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Analytic Solutions for Seismic Travel Time and Ray Path Geometry Through Simple Velocity Models

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Analytic Solutions for Seismic Travel Time and Ray Path Geometry Through Simple Velocity Models

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Abstract

Analytic solutions are described for the geometry and travel time of infinite frequency rays through radially symmetric 1D Earth models characterized by an inner sphere where the velocity distribution is given by the function $V(r) = A - Br^2$, optionally surrounded by some number of spherical shells of constant velocity. The mathematical basis of the calculations is described, sample calculations are presented, and results are compared to the Taup Toolkit of Crotwell et al. (1999).

These solutions are useful for evaluating the fidelity of sophisticated 3D travel time calculators and in situations where performance requirements preclude the use of more computationally intensive calculators.

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INTRODUCTION

The geometry of ray paths through realistic Earth models can be extremely complex due to the vertical and lateral heterogeneity of the velocity distribution within the models. Calculation of high fidelity ray paths and travel times through these models generally involves sophisticated algorithms that require significant assumptions and approximations. To test such algorithms it is desirable to have available analytic solutions for the geometry and travel time of rays through simpler velocity distributions against which the more complex algorithms can be compared. Also, in situations where computational performance requirements prohibit implementation of full 3D algorithms, it may be necessary to accept the accuracy limitations of analytic solutions in order to compute solutions that satisfy those requirements.

In this paper, solutions are described for the geometry and travel time of infinite frequency rays through radially symmetric 1D Earth models characterized by an inner sphere where the velocity distribution is described by the function $V(r) = A - Br^2$, optionally surrounded by some number of spherical shells of constant velocity. If the surface of the inner sphere were to correspond with the Moho, then the computed rays correspond to PmP and Pn. The mathematical basis of the calculations is described, sample calculations are presented, and results are compared to the Taup Toolkit of Crotwell et al. (1999).

It should be noted that most of the solutions presented are only quasi-analytic. Exact, closed form equations are derived but computation of solutions to specific problems generally require application of numerical integration or root finding techniques, which, while approximations, can be calculated to very high accuracy. Tolerances are set in the numerical algorithms such that computed travel time accuracies are better than 1 microsecond.

METHOD

Four scenarios are considered. First, we consider cases where both the source and receiver are located at the surface of the inner sphere, with no surrounding constant velocity shells. Second, the source and receiver are located at the surface of the outer-most of N constant velocity shells surrounding the inner sphere. The third and fourth scenarios we consider are similar to the second, only the source is located below the surface of the inner sphere, or within one of the constant velocity shells. The manner in which reflections are handled is described last.

Inner Sphere Only

Consider a source, S, and receiver, P_0, located at the surface of a sphere of radius R_0 in which the velocity distribution is given by $V(r) = A - Br^2$, where r is radial distance from the center of the sphere (Figure 1a). Given the angular distance from the source to the receiver, Δ, we wish to find the radius of the ray path as a function of angular distance from the receiver and the travel time from the source to the receiver, T.

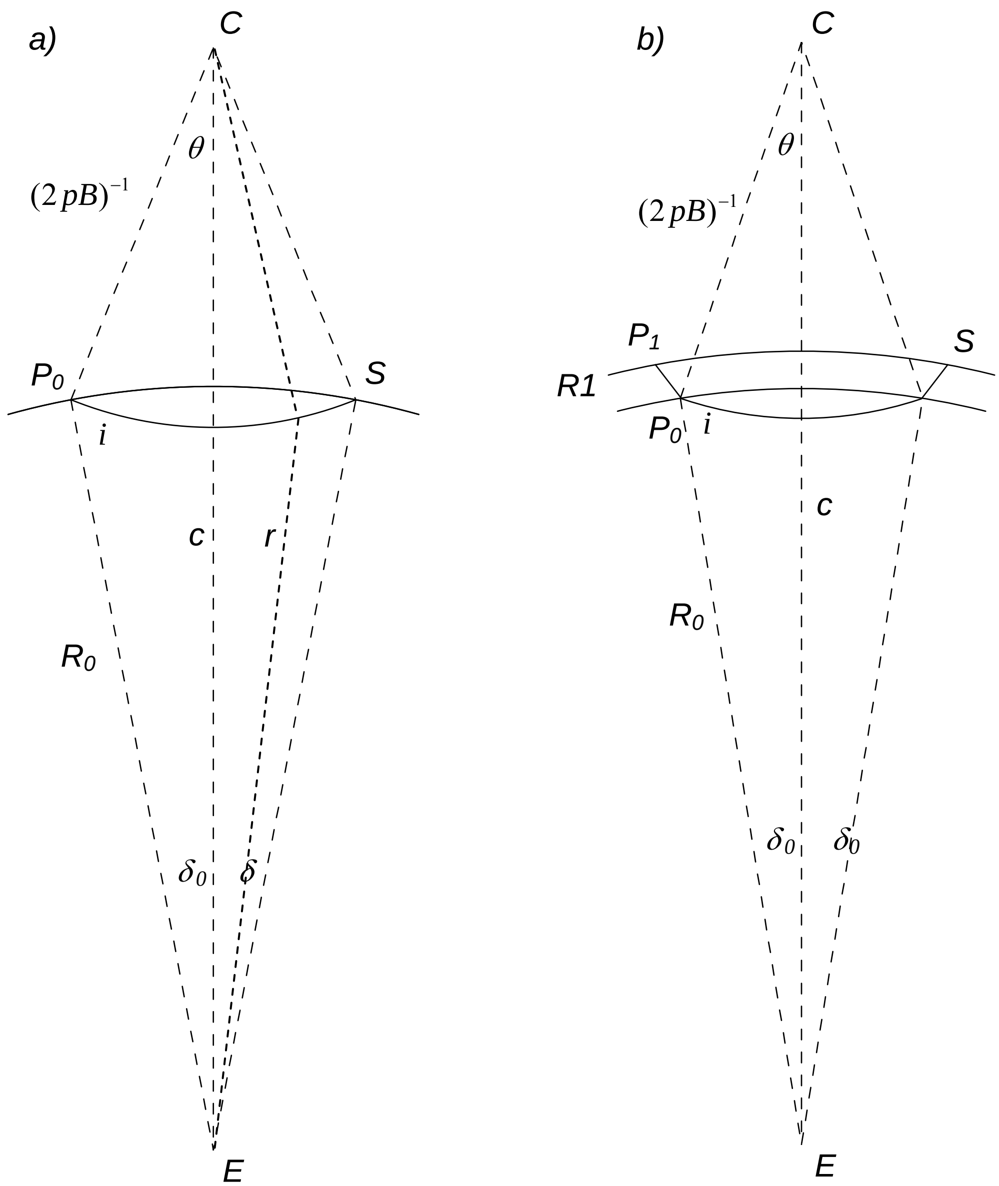

Figure 1 – Geometry of ray path for scenarios where both the source and receiver reside at the surface of the model. a) inner sphere only. b) one outer shell.

From the information provided, the following are true:

Given the velocity at the surface of the sphere, V_0, and the velocity gradient with respect to radius at the surface of the sphere, $g_0 = dV/dr$, then

$$B = -g_0 / 2R_0 \tag{1}$$

and

$$A = V_0 + BR_0^2 \tag{2}$$

The ray path from the source to the receiver, r, is described by the arc of a circle with radius $(2pB)^{-1}$, (Bullen and Bolt, 1985), where p is the ray parameter

$$p = R_0 \sin i / V_0 \tag{3}$$

and i is the incidence angle of the ray immediately below the surface of the sphere.

Applying the Law of Sines (Zwillinger, 2003) in triangle ECP_0,

$$2Bp \sin \delta_0 = \frac{\sin\theta}{R_0} \tag{4}$$

where δ_0 is the angular distance from the turning point of the ray to the surface of the sphere.

Since the angles of a triangle sum to π,

$$\delta_0 + \theta + i + \frac{\pi}{2} = \pi \tag{5}$$

Combining these equations and eliminating θ, we find that

$$i = \tan^{-1}\left[\left(\left(1 + 2BR_0^2 / V_0\right)\tan\delta_0\right)^{-1}\right] \tag{6}$$

Applying the Law of Cosines (Zwillinger, 2003) in triangle ECP_0, we can deduce that c, the vertical distance from the center of the Earth to the center of the circular arc that defines the ray path is

$$c = \left((2Bp)^{-2} + A/B\right)^{1/2} \tag{7}$$

Applying the Law of Cosines in triangle E-C-$r(\delta)$, we find that the radius of a point on the ray as function of angular distance from the turning point of the ray is

$$r(\delta) = c\cos\delta - \left((2Bp)^{-2} - c^2\sin^2\delta\right)^{1/2} \tag{8}$$

where δ is angular distance from the turning point of the ray.

The travel time from the turning point of the ray to the surface of the sphere, T_0 is given by

$$T_0 = \int_0^{\delta_0} \frac{ds}{V} \tag{9}$$

where ds is a small increment of the ray path. Given that

$$ds = \frac{r d\delta}{\sin i} \tag{10}$$

we can use the definition of the ray parameter, Equation 3, to find that

$$T_0 = \int_0^{\delta_0} \frac{r^2}{p(A - Br^2)^2} d\delta \tag{11}$$

Equation 8 is substituted into Equation 11 and the integral is solved numerically.

Since the source and receiver are both located at the surface of the sphere the problem is symmetric about the turning point of the ray so

$$\delta_S = \delta_P = \delta_0 = \Delta / 2 \tag{12}$$

where δ_S and δ_P are the angular distances from the turning point of the ray to the source and receiver, respectively. The total travel time from the source to the receiver is

$$T = T_P + T_S = 2T_0 \tag{13}$$

Constant Velocity Shells

A more complicated situation arises if we wish to also consider one or more constant velocity shells which surround the inner sphere described above (Figure 1b). We are given N shells, each extending outward from radius R_{j-1} to radius R_j, and each characterized by a constant velocity V_j. The receiver is now located at P_N, and P_j; $j=0, N-1$ represent points where the ray pierces subsurface interfaces. The source, S, like the receiver, is located at the surface of the N^{th} shell. Note that in Figure 1b, only one constant velocity layer is shown (N=1).

In this case, the solutions for $r(\delta)$ and T as functions of source-receiver separation, Δ, are difficult but solutions in terms of ray parameter, p, are readily obtained. If Δ is known but p is not, then an appropriate root finding algorithm can be implemented that systematically varies an initial estimate of p until a solution is produced that matches the known Δ to some acceptable tolerance. Routine zbrent from Numerical Recipes in C++ (Press et al., 2002) works well for this purpose.

Given ray parameter, p, equations 3.66 and 3.68 in Lay and Wallace (1995) are integrated across each shell to obtain the horizontal offset of the ray and the travel time as it traverses the shell. The total horizontal offset across all N constant velocity shells is given by

$$\delta_{Shells} = \sum_{j=1}^{N} \left[\cos^{-1}(pV_j / R_j) - \cos^{-1}(pV_j / R_{j-1}) \right] \tag{14}$$

and the total one-way travel time across the shells is given by

$$T_{Shells} = \sum_{j=1}^{N} \left[\sqrt{R_j^2 / V_j^2 - p^2} - \sqrt{R_{j-1}^2 / V_j^2 - p^2} \right] \tag{15}$$

Given the ray parameter, p, we can apply Equation 7 to obtain a value for c, the distance from the center of the model to the center of the circular arc that describes the ray path. Then applying the Law of Sines to triangle ECP_0 (Figure 1b), we find that

$$\delta_0 = \sin^{-1}\left(\frac{\sqrt{1-(pV_0 / R_0)^2}}{2Bpc} \right) \tag{16}$$

Since the source is located at the same radius as the receiver, $\delta_S = \delta_P$, the total distance from receiver to the source, Δ, is given by

$$\Delta = 2(\delta_{Shells} + \delta_0) \tag{17}$$

and the total travel time, T, by

$$T = 2(T_{Shells} + T_0) \tag{18}$$

To find the radius of the ray at some distance from the turning point of the ray, $r(\delta)$, we use Equation 8 when the ray is in the inner sphere ($\delta \le \delta_0$). When the ray is in the j^{th} outer shell, $\delta_{j-1} < \delta \le \delta_j$, we need to interpolate the radius of the ray at the appropriate distance. Consider triangle E-$r(\delta)$-P_{j-1} illustrated in Figure 2. E is the center of the model and P_{j-1} is the ray pierce point on interface j-1.

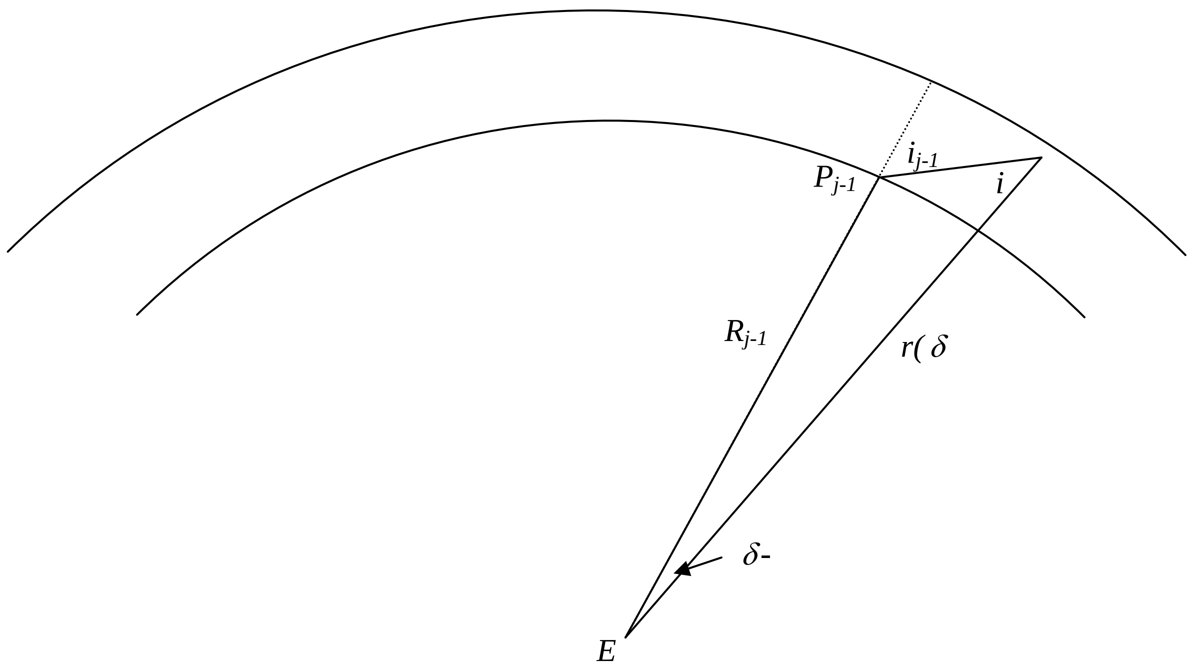

Figure 2 – Illustration of triangle E-$r(\delta)$-P_{j-1}, used to find the horizontal offset, $\delta - \delta_{j-1}$, of point $r(\delta)$ relative to point P_{j-1}.

From the definition of the ray parameter

$$i_{j-1} = \sin^{-1}\left(\frac{pV_j}{R_{j-1}}\right) \tag{19}$$

Because the angles of a triangle sum to π,

$$i = i_{j-1} - (\delta - \delta_{j-1}) \tag{20}$$

and from the Law of Sines

$$r(\delta) = \frac{\sin i_{j-1}}{\sin(i_{j-1} + \delta_{j-1} - \delta_j)} R_{j-1} \tag{21}$$

Subsurface Sources

The solutions presented thus far have all assumed that the source and receiver are both located at the surface of the model. For subsurface sources these solutions need to be modified since now $\delta_S \neq \delta_P$ (Figure 3). The approach is to first estimate *p* and then find $\delta_P = \delta_0 + \delta_{Shells}$ using Equations 14 and 16. Given $\delta_S = \Delta - \delta_P$, calculate R_S, the radius of the source. If the source is in the inner sphere ($\delta_S \leq \delta_0$), then R_S is found using Equation 8 with $\delta = \delta_S$. If the source is in the j^{th} shell surrounding the inner sphere ($\delta_{j-1} < \delta_S \leq \delta_j$), then R_S is found using Equation 21 with $\delta = \delta_S$. A root-finding algorithm such as zbrent (Press et al., 2002) is then implemented to find the value *p* that minimizes the difference between the calculated and known values of source radius.

After the optimal ray parameter, *p*, has been identified by the root-finder, the calculation of the total travel time depends on the source location. If the source is located in the inner sphere then the total travel time is given by

$$T = T_{Shells} + T_0 \pm \int_0^{\delta_S} \frac{r^2}{p(A - Br^2)^2} d\delta \tag{22}$$

The integral in Equation 22 is added to the first two terms if the ray leaves the source in a downgoing direction ($\Delta > \delta_P$), and subtracted if the ray leaves the source in an upgoing direction ($\Delta < \delta_P$).

If the source is located in the j^{th} shell surrounding in the inner sphere ($\delta_{j-1} < \delta_S \leq \delta_j$), then the total travel time is given by

$$\begin{aligned} T &= T_{Shells} + 2T_0 \\ &+ \sum_{k=1}^{j-1} \left[\sqrt{R_k^2 / V_k^2 - p^2} - \sqrt{R_{k-1}^2 / V_k^2 - p^2} \right] \\ &+ \sqrt{R_S^2 / V_j^2 - p^2} - \sqrt{R_{j-1}^2 / V_j^2 - p^2} \end{aligned} \tag{23}$$

where the first term represents the travel time from the surface of the inner sphere up to the receiver, the second term represents the travel time through the inner sphere, the summation represents the travel time through all the complete shells beneath the source and the last two terms represent the travel time from the source down to the bottom of the shell in which it resides.

The radius of the ray as a function of distance from the turning point, *r(δ)*, is still given by Equation 8 for $\delta \leq \delta_0$ or by Equation 21 for $\delta > \delta_0$.

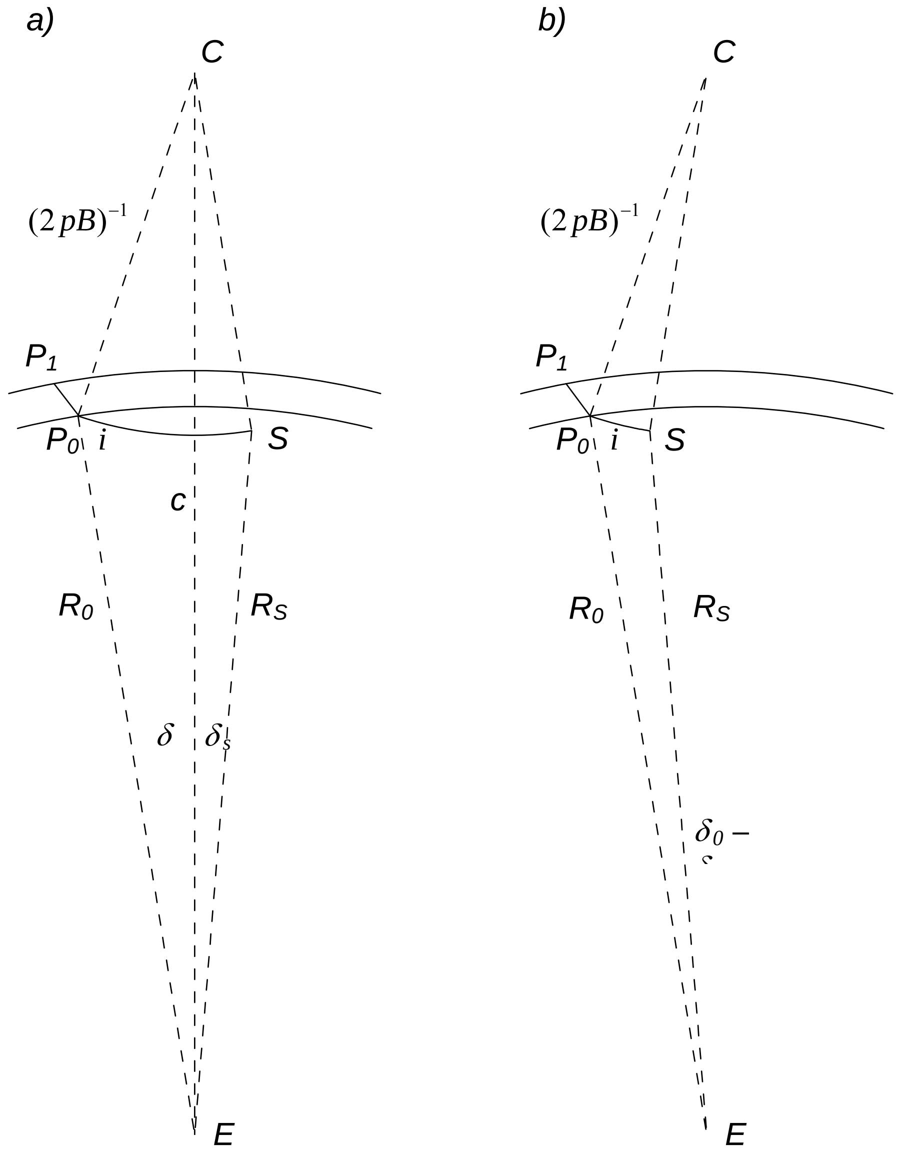

Figure 3 - Geometry of ray path for scenarios where the source resides below the surface of the inner sphere, a) ray leaves the source in the downgoing direction, b) in upgoing direction.

Reflections

In situations where the source resides in one of the constant velocity shells (Figure 1b), and the source-receiver separation is less than some threshold value, Δ_{min}, it will be impossible for energy from the source to be refracted into the inner sphere and still arrive at the receiver. This occurs when the ray parameter p exceeds a critical value p_{max} given by

$$p_{\max} = R_0 / V_0 \tag{24}$$

To evaluate Δ_{min} we set $p = p_{\max}$ and find the horizontal offset from the turning point of the ray to the source using

$$\delta_{Soffset} = \sum_{j=1}^{k-1} \left[\cos^{-1}(pV_j / R_j) - \cos^{-1}(pV_j / R_{j-1}) \right] + \cos^{-1}(pV_k / R_S) - \cos^{-1}(pV_k / R_{k-1}) \tag{25}$$

where k is the index of the shell in which the source resides.

Note that if the source resides at the surface of the model then $\delta_{Soffset} = \delta_{Shells}$. Then

$$\Delta_{\min} = \delta_{Shells} + \delta_{Soffset} \tag{27}$$

There are several options on how to proceed when $\Delta < \Delta_{\min}$. The first is to simply not return any solutions, thereby indicating that refraction into the inner sphere is not possible. Another option would be to compute and return solutions for rays that bottom in one of the outer shells. The option adopted here is to return solutions for the ray that reflects off the surface of the inner sphere since the travel times and slowness for this ray are continuous with the travel times for the refracted ray at $\Delta = \Delta_{\min}$. This is accomplished by using a root-finding technique to find the value of p such that $\delta_{Shells} + \delta_{Soffset} = \Delta$. Once the optimal value of p has been identified, the radius of the ray as a function of distance, $r(\delta)$, can be found using Equation 21 and the total travel time computed using Equation 23 with $T_0 = 0$.

SAMPLE CALCULATIONS

In the following sections, sample calculations are presented for each of 4 different situations. Travel times are believed to be accurate to the precision specified.

Single Layer Model with Sources at the Surface

Layer	Radius (km)	Depth (km)	Velocity (km/sec)
0	6371.0000	0.0000	8.0000

Gradient : 0.003000 sec^{-1}
A : 17.556500000000 km/sec
B : 2.354418458641e-07 $(sec\ km)^{-1}$

Source depth = 0.0000 km

Distance (deg)	Travel Time (sec)	Ray Parameter (sec/radian)	Max Depth (km)
1.0	13.897340	796.026907	0.821960
2.0	27.782544	794.985151	3.285365
3.0	41.643564	793.257243	7.382818
4.0	55.468525	790.855534	13.102097
5.0	69.245808	787.796968	20.426305
6.0	82.964128	784.102748	29.334077
7.0	96.612604	779.797935	39.799840
8.0	110.180824	774.910982	51.794109
9.0	123.658895	769.473231	65.283821
10.0	137.037498	763.518385	80.232697
11.0	150.307913	757.081968	96.601619
12.0	163.462056	750.200794	114.349019
13.0	176.492492	742.912450	133.431271
14.0	189.392444	735.254818	153.803077
15.0	202.155798	727.265626	175.417842
16.0	214.777091	718.982057	198.228033
17.0	227.251505	710.440396	222.185518
18.0	239.574842	701.675737	247.241883
19.0	251.743507	692.721740	273.348718
20.0	263.754478	683.610437	300.457879
21.0	275.605275	674.372089	328.521723
22.0	287.293931	665.035086	357.493310
23.0	298.818959	655.625883	387.326583
24.0	310.179314	646.168976	417.976513
25.0	321.374365	636.686907	449.399232
26.0	332.403854	627.200292	481.552130
27.0	343.267868	617.727872	514.393934
28.0	353.966806	608.286580	547.884776
29.0	364.501345	598.891622	581.986226
30.0	374.872416	589.556563	616.661327

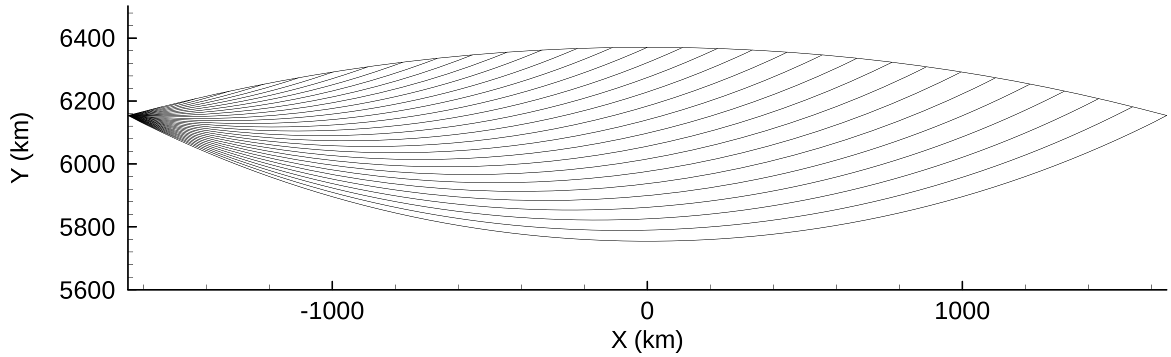

Figure 4 – Ray geometry for a single layer model with sources at the surface.

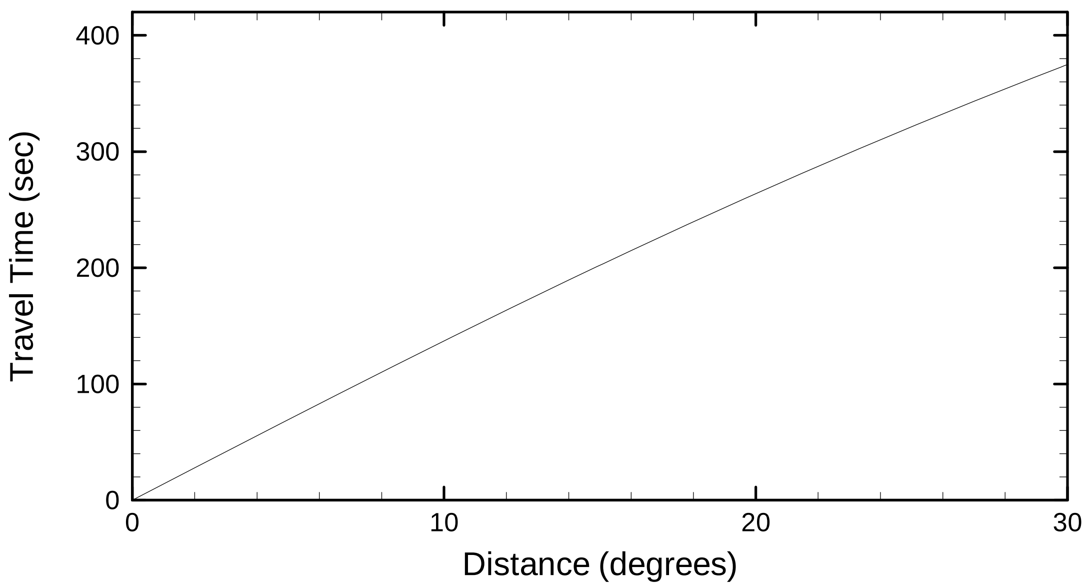

Figure 5 – Travel time curve for a single layer model with sources at the surface.

Two Layer Model with Sources at the Surface

Layer	Radius (km)	Depth (km)	Velocity (km/sec)
1	6371.0000	0.0000	6.0000
0	6271.0000	100.0000	8.0000

Gradient : 0.003000 sec^{-1}
A : 17.406500000000 km/sec
B : 2.391963004306e-07 $(sec\ km)^{-1}$

Source depth = 0.0000 km

Distance (deg)	Travel Time (sec)	Ray Parameter (sec/radian)	Max Depth (km)
0.2	33.535557	115.516367	100.000000
0.4	34.135041	226.974964	100.000000
0.6	35.111440	330.993828	100.000000
0.8	36.434462	425.298025	100.000000
1.0	38.067981	508.807981	100.000000
1.2	39.973945	581.454282	100.000000
1.4	42.115375	643.866502	100.000000
1.6	44.458254	697.064142	100.000000
1.8	46.972441	742.216818	100.000000
2.0	49.631902	780.488131	100.000000
2.2	52.367558	783.864056	100.026122
2.4	55.103702	783.826363	100.116097
2.6	57.839667	783.761742	100.270367
2.8	60.575360	783.670149	100.489063
3.0	63.310686	783.551547	100.772307
4.0	76.978508	782.552685	103.160535
5.0	90.622993	780.877737	107.176295
6.0	104.232375	778.531341	112.825272
7.0	117.795014	775.523451	120.107012
8.0	131.299490	771.869312	129.014568
9.0	144.734694	767.589284	139.534324
10.0	158.089915	762.708507	151.646009
11.0	171.354919	757.256422	165.322923
12.0	184.520019	751.266184	180.532326
13.0	197.576135	744.773979	197.235998
14.0	210.514839	737.818305	215.390898
15.0	223.328389	730.439231	234.949910
16.0	236.009752	722.677684	255.862607
17.0	248.552613	714.574780	278.076025
18.0	260.951369	706.171223	301.535399
19.0	273.201121	697.506786	326.184841
20.0	285.297655	688.619878	351.967959
21.0	297.237409	679.547196	378.828392
22.0	309.017448	670.323465	406.710272
23.0	320.635423	660.981252	435.558610
24.0	332.089534	651.550849	465.319609
25.0	343.378489	642.060217	495.940912
26.0	354.501463	632.534977	527.371797
27.0	365.458059	622.998445	559.563315
28.0	376.248268	613.471692	592.468391
29.0	386.872428	603.973637	626.041883
30.0	397.331191	594.521152	660.240617

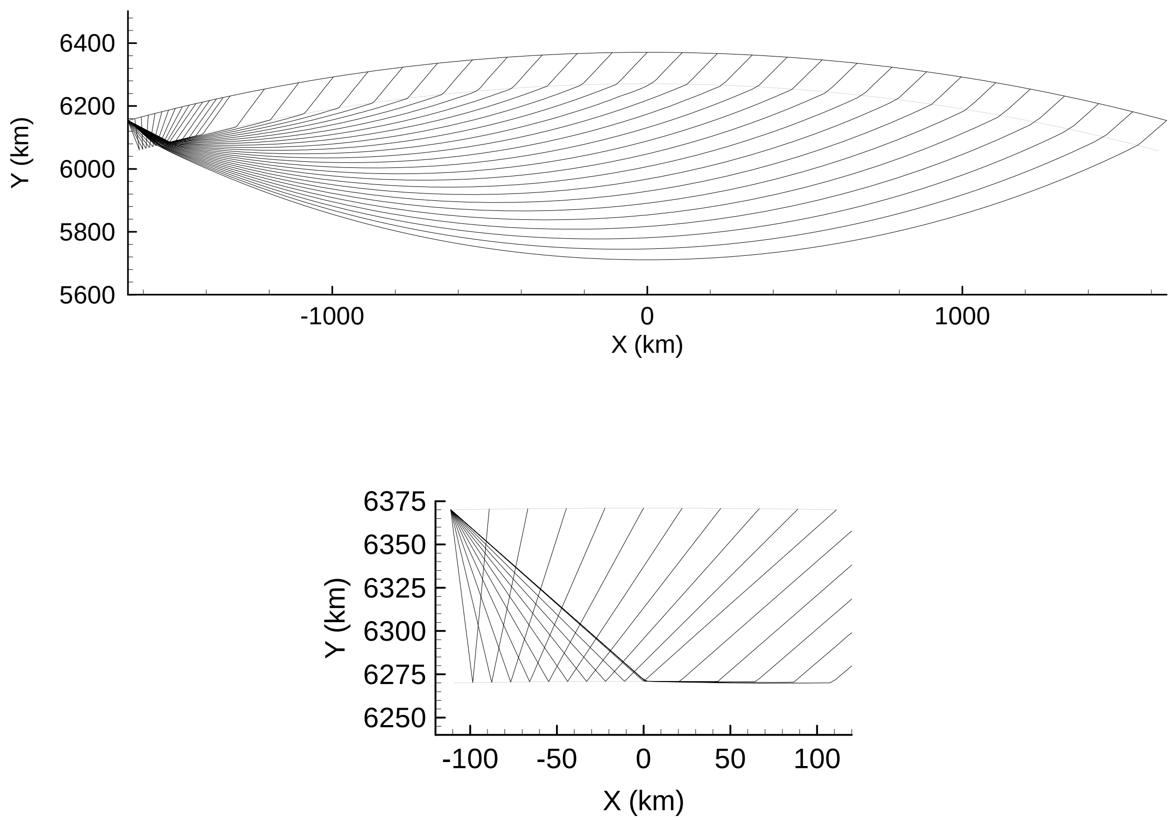

Figure 6 – Ray geometry for a two layer model with sources at the surface.

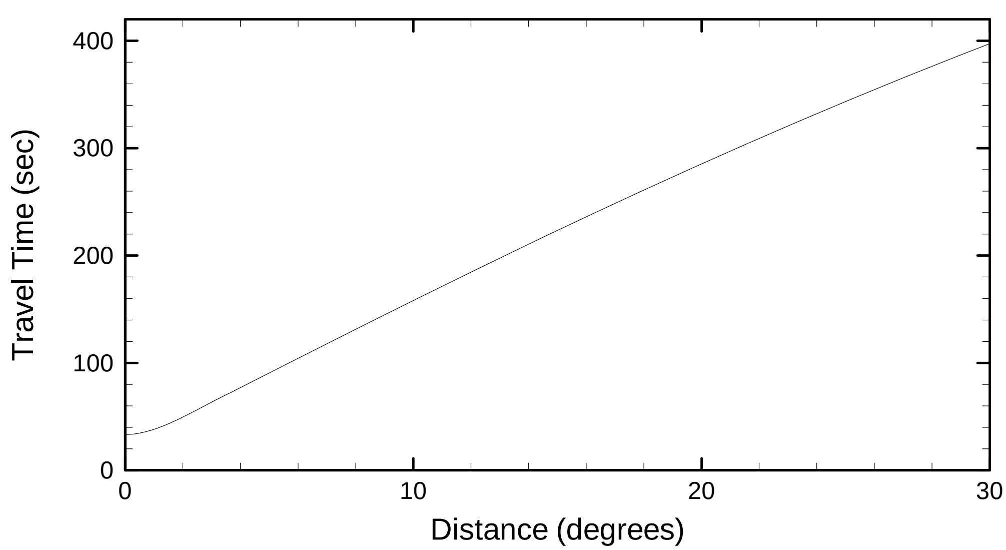

Figure 7 – Travel time curve for a two layer model with sources at the surface.

Three Layer Model with Sources in Outer Shell

Layer	Radius (km)	Depth (km)	Velocity (km/sec)
2	6371.0000	0.0000	4.0000
1	6321.0000	50.0000	6.0000
0	6271.0000	100.0000	8.0000

Gradient : 0.003000 sec^{-1}
A : 17.406500000000 km/sec
B : 2.391963004306e-07 $(sec\ km)^{-1}$

Source depth = 20.0000 km

Distance (deg)	Travel Time (sec)	Ray Parameter (sec/radian)	Max Depth (km)
0.2	36.929399	149.911327	100.000000
0.4	37.704825	292.637105	100.000000
0.6	38.957254	422.433074	100.000000
0.8	40.634792	535.831091	100.000000
1.0	42.677571	631.680334	100.000000
1.2	45.025052	710.634263	100.000000
1.4	47.621108	774.454730	100.000000
1.6	50.354586	783.865723	100.022143
1.8	53.090739	783.829974	100.107477
2.0	55.826721	783.767343	100.256995
3.0	69.500764	783.050470	101.969743
4.0	83.156428	781.661613	105.295193
5.0	96.782021	779.605933	110.234782
6.0	110.365987	776.893556	116.784528
7.0	123.896988	773.539483	124.934865
8.0	137.363989	769.563373	134.670603
9.0	150.756338	764.989215	145.971008
10.0	164.063833	759.844892	158.810012
11.0	177.276792	754.161663	173.156547
12.0	190.386104	747.973581	188.974961
13.0	203.383269	741.316875	206.225528
14.0	216.260437	734.229324	224.864997
15.0	229.010426	726.749639	244.847180
16.0	241.626731	718.916885	266.123532
17.0	254.103532	710.769945	288.643732
18.0	266.435678	702.347051	312.356223
19.0	278.618680	693.685375	337.208720
20.0	290.648684	684.820693	363.148660
21.0	302.522449	675.787122	390.123616
22.0	314.237311	666.616911	418.081642
23.0	325.791151	657.340303	446.971578
24.0	337.182361	647.985447	476.743299
25.0	348.409806	638.578351	507.347923
26.0	359.472786	629.142890	538.737970
27.0	370.370998	619.700826	570.867494
28.0	381.104502	610.271872	603.692173
29.0	391.673685	600.873771	637.169371
30.0	402.079226	591.522384	671.258183

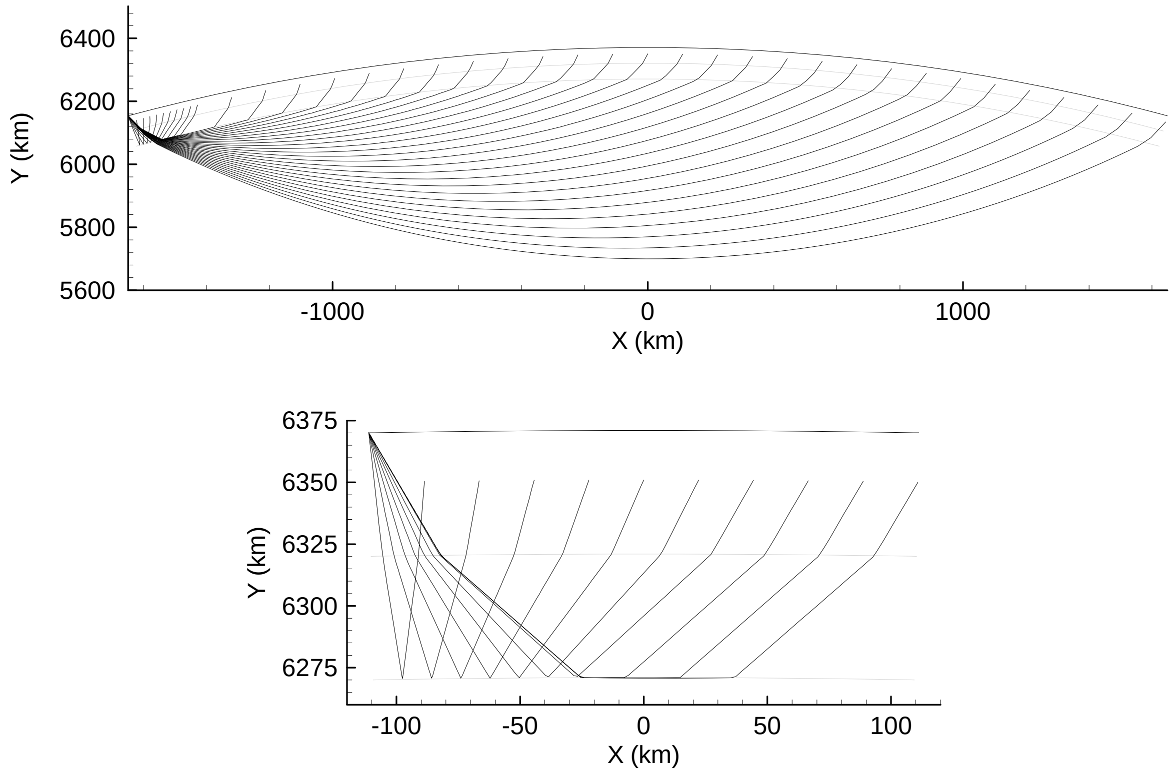

Figure 8 – Ray geometry for a three layer model with sources in outermost shell.

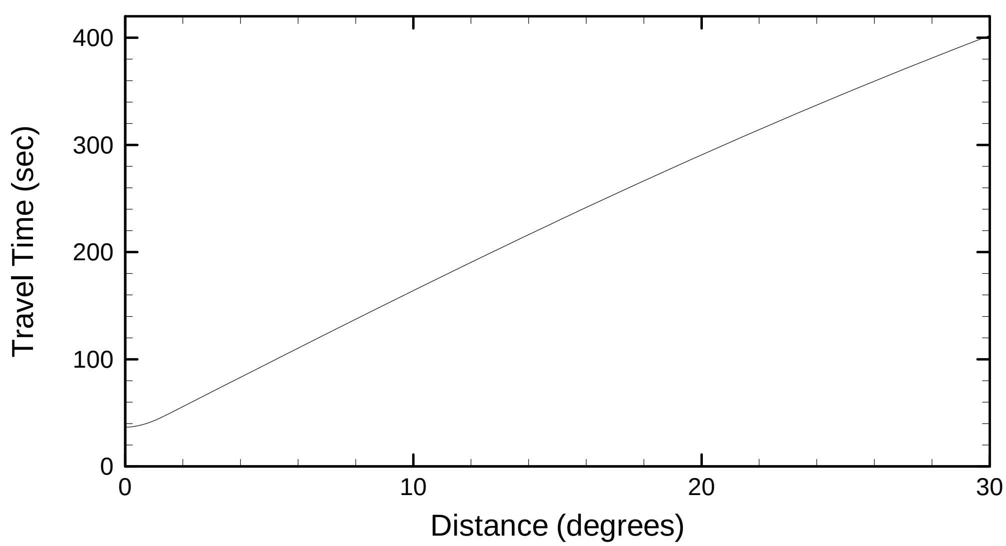

Figure 9 – Travel time curve for a three layer model with sources in outermost shell.

Three Layer Model with Sources in Inner Sphere

Layer	Radius (km)	Depth (km)	Velocity (km/sec)
2	6371.0000	0.0000	4.0000
1	6321.0000	50.0000	6.0000
0	6271.0000	100.0000	8.0000

Gradient : 0.003000 sec^{-1}
A : 17.406500000000 km/sec
B : 2.391963004306e-07 $(sec\ km)^{-1}$

Source depth = 120.0000 km

Distance (deg)	Travel Time (sec)	Ray Parameter (sec/radian)	Max Depth (km)
0.2	23.686604	205.648370	120.000000
0.4	24.732280	388.041411	120.000000
0.6	26.351100	532.524570	120.000000
0.8	28.400852	635.022158	120.000000
1.0	30.739788	699.376437	120.000000
1.2	33.249881	734.987898	120.000000
1.4	35.850970	753.286853	120.000000
1.6	38.498738	762.782742	120.000000
1.8	41.171305	767.995938	120.000000
2.0	43.857873	771.036303	120.000000
3.0	57.368117	775.469654	120.000000
4.0	70.901627	775.006363	121.363397
5.0	84.412703	773.074309	126.069717
6.0	97.881871	770.248938	132.986204
7.0	111.295290	766.703685	141.722768
8.0	124.641155	762.517691	152.122041
9.0	137.908749	757.741797	164.098952
10.0	151.088143	752.417447	177.594244
11.0	164.170105	746.583060	192.558219
12.0	177.146060	740.276311	208.944361
13.0	190.008094	733.534864	226.706689
14.0	202.748948	726.396475	245.798697
15.0	215.362024	718.898826	266.173008
16.0	227.841378	711.079236	287.781382
17.0	240.181713	702.974362	310.574887
18.0	252.378361	694.619888	334.504147
19.0	264.427269	686.050264	359.519622
20.0	276.324973	677.298478	385.571882
21.0	288.068569	668.395867	412.611869
22.0	299.655685	659.371989	440.591136
23.0	311.084449	650.254522	469.462056
24.0	322.353456	641.069213	499.178003
25.0	333.461731	631.839858	529.693503
26.0	344.408699	622.588311	560.964358
27.0	355.194150	613.334520	592.947748
28.0	365.818205	604.096581	625.602298
29.0	376.281286	594.890814	658.888138
30.0	386.584084	585.731837	692.766934

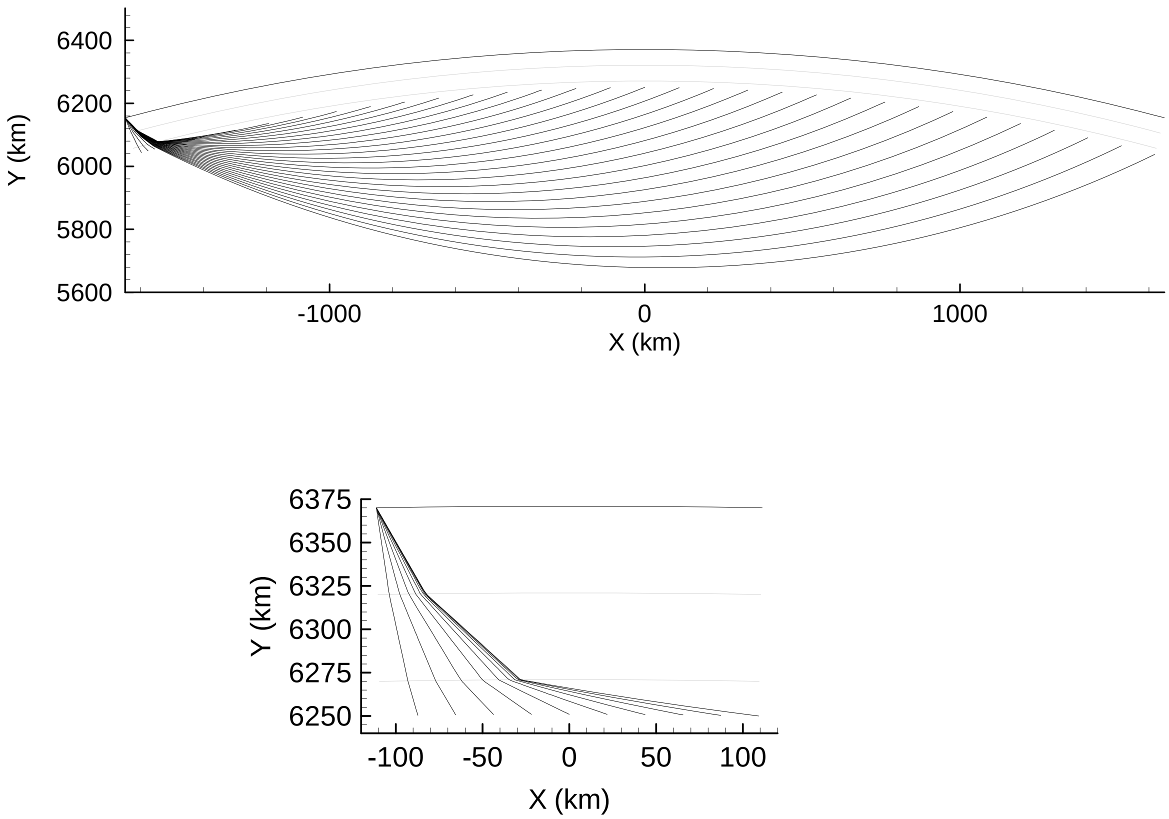

Figure 10 – Ray geometry for a three layer model with sources in the inner sphere.

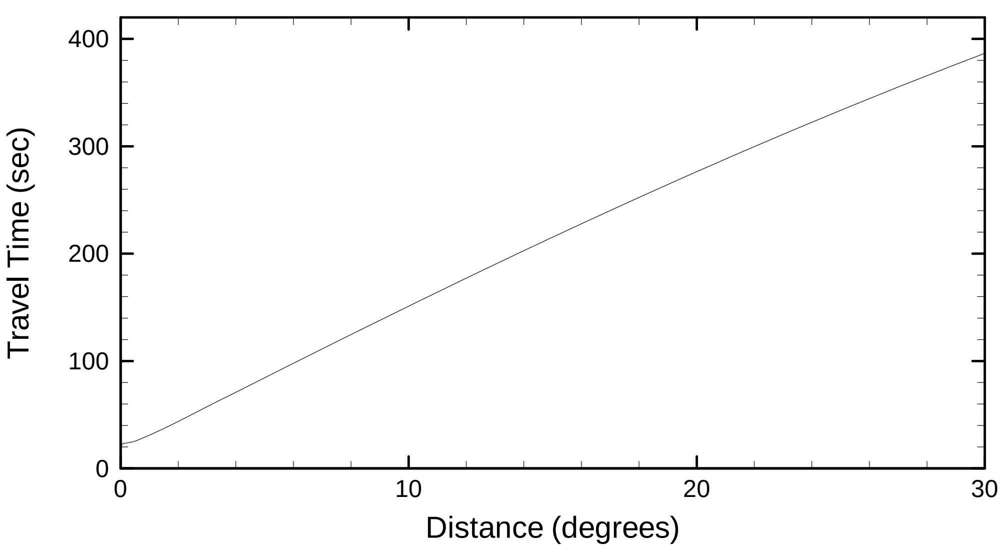

Figure 11 – Travel time curve for a three layer model with sources in the inner sphere.

COMPARISON WITH TAUP

To verify that computed analytic solutions are correct, results were compared with travel times calculated using the Taup Toolkit software package (Crotwell et al., 1999). A Taup Toolkit model was constructed with a 100 km thick crust consisting of two 50 km thick layers, the upper and lower crustal layers having velocities of 6 and 7 km/sec, respectively. The top of the mantle had a velocity of 8 km/sec and a velocity gradient of 0.003 sec^{-1}. The Taup Toolkit model file had velocities defined by the function $V(r) = A - Br^2$ every 50 km from the Moho at 100 km depth to the outer core boundary at approximately 2900 km. Travel time curves were constructed using the Taup_Curve utility for source depths of 0, 25 and 150 km.

The differences between the Taup Toolkit travel times and those computed using the algorithm described in this document were less than 2.5 milliseconds for the distance range from 0º to 80 º. Taup Toolkit documentation states that the expected accuracy of Taup Toolkit travel times is approximately 10 milliseconds.

SUMMARY

Analytic solutions for the ray geometry and travel time for seismic rays through some simple velocity models have been derived and sample calculations presented. Results obtained using these solutions are compared to results obtained using the Taup Toolkit software package (Crotwell et al., 1999). These solutions are useful for evaluating the fidelity of sophisticated 3D travel time calculators and in situations where performance requirements preclude the use of more computationally intensive calculators.

REFERENCES

Bullen, K. E. and B. A. Bolt, 1985, *An Introduction to the Theory of Seismology*, Cambridge University Press.

Crotwell, H.P., T.J. Owens, and J. Ritsema, 1999, The TauP ToolKit: Flexible seismic traveltime and raypath utilities, Seism. Res. Lett., 70, 154-160.

Lay, T. and T. C. Wallace, 1995, *Modern Global Seismology*, Academic Press.

Press, W. H., S. A. Teukolsky, W. T. Vetterling and B. P. Flannery (2002), *Numerical Recipes in C++, The Art of Scientific Computing*, 2nd Edition, Cambridge University Press.

Zwillinger, D., 2003, *CRC Standard Mathematical Tables and Formulae,* Chapman & Hall/CRC.

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