.5.2. Random Uncertainty

Random uncertainty is defined as uncertainty that affects only a single member of a data set.

A.5.2.1. Counting Statistics:

The relative uncertainty in each measurement shot due to counting statistics is estimated assuming Poisson Statistics:

$$\hat{\sigma}_{(relative\ tot-bkg)} = \frac{\sqrt{\left(tot + R^2 \times pkbg\right)_{sample} + \left(tot + R^2 \times pkbg\right)_{background}}}{\left(tot - R \times pkbg\right)_{sample} - \left(tot - R \times pkbg\right)_{background}}$$

A.5.2.2. Self attenuation correction (Applicable to line and point sources)

The uncertainty in the self attenuation correction is considered a random uncertainty that is an extension of the counting statistics. The correction for self attenuation differs for each individual measurement and is a function of the measured linear concentration. As stated in LA-13600-MS, *Achieving Higher Accuracy in the Gamma-Ray Spectroscopic Assay of Holdup*, P.A. Russo, Los Alamos National Laboratory, September 2000.

The absolute uncertainty due to the random components in the corrected mass is as follows:

Line Sources:

$$\hat{\sigma}_{L_{SAcorrPu}} = \frac{L_{Pu}}{1 - \mu / \rho \times \left(\rho x\right)_{meas}} \hat{\sigma}_{(relative\ tot-bkg)}$$

Point sources:

$$\hat{\sigma}_{P_{SAcorrPu}} = \frac{P_{Pu}}{1 - \mu / \rho \times \left(\rho x\right)_{meas}} \hat{\sigma}_{(relative\ tot-bkg)}$$

A.5.2.3. Combined Random Uncertainty at Each Measurement Location (Point Sources):

The combined absolute random uncertainty at each location is a propagation of the distance uncertainty and the self attenuation correction. The distance is included as a random uncertainty with a point source because counts from multiple sides reduces the overall bias due to distance:

$$\hat{\sigma}(combined\ random)_{P_{SAcorrPu}} = P_{SAcorrPu} \sqrt{\left(\frac{\hat{\sigma}_{P_{SAcorrPu}}}{P_{SAcorrPu}}\right)^2 + 4\hat{\sigma}_d^2}$$

A.5.3. Uncertainty in the plutonium mass at each measurement location

The uncertainty in the measured value at each location is the summation of all the random and systematic uncertainties as applied to each measurement location, exclusive of self-attenuation effects. (Note : This value is used only for the test in section A.4.2.2).

Line sources:

$$\hat{\sigma}_{L_{Pu}} = \sqrt{\hat{\sigma}_f^2 + \hat{\sigma}_K^2 + \hat{\sigma}_{CF(AT)_{long-term}}^2 + \hat{\sigma}_{CF(AT)_i}^2 + \hat{\sigma}_{ICF}^2 + \hat{\sigma}_{method}^2 + \hat{\sigma}_d^2 + \hat{\sigma}_{relativetot-bkg}^2}$$

Point Sources:

$$\hat{\sigma}_{P_{Pu}} = \sqrt{\hat{\sigma}_f^2 + \hat{\sigma}_K^2 + \hat{\sigma}_{CF(AT)_{long-term}}^2 + \hat{\sigma}_{CF(AT)_i}^2 + \hat{\sigma}_{ICF}^2 + \hat{\sigma}_{method}^2 + 4\hat{\sigma}_d^2 + \hat{\sigma}_{relativetot-bkg}^2}$$

A.6. Plutonium Mass on a Surface, Length or Point Item

A.6.1. Area Source

A.6.1.1. Average Surface Concentration

The average surface concentration of plutonium for each surface of an area source in grams per square is calculated as follows:

$$\overline{S}_{Pu} = \frac{\sum_i (S_{Pu} \times w_i)}{\sum_i w_i}$$

where:

i is the individual measurement location, and

w is the weighting factor assigned to each measurement location.

A.6.1.2. Surface Mass

The mass of plutonium for each surface of an area source is calculated as follows:

$$m_j = Area \times \overline{S}_{Pu}$$

where:

m_j is the mass of plutonium present on the surface in grams,

Area is the surface area in square inches, and

$\overline{S}_{Pu}$ is the average surface concentration of plutonium.

A.6.1.3. Random Uncertainty

A.6.1.3.1. Uncertainty in the Average Surface Concentration

The relative standard deviation of the mean (of the surface concentration) is calculated as follows:

$$\hat{\sigma}(\text{random})_{\bar{S}_{Pu}} = \frac{s/\sqrt{N}}{S_{Pu}}$$

where:

s is the standard deviation of individual measurements of S_{Pu} in grams per square inch, and

N is the total number of measurements taken.

A.6.1.3.1.1. In cases where the entire surface is represented by a single measurement, $\hat{\sigma}(random)_{\bar{S}_{Pu}}$ is taken to be equivalent to

$$\hat{\sigma}_{(relative\ tot-bkg)}$$

A.6.1.3.2. Relative Random Uncertainty in the Surface Mass:

The relative random uncertainty in the mass of the plutonium present on each surface is calculated as follows by including the relative random uncertainty in the surface area measurement:

$$\hat{\sigma}(random)_{m_j} = \sqrt{\hat{\sigma}(random)_{\bar{S}_{Pu}}^2 + \hat{\sigma}_{Area}^2}$$

The relative random uncertainty in the surface area measurement, σ_{Area}, is based on uncertainties in the ability to measure an item. An example relates to the ability to calculate the inside surface area of an object which contains an external frame and external shielding walls.

A.6.1.3.3. Absolute Random Uncertainty in the Surface Mass:

The absolute random uncertainty in the mass of the plutonium present on each surface is calculated as follows:

$$\hat{\sigma}(absolute\ random)_{m_j} = m_j \times \hat{\sigma}(random)_{m_j}$$

A.6.2. Line Source

A.6.2.1. Average Linear Concentration

The average linear concentration for each line segment of a line source in grams per inch will be calculated as follows:

$$\overline{L}_{SAcorrPu} = \frac{\sum_i \left(L_{SAcorrPu} \times w_i \right)}{\sum_i w_i}$$

where:

i is the individual measurement location, and

w is the weighting factor assigned to each measurement location.

A.6.2.2. Line Mass

The total mass of plutonium for each segment of the line source is calculated as follows:

$$m_j = \overline{L}_{SAcorrPu} \times length$$

where:

length is the total length of the line segment.

A.6.2.3. Random Uncertainty

A.6.2.3.1. Standard Deviation of the Mean

The absolute standard deviation of the mean (of the linear concentration) is calculated as follows:

$$\hat{\sigma}_{\bar{x}} = \frac{s}{\sqrt{N}}$$

where:

s is the standard deviation of individual measurements of $L_{SAcorrPu}$ in grams per inch, and

N is the total number of measurements taken.

A.6.2.3.1.1. If there is only a single measurement taken of the line source, then the absolute standard deviation of the mean is not applicable.

A.6.2.3.2. Absolute Random Uncertainty in the Linear Concentration

The absolute relative uncertainty due to the self attenuation correction in the mean corrected linear concentration in grams per inch is calculated assuming independent variables:

$$\hat{\sigma}(random)_{\overline{L.SAcorrPu}} = \sqrt{\left(\frac{\sqrt{\sum_i \hat{\sigma}^2_{L.SAcorrPu}}}{N}\right)^2 + \hat{\sigma}^2_{\overline{x}}}$$

When there is only a single measurement of the item, the absolute random uncertainty in the plutonium mass is calculated as follows:

$$\hat{\sigma}(random)_{\overline{L.SAcorrPu}} = \sqrt{\left(\frac{\sqrt{\sum_i \hat{\sigma}^2_{L.SAcorrPu}}}{N}\right)^2}$$

A.6.2.3.3. Relative Random Uncertainty in the Line Mass

The relative random uncertainty in total mass of plutonium for each segment of the line source is calculated as follows:

$$\hat{\sigma}(random)_{m_j} = \sqrt{\left(\frac{\hat{\sigma}(random)_{\overline{L.SAcorrPu}}}{\overline{L}_{SAcorrPu}}\right)^2 + \hat{\sigma}^2_{length}}$$

The relative random uncertainty in the line length measurement, $\hat{\sigma}_{length}$, is based on uncertainties in the ability to measure an item. An example relates to the ability to calculate the length of an item where the ends are elbows that are not included in the item.

A.6.2.3.4. Absolute Random Uncertainty in the Line Mass

The absolute random uncertainty in the mass of the plutonium present on each line segment is calculated as follows:

$$\hat{\sigma}(absolute\ random)_{m_j} = m_j \times \hat{\sigma}(random)_{m_j}$$

A.6.3.   Point Source

A.6.3.1.   Mass of a Point Source Item

The mass of plutonium in grams for each point source item will be calculated as follows:

$$m_j = \frac{\sum_i P_{SAcorrPu}}{N}$$

where:

N is the number of measurements made on the item.

A.6.3.2.   Random Uncertainty

A.6.3.2.1.   Standard Deviation of the Mean

The absolute standard deviation of the mean (of the measured mass) is calculated as follows:

$$\hat{\sigma}_x = \frac{s}{\sqrt{N}}$$

where:

s is the standard deviation of individual measurements of $P_{SAcorrPu}$ in grams, and

N is the total number of measurements taken.

A.6.3.2.1.1.   If there is only a single measurement taken of the point source, then the absolute standard deviation of the mean is not applicable.

A.6.3.2.2.   Absolute Random Uncertainty

The combined absolute random uncertainty in the plutonium mass is calculated as follows:

$$\hat{\sigma}(absolute\ random)_{m_j} = \sqrt{\left(\frac{\sqrt{\sum_i \hat{\sigma}(combined\ random)^2 P_{SAcorrPu}}}{N}\right)^2 + \hat{\sigma}_{\bar{x}}^2}$$

When there is only a single measurement of the item, the combined absolute random uncertainty in the plutonium mass is calculated as follows:

$$\hat{\sigma}(absolute\ random)_{m_j} = \sqrt{\left(\frac{\sqrt{\sum_i \hat{\sigma}(combined\ random)^2 P_{SAcorrPu}}}{N}\right)^2}$$

A.6.3.2.3.   Relative Random Uncertainty

The relative random uncertainty in plutonium mass is calculated as follows:

$$\hat{\sigma}(random)_{m_j} = \frac{\hat{\sigma}(absolute\ random)_{m_j}}{m_j}$$

A.7. Overall System Plutonium Content

A.7.1. Overall Plutonium Mass

The overall plutonium mass present in the system is the sum of the mass contained on each surface, line segment and point source:

$$m_{Pu} = \sum_j m_j$$

where:

j is each surface, line segment or point source measured.

Plutonium mass values, m_{Pu}, which are zero or negative are assigned a value of 0.01 grams. The plutonium mass is rounded to the nearest integer for reporting purposes.

A.7.2. Absolute Random Uncertainty

The overall absolute random uncertainty of the measurement is calculated as follows, assuming independent variables:

$$\hat{\sigma}(random)_{m_{Pu}} = \sqrt{\sum_j \hat{\sigma}(absolute\ random)^2_{m_j}}$$

A.7.3. Overall Absolute Uncertainty

The overall absolute uncertainty of the plutonium mass at one standard deviation is a combination of the random and systematic components:

$$\hat{\sigma}(overall)_{m_{Pu}} = \sqrt{\hat{\sigma}(random)^2_{m_{Pu}} + \hat{\sigma}(systematic)^2_{m_{Pu}}}$$

The uncertainty is rounded to the nearest integer for reporting purposes.

A.7.4. Criticality Value

The approximate 95% confidence interval upper limit is calculated from the overall uncertainty and is called the criticality value (CV):

$$CV_{m_{Pu}} = m_{Pu} + 2 \times \hat{\sigma}(overall)_{m_{Pu}}$$

The criticality value is rounded to the nearest integer for reporting purposes.

A.8.   Overall System Plutonium-239 Content

A.8.1.   Overall Plutonium-239 Mass

The total mass of plutonium-239 for the system is calculated as follows:

$$m_{Pu-239} = m_{Pu} \times f$$

The plutonium mass is rounded to the nearest integer for reporting purposes.

A.8.2.   Overall Absolute Uncertainty

The overall absolute uncertainty of the plutonium-239 mass at one standard deviation is estimated using the following formula:

$$\hat{\sigma}(overall)_{m_{Pu-239}} = \hat{\sigma}(overall)_{m_{Pu}} \times f$$

This method slightly overestimates the uncertainty in the plutonium-239 mass due to accounting for the isotopic fraction uncertainty in $\hat{\sigma}(overall)_{m_{Pu}}$. The uncertainty is rounded to the nearest integer for reporting purposes.

A.8.3.   Criticality Value

The criticality value of Pu-239 is calculated from the overall uncertainty:

$$CV_{m_{Pu-239}} = m_{Pu-239} + 2 \times \hat{\sigma}(overall)_{m_{Pu-239}}$$

The criticality value is rounded to the nearest integer for reporting purposes.