Estimates of HE-LHC beam parameters at different injection energies

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Abstract

A future upgrade to the LHC envisions increasing the top energy to 16.5 TeV and upgrading the injectors. There are two proposals to replace the SPS as the injector to the LHC. One calls for a superconducting ring in the SPS tunnel while the other calls for an injector (LER) in the LHC tunnel. In both scenarios, the injection energy to the LHC will increase. In this note we look at some of the consequences of increased injection energy to the beam dynamics in the LHC.

I. INTRODUCTION

The present design luminosity of the LHC is 10^{34} cm^{-2} s^{-1} at a top energy of 7 TeV. Upgrades to the LHC are envisaged to proceed in two steps – first the luminosity will be increased (the so-called HL-LHC) and next the top energy more than doubled to 16.5 TeV (the so-called HE-LHC). This ambitious program will also require upgrades to many of the injectors, especially the SPS. One plan is to build a fast cycling superconducting accelerator (the S-SPS) in the same tunnel as the SPS. This new injector would accept beams from the SPS and accelerate them to 1 or 1.3 TeV before extraction to the LHC. Another recently proposed option [1] is to build an injector (the LER) in the LHC ring with transmission line style magnets, similar to the ones proposed for the VLHC. The LER will be capable of accelerating beams to 1.65 TeV. There are many significant differences between the two options both in construction and beam issues in the two accelerators. One of the major beam issues is the maximum beam energy that can be injected into the LHC. In this note we will consider the impact of a change in the beam injection energy on beam dynamics in the LHC. No detailed simulations will be done but simple formulas will be used to extract the energy dependence of the relevant quantities. Table 1 shows the values of some basic LHC beam parameters.

Table 1: Input parameters of the LHC

Parameter	Value
Nominal Bunch intensity	1.15×10^{11}
Number of bunches	2808
Transverse emittance (1 σ) [mm-mrad]	3.75
Longitudinal emittance (4 σ) [eV-sec]	1.0
Rf frequency [MHz]	400
RF voltage at injection [MV]	8.0

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Higher injection energy will affect several key beam dynamics issues. These include (1) Dynamic Aperture, (2) Persistent current decay and snapback, (3) Instabilities, (4) Electron cloud, (5) Intra-beam scattering, (6) Synchrotron radiation, (7) Rest-gas scattering and possibly others.

II. DYNAMIC APERTURE

At injection energy, the field quality of the superconducting dipoles in the main arcs poses the strongest limits to the dynamic aperture. Increasing the beam energy will help to increase the dynamic aperture for two reasons

- the beam size decreases with increasing energy as $1/\sqrt{\gamma}$, hence the physical aperture will be larger when measured in units of the rms beam size
- the field quality improves with increasing energy. This will also result in a larger dynamic aperture.

As an example, increasing the beam energy from 0.45 TeV to 1.65 TeV in the LHC will reduce the beam size by 1.9 times. Quantifying the increase due to the second effect will require field quality measurements at different energies and particle tracking. For a cruder estimate, scaling laws could be applied to estimate the impact on the dynamic aperture if the multipole errors in the magnets at higher energies are known.

III. PERSISTENT CURRENT DECAY AND SNAPBACK

At the injection plateau, persistent currents in the superconducting magnets decay with time. This decay in the main field is also accompanied by decays in the multipole components, specifically the sextupole (b_3) and decapole (b_5) components in the main dipoles. The time constant for this decay depends on several factors including the initial magnetic field, the cable, the magnet's history etc. A quantitative model for this decay does not seem to exist [2]. The persistent currents in the LHC magnets decay by about a third [3] during the injection plateau which lasts several minutes. At the start of the ramp, the fields snap back to their initial values on a much shorter time scale, of the order of seconds in the LHC. It is estimated that b_3 changes by more than 3 units during this process leading to a chromaticity change of more than 150 units. In addition to the chromaticity change which is the dominant effect on the beam, there are also smaller relative changes in the orbit, tunes, beta beats and collimation efficiency. Correction algorithms have been devised to ensure that these changes have little impact on the beam.

At higher injection energy and hence higher fields, the persistent currents decay at a slower rate. Consequently the amount of snapback will be smaller and the impact on the beam will also be reduced.

Measurements of the sextupole current in a dipole magnet with initial field 1.2T corresponding to an energy of 1TeV showed that the persistent current decay was reduced by a factor of 2.6 [5]. One expects that at a higher field of 2T corresponding to

1.65 TeV, the decay would be even less. This would help reduce the setup time, overall turn around time between luminosity stores and help increase the integrated luminosity.

IV. INSTABILITIES

Here we will estimate the impact of raising the injection energy on the important instabilities at injection. The formulae in this section are taken from reference [4].

IV.A Direct space charge tune shift and tune spread

The direct space charge creates an incoherent tune spread with smaller tune shifts for larger amplitude particles. The tune shift at small amplitudes is

$$\Delta Q_{dsc} = \frac{N_b r_p}{4\pi B \varepsilon_N} \frac{1}{\gamma^2} \qquad (1)$$

Notation: N_b = bunch intensity, r_p = classical proton radius, *B* = bunching factor, ε_N = normalized transverse emittance, γ is the relativistic factor.

This direct space charge effect decreases rapidly with the inverse square of the energy. At 450 GeV, this tune shift and tune spread is ~0.001 and comparable to the tune spread from the lattice nonlinearities. This tune spread may also help contribute to the Landau damping of mode numbers higher than the rigid dipole mode [6]. The fact that the space charge tune spread will be an order of magnitude smaller at 1.65 TeV should not be an issue since there is a transverse feedback system in the LHC.

IV.B Laslett tune shift

Due to the image current induced on the beam pipe and ferromagnetic magnet poles; all particles suffer a tune shift with opposite sign in the two transverse planes. In the vertical plane, the shift is

$$\Delta Q_{y,Laslett} = -\frac{N_b k_b r_p \beta_{av}}{\pi} \left(\frac{\varepsilon_1}{h^2} + \frac{\varepsilon_2}{g^2}\right) \frac{1}{\gamma} \qquad (2)$$

Notation: k_b = Number of bunches, β_{av} = average beta function around the ring, ε_1, ε_2 = Electric and magnetic Laslett coefficients which depend on the geometry, *h* = half-height of beam pipe, g = half the distance between magnet poles

At 450 GeV, this shift is about 0.01 which is significant. This tune shift is likely compensated by the tuning quadrupoles. The Laslett tune shift decreases inversely with the energy and will require smaller changes in the tuning quadrupoles at higher energy.

IV.C Space charge impedance

Space charge also creates a capacitive coupling impedance that depends on the sizes of the beam pipe and the beam. Assuming a circular beam pipe of radius *b* and beam radius $a=\sqrt{\beta_{av}\varepsilon}$, the longitudinal and transverse coupling impedances are

$$Z_L(\omega) = -j\frac{\omega R Z_0}{2c}[1+2\ln(\frac{b}{a})]\frac{1}{\gamma^2} \quad \propto \frac{1}{\gamma^2}$$

$$Z_T(\omega) = -jZ_0 R(\frac{1}{2a^2} - \frac{1}{b^2})\frac{1}{\gamma^2} \quad \propto \frac{1}{\gamma}$$

assuming $a << b$ (3)

Notation: R is the average machine radius, Z_0 = 376.73 Ohms is the impedance of free space. These give rise to coherent tune shifts in the longitudinal and transverse planes which can be estimated using these impedances as effective impedances.

IV.D Coherent tune shifts due to space charge

The longitudinal complex tune shift in the presence of complex impedances is

$$\Delta Q_L^{(m,n)} = -j\frac{|n|}{|n|+1}\frac{Q_s I_b}{3h_{rf}V_{rf}}(\frac{2\pi R}{L_s})^3(\frac{Z_L}{n})_{eff}^{(m,n)} \propto (\frac{Z_L}{n})_{eff}^{(m,n)}\gamma^{1/4} \quad (4)$$

Notation: n=±1, ±2, is the azimuthal mode number and m=0, 1,k_b-1 is the coupled bunch mode number. Q_s is the synchrotron tune, I_b is the bunch current, and $L_s = 4\sigma_s$ is the full bunch length.

In extracting the energy dependence, we used the relations $Q_s \propto \gamma^{-1/2}$ while $L_s \propto \gamma^{-1/4}$. The dependence of the effective impedance on the energy is determined mainly by the frequency dependent impedance but it is also modified by the bunch spectrum.

The transverse complex tune shift is

$$\Delta Q_T^{(m,n)} = \frac{j}{|n|+1}\frac{I_b R}{2(E/e)L_s}\beta_{av}(Z_T)_{eff}^{(m,n)} \propto (Z_T)_{eff}^{(m,n)}\frac{1}{\gamma^{3/4}} \quad (5)$$

Here (E/e) is the beam energy in volts.

The effective impedances are found by averaging the frequency dependent impedances over the bunch mode spectrum. The relative transverse tune shifts $\Delta Q_T^{(m,n)}$/{ Q_β }, where { Q_β } is the fractional part of the betatron tune, for some low order modes at 450 GeV are of the order of 10^{-3} and the relative longitudinal tune shifts for low order modes are of the order of 10^{-4}. At higher energies, these shifts become even smaller and are negligible.

IV.E Longitudinal microwave instability threshold intensity

The threshold for the longitudinal microwave instability is

$$I_b^{th} \approx \frac{3}{2}\frac{h_{rf}V_{rf}}{(Z_L/n)_{eff}}(\frac{L_s}{2\pi R})^3 \propto \frac{1}{\gamma^{3/4}}\frac{1}{(Z_L/n)_{eff}} \quad (6)$$

The threshold intensity decreases with increasing energy so this would be an argument against higher injection energy if this threshold is close to realistic bunch intensities. As it turns out, the threshold is far above ultimate intensities in the LHC so this does not pose a concern.

IV.F Loss of Landau damping against longitudinal instabilities

The longitudinal coherent tune shifts should not exceed the longitudinal tune spread for longitudinal Landau damping to be maintained. Requiring that the tune shift be less than (tune spread)/4 imposes the intensity threshold

$$I_b^{th} \approx \frac{3\pi^2}{32} \frac{h_{rf}^3 V_{rf}}{\mathrm{Im}[(Z_L/n)_{eff}^{bb}]} \left(\frac{L_s}{2\pi R}\right)^5 \propto \frac{1}{\gamma^{5/4}} \frac{1}{\mathrm{Im}[(Z_L/n)_{eff}^{bb}]} \qquad (7)$$

This threshold also decreases with increasing energy and faster than the threshold for the longitudinal microwave instability. The longitudinal effective broadband impedance has a negligibly small dependence on energy. Among all the instabilities considered here, this instability imposes the smallest threshold intensity at 7 TeV of about 9.1×10^{11} particles/bunch, assuming an effective broadband impedance of 0.076 mΩ. This threshold is still sufficiently above feasible intensities that a dedicated longitudinal feedback system was not considered necessary [7].

IV.G Transverse Mode Coupling Instability

This occurs when two neighboring head-tail modes coalesce. The threshold is given by

$$I_b^{th} \approx \frac{2 Q_s E}{e \beta_{av} \mathrm{Im}[Z_T]_{eff}} \frac{L}{R} \propto \frac{\gamma^{1/4}}{\mathrm{Im}[Z_T]_{eff}} \qquad (8)$$

Unlike the other instabilities, the threshold for this instability increases with energy. This is especially beneficial since the TMCI instability has the lowest threshold among the three instabilities at 450 GeV.

The CERN design report [8] states that the imaginary part of the effective broad-band impedance is 1.34 MΩ/m at 450 GeV and rises to 2.67 MΩ/m at 7 TeV. This is a fairly slow increase, hence in the absence of a detailed knowledge of the impedance values, we will use the value of 1.5 MΩ/m at all injection energies.

IV.G Coupled bunch resistive wall instability

Coupled bunch instabilities can be driven by the narrow band transverse resistive wall impedances. Given the impedance Z_T at the frequency of the lowest unstable mode, the growth time of the instability can be found from

$$\frac{1}{\tau_{RW}} = \frac{N_b k_b r_0 \omega_0^2}{2\pi\gamma c Z_0} \beta_{av} \mathrm{Re}[Z_T(\omega)] \quad \propto \quad \frac{\mathrm{Re}[Z_T(\omega)]}{\gamma} \tag{9}$$

In the LHC design report [8], the magnitude of the real part of the transverse impedance at the lowest unstable mode,8 kHz, increases from 57 MOhm/m at 450 GeV to 145 MOhm/m at 7 GeV, almost a factor of 3 over an energy that increases by nearly a factor of 16. Without a detailed impedance model, it is not possible to calculate the impedance at the higher energies but the increase will be slower than a linear growth. The growth time of the vertical instability will therefore increase from the present estimate of 27 msec at 450 GeV [8] as the energy is increased to 1.65 TeV but by less than a factor of 4. The requirements of the transverse feedback system will therefore be relaxed at the higher energies.

IV.I Summary of instabilities

Table 2: Tune shifts, impedances and threshold at different energies

	Energy scaling	450 GeV	1 TeV	1.65TeV
RMS bunch length [cm]	$E^{-1/4}$	11.24	9.23	8.15
RMS energy spread	$E^{-3/4}$	4.72×10^{-4}	2.58×10^{-4}	1.77×10^{-4}
Direct space charge tune shift	E^{-2}	-1.54×10^{-3}	-3.8×10^{-4}	-1.58×10^{-4}
Laslett tune shift	E^{-1}	-1.42×10^{-2}	-6.4×10^{-3}	-3.88×10^{-3}
Space charge transverse impedance[MΩ/m]	E^{-1}	-j 6.71	-j 3.03	-j 1.83
Space charge long. Impedance [mΩ]	E^{-2}	-j 6.04	-j 1.36	-j 0.528
Microwave threshold intensity	$E^{-3/4}$	1.14×10^{13}	6.3×10^{12}	4.3×10^{12}
Landau damping threshold intensity (longitudinal)	$E^{-5/4}$	2.5×10^{12}	9.5×10^{11}	5.1×10^{11}
TMCI threshold intensity	$E^{1/4}$	3.0×10^{12}	3.7×10^{12}	4.2×10^{12}

Comments:

- The rms energy spread decreases rapidly with increasing energy – at 1.65 TeV the spread is less than half the value at 450 GeV. This should make injection easier, reduce the sensitivity to chromatic errors, help collimation efficiency, improve the lifetime and may also improve the ramp efficiency.
- The space charge tune shifts (direct and Laslett) as well the space charge impedances fall with increasing energy. Space charge related issues may not be a factor in the range of higher injection energies considered.
- The space charge impedances are capacitive, indicated by the (-j)
- Threshold intensities are quoted for the bunch intensities. The variation of the threshold intensities with energy does not include the slower dependence of the

effective impedances on the energy. It also assumes parameters such as the rf voltage are held constant.

- The microwave instability threshold intensity and the longitudinal Landau damping threshold intensity are found by assuming $(Z_L/n)\_eff = 0.1\ \Omega$, The TMCI threshold intensity is found assuming a transverse impedance $Z_T = 1.5\ M\ \Omega/m$.

  For more accurate values of the LHC impedances, appropriate corrections should be made.
- The thresholds for the longitudinal microwave and the loss of Landau damping against longitudinal instabilities fall with increasing energy. The threshold for the loss of Landau damping is the more critical. However even at 1.65 TeV, the threshold intensity at $5.1\text{x}10^{11}$ is well above ultimate intensity. Note also that the realistic value of the effective longitudinal broad-band impedance may be smaller than 0.1 Ohms as assumed.
- The TMCI threshold by contrast increases with energy.

V. ELECTRON CLOUD

At injection energies in the range 450 GeV-1.65 TeV, the synchrotron radiation photons are not energetic enough to produce photoelectrons when they strike the beam chamber. Instead, electrons are produced either by gas ionization or lost protons. The electron cloud density should be smaller than at 7 TeV. However the wakefields from the electron cloud may have a stronger effect at lower energies.

Threshold e-cloud density for the single bunch instability is given by [8]

$$\rho_{e,thr} \approx \frac{2\gamma Q_s}{\pi\beta r_p C} \propto \gamma^{1/2} \qquad (10)$$

Increasing the injection energy will also beneficially increase the threshold electron density for the instability to develop.

Rise time of instability from the coupled bunch instability is [8]

$$\tau_{e,CB} \approx \frac{\gamma}{2\pi r_p c\beta\rho_e} \propto \frac{\gamma}{\rho_e} \qquad (11)$$

where ρ_e is the electron cloud density assumed to be above threshold for the instability. This rise time increases with energy, thus a higher injection energy may slow down the growth of the coupled bunch instability due to the electron cloud.

A recent study [9], reporting numerical simulations and an experimental measurement in the SPS, suggests that the threshold for the electron cloud instability may not have the simple behaviour predicted by the above expressions. Instead the study claims that the smaller transverse beam size at higher energies enhances the electron cloud pinch and the smaller synchrotron tune implies longer damping times. The study suggests that the intensity threshold reaches a constant value above a certain energy. It is not yet clear if these results are true in general and in particular if they apply to the LHC at the injection energies of interest.

VI. SYNCHROTRON RADIATION

Table 3 shows some of the relevant parameters related to synchrotron radiation from the beam at different injection energies.

Table 3: Some synchrotron radiation parameters

	450 GeV	1 TeV	1.65 TeV
Synchrotron radiation power from beam per ring [W]	0.066	1.61	11.9
Energy loss per particle per turn [eV]	0.114	2.78	20.6
Critical photon energy [eV]	0.0117	0.128	0.574
Average photon energy [eV]	0.0036	0.039	0.177
Transverse damping time [hrs]	97537	8896	1980

Effects of synchrotron radiation should not have a significant effect on the beam at these injection energies. The critical energy even at 1.65 TeV is well below the work function of photo-electrons (~40 eV) so there should be no significant increase in photo-electrons at 1.65 TeV.

VII. INTRA-BEAM SCATTERING

Intra-beam scattering contributes to emittance growth, the growth of the longitudinal emittance at high energy is mainly due to intra-beam scattering. To estimate the energy dependence, we will use the simplified expressions from reference [10]. The growth rate of the energy spread is given by

$$\frac{1}{T_p} \equiv \frac{1}{\sigma_p}\frac{d\sigma_p}{dt} \approx \frac{r_0^2 cN(\log)}{16\gamma^{3/2}(\varepsilon_{x,N}\varepsilon_{y,N})^{3/4}\sigma_s\sigma_p^3} < \sigma_H f(\beta_x, \beta_y, \frac{\varepsilon_{x,N}}{\varepsilon_{y,N}}) >$$

$$\frac{1}{\sigma_H^2} = \frac{1}{\sigma_p^2} + (\frac{H_x}{\varepsilon_{x,N}} + \frac{H_y}{\varepsilon_{y,N}})\gamma \qquad (12)$$

$$H_{x,y} = \frac{1}{\beta_{x,y}}[\eta_{x,y}^2 + (\beta_{x,y}\eta'_{x,y} - \frac{1}{2}\beta'_{x,y}\,\eta_{x,y})^2]$$

Notation: $\varepsilon_{x,N}$, $\varepsilon_{y,N}$ are the transverse normalized emittances, (log) is the Coulomb logarithm, *f* is a function of its arguments and does not depend on the energy, <> represents an average over the lattice, $\eta_{x,y}$ and $\eta'_{x,y}$ are the dispersion and its slope.

The energy dependence of σ_H is determined by the relative magnitudes of the first term and the two terms in parenthesis. At 450 GeV, these are of comparable magnitude. At higher energies the first term will determine the scaling of σ_H since it scales as $\gamma^{3/2}$ while the other terms scale as γ. Hence we find that $\sigma_H \sim \gamma^{-3/4}$. Thus the energy scaling of the growth rate is

$$\frac{1}{T_p} \propto \gamma^{-3/2}\sigma_s^{-1}\sigma_p^{-3}\sigma_H \sim \gamma^{-3/2}\gamma^{1/4}\gamma^{9/4}\gamma^{-3/4} \sim \gamma^{1/4} \qquad (13)$$

The transverse growth rates are given by

$$\frac{1}{T_{x,y}} \equiv \frac{1}{\sigma_{x,y}} \frac{d\sigma_{x,y}}{dt} \approx \frac{\sigma_p^2 < H_{x,y} > \gamma}{\varepsilon_{x,y;N}} \frac{1}{T_p} \sim \gamma^{-1/4} \quad (14)$$

The longitudinal emittance growth rate will decrease as $\gamma^{1/2}$ while the horizontal emittances growth rate will fall as $\gamma^{-1/2}$. Hence increasing the injection energy from 450 GeV to 1.65 TeV will increase the longitudinal emittance growth rate 1.9 times while the transverse emittance growth rate will fall by the same factor. The smaller transverse emittance at the start of the ramp may improve the ramp efficiency.

VIII. REST-GAS SCATTERING

The lifetime due to inelastic scattering with the residual gas will not be much affected at higher energy. The lifetime is determined by the cross-section of inelastic nuclear scattering and the gas density. This cross-section changes very little over the considered range of injection energies.

However the emittance growth rate due to elastic multiple scattering falls with increasing energy as 1/γ, so the emittance growth rate due to this effect will be abut a factor of 4 smaller at 1.65 TeV compared to 450 GeV.

IX. CONCLUSIONS

The most important factors in favor of increasing the injection energy are the increases in dynamic aperture, reduction in the persistent current decay and thereby the reduction in the setup time, the reduction in the energy spread and the reduction in the transverse emittance growth rate due to intra-beam and rest-gas scattering. The TMCI threshold intensity will also increase with increasing energy but the threshold intensity is about an order of magnitude above feasible bunch intensities. The microwave instability threshold decreases with energy but its threshold is also at least an order of magnitude above feasible intensities and not likely to be an issue. Effects due to space charge and synchrotron radiation will also likely play no role. Theoretical arguments suggest that electron cloud effects will also be less harmful at higher energy but the experimental situation is not so clear.

The negative impacts at higher injection energy are the smaller threshold for the loss of longitudinal Landau damping and the increase in the longitudinal emittance growth rate due to intra-beam scattering. At 1.65 TeV with a somewhat pessimistic estimate for the longitudinal impedance, the threshold for Landau damping is 5.1×10^{11} particles, comfortably above foreseen intensities for the upgrades. The higher longitudinal emittance growth rate at injection may not result in a longer bunch at top energy since the energy swing from injection to top energy is smaller which may reduce any blow up effects during the acceleration.

As mentioned in the introduction, two proposals for new injectors to the LHC have been presented. They differ in their maximum energy, likely reliability, ease of operation, cost and other details – a detailed comparison can be found in [1]. One way of deciding between the two is to ask whether the differences between injecting at 1 TeV or at 1.65

TeV are important. The momentum spread is more than 30% smaller at the higher energy and this itself is significant. Dynamic aperture will be higher and the setup time will be shorter at 1.65 TeV but quantitative results need dedicated simulations and measurements. Experiments to settle the dependence of electron cloud issues on energy will also be very useful.

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