Analysis on achieving a minimum bunch length in LCLS Bunch Compressor One

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An ultra-short bunch is required by different applications in many aspects. In this paper, the condition to achieve a minimum bunch length at the Linac Coherent Light Source (LCLS) [1] bunch compressor one (BC1) is analyzed analytically and evaluated by simulation. The space charge, wake field and coherent synchrotron radiation (CSR) effects are not discussed here.

1. Analytical derivation

First of all, some general formulae are derived and presented on the longitudinal motion of the electron bunch [2]. We recall the key point of magnetic bunch compression which is the compression method discussed here. A z-correlated energy modulation (chirp) is established by either RF structure or something similar. Then one lets the bunch passing by a dispersive region, in which particles' pathlength is dependent on their energy. In that case, particles in the head and tail of the bunch both move toward the center relatively, so the bunch length is shorten.

Let us assume that the chirp on the bunch is established by a S-band acceleration and a harmonic x-band deceleration, which is just like the case of LCLS BC1. After this process, the z-correlated energy offset of any particle in the bunch can be expressed as

$$\delta(z) = \delta_i \frac{E_{i0}}{E_{f0}} + h_1 z + h_2 z^2 + h_3 z^3 + ... \tag{1}$$

where δ_i denotes the initial un-correlated energy offset, E_{i0} central energy before RF acceleration, E_{f0} central energy after RF acceleration, h_1 the first order chirp, h_2 second order chirp, and h_3 third order chirp.

One could easily derive that the chirp up to third order can be expressed as in the following formulae.

$$h_1 = -\frac{keV_0 \sin\phi}{E_{f0}} \approx -k\tan\phi \tag{2}$$

$$h_2 = -\frac{k^2 eV_0 \cos\phi}{2E_{f0}} \tag{3}$$

$$h_3 = -\frac{k^3 eV_0 \sin\phi}{6E_{f0}} \tag{4}$$

where e denotes the electron charge.

After passing by the dispersive region, the longitudinal coordinate (relative to the bunch center) of any particle can be expressed as

$$z_f(\delta) = z_i + b_1\delta + b_2\delta^2 + b_3\delta^3 + ... \tag{5}$$

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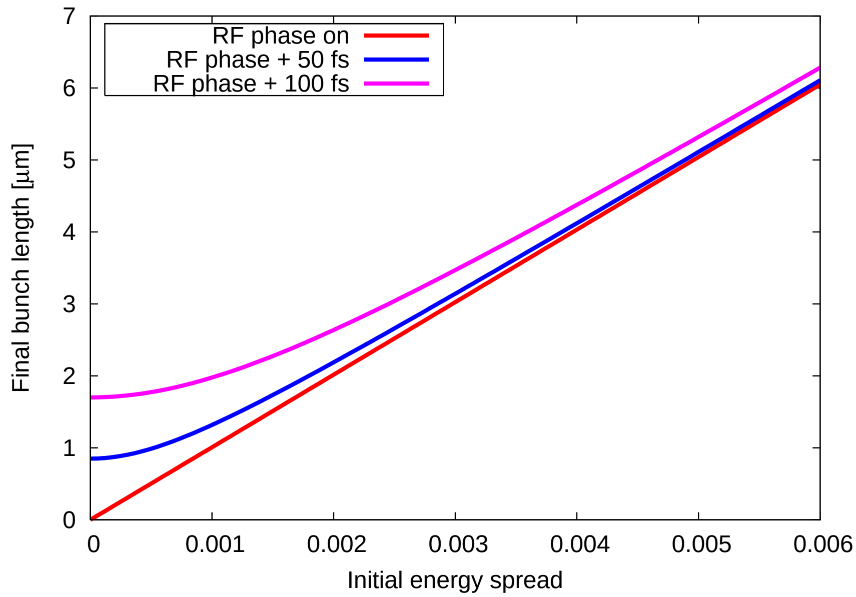

Figure 1: Minimum final bunch length as a function of the initial energy spread, without or with RF phase jitter.

where z_i denotes the initial longitudinal coordinate relative to the bunch center, b_1 the first order longitudinal dispersion, b_2 the second order dispersion, and b_3 the third order dispersion.

Assume the bunch maintains a Gaussian distribution in longitudinal direction, the RMS bunch length can be calculated through the integration, as expressed in formula (6).

$$\sigma_z{}^2 = \int\int z_f{}^2(z,\delta) \cdot f(z,\delta) dz d\delta \tag{6}$$

where $f(z,\delta)$ denotes a Gaussian distribution in both z and δ.

1.1. First order

Consider only first order terms, the z-correlated energy offset and the final longitudinal coordinate is shown in formulae (7) and (8).

$$\delta(z) = \delta_i \frac{E_{i0}}{E_{f0}} + h_1 z \tag{7}$$

$$z_f(\delta) = z_i + b_1\delta \tag{8}$$

From formulae (6)-(8), one can write down the final RMS bunch length as expressed in formula (9).

$$\sigma_z{}^2 = a^2 R_{56}^2 \sigma_{\delta i}^2 + (1 + h_1 R_{56})^2 \cdot \sigma_{z,i}{}^2 \tag{9}$$

where $a = \frac{E_{i0}}{E_{f0}}$ denotes the energy ratio, $\sigma_{z,i}$ initial bunch length.

It is observed that by choosing a proper RF phase (for a fixed RF frequency) and R_{56}, one can make the second term equal zero in formula (9), $1 + h_1 R_{56} = 0$. In this case, the final bunch length only depends on the energy ratio, the initial un-correlated energy spread and the first order dispersion R_{56}.

For LCLS BC1, the initial beam energy from the injector is $7MeV$, with a RMS energy spread of 0.2%. The chicane BC1 has a first order dispersion of $R_{56} = -36mm$, which is operated at $250MeV$. For the initial RMS bunch length, here we assume a smaller value, which is $300\mu m$. In Figure 1, the minimum final bunch length is plotted as a function of the initial energy spread.

1.2. Higher order

Consider up to second order terms in the bunch compressor while still assume a linearized bunch, the z-correlated energy offset and the final longitudinal coordinate is shown in formulae (10) and (11).

$$\delta(z) = \delta_i \frac{E_{i0}}{E_{f0}} + h_1 z \tag{10}$$

$$z_f(\delta) = z_i + b_1\delta + b_2\delta^2 + b_3\delta^3 \tag{11}$$

Insert formulae (10) and (11) into formula (6), neglect the un-correlated energy spread δ_i for the higher order terms $b_2\delta^2$ and $b_3\delta^3$, and do the integration, one finds the final bunch length for this case is

$$\sigma_z{}^2 = a^2 R_{56}^2 \sigma_{\delta i}^2 + (1+h_1 R_{56})^2 \cdot \sigma_{z,i}{}^2 + 3 \cdot \left(h_1^4 T_{566}^2 + 2U_{5666} h_1^3 (1+h_1 R_{56}) + (...) \cdot \sigma_{\delta i}\right) \cdot \sigma_{z,i}{}^4 + 15 \cdot h_1^6 U_{5666}^2 \cdot \sigma_{z,i}{}^6 \tag{12}$$

where T_{566} denotes the second order dispersion, U_{5666} the third order dispersion.

Under the optimal compression condition $1 + h_1 R_{56} = 0$, from formula (12) one finds that the second order and third order dispersion always make the final bunch length longer, given an initial linearized bunch from harmonic RF. An example for LCLS BC1 (initial RMS bunch length of $300\mu m$) is shown in Figure 2. It is observed that the second order dispersion T_{566} always make the minimum bunch length longer, and for that specified case the third order effects can be negligible. A good agreement is achieved between the analytical formulae and LiTrack [3] simulation results, as shown in Figure 2.

In Figure 3 the longitudinal phase space is illustrated in details. After the linear energy modulation is established according to formula (10), the phase space is shown in the left plot of Figure 3, with a peak current of 0.1 kA. Under optimal compression condition and only consider first order dispersion R_{56}, the bunch is linearly compressed to a final RMS length of $1\mu m$, with a peak current of 30 kA, as shown in the middle plot. With both R_{56} and T_{566} included, the final RMS bunch length under same condition is $5\mu m$, and the longitudinal phase space has a 'C' shape (hyperbolic shape dominated by second order effect).

1.3. Residual second order chirp

On the other hand, if there is a residual second order chirp left on the bunch, it is possible to suppress the impact from second order dispersion, under some circumference. Let us assume, the z-correlated modulation is now shown in formula (13).

$$\delta(z) = a\delta_i + h_1 z + h_{2,r} z^2 \tag{13}$$

where $h_{2,r}$ denotes the residual second order chirp.

Similarly, given an un-correlated initial energy spread $\sigma_{\delta i}$ and the optimal compression condition $1 + h_1 R_{56} = 0$, the contribution to z^4 from all the cross terms is zero, as illustrated in [2]. Insert formulae (13) and (11) into formula (6) and do the integration, similarly one finds the final bunch length for this new case is (keep up to second order terms)

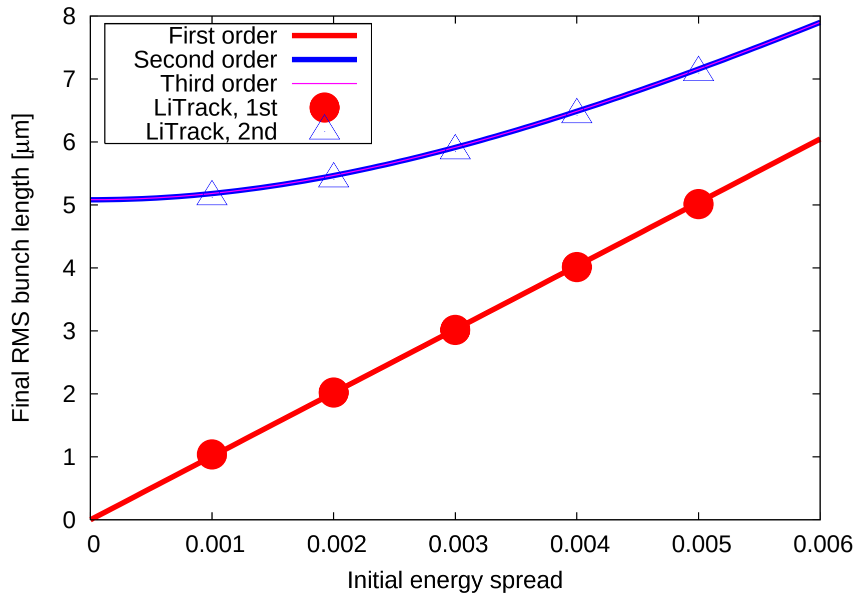

Figure 2: Minimum final bunch length as a function of the initial energy spread, consider up to first, second or third order effects. For a perfectly linearized bunch plus a four bend chicane, the third order effect can be negligible. The optimal compression condition $1 + h_1 R_{56} = 0$ is fulfilled. Good agreement between the analytical formulae and LiTrack simulation.

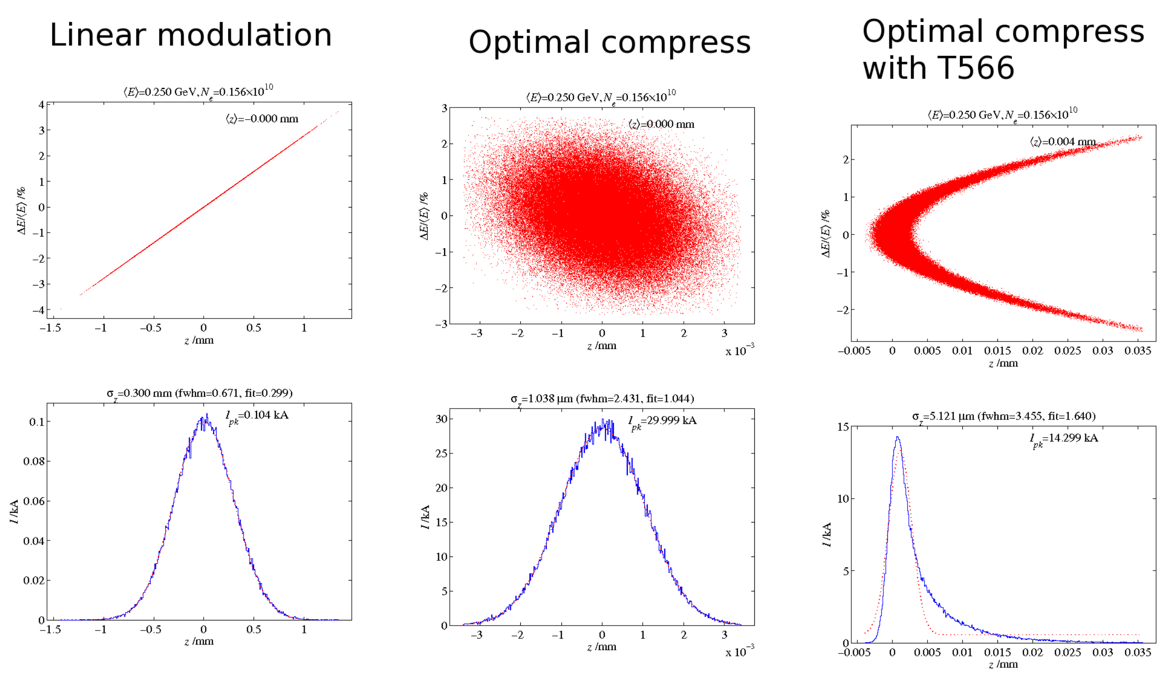

Figure 3: Longitudinal phase space and current (RMS and FWHM fitting bunch length) for 3 cases from Litrack simulation: initial condition with linear energy modulation; optimal compression with only R_{56}, a final bunch length of $1\mu m$ (FWHM fit); optimal compression with both R_{56} and T_{566}, a final bunch length of $1.7\mu m$ (FWHM fit).

$$\sigma_z{}^2 = a^2 R_{56}^2 \sigma_{\delta i}^2 + (1 + h_1 R_{56})^2 \cdot \sigma_{z,i}{}^2 + 3 \cdot (h_{2,r} \cdot R_{56} + T_{566} \cdot h_1^2)^2 \cdot \sigma_{z,i}{}^4 \tag{14}$$

where T_{566} denotes the second order dispersion.

One finds that the minimum bunch length condition reads (given that the second term on the right side of formula (14) is already zeroed, $1 + h_1 R_{56} = 0$)

$$h_{2,r} = -\frac{T_{566}}{R_{56}} \cdot h_1^2 \tag{15}$$

Again consider the LCLS BC1 which is a four bend chicane, one can easily find the second order dispersion by Taylor series, as shown below.

$$T_{566} = -\frac{3}{2} R_{56} \tag{16}$$

From the above two formulae, one find the minimum bunch length condition reads

$$h_{2,r} = \frac{3}{2} \cdot h_1^2 \tag{17}$$

Next step let us find out if one can get such a residual chirp from the harmonic RF linearization process.

1.4. Harmonic RF linearization

To linearize the RF curvature on the energy chirp, a harmonic RF section is required which has a higher frequency. The energy offset of any particle after passing by these two RF section is expressed in the following formula, where the third term on the right side is from the harmonic RF.

$$\delta(z) = \delta_i \frac{E_{i0}}{E_{f0}} + \frac{eV_0 \cos(\phi + kz_i)}{E_{f0}} + \frac{eV_h \cos(\phi_h + k_h z_i)}{E_{f1}} \tag{18}$$

where V_h denotes the RF voltage of harmonic RF, E_{f1} central energy after RF acceleration and deceleration with harmonic RF, V_h harmonic RF voltage, ϕ_h harmonic RF phase, $k_h = \frac{2\pi f_h}{c}$ the harmonic RF wave number, f_h the RF frequency and c speed of light, δ_i the initial un-correlated energy offset, E_{i0} central energy before RF acceleration, E_{f0} central energy after RF acceleration, V_0 RF voltage, ϕ RF phase, $k = \frac{2\pi}{\lambda}$ the RF wave number, λ the RF wave length and z_i particle's longitudinal coordinate relative to the bunch center.

Up to second order, it is easy to write down the overall energy chirp, after passing by these two RF sections, as shown in the formulae below.

$$h_1 = h_{1s} + h_{1h} = -\frac{keV_0 \sin\phi}{E_{f0}} - \frac{k_h eV_h \sin\phi_h}{E_{f1}} \tag{19}$$

$$h_2 = h_{2s} + h_{2h} = -\frac{k^2 eV_0 \cos\phi}{2E_{f0}} - \frac{k_h^2 eV_h \cos\phi_h}{2E_{f1}} \tag{20}$$

where h_{1s} denotes the first order chirp, h_{2s} second order chirp (the name 's' reflects that it is assumed to be s-band as for the normal LCLS case), h_{1h} the first order harmonic chirp, h_{2h} second order harmonic chirp.

It is observed that two conditions need to be fulfilled, in order to generate a linearized bunch up to second order. One is to make use of a higher frequency harmonic RF, and the voltage of the harmonic RF is inversely proportional

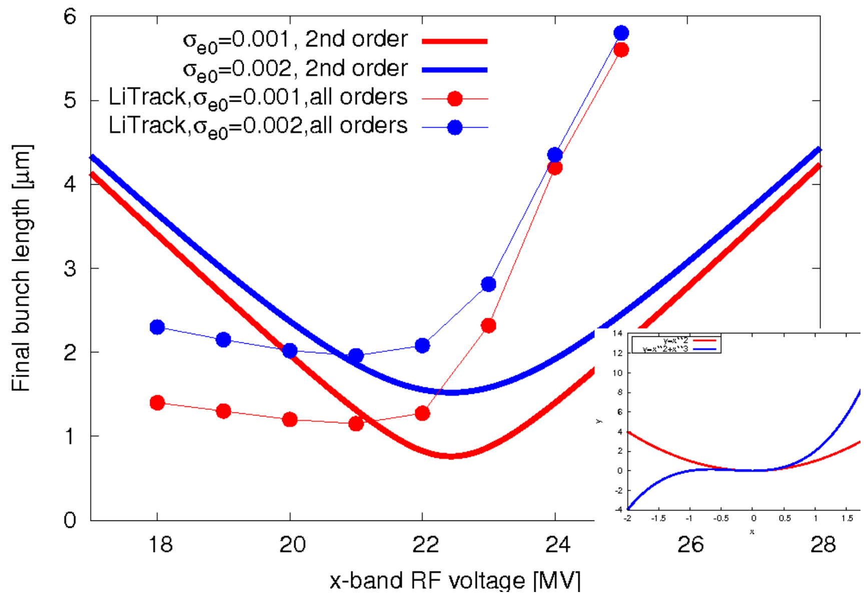

Figure 4: Minimum final bunch length as a function of the x-band RF voltage, for two different initial un-correlated energy spread. Dots represent simulation results from LiTrack where the energy chirp to any order is included, while curves are plotted from formulae (14) and (23) where the terms up to second order is kept.

to the square of its frequency f_h^2. The second one is that one need to choose a decelerating phase for the harmonic RF.

A simple relation to zero the overall second order energy chirp is shown in formula (21). One observes that for a given RF frequency of both RF sections and fixed design energy E_{f0} plus some specified initial energy E_{i0}, the energy loss from the harmonic RF $V_h \cos\phi_h$ equals a constant.

$$V_h \cos\phi_h \left(E_{f0} \cdot k_h^2 + k^2 \cdot (E_{f0} - E_{i0})\right) = -E_{f0} k^2 \cdot (E_{f0} - E_{i0}) \tag{21}$$

Let us consider a special case with $\phi_h = 180°$, in that case formula (19) is simplified to be formula (22). In that case, the linear energy chirp is only dependent on the RF acceleration in the first RF section, and has no relation with the harmonic RF.

$$h_1 = h_{1s} + h_{1h} = -\frac{keV_0 \sin\phi}{E_{f0}} \tag{22}$$

At the same time, the second order energy chirp now is

$$h_2 = h_{2s} + h_{2h} = -\frac{k^2 eV_0 \cos\phi}{2E_{f0}} + \frac{k_h^2 eV_h}{2(E_{f0} - V_h)} \tag{23}$$

There is a unique harmonic RF voltage V_{h0} to zero the second order chirp h_2. For a lower voltage than V_{h0}, the second order chirp h_2 goes negative, and it is the opposite for a higher voltage.

As discussed in section 1.2, one need to further increase the harmonic RF voltage V_h so that the condition $h_{2,r} = \frac{3}{2} \cdot h_1^2$ is fulfilled, in order to achieve a minimum bunch length.

Let us take some real parameters and apply the above derivations. The initial electron energy is $5MeV$ from the photoinjector, which is then accelerated up to roughly $265MeV$ through a $3GHz$ RF section with a RF phase of $-22°$. After that the electron bunch passes by a $12GHz$ harmonic RF section and decelerated at a RF phase

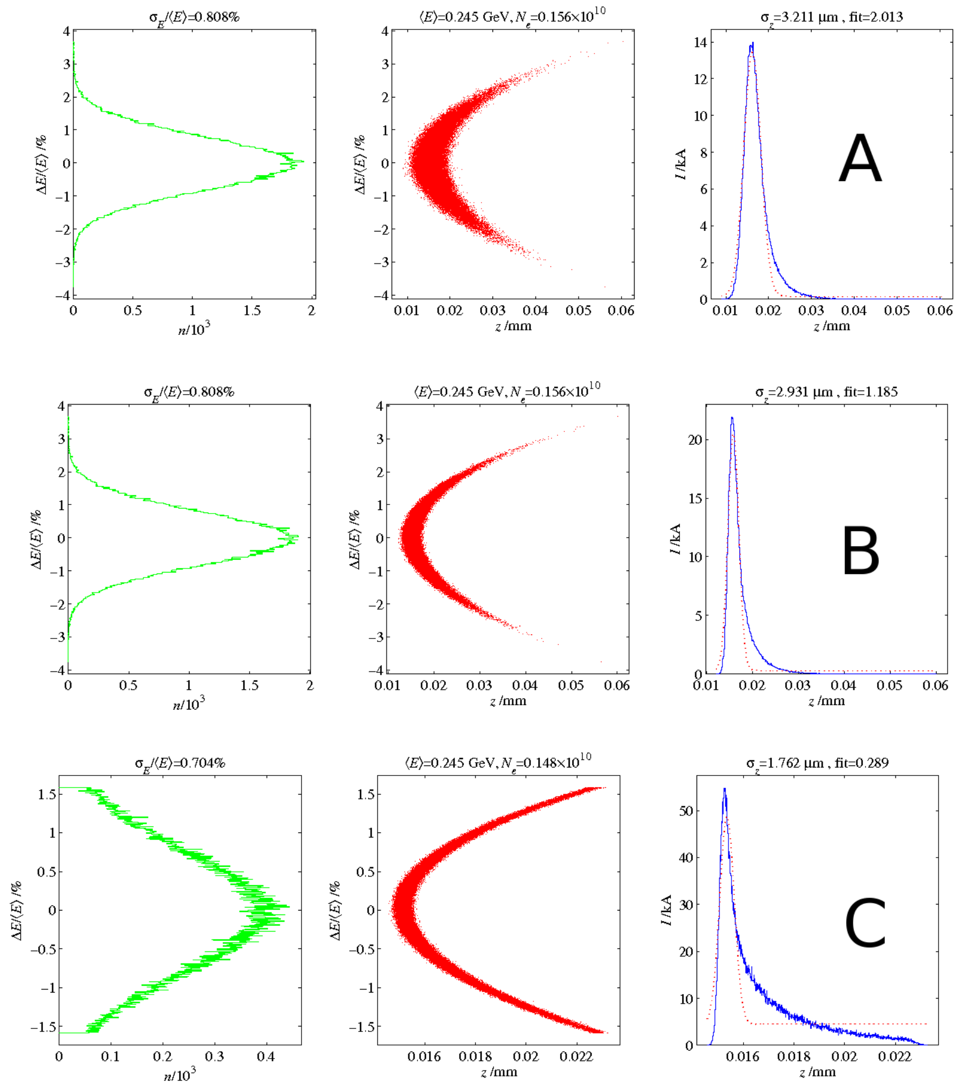

Figure 5: Final longitudinal phase space after chicane for three cases: (A) initial un-correlated energy spread of 0.2%; (B) initial un-correlated energy spread of 0.1%; (C) initial un-correlated energy spread of 0.02%.

of $-180°$. Finally the beam passes by a four bend chicane which has a first order dispersion of $R_{56} = -37mm$. The final bunch length as a function of the harmonic RF voltage is shown in Figure 4 by curves, with two different initial un-correlated energy spread, according to formulae (14) and (23). Tracking simulation is also performed in LiTrack [3], with the same parameters and the results are shown in Figure 4 with dots plus curves. It is observed that considering up to second order terms, the curve has a hyperbolic shape and the minimum bunch length is achieved at a harmonic RF voltage of 22.5 MV. With all the terms included up to any order (as done in the Litrack simulation), the shape of the curve is deformed mainly by third order (cubic) term, and the minimum bunch length location shifts to 20-21 MV.

In the case that one manages to zero (minimize) the second and third terms on the right side of formula (14), the minimum final bunch length is dominanted by the initial un-correlated energy spread, given a fixed R_{56} from the bunch compressor. That is confirmed by the LiTrack simulation results as shown in Figure 5, for three cases (A),

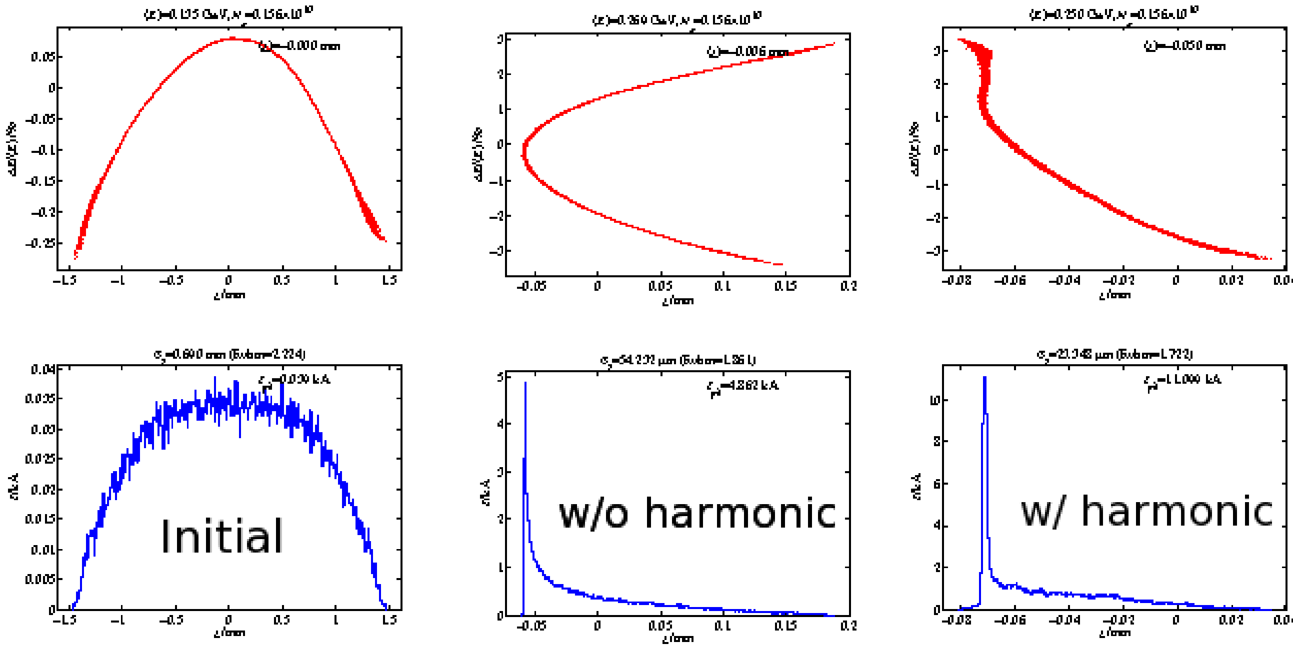

Figure 6: Left: initial beam at a beam energy of 135 MeV; middle: after BC1, without harmonic RF, at a beam energy of 269 MeV, peak current of 5 kA; Right: after BC1, harmonic RF voltage equals 19 MV with a RF phase of -180 degree, at a beam energy of 250 MeV, peak current of 10 kA.

(B) and (C). If the initial un-correlated energy spread can be decreased from 0.2% to 0.02%, then it is possible to achieve a bunch length of $0.3\mu m$, instead of $2\mu m$ (from Full Width at Half Maximum calculation, FWHM).

2. LCLS BC1

For LCLS BC1, the condition to achieve a minimum bunch length is more complicated, due to longitudinal dispersion in the first dogleg, coherent synchrotron radiation (CSR), wake field and space charge effects.

An example is shown in Figure 6, with first dogleg, wake field and space charge effects in consideration. The simulation starts at a beam energy of 135 MeV, a bunch charge of 250 pC and a RMS bunch length of $690\mu m$. The initial bunch distribution is read in from an external file 'OTR2-250pC-690um.zd', which has been generated in an earlier simulation that considers space charge and velocity bunching effects. The energy spread introduced by the laser heater is decreased to 2×10^{-5}. The beam then passes by the first dogleg which has $R_{56} = 6.3mm$ and $T_{566} = 140mm$, and is accelerated to $269MeV$ through an $8.78m$ long s-band RF section (with wake fields), on a RF phase of -36 degree. The next two sections are a $0.6m$ long x-band harmonic RF section (with wake fields) and the first bunch compressor. The x-band RF phase is set to be -180 degree so that it does not change the first order energy chirp. It is found that under these specified conditions, a maximum current of 10 kA after BC1 is achieved with a harmonic RF voltage of 19 MV, as shown in Figure 6.

There are other possible configurations to achieve a shorter bunch length and higher current. For example, one can simply change the harmonic RF phase to be '-183' degree, and keep the same final energy. In that case the peak current is increased to be 20 kA.

Further detailed study is needed, to figure out the minimum bunch length condition for LCLS BC1, with the wake field and also CSR effects in consideration.

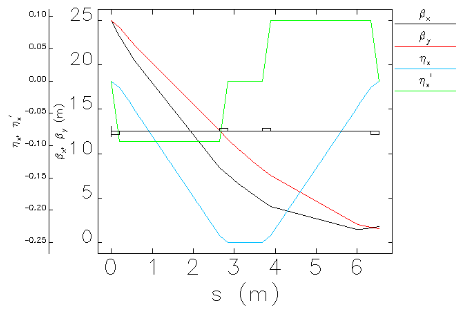

Figure 7: LCLS BC1 optics.

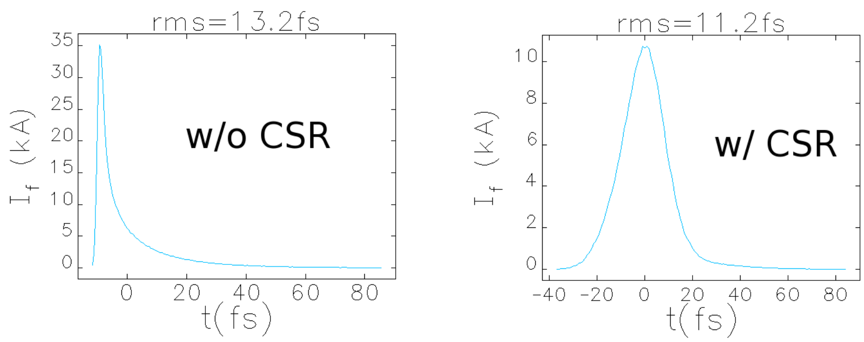

Figure 8: Electron beam current.

3. Elegant simulation

Some preliminary simulations have been performed in Elegant [4], to roughly evaluate the impact of Coherent Synchrotron Radiation (CSR) effect. The LCLS BC1 optics being used is shown in Figure 7.

The beam and optics parameters are similar as introduced in the previous sections. Here an electron bunch with linearized energy chirp is generated in Elegant and then passes by the four dipole chicane (LCLS BC1). The value of the chirp is chosen so that the optimal compression condition $1 + h_1 R_{56} = 0$ is fulfilled. As the bunch length is short in the third and fourth dipole magnets, also in the drift space, the CSR effect plus phase space smear effect may influence the available peak current and transverse emittance. In Figure 8, the current distribution is compared between two cases, with and without CSR effect.

At the same time, the final longitudinal phase space is also compared between these two cases, as shown in Figure 9. The transverse emittance is increased from $0.5\mu m$ to $7\mu m$ with CSR. A larger un-correlated initial energy spread can provide 'Landau damping' and partially suppress the CSR impact. However, it also degrades the available peak current.

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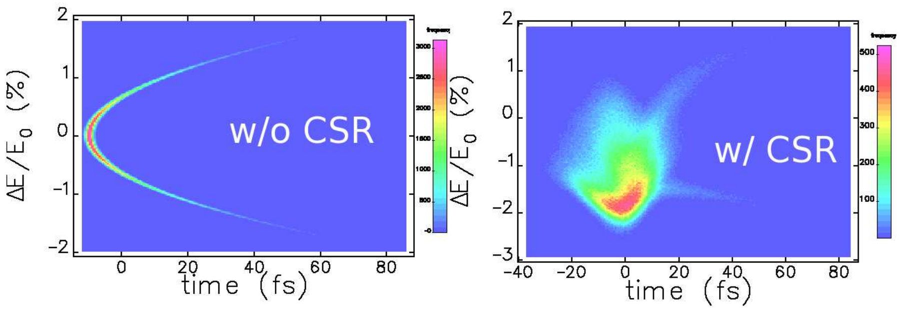

Figure 9: Longitudinal phase space.

References

[1] Linac Coherent Light Source Conceptual Design Report, SLAC-R-593, SLAC (2002); P. Emma et al., Nature Photonics **4**, 641-647 (2010)

[2] Yipeng Sun etc., SLAC-pub-14327, 2010.

[3] K.L.F. Bane and P. Emma, "Litrack: A Fast Longitudinal Phase Space Tracking Code with Graphical User Interface", Proceedings of the 2005 Particle Accelerator Conference (PAC 2005), 16-20 May 2005, pp. 4266-4268.

[4] M. Borland, "elegant: A Flexible SDDS-Compliant Code for Accelerator Simulation," Advanced Photon Source LS-287, September 2000.