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Long-Range Lepton Flavor Interactions and Neutrino Oscillations

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Recent results from the MINOS accelerator neutrino experiment suggest a possible difference between ν_μ and $\bar{\nu}_\mu$ disappearance oscillation parameters, which one may ascribe to a new long-distance potential acting on neutrinos. As a specific example, we consider a model with gauged $B - L_e - 2L_\tau$ number which contains an extremely light new vector boson, $m_{Z'} < 10^{-18}$ eV and extraordinarily weak coupling $\alpha' \lesssim 10^{-52}$. In that case, differences between $\nu_\mu \rightarrow \nu_\tau$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$ oscillations can result from a long-range potential due to neutrons in the Earth and the Sun that distinguishes ν_μ and ν_τ on Earth, with a potential difference of $\sim 6 \times 10^{-14}$ eV, and changes sign for anti-neutrinos. We show that existing solar, reactor, accelerator, and atmospheric neutrino oscillation constraints can be largely accommodated for values of parameters that help explain the possible MINOS anomaly by this new physics, although there is some tension with atmospheric constraints. A long-range interaction, consistent with current bounds, could have very pronounced effects on atmospheric neutrino disappearance in the 20 - 50 GeV range that will be studied with the IceCube DeepCore array, currently in operation, and can have a significant effect on future high-precision long-baseline oscillation experiments which aim for $\pm 1\%$ sensitivity, in ν_μ and $\bar{\nu}_\mu$ disappearance, separately. Together, these experiments can extend the reach for new long-distance effects well beyond current bounds and test their relevance to the aforementioned MINOS anomaly. We also point out that long-range potentials originating from the Sun could lead to annual modulations of neutrino data at the percent level, due to the variation of the Earth-Sun distance. A similar phenomenology is shown to apply to other potential new gauge symmetries such as $L - 3L_\tau$ and $B - 3L_\tau$.

I. INTRODUCTION

Neutrino flavor oscillation experiments have provided some of the most direct and robust indications of physics beyond the Standard Model (SM). Solar, atmospheric, reactor, and accelerator data all point to the conclusion that at least 2 active neutrinos have tiny but non-zero masses of up to order 0.1 eV, whose generation requires extending the SM. We refer the interested reader to Refs. [1, 2], for a review of the extensive literature on neutrino oscillation physics. Given the smallness of neutrino mass differences, even minute perturbations to the time evolution of flavor eigenstates, caused by feeble differences of interactions of neutrinos with background sources, can produce measurable departures from vacuum oscillations. For example, these effects can be caused by the short-distance electroweak interactions of neutrinos with solar or terrestrial electrons, referred to as the Mikheev-Smirnov-Wolfenstein (MSW) effect [3, 4]. The sensitivity of neutrino oscillations to such small effects makes them a good probe of new physics that violates ν_e - ν_μ - ν_τ universality [5]. Hence it is interesting to look for unexpected effects in neutrino data.

Recently, measurements at the MINOS experiment [6] have resulted in different inferred values for differences of squared masses and mixing angles

$$|\Delta m_{23}^2| = 2.35_{-0.08}^{+0.11} \times 10^{-3} \text{ eV}^2; \sin^2(2\theta_{23}) = 1.00 \quad (1)$$

[where $\sin^2(2\theta_{23}) = 1.00$ is the best fit value, while $\sin^2(2\theta_{23}) > 0.91$ at 90% confidence level] and

$$|\Delta \bar{m}_{23}^2| = 3.36_{-0.40}^{+0.45} \times 10^{-3} \text{ eV}^2; \sin^2(2\bar{\theta}_{23}) = 0.86 \pm 0.11 \quad (2)$$

in ν_μ and $\bar{\nu}_\mu$ disappearance, respectively. The above MINOS results have revived some interest in long-range interactions (LRIs) [7] that can cause disparities between neutrinos and anti-neutrinos. For other related works on the MINOS anomaly, see, for instance, Refs. [8–10].

The possibility of new long-range forces was discussed in the pioneering work of Ref. [11], and subsequently considered as an alternative way to explain apparent CP violating effects in K meson decays [12, 13]. Note that the disparity in the oscillation parameters for neutrinos and anti-neutrinos, as suggested by the MINOS results (1) and (2), can be ascribed to an apparent violation of CPT [8, 14]. However, in what follows we will assume that CPT is conserved in vacuo and consider the possibility that the MINOS result could be a hint of a new LRI. Eötvös-type [15] tests of gravity place stringent bounds on these interactions [11], constraining their “fine structure constant” $\alpha' \leq 10^{-49}$ (electron coupling) and $\alpha' \leq 10^{-47}$ (nucleon coupling) [16, 17]. This suggests an astrophysical source with a large number of particles is needed, for sizable long-range effects. Long-range interactions in neutrino oscillations were considered in Refs. [18–21]; see also Ref. [22]. The long-range vector interaction yields equal and opposite potentials for leptons and anti-leptons. This can then manifest itself as a difference in the properties of neutrinos and anti-neutrinos in terrestrial oscillation experiments, caused by the collective effect of particles in the Sun and the Earth charged under a new $U(1)'$ gauge symmetry. The correspond-

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ing effective fine structure constant must be extremely small, $\lesssim \mathcal{O}(10^{-49} - 10^{-47})$, as required by precision tests of gravity [23]. For comparison, note that the effective gravitational coupling between two protons is of order $\alpha_g \sim G_N m_p^2 \sim 10^{-38}$, where G_N is Newton's constant and m_p is the proton mass. Here, it is assumed that the associated Z' vector boson has a mass $m_{Z'} < 1/R_{\text{AU}}$, where $R_{\text{AU}} = 1 \text{ AU} \simeq 1.50 \times 10^8 \text{ km} \sim 10^{18} \text{ eV}^{-1}$ is the mean Earth-Sun distance. (Later, we will limit our discussion to values of $m_{Z'}$ that are not far below 10^{-18} eV , in order to exclude contributions from the rest of the Galaxy.)

Before going further, we would like to make a few comments regarding the results (1) and (2). First, the suggested MINOS anomaly is not at statistically significant levels, being at most a 2-sigma effect. In addition, the available atmospheric data from MINOS yield the best fit [24] (2-state mixing)

$$|\Delta m^2| - |\Delta \bar{m}^2| = 0.4_{-1.2}^{+2.5} \times 10^{-3} \text{ eV}^2 \quad (3)$$

which does not support the above accelerator results, and, while also statistically limited, very mildly prefers an opposite sign for the effect. Taken together, the above considerations do not provide a strong case for invoking new physics. Nevertheless, we find the MINOS accelerator data sufficiently intriguing to motivate an examination of the prospects for probing long-range leptonic forces at current and future experiments, as detailed below.

In what follows, we will discuss the possibility of attributing the aforementioned MINOS anomaly to a LRI potential, generated by the neutrons in the Earth and the Sun. We will show that the existing bounds from neutrino oscillation data do not exclude such an interpretation. We use our approximate fit as a benchmark for potentially interesting values of parameters and estimate the reach of current and future experiments for the new LRI. We find that the IceCube DeepCore array [25], which is currently in operation, can provide an excellent probe of the benchmark model parameters and reach well-beyond them. We point out that long-range potentials generated by solar particles will inevitably lead to *annual modulation* of neutrino oscillations at Earth, due to the variation of the Earth-Sun distance. The large event sample expected at DeepCore seems sufficient to uncover a possible effect at the 1% level, statistically. Observation of such modulations can provide a distinct clue as to the solar contribution to the LRI and set a lower bound on its range. We will also consider long-baseline experiments, such as those envisioned for the Deep Underground Science and Engineering Laboratory (DUSEL) [26], to discover or constrain various effects of the LRI. We find that the expected capabilities of these experiments would allow them to probe the difference between the oscillation parameters of neutrinos and anti-neutrinos, induced by LRIs, which is a key signal for this type of new physics. Although we concentrate on the case of a potential generated by neutrons, our results and

analysis carry over to other interesting scenarios where, for example, electrons or the total baryon number of the Sun and Earth may be responsible for the LRI.

We will next briefly present the basic setup and formalism used in our work. Section III will contain our analysis and results. Our concluding remarks will be presented in section IV.

II. FORMALISM

Let us consider the addition of a general anomaly free $U(1)'$ gauge quantum number (for vectorial representations) to the SM [27, 28]

$$\mathcal{Q} = a_0(B-L) + a_1(L_e - L_\mu) + a_2(L_e - L_\tau) + a_3(L_\mu - L_\tau), \quad (4)$$

where B and L are baryon and lepton numbers, respectively, while $L_\ell, \ell = e, \mu, \tau$ are lepton flavor numbers, and $a_i, i = 0, 1, 2, 3$, are arbitrary constants. For our primary example, we will set $a_1 = a_2 = 0$ and for definiteness take $a_0 = a_3 = 1$. However, any values of a_0 and a_3 will lead to the same neutrino oscillation phenomenology for a fixed coupling between $B-L$ and $L_\mu - L_\tau$. In this combination of quantum numbers,

$$\mathcal{Q} = (B-L) + (L_\mu - L_\tau) = B - L_e - 2L_\tau, \quad (5)$$

$(B-L)$ is associated with the source of the new potential, while $(L_\mu - L_\tau)$ provides a contribution to the relevant neutrino oscillation $\nu_\mu - \nu_\tau$. Our choice for \mathcal{Q} in Eq. (5), as we will later argue, is less constrained by experiments than the previously studied $L_e - L_\ell, \ell = \mu, \tau$, cases. It also follows that the LRI potential due to $B-L$ that we consider is generated by the total neutron number, since the contributions of electrons and protons cancel.

Our charge assignment provides a simple way of achieving the effective coupling in Ref. [7], where the microscopic origin of the requisite interactions is a mixing between a Z' associated with $L_\mu - L_\tau$ number and the Z boson of the SM. In principle, one could also imagine a mixing between two Z' states associated with, say, $B-L$ and $L_\mu - L_\tau$, where an appropriate choice of mixing parameters will yield the effective scenario adopted here. Given that our main purpose in this work is to elucidate the relevant phenomenology, without reference to a particular underlying theoretical context, our choice of the gauged quantum number captures all the relevant key features for our analysis, while avoiding unnecessary complications. Note that as long as one of the anomaly-free quantum numbers is carried by a main constituent of solar or terrestrial matter, with the other lepton flavor number differences, one can build models that result in qualitatively similar effects. Indeed, we later consider models with gauged $L-3L_\tau$ and $B-3L_\tau$ that exhibit essentially the same phenomenology as that of $B-L_e-2L_\tau$, but with even smaller gauge couplings.

The range of the interaction corresponding to charge \mathcal{Q} is determined by the mass $m_{Z'}$ of the force carrier Z' .

Since we are interested in the effect of a large number of particles, we assume that $m_{Z'} \lesssim 10^{-18}$ eV so that the neutrons both in the Earth and the Sun can contribute. However, we will not consider $m_{Z'} \ll 10^{-18}$ eV so that our assumed LRI does not extend far beyond the solar system and the contribution of the rest of the galaxy can be ignored. The resulting potential felt by neutrinos on the Earth is then given by

$$V_n = \alpha' \left(\frac{N_n^\oplus}{R_\oplus} + \frac{N_n^\odot}{R_{\text{ES}}} \right) = 2.24 \times 10^{-12} \text{ eV} \\ \times \left(\frac{\alpha'}{10^{-50}} \right) \left[0.25 + \left(\frac{R_{\text{AU}}}{R_{\text{ES}}} \right) \right], \quad (6)$$

using the estimated solar neutron fraction $Y_n^\odot = 1/7$ (*i.e.* $N_p^\odot/N_n^\odot \simeq 6$, where N_p^\odot is the number of protons in the Sun) and $Y_n^\oplus = 1/2$ in the Earth. In Eq. (6), $N_n^\oplus = 1.78 \times 10^{51}$ and $N_n^\odot = 1.70 \times 10^{56}$ are numbers of neutrons in the Earth and the Sun, respectively, $R_\oplus = 6.4 \times 10^3$ km is the Earth's radius, and R_{ES} is the variable distance of the Earth from the Sun. We note that R_{ES} attains its maximum value $R_{\text{ES}}^a \simeq 1.52 \times 10^8$ km at the aphelion (\sim July 4) and its minimum value $R_{\text{ES}}^p \simeq 1.47 \times 10^8$ km at the perihelion (\sim January 4).

The ratio of the potential V_n^\oplus at the Earth's surface from its neutrons to V_n^\odot from solar neutrons is given by

$$\frac{V_n^\oplus}{V_n^\odot} \approx \frac{1}{4}. \quad (7)$$

Thus, the contribution of the Earth-generated potential is sub-dominant but not negligible. Note that if electrons are the source of the long-range potential, one can show that the solar contribution is roughly 24 times larger than that generated by the Earth [18, 21].

As discussed in Refs. [5, 18, 19], the ν_μ survival probability in the 2 flavor $\nu_\mu - \nu_\tau$ oscillation [for $\sin^2(2\theta_{13}) \simeq 0$] is given by

$$\tilde{P}_{\mu\mu} = 1 - \sin^2(2\tilde{\theta}_{23}) \sin^2 \left(\frac{\Delta\tilde{m}_{23}^2 L}{4E_\nu} \right), \quad (8)$$

where

$$\Delta\tilde{m}_{23}^2 = \Delta m_{23}^2 \sqrt{[\xi - \cos(2\theta_{23})]^2 + \sin^2(2\theta_{23})} \quad (9)$$

and

$$\sin^2(2\tilde{\theta}_{23}) = \frac{\sin^2(2\theta_{23})}{[\xi - \cos(2\theta_{23})]^2 + \sin^2(2\theta_{23})}. \quad (10)$$

Here, the symbols that are tilde-free denote vacuum quantities, and

$$\xi \equiv -\frac{2W_\tau E_\nu}{\Delta m_{23}^2}, \quad (11)$$

with $W_\tau = Q_\tau V_n$ the potential energy for ν_τ ; $Q_\tau = -2$ is the charge of ν_τ , in our model. One can obtain the

$\bar{\nu}_\mu$ survival probability from the above expressions by $\xi \rightarrow -\xi$, and there is a degeneracy if both Δm_{23}^2 and $\cos(2\theta_{23})$ change sign. Note that if $\sin^2(2\theta_{23}) = 1$ the formalism yields the same results for ν and $\bar{\nu}$. It should also be noted that the $\Delta\tilde{m}_{23}^2$ and $\sin^2(2\tilde{\theta}_{23})$ are energy-dependent for $\alpha' \neq 0$ and deviations from the vacuum values tend to increase with energy.

The $\sin^2(2\tilde{\theta}_{23})$ and $\Delta\tilde{m}_{23}^2$ measure the depth and location ($E_\nu \approx \Delta\tilde{m}_{23}^2 L/2\pi$) of the first oscillation minimum in the survival probability $\tilde{P}_{\mu\mu}$ versus energy E_ν . With $X \equiv |\xi - \cos(2\theta_{23})|$, in the limit of $X \simeq 0$ (resonance condition), we have $\sin^2(2\tilde{\theta}_{23}) \simeq 1$ and $\Delta\tilde{m}_{23}^2 \simeq \Delta m_{23}^2 \sin(2\theta_{23})$. As X increases, $\sin^2(2\tilde{\theta}_{23})$ decreases and $\Delta\tilde{m}_{23}^2$ increases. For a negligibly small $\cos(2\theta_{23})$ where $X \simeq |\xi| = |-2Q_\tau V_n E_\nu/\Delta m_{23}^2|$, $\sin^2(2\tilde{\theta}_{23})$ decreases and $\Delta\tilde{m}_{23}^2$ increases with E_ν for both ν and $\bar{\nu}$, for a given Δm_{23}^2 . For a sizable $\cos(2\theta_{23})$, X may increase/decrease with E_ν depending on the relative sign of the two terms in X as long as $|\xi| < |\cos(2\theta_{23})|$. This means deviations from the standard oscillations are different for ν and $\bar{\nu}$. One of them might have accidental cancellation in X , and the deviation from the standard oscillation can be much larger for one of them. If a new potential (or in general X) is too large, $\sin^2(2\tilde{\theta}_{23}) \approx 0$ and the oscillation would be quenched.

III. PHENOMENOLOGY

In this section, we will examine the implications of new LRIs for current and future experiments. As a guide for our following discussion, we first derive, for the $U(1)'$ of Eq.(5) where neutrons are responsible for distinguishing ν_μ and ν_τ , an approximate bound on α' based on the MINOS ν_μ disappearance data (which dominate the statistics [6]). At 3-sigma, we roughly get $\alpha' < 5 \times 10^{-52}$, assuming $\cos(2\theta_{23}) = 0$ [corresponding to the best fit value $\sin^2(2\theta_{23}) = 1$ without new physics]. However, in order to address the disparity between the parameters of neutrinos and antineutrinos suggested by the MINOS results (1) and (2), we will consider allowing $\cos(2\theta_{23}) \neq 0$ within the LRI scenario. We will next perform an approximate fit of the aforementioned MINOS results, obtained at a baseline of $L = 735$ km, within our reference model. Given the low statistical significance of the anti-neutrino results (~ 100 events [6]), the fit is dominated by the neutrino data points (~ 2000 events [6]). For convenience, we employ the simplified MINOS data used in Ref. [10], subtracting the neutral current background from the oscillated signals. For the entire neutrino and anti-neutrino data set (23 bins) we find the best fit vacuum parameters:

$$\Delta m_{23}^2 = 2.4 \times 10^{-3} \text{ eV}^2; \quad \sin^2(2\theta_{23}) = 0.89 \quad (12)$$

and

$$\alpha' = 1.0 \times 10^{-52}, \quad (13)$$

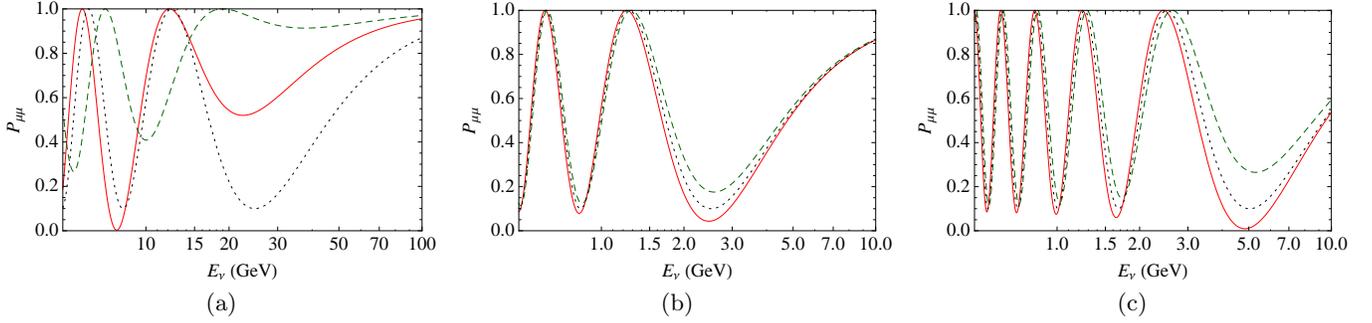


FIG. 1: Survival probability $P_{\mu\mu}$ for ν_μ and $\bar{\nu}_\mu$ with E_ν without (black dotted curve) and with the LRI (red solid curve for ν_μ and green dashed curve for $\bar{\nu}_\mu$), for $\Delta m_{23}^2 = 2.4 \times 10^{-3} \text{ eV}^2$, $\sin^2(2\theta_{23}) = 0.9$, and $\alpha' = 1.0 \times 10^{-52}$. Typical values for the baselines have been chosen: (a) $L = 2 \times 6400 \text{ km}$ (DeepCore), (b) $L = 1300 \text{ km}$ (DUSEL), and (c) $L = 2 \times 1300 \text{ km}$. The neutrino and anti-neutrino survival probabilities are different from each other in the presence of the LRI, since $\sin^2(2\theta_{23}) \neq 1$.

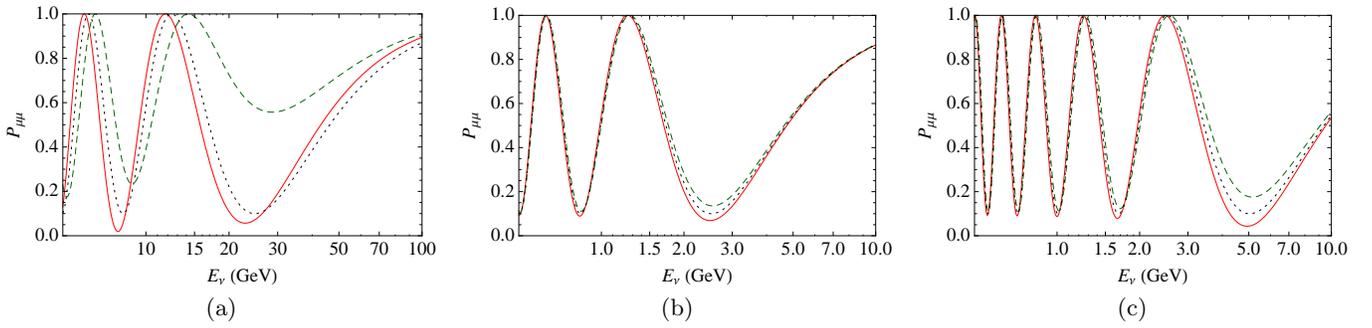


FIG. 2: Same as Fig. 1 except for $\alpha' = 0.5 \times 10^{-52}$.

with $\chi^2 = 20.4$ for 20 degrees of freedom. The above fit represents the preferred values of parameters in the presence of the LRI, although we find that the goodness of fit is basically the same as the standard oscillations with no new physics; this was also the case in Ref. [7], where a fit but with a larger effective coupling was obtained. However, the parameters in Eqs. (12) and (13) capture the implications of a new physics effect on the ν_μ and $\bar{\nu}_\mu$ data. Next, we will examine the implications of existing bounds for our fit to the MINOS results.

First, let us consider the constraints from the solar and KamLAND data. The bound obtained in Refs. [20, 21] by comparing KamLAND and solar neutrino data leads roughly, for our neutron (rather than electron) based potential, to the increased bound $\alpha' < 6 \times 2.5 \times 10^{-53} / \cos(2\theta_{23}) = 1.5 \times 10^{-52} / \cos(2\theta_{23})$ at 3 sigma; the factor of 6 comes from $N_n^\odot / N_e^\odot \simeq 1/6$, with N_e^\odot the number of electrons in the Sun. For our value of $\cos(2\theta_{23}) \simeq 0.3$, the bound becomes $\alpha' < 5 \times 10^{-52}$ which is about the same as the rough MINOS bound given above and easily satisfied by our new physics scenario. To see how the quantity $\cos(2\theta_{23})$ enters into the bound with our choice of gauged quantum number, note that the solar neutrino oscillations can be described by two flavors: $\nu_e \leftrightarrow \nu_x$, where $\nu_x \equiv \cos\theta_{23}\nu_\mu - \sin\theta_{23}\nu_\tau$, assuming $\theta_{13} \rightarrow 0$ [the

present bound is $\sin^2(2\theta_{13}) < 0.15$, at 90% confidence level [2]]. A third eigenstate $\nu_y \equiv \sin\theta_{23}\nu_\mu + \cos\theta_{23}\nu_\tau$ decouples in this limit (for more details see Ref. [21]). With our choice of $Q = B - L_e - 2L_\tau$, we get

$$\langle \nu_e | H_{\text{LRI}} | \nu_e \rangle \propto Q_e \langle \nu_e | \nu_e \rangle = -1 \quad (14)$$

$$\langle \nu_x | H_{\text{LRI}} | \nu_x \rangle \propto Q_x \langle \nu_x | \nu_x \rangle = -2 \sin^2 \theta_{23}, \quad (15)$$

where H_{LRI} is the contribution of the LRI to the Hamiltonian. Since subtracting a matrix proportional to the identity in the evolution equation does not alter the oscillations, we have effectively

$$H_{\text{LRI}}^{\text{eff}} \propto \begin{pmatrix} 0 & 0 \\ 0 & \cos 2\theta_{23} \end{pmatrix} \quad (16)$$

for $\nu_e - \nu_x$ oscillations, which yields the aforementioned suppression by $\cos(2\theta_{23})$.

We now turn to the atmospheric constraints, which turn out to be the tightest. The effects of new physics on neutrino oscillations are often discussed in terms of coefficients $\varepsilon_{\ell\ell'}$ [29] which parametrize the strength of the additional contributions in units of $\sqrt{2}G_F n_e$, the standard MSW matter effect, with G_F the Fermi constant and n_e the electron number density of the relevant medium. The analyses in Ref. [29] yields, at the

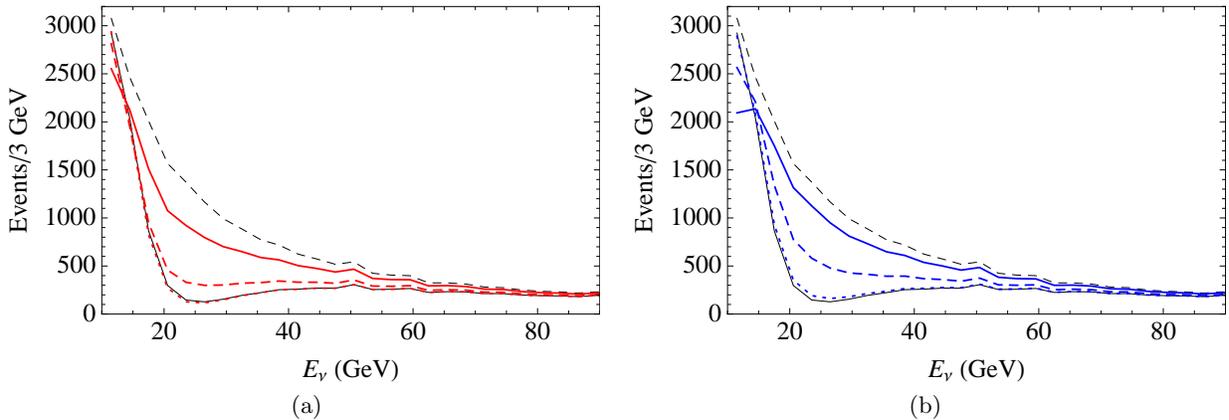


FIG. 3: Atmospheric neutrinos per year (per 3 GeV) with E_ν at the IceCube DeepCore, with a simplifying assumption that all neutrinos traverse an exactly vertical path. Shown are the unoscillated case (top black dashed curves), and the case of no new physics (bottom thin solid black curves), as well as the cases $\alpha' = 1.0, 0.5, 0.1 \times 10^{-52}$ corresponding to thick solid, dashed, and dotted curves, respectively. The vacuum parameters are $\sin^2(2\theta_{23}) = 0.9$ with (a) $\Delta m_{23}^2 = 2.4 \times 10^{-3} \text{ eV}^2$ and (b) $\Delta m_{23}^2 = -2.4 \times 10^{-3} \text{ eV}^2$.

95% confidence level, the upper bound $\varepsilon_{\tau\tau} \lesssim 0.2$. For atmospheric neutrinos traveling through the Earth, assuming an average density of roughly 6 g cm^{-3} , corresponding to $n_e \approx 1.4 \times 10^{10} \text{ eV}^3$, that constraint implies $W_\tau = \varepsilon_{\tau\tau} \sqrt{2} G_F n_e < 4.6 \times 10^{-14} \text{ eV}$. Our MINOS fit corresponds to $W_\tau \approx 5.6 \times 10^{-14} \text{ eV}$. The value of W_τ from the LRI is however approximately constant throughout the Earth, due to the dominance of the solar contribution, whereas the density of the Earth, sampled by neutrinos traveling along its diameter, varies from roughly 12 g cm^{-3} in the core to 5 g cm^{-3} in the mantle, giving an average of about 8.5 g cm^{-3} which corresponds to a somewhat larger $W_\tau \sim 6 \times 10^{-14} \text{ eV}$. Therefore, while our fit appears to have some tension with the atmospheric constraint, a more detailed analysis is called for, in order to pinpoint a precise constraint on the contribution from the LRI we have assumed. Henceforth, we will adopt $\alpha' = 1.0 \times 10^{-52}$ as a plausible benchmark value for further exploration by experiments, where the potential MINOS anomaly suggests that new physics may appear. For our numerical illustrations, we will use vacuum parameters $\Delta m_{23}^2 = 2.4 \times 10^{-3} \text{ eV}^2$ and $\sin^2(2\theta_{23}) = 0.9$, which are motivated by our fit results in Eq. (12) and are also reasonable in the standard scenario without new physics. In this way, we can focus on the effect of the LRI potential on neutrino oscillations, given specific vacuum parameters that could in principle be determined with high precision from other measurements.

We also mention that the same type of study outlined above can easily be applied to other $U(1)'$ gauge groups with different quantum numbers and LRIs. Two interesting examples, gauged $L - 3L_\tau$ and $B - 3L_\tau$, turn out to have essentially the same effect on MINOS as well as atmospheric and long baseline neutrinos as our $B - L_e - 2L_\tau$ example, but with smaller values for α' of 1.3×10^{-53} and 1.1×10^{-53} , respectively. Here, the smaller values of α'

are compensated by the larger numbers of electrons and baryons.

With the completion of the DeepCore array, the IceCube experiment will be able to probe atmospheric neutrino oscillations for energies of order 10 GeV and above, well beyond the typical reach of the SuperKamiokande detector. The expected large statistics, of order 10^5 events per year [25], make DeepCore an interesting probe of LRI using atmospheric neutrino data, which we now consider.

Fig. 1(a) shows the ν_μ survival probability $P_{\mu\mu}$ versus energy E_ν without (black dotted curve) and with the LRI, where the red solid and green dashed curves correspond to ν_μ and $\bar{\nu}_\mu$, respectively. We have assumed the Earth diameter as the baseline, $L = 2 \times 6400 \text{ km}$, as a typical value relevant to the DeepCore array. $\Delta m_{23}^2 = 2.4 \times 10^{-3} \text{ eV}^2$, $\sin^2(2\theta_{23}) = 0.9$, and $\alpha' = 1.0 \times 10^{-52}$ have been chosen for illustration purposes. The sign of Δm_{23}^2 has been chosen according to our fit in Eq. (12), and for definiteness we will choose $\cos(2\theta_{23}) > 0$ throughout our analysis. We see that the LRI distinguishes neutrinos and anti-neutrinos for $\sin^2(2\theta_{23}) \neq 1$. The deviation of the LRI from the standard scenario becomes significant for $E_\nu \gtrsim 15 \text{ GeV}$ for neutrinos, whereas the effect is quite large for anti-neutrinos over the entire range of energies considered here.

Fig. 1(b) shows a similar plot for $L = 1300 \text{ km}$, typical of the baseline for DUSEL experiments, which happens to be approximately 1/10 of the Earth diameter, with the other parameters as in panel (a). Deviations from standard oscillations do not become as significant as in case (a) for the same value of E_ν/L , due to the additional energy dependence of the LRI parameters in Eqs. (9) and (10). The difference between the ν_μ and $\bar{\nu}_\mu$ signal is larger than their individual deviation from standard oscillations and can be a potentially distinct signal.

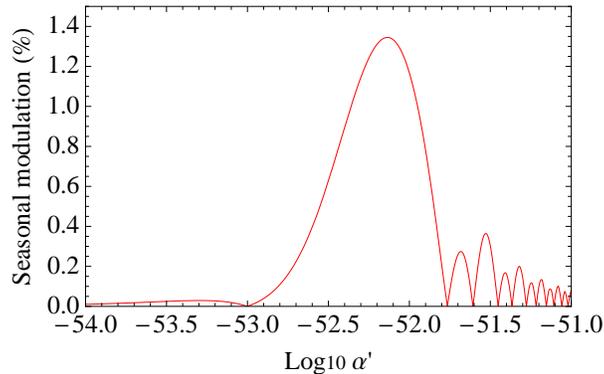


FIG. 4: Percentage of annual modulation $|(N_a - N_p)/(N_a + N_p)|$ in atmospheric neutrino oscillation at DeepCore versus the LRI coupling α' , for $20 < E_\nu$ (GeV) < 40 around aphelion and perihelion. For each season, 120 days have been included and vertical paths have been assumed. Other parameter values are the same as those of Fig. 3(a).

Fig. 1(c) shows the effect of increasing the baseline by a factor of two, $L = 2 \times 1300$ km, compared to the case presented in Fig. 1(b). For the given energy range, as expected from Eq. (8), more oscillations will take place. We also see that the effect of the LRI on oscillations is much more prominent, but at about twice the value of E_ν compared to that in Fig. 1(b). It should be noted that merely increasing the baseline by a factor of two compared to the case in Fig. 1(b) would result in a reduction by 1/4 in the flux and hence the event rate, all other factors being equal.

The results in Fig. 1 suggest that it might be easier to see the LRI effect in the DeepCore experiment than at a long-baseline experiment. On the other hand, long-baseline experiments, unlike the DeepCore, can in principle detect the asymmetry in the ν_μ and $\bar{\nu}_\mu$ oscillations for $\sin^2(2\theta_{23}) \neq 1$, which is a key feature of the LRI assumed here.

Fig. 2 shows the same qualitative features for a smaller coupling $\alpha' = 0.5 \times 10^{-52}$; as expected, the effect is less pronounced. In any event, the plots suggest that for $E_\nu \gtrsim 15$ GeV DeepCore can be quite sensitive to the new physics. Note that the effect on ν_μ and $\bar{\nu}_\mu$ can be interchanged by changing the sign of Δm_{23}^2 .

Fig. 3(a) illustrates the effect of the LRI on the number of events (for a one-year run and per 3 GeV energy bins) in ν_μ and $\bar{\nu}_\mu$ disappearance experiments at the DeepCore array with $L = 2 \times 6400$ km. The top (dashed) curve corresponds to no oscillations. The thick (red) solid, dashed, and dotted curves are for $\alpha' = 1.0, 0.5, 0.1 \times 10^{-52}$, respectively and $\alpha' = 0$ (no new physics) is presented by the thin (black) solid curve at the bottom. The curve for the no-oscillation case has been adopted from Ref. [25], where an acceptance solid angle of 1.6π sr has been assumed. For simplicity, we work in the limit of vertically traveling neutrinos. We also assume the ratio of ν_μ to $\bar{\nu}_\mu$ events with no oscillations to be weighted 2 : 1, reflecting a good approximation for the ratio of the cross sections [30]. We expect that our approximations would provide a

fair estimate of the size of the effect. The vacuum parameters are $\sin^2(2\theta_{23}) = 0.9$ and $\Delta m_{23}^2 = 2.4 \times 10^{-3}$ eV², for all cases, to illustrate the effect of the LRI on neutrino oscillations. We see that except for the smallest value of α' the other cases are very distinct from the no new physics case, for $20 \lesssim E_\nu$ (GeV) $\lesssim 40$. Over this range of energies, the no-new-physics case yields roughly 1400 events and for $\alpha' = 0.1 \times 10^{-52}$ the number of events changes by about 70. Hence, statistically, a percent-level measurement, which might require 7 years of data, could in principle reach one order of magnitude below our benchmark value of $\alpha' = 1.0 \times 10^{-52}$, at the 5 sigma level.

Fig. 3(b) is the same as Fig. 3(a) except that $\Delta m_{23}^2 = -2.4 \times 10^{-3}$ eV², interchanging ν and $\bar{\nu}$, is chosen. The thick (blue) solid, dashed, and dotted lines correspond to the same values of α' as in Fig. 3(a) and the thin solid curve at the bottom represents no new physics. We see that the effect of the LRI is now distinct for all values of α' . For $20 \lesssim E_\nu$ (GeV) $\lesssim 40$, the number of events for the new-physics case with $\alpha' = 0.1 \times 10^{-52}$ differs from that of the no-new-physics case by about 200. We see that even a several-percent-level measurement could in principle reach one order of magnitude below our benchmark value of $\alpha' = 1.0 \times 10^{-52}$. Given the large statistical samples expected at DeepCore, our estimates suggest that values of α' around one order of magnitude below that of our benchmark α' could potentially be probed by this experiment.

Fig. 4 shows our estimate for the size of the annual modulation of atmospheric neutrino oscillations at DeepCore, as a function of the LRI coupling α' . Here, the vertical axis is $|(N_a - N_p)/(N_a + N_p)|$, where N_a and N_p are the numbers of events associated with aphelion and perihelion, respectively. An energy cut of $20 < E_\nu$ (GeV) < 40 has been implemented and 120 days have been included around each apsis. For our estimates, we have simulated the variation in R_{ES} by a sinusoidal function. This approximation of Earth's true Keplerian orbit captures the main effect we would like to illustrate,

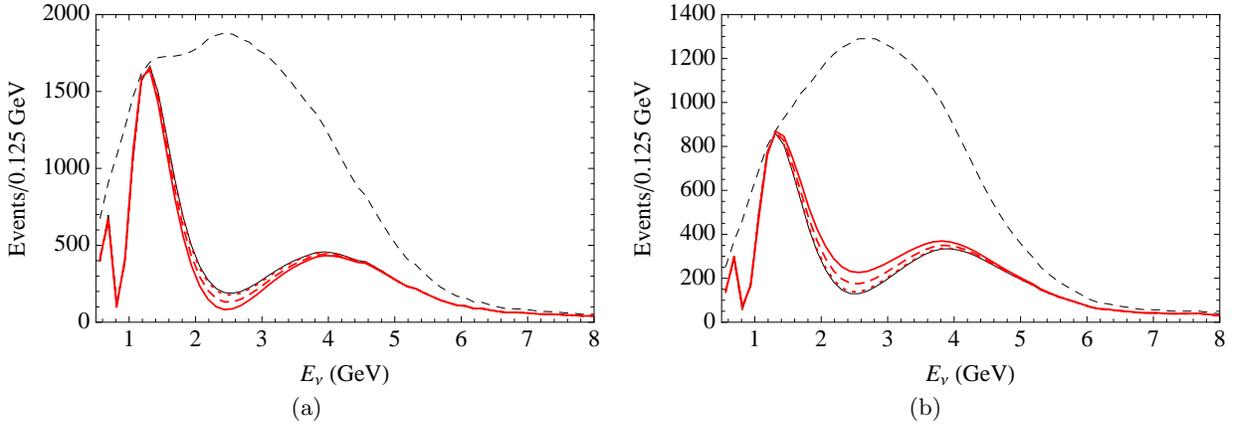


FIG. 5: The number of (a) neutrino and (b) anti-neutrino events for a 5-year run (per 0.125 GeV) versus E_ν , in a long-baseline experiment with $L = 1300$ km (DUSEL). The unoscillated case (top black dashed curves), and the case of no new physics (thin solid black curves) are displayed, as well as the cases with $\alpha' = 1.0, 0.5, 0.1 \times 10^{-52}$ corresponding to thick solid, dashed, and dotted curves, respectively. The vacuum parameters are $\sin^2(2\theta_{23}) = 0.9$ and $\Delta m_{23}^2 = 2.4 \times 10^{-3} \text{ eV}^2$.

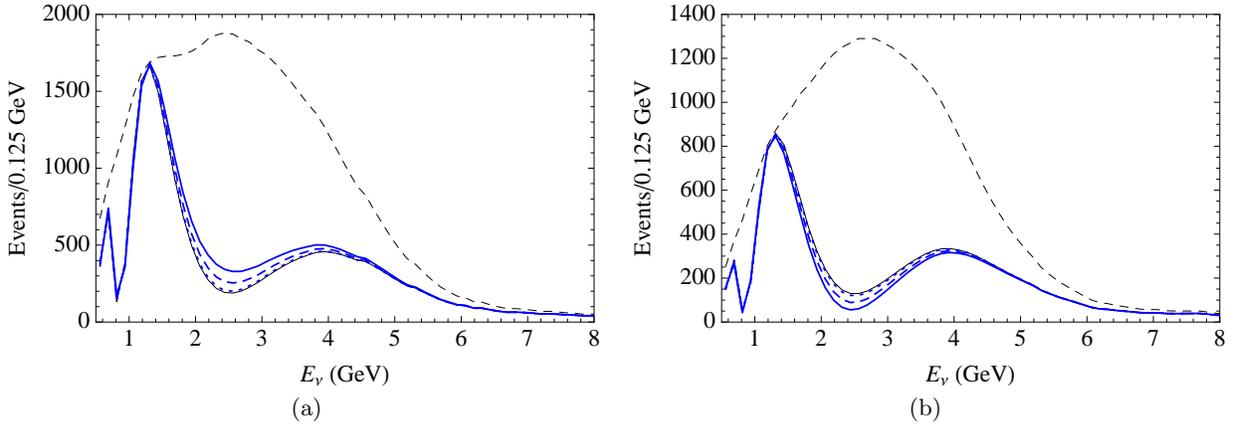


FIG. 6: Same as Fig. 5 except for $\Delta m_{23}^2 = -2.4 \times 10^{-3} \text{ eV}^2$.

at the level of our analysis. The same parameter values as in Fig. 3(a), *i.e.* $\Delta m_{23}^2 = 2.4 \times 10^{-3} \text{ eV}^2$ and $\sin^2(2\theta_{23}) = 0.9$, as well as vertical neutrino paths are assumed. The total number of events ($N_a + N_p$) per year for $\alpha' = 1.0 \times 10^{-52}$ and $\alpha' = 0.5 \times 10^{-52}$ are about 3500 and 1600, respectively, and the seasonal modulations are near 1.2% for the former and 1.1% for the latter. Our estimates then suggest that depending on the value of α' , 3-5 years of data could yield the necessary statistics to measure such levels of modulation. We have not accounted for atmospheric neutrino flux uncertainties, which can be as much as 10%. However, the $(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$ flux ratio, which is proportional to our result, can be known much more precisely and would have uncertainties at the 1% level [31]. The large amount of statistics expected at DeepCore makes per-cent level measurements a realistic possibility [32]. We may also expect that a more detailed and optimized analysis of the real data, using the predicted time evolution of a stable flux ratio, could allow

for a larger observed effect. Hence, our estimate suggests that DeepCore could be sensitive to annual modulations, when the solar source particles dominate the LRI potential, as we have assumed in this work.

Fig. 5 shows the predictions for the number of events per 0.125 GeV energy bins, over a 5-year run, for (a) ν_μ and (b) $\bar{\nu}_\mu$ long-baseline disappearance experiments, *e.g.* at DUSEL, with $L = 1300$ km. We have taken the unoscillated beam profile (dashed black curves at the top) from Ref. [33], corresponding to a 200 kt water Čerenkov detector. The vacuum parameters are $\sin^2(2\theta_{23}) = 0.9$ and $\Delta m_{23}^2 = 2.4 \times 10^{-3} \text{ eV}^2$. The thin black solid curves correspond to no new physics. The thick red solid, dashed, and dotted curves again correspond to $\alpha' = 1.0, 0.5, 0.1 \times 10^{-52}$. We see that the largest two values of α' yield predictions that are distinct from no new physics, for $2 \lesssim E_\nu \text{ (GeV)} \lesssim 3$ in both neutrino and anti-neutrino cases. We see that the LRI leads to distinct effects for ν_μ and $\bar{\nu}_\mu$, which is a key signature

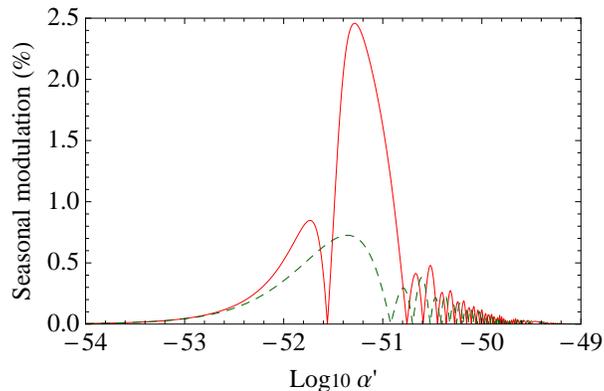


FIG. 7: Percentage of annual modulation $|(N_a - N_p)/(N_a + N_p)|$ for a 5-year run versus the LRI coupling α' , for $2 < E_\nu$ (GeV) < 3 around aphelion and perihelion. In each season, 180 days have been included, for ν_μ (red solid curve) and $\bar{\nu}_\mu$ (green dashed curve). The same parameter values as Fig. 5 are used.

of this new physics. However, the smallest value of α' does not yield a visible effect on these plots and may be difficult to reach at these experiments.

Fig. 6 contains the same information, except for $\Delta m_{23}^2 = -2.4 \times 10^{-3} \text{ eV}^2$, with the same conventions (new physics contributions are now displayed with blue thick lines). The same qualitative features as in the previous case with $\Delta m_{23}^2 > 0$ are present and reaching $\alpha' \sim 0.1 \times 10^{-52}$ seems to be difficult here as well.

Fig. 7 shows the seasonal modulation for a 5-year run of the DUSEL as a function of the LRI coupling α' for ν_μ (red solid curve) and $\bar{\nu}_\mu$ (green dashed curve). We take $2 < E_\nu$ (GeV) < 3 , with 180 days around each apsis. The same parameter values as in Fig. 5, $\sin^2(2\theta_{23}) = 0.9$, $\Delta m_{23}^2 = 2.4 \times 10^{-3} \text{ eV}^2$, are assumed. For $\alpha' = 1.0 \times 10^{-52}$, the total number of events ($N_a + N_p$) are about 1100 (ν_μ) and 2100 ($\bar{\nu}_\mu$) with the modulation at 0.5% and 0.4%, respectively. For $\alpha' = 0.5 \times 10^{-52}$, the modulation is at 0.2%, for the total number of events about 1500 (ν_μ) and 1700 ($\bar{\nu}_\mu$), respectively. We assume the beam flux is constant over time. While our analysis is only a rough estimate, we may conclude that observing the modulation at the per-cent level would require experiments (beams and detectors) with somewhat more enhanced capabilities, compared to those assumed for this analysis.

IV. SUMMARY AND CONCLUSIONS

In this work, we examined the effect of long-range interactions on neutrino oscillation experiments, motivated by the recent accelerator data from MINOS. These data are not conclusive, but could be suggesting that the oscillation parameters for neutrinos and anti-neutrinos may be distinct. Such an effect, if confirmed with more data in the future, could in principle be caused by long range interactions coupled to neutrinos [7, 18, 19]. As an illustrative example, we considered a $U(1)'$ model with an

ultra light gauge boson coupled to $(B - L) + (L_\mu - L_\tau) = B - L_e - 2L_\tau$ that captures the key aspects of the requisite phenomenology. Such an effective interaction can also arise in other ways, for example through gauge boson mixing from two separate sectors [7]. The main required features are an interaction with a range of order 1 AU and coupling to both stellar matter and neutrinos, with the resulting potential characterized by a fine structure constant $\alpha' \leq 10^{-52}$. We pointed out that when the Sun is the dominant source of the long-range potential, the effect on neutrino oscillations at Earth will exhibit annual modulations, due to the variable distance between the Earth and the Sun.

We performed an approximate fit to the MINOS data within our reference model which gave a qualitative description of the data. Our fit results accommodate the current experimental bounds on non-standard contributions to neutrino oscillation data, although they show some tension with atmospheric bounds. However, for benchmark parameters motivated by our MINOS fit, we show that ongoing and future experiments could detect large effects due to the LRI potential, or else significantly further constrain such new physics. In particular, we estimated that the currently operational IceCube DeepCore array can reach well beyond, by about an order of magnitude, the benchmark parameter space suggested by our MINOS fit, given about one year of atmospheric neutrino data at a typical baseline of $L = 2 \times 6400 \text{ km}$, for $20 \lesssim E_\nu$ (GeV) $\lesssim 50$. In addition, the large statistics afforded by DeepCore seem sufficient to detect a per-cent level modulation of neutrino oscillations with 3-5 years of data, providing key evidence for the solar source of the long-range potential.

While DeepCore is only sensitive to the sum of neutrino and anti-neutrino events, they can be separately probed at future precision long-baseline experiments, such as those at a future DUSEL facility. With typical assumptions about the capabilities of such experiments, we showed that the values of parameters motivated by

the MINOS results will be well-covered, with about 5 years of data. These experiments can in principle uncover an asymmetry in the properties of neutrinos and anti-neutrinos, which is a key feature of the long-range interactions we have considered. Our simple estimates suggest the annual modulation of the data may not be easily accessible in these experiments unless their capabilities are somewhat more enhanced compared to our reference values.

For $U(1)$'s with gauged $L-3L_\tau$ and $B-3L_\tau$, we found essentially the same phenomenology but with couplings α' about an order of magnitude smaller because of the larger sources of electrons and protons in the Sun, compared to neutrons. In all cases, DeepCore and future long baseline neutrino oscillation experiments are expected to push the α' sensitivity about an order of magnitude beyond our benchmark values.

Before closing, we would like to comment on some related possibilities that could be of interest in the context of the new physics considered here. First of all, in our discussion of annual modulation, we mainly considered future data. However, the Super-Kamiokande atmospheric data could already contain the annual modulation signal in the ratio of the muon and electron neutrinos and a dedicated analysis could be of interest in view of the MINOS anomaly. Also, while we largely focused on a particular type of long-range interaction, interesting motivation can be found in other possibilities. For example, the $B-x_\ell L_\ell$ type of model, where the lepton sector has flavor-dependent couplings, can contain residual discrete symmetries that stabilize the proton and a dark matter candidate [28]. This could provide possible new connections between neutrino experiments and other areas of

particle physics. It is conceivable that dark matter is charged under a new long-range force [34], via a generalization of the setup in Ref. [35], or as in Ref. [36]. Then, new effects could perhaps arise from dark matter trapped in the Sun or the Earth, in such scenarios. (While the modulation observed at the DAMA/LIBRA experiment [37] might be explained by the motion of the Earth through the dark matter halo of the Galaxy, we find the relative proximity of the dates of the apsides to the extrema observed by this experiment intriguing. A long-range potential from solar sources that also acts on dark matter could in principle affect the interpretation and implications of these measurements.)

Hence, we conclude that current and future atmospheric and long-baseline neutrino oscillation experiments will provide an opportunity to probe long-range interactions whose feebleness generally excludes other experimental search avenues. Together, such experiments, like those at the DeepCore array and DUSEL, can test the relevance of these interactions to the suggested MINOS anomaly, or place more stringent bounds on their parameters. Detection of a new long-range force would be an important discovery with an immense impact on our view of fundamental physics.

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