Original Approaches for Solving Electromagnetic Interference Problems

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***Abstract*—The accurate calculation of the current densities induced in layered soil by HV transmission lines in normal conditions is one of the most important steps in the study of the electromagnetic interference between transmission lines and underground metallic structures (i.e. metallic gas pipelines). In electromagnetic interference problems, the best way to investigate the soil's behavior as conducting media is to determine the current distribution within ground. The aim of the present paper is to examine the level of influence that soil layers with different resistivity have on the induced current densities. New analytical formulas for the induced current densities in the two-layer soil case are derived. The determined formulas contain semi-infinite integral terms which are calculated through a stable and efficient numerical integration scheme, in order to overcome the problems arising from the oscillate form of the infinite integrals. In the second part of the paper, the author's contribution relates to the exposure and implementation of a robust Monte Carlo simulation method, in an original approach, for solving ill-posed synthesis magnetic or electric field problems.**

***Index Terms*—electromagnetic interference, induced current densities, induced voltages, numerical integration, ill-posed problems**

I. INTRODUCTION

Solving problems that imply electromagnetic fields generated by high voltage power lines (HVPL) that cause electromagnetic interference on underground metallic structures (UMS) tends to be a complex issue. There are a great number of coexistent areas between HVPL and UMS. Almost any attempt to simulate problems involving currents circulating outside phase conductors or induced currents by inductive effect (in soil, neutral ground wires, metallic pipelines) should take into account many aspects regarding the soil properties, electromagnetic fields and interferences. [1-3].

There are a number of compatibility problems that must be carefully considered before a conclusion is drawn, in order to share new or existing rights of way.

The eddy currents induced in the metallic neighborhoods of the AC systems in the soil lead to important supplementary power looses. One cannot neglect the presence of field currents in the soil (considering a dissipative medium) and, thus, must study the current diffusion in soil in quasi-stationary regime. [4-5]

The soil structure is an important parameter that affects the level of the interference problem. The paper investigates the influence of a soil structure composed of two layers with different resistivity, both horizontally, in respect with the inductive interference HVPL – UMS.

In real cases, the ground is composed of several layers with different resistivity. The results of the parametric analysis performed in [6] showed that the inductive interference levels are also influenced by the soil structure.

A purpose of the paper is to examine the level of influence that soil layers with different resistivity have on the induced current densities, on levels, due to the presence of a nearby high voltage power line. New analytical formulas for the induced current densities in the two-layer soil case are derived.

The accurate calculation of the current densities induced in layered soil by a HV transmission line in normal conditions is the first and most important step in the study of the electromagnetic interference between transmission lines and underground metallic structures (i.e. metallic gas pipelines). [7]

The paper starts from the assumption that there is still a lack of analytic clear formulations for the case of the current densities induced in multi-layered soil. The determined formulas contain semi-infinite integral terms that are evaluated by a stable and efficient numerical integration scheme, in order to overcome efficiently the problems arising from the oscillatory form of the infinite integrals.

II. ANANALYTICAL SOLUTION OF THE INDUCED CURRENT DENSITIES IN LAYERED SOIL

A. Proposed approach

In order to calculate the current distribution in the soil, let us assume the system shown in Fig 1, composed of an overhead conductor, parallel to the earth's surface, and a pipeline placed into the soil.

There are many efforts to solve the problem in the reference materials, but usually the authors make the approximations derived from the circuits' theory. Many physical events cannot be taken into account using the elements with concentrated parameters, i.e. grounding resistors, capacitors and inductances to describe the behavior into the soil. All these calculations are more or less accurate for basic frequency, but not so exact for higher harmonics. [5]

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To avoid mathematical difficulties, they introduced some simplifications, which conceal the physical picture of the problem. The skin effect in the soil, especially for different earth's resistivities, is hidden in empiric formulas, diagrams and nomograms, usually applied in power engineering. For this reason, it has been decided to take the physical base as an essential electromagnetic starting-point and the applied mathematical methods will be only the consequence of such a treatment.

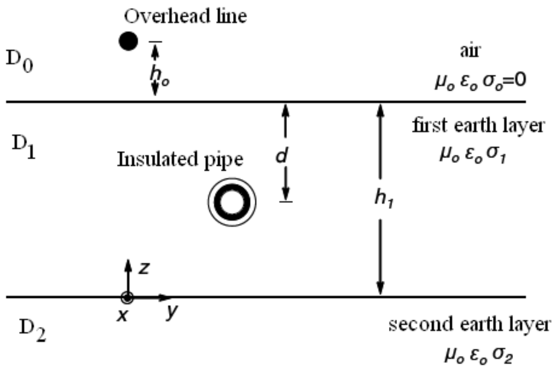


Figure 1. Geometric configuration of an overhead line and an underground metallic pipeline in a two-layer earth (soil).

The medium is considered linear, homogenous and isotropic, and the problem is to find the analytical expression of the magnetic vector potential, the induced current density and the power losses in some D domains. We assume the feeding electrical current has the expression:

$$i(t) = I\sqrt{2}\sin(\omega \cdot t), \quad \underline{I} = I. \tag{1}$$

The angular frequency is considered sufficiently low to assume the quasi-stationary magnetic regime and neglect the displacement current. [8]

Using Maxwell equations for a quasi-stationary magnetic state, we obtain the linear diffusion equation:

$$\operatorname{grad}\left(\operatorname{div}\overline{A}\right) - \Delta\overline{A} = -\mu_0\sigma\frac{\partial\overline{A}}{\partial t} - \mu_0\sigma\cdot\nabla V \tag{2}$$

In the above relation (2), we impose the Lorentz gauge condition $\operatorname{div}\overline{A} = -\mu_0\sigma V$, and we obtain a Helmholtz equation:

$$\Delta\overline{A} = \mu_0\sigma\left(\partial\overline{A}/\partial t\right) \Rightarrow \Delta\underline{\overline{A}} = j\omega\mu_0\sigma\underline{\overline{A}} \tag{3}$$

Taking into account the symmetry of the problem $\overline{A} = A(x,y)\cdot\overline{k}$, we rewrite the above relation: $\Delta\underline{A} = j\omega\mu_0\sigma\underline{A}$.

It is then denoted:

$\gamma = \sqrt{j\omega\mu_0\sigma} = \alpha(1+j); \quad \alpha = \dfrac{1}{\delta} = \sqrt{\pi f\mu_0\sigma}$ (the inverse of the skin depth), so finally a typical Helmholtz type equation is obtained:

$$\Delta\underline{A} = \gamma^2\cdot\underline{A} \tag{4}$$

Considering D_0 the domain between 0 and h_0 and, eventually, the domain above the wire, for not very high values of y; D_1 the domain between y=0 and y=-h_1 and D_2 the domain for $y \in (-\infty, -h_1)$, the equations are:

$$\begin{cases} \Delta\underline{A} = 0 & D_0\left(\sigma_0 = 0\right) \\ \Delta\underline{A} = \gamma_1^2\cdot\underline{A} & D_1\left(\gamma_1^2 = j\omega\mu_0\sigma_1\right) \\ \Delta\underline{A} = \gamma_2^2\cdot\underline{A} & D_2\left(\gamma_2^2 = j\omega\mu_0\sigma_2\right) \end{cases} \tag{5}$$

B. Analytical expressions of the induced magnetic vector potentials in each layer

For solving the Helmholtz equation $\Delta\underline{A} = \gamma^2\cdot\underline{A}$, the method of variable separation is applied, thus the solutions could be particularized for the three domains. The equation $\dfrac{\partial^2\underline{A}}{\partial x^2} + \dfrac{\partial^2\underline{A}}{\partial y^2} = \gamma^2\underline{A}$ is reshaped as $\underline{A} = P(x)\cdot Q(y)$, where P(x) and Q(y) are complex functions. The complex equation becomes:

$$Q(y)\frac{\partial^2 P(x)}{\partial x^2} + P(x)\frac{\partial^2 Q(y)}{\partial y^2} = \gamma^2 P(x)Q(y) \tag{6}$$

Taking into account that $B_x = \partial A/\partial y$ is an even function in x, P(x) must be also even. The magnetic vector B projected on Ox is the same, therefore B_x is an even function so A_x is even; $B_y = -\partial A/\partial x$ is an odd function in x, and also is Q(y). The convenient solutions are (7):

$$P(x) = a\cdot\sin(mx) + b\cdot\cos(mx);\ Q(y) = c\cdot e^{\left(\sqrt{m^2+\gamma^2}\right)y} + d\cdot e^{-\left(\sqrt{m^2+\gamma^2}\right)y}$$

Taking into account that P(x) must be even, it results a=0, and we could determine the solution for the magnetic vector potential of the i domain (i=0, 1, and 2) (8):

$$\underline{A}_i(x,y) = \int_0^\infty\left[C_{1i}(m)e^{\left(\sqrt{m^2+\gamma_i^2}\right)y} + C_{2i}(m)e^{-\left(\sqrt{m^2+\gamma_i^2}\right)y}\right]\cos(mx)dm$$

where $\gamma_i^2 = j\omega\mu_0\sigma_i$, with $\sigma_0 = 0$.

For the D_0 domain, the Ox axis $\gamma_0 = 0$ and we obtain (9):

$$\underline{A}_0(x,y) = \int_0^\infty\left[C_{10}(m)\cdot e^{m\cdot y} + C_{20}(m)\cdot e^{-m\cdot y}\right]\cdot\cos(mx)\cdot dm$$

For the D_1 domain, $y \in [-h_1, 0]$, $\gamma_1^2 = j\omega\mu_0\sigma_1$, we have (10):

$$\underline{A}_1(x,y) = \int_0^\infty\left[C_{11}(m)\cdot e^{\left(\sqrt{m^2+\gamma_1^2}\right)y} + C_{21}(m)\cdot e^{-\left(\sqrt{m^2+\gamma_1^2}\right)y}\right]\cdot\cos(mx)dm$$

For the D_2 domain, $y \in (-\infty, -h_1)$, $\gamma_2^2 = j\omega\mu_0\sigma_2$, we obtain (11):

$$\underline{A}_2(x,y) = \int_0^\infty\left[C_{12}(m)\cdot e^{\left(\sqrt{m^2+\gamma_2^2}\right)y} + C_{22}(m)\cdot e^{-\left(\sqrt{m^2+\gamma_2^2}\right)y}\right]\cdot\cos(mx)dm$$

From $\underline{B}_{xi}(x,y) = \dfrac{\partial\underline{A}_i(x,y)}{\partial y}$, $\underline{B}_{yi}(x,y) = -\dfrac{\partial\underline{A}_i(x,y)}{\partial x}$ it results:

$$\underline{B}_{xi}(x,y) = \int_0^\infty\left[C_{1i}(m)e^{\left(\sqrt{m^2+\gamma_i^2}\right)y} - C_{2i}(m)e^{\left(-\sqrt{m^2+\gamma_i^2}\right)y}\right]\sqrt{m^2+\gamma_i^2}\cos(mx)dm \tag{12}$$

$$\underline{B}_{yi}(x,y) = \int_0^\infty\left[C_{1i}(m)e^{\left(\sqrt{m^2+\gamma_i^2}\right)y} + C_{2i}(m)e^{\left(-\sqrt{m^2+\gamma_i^2}\right)y}\right]\cdot m\cdot\sin(mx)\cdot dm \tag{13}$$

For $y \to -\infty$, the magnetic field is zero, so the constant $C_{22}(m) = 0$. The components B_{x0}, B_{y0} are given through wire superposition and by the semi-space current. For $y \in [0, h]$ and for x fixed, if y is increased (between 0 and h), the component due to the wire is increasing and the other component, accordingly to the semi-space, is decreasing. Considering only the conductor without the influence of the semi space (supposing the semi-space does not exist), with the known formulas we obtain (14):

$$\int_0^\infty C_{10}(m) e^{my} m \cos(mx) dm \equiv \int_0^\infty \frac{\mu_0 \underline{I}}{2\pi} e^{-(h-y)m} \cos(mx) dm \quad (14)$$

so the constant $C_{10}(m)$ is expressed by:

$$C_{10}(m) = \frac{\mu_0 \underline{I}}{2\pi} \cdot \frac{e^{-mh}}{m} \quad (15)$$

At the surface of separation (y = 0) the normal component of the magnetic induction is conserved $B_{y0}(x,0) = B_{y1}(x,0)$ and it results: $C_{10}(m) + C_{20}(m) = C_{11}(m) + C_{21}(m)$.

At the interface (y = 0) the tangential component of the magnetic field intensity is conserved:

$$\frac{1}{\mu_0} B_{x0}(x,0) = \frac{1}{\mu_0} B_{x1}(x,0) \quad (16)$$

yields:

$$[C_{10}(m) - C_{20}(m)] \cdot m = [C_{11}(m) - C_{21}(m)] \cdot \sqrt{m^2 + \gamma_1^2} \quad (17)$$

At the separation surface between the two layers (y=-h1) the normal component of the magnetic induction is conserved $B_{y1}(x, -h_1) = B_{y2}(x, -h_1)$ and results (18):

$$\left[C_{11}(m) \cdot e^{-\sqrt{m^2+\gamma_1^2} \cdot h_1} + C_{21}(m) \cdot e^{\sqrt{m^2+\gamma_1^2} \cdot h_1}\right] = C_{12}(m) \cdot e^{-\sqrt{m^2+\gamma_2^2} \cdot h_1}$$

The constant $C_{22}(m)$=0, for any value of m, and for $y \to -\infty$ we have $e^{-\sqrt{m^2+\gamma_2^2} \cdot y} \to \infty$, which is not possible.

At the separation surface between the two layers (y=-h_1) the tangential component of the magnetic field intensity is conserved: $\frac{1}{\mu_0} B_{x1}(x, -h_1) = \frac{1}{\mu_0} B_{x2}(x, -h_1)$, and we obtain:

$$\left[C_{11}(m) e^{-\sqrt{m^2+\gamma_1^2} \cdot h_1} - C_{21}(m) e^{\sqrt{m^2+\gamma_1^2} \cdot h_1}\right] \sqrt{m^2+\gamma_1^2} = C_{12}(m) e^{-\sqrt{m^2+\gamma_2^2} \cdot h_1} \sqrt{m^2+\gamma_2^2}$$

(19)

Thus, we gain a relation between constants:

$$2C_{10}(m) = C_{11}(m)\left(1 + \sqrt{1 + \frac{\gamma_1^2}{m^2}}\right) + C_{21}(m)\left(1 - \sqrt{1 + \frac{\gamma_1^2}{m^2}}\right) \quad (20)$$

By solving the system of the above equations (19) and (20):

$$\begin{aligned} 2 \cdot C_{11}(m) e^{-\sqrt{m^2+\gamma_1^2} \cdot h_1} &= C_{12}(m) e^{-\sqrt{m^2+\gamma_2^2} \cdot h_1} \left(1 + \frac{\sqrt{m^2+\gamma_2^2}}{\sqrt{m^2+\gamma_1^2}}\right) \\ 2 \cdot C_{21}(m) e^{\sqrt{m^2+\gamma_1^2} \cdot h_1} &= C_{12}(m) e^{-\sqrt{m^2+\gamma_2^2} \cdot h_1} \left(1 - \frac{\sqrt{m^2+\gamma_2^2}}{\sqrt{m^2+\gamma_1^2}}\right) \end{aligned} \quad (21)$$

and substituting the $C_{11}(m)$ and $C_{21}(m)$ in $C_{10}(m)$, and denoting $\beta = e^{-(k_1+k_2) \cdot h_1} \cdot \left(1 - \frac{k_2}{k_1}\right)$, $\alpha = e^{(k_1-k_2) h_1}\left(1 + \frac{k_2}{k_1}\right)$, $k_1 = \sqrt{m^2 + \gamma_1^2}$, $k_2 = \sqrt{m^2 + \gamma_2^2}$, we finally get the constants:

$$\begin{aligned} &C_{10}(m) = C_{20}(m) = \frac{\mu_0 \cdot \underline{I} \cdot e^{-mh}}{2 \cdot \pi \cdot m}; \; C_{22}(m) = 0 \\ &C_{11}(m) = 2 \cdot C_{10}(m) \cdot \frac{\alpha}{\alpha + \beta} = \frac{\mu_0 \cdot \underline{I} \cdot e^{-mh}}{\pi \cdot m} \cdot \frac{\alpha}{\alpha + \beta} \\ &C_{21}(m) = 2 \cdot C_{10}(m) \cdot \frac{\beta}{\alpha + \beta} = \frac{\mu_0 \cdot \underline{I} \cdot e^{-mh}}{\pi \cdot m} \cdot \frac{\beta}{\alpha + \beta} \\ &C_{12}(m) = 4 \cdot C_{10}(m) \cdot \frac{1}{\alpha + \beta} = \frac{2 \cdot \mu_0 \cdot \underline{I} \cdot e^{-mh}}{\pi \cdot m} \cdot \frac{1}{\alpha + \beta} \end{aligned} \quad (22)$$

Therefore, in the D_0 domain the expression of the magnetic vector potential is:

$$\underline{A}_0(x, y) = \int_0^\infty \frac{\mu_0 \cdot \underline{I} \cdot e^{-mh}}{2 \cdot \pi \cdot m} \cdot \left(e^{my} + e^{-my}\right) \cdot \cos(mx) \cdot dm \quad (23)$$

For the D_1 and D_2 domains, the expressions of the magnetic vector potentials are (24), and (25):

$$\underline{A}_1(x, y) = \int_0^\infty \frac{\mu_0 \cdot \underline{I} \cdot e^{-mh}}{\pi \cdot m} \cdot \frac{e^{\left(\sqrt{m^2+\gamma_1^2} - \sqrt{m^2+\gamma_2^2}\right) \cdot h_1}\left(1 + \frac{\sqrt{m^2+\gamma_2^2}}{\sqrt{m^2+\gamma_1^2}}\right) e^{\left(\sqrt{m^2+\gamma_1^2}\right) \cdot y} + e^{-\left(\sqrt{m^2+\gamma_1^2} - \sqrt{m^2+\gamma_2^2}\right) \cdot h_1}\left(1 - \frac{\sqrt{m^2+\gamma_2^2}}{\sqrt{m^2+\gamma_1^2}}\right) e^{-\left(\sqrt{m^2+\gamma_1^2}\right) \cdot y}}{e^{\left(\sqrt{m^2+\gamma_1^2} - \sqrt{m^2+\gamma_2^2}\right) \cdot h_1}\left(1 + \frac{\sqrt{m^2+\gamma_2^2}}{\sqrt{m^2+\gamma_1^2}}\right) + e^{-\left(\sqrt{m^2+\gamma_1^2} - \sqrt{m^2+\gamma_2^2}\right) \cdot h_1}\left(1 - \frac{\sqrt{m^2+\gamma_2^2}}{\sqrt{m^2+\gamma_1^2}}\right)} \cos(mx) dm$$

$$\underline{A}_2(x, y) = \int_0^\infty \frac{2 \cdot \mu_0 \cdot \underline{I} \cdot e^{-mh}}{\pi \cdot m} \cdot \frac{e^{\left(\sqrt{m^2+\gamma_2^2}\right) \cdot y}}{e^{\left(\sqrt{m^2+\gamma_1^2} - \sqrt{m^2+\gamma_2^2}\right) \cdot h_1} \cdot \left(1 + \frac{\sqrt{m^2+\gamma_2^2}}{\sqrt{m^2+\gamma_1^2}}\right) + e^{-\left(\sqrt{m^2+\gamma_1^2} - \sqrt{m^2+\gamma_2^2}\right) \cdot h_1} \cdot \left(1 - \frac{\sqrt{m^2+\gamma_2^2}}{\sqrt{m^2+\gamma_1^2}}\right)} \cdot \cos(mx) \cdot dm$$

C. Analytical expressions of the induced current densities in each layer

Taking into consideration the above expressions for the magnetic vector potential, after the substitution of the constants and the relation between the magnetic vector potential and the induced current density $\underline{J}_i(x,y) = -\sigma_i \cdot \frac{\partial \underline{A}_i(x,y)}{\partial t}$, we obtain the final expressions of the induced current densities in the two-soil layers:

$$\underline{J}_1(x,y) = \int_0^{\infty} -\frac{j\omega\sigma_1\mu_0 \underline{I} e^{-mh}}{\pi \cdot m}\left(\frac{\alpha e^{\left(\sqrt{m^2+\gamma_1^2}\right)y}}{\alpha+\beta} + \frac{\beta e^{-\left(\sqrt{m^2+\gamma_1^2}\right)y}}{\alpha+\beta}\right)\cos(mx)dm$$

$$\underline{J}_2(x,y) = \int_0^{\infty} -\frac{j\omega\sigma_2\mu_0 \underline{I} e^{-mh}}{\pi \cdot m}\left(\frac{2}{\alpha+\beta}\cdot e^{\left(\sqrt{m^2+\gamma_2^2}\right)y}\right)\cdot\cos(mx)\cdot dm \tag{26}$$

The ratio between the two induced current densities in each layer is (27):

$$\left|\frac{\underline{J}_1(x,y_1)}{\underline{J}_2(x,y_2)}\right| \cong \left|\frac{\sigma_1}{\sigma_2}\cdot\frac{\alpha\cdot e^{\left(\sqrt{m^2+\gamma_1^2}\right)y_1} + \beta\cdot e^{-\left(\sqrt{m^2+\gamma_1^2}\right)y_2}}{2\cdot e^{\left(\sqrt{m^2+\gamma_2^2}\right)y_2}}\right| =$$

$$= \frac{\sigma_1}{\sigma_2}\left|\frac{e^{(k_1-k_2)\cdot h_1}\cdot\left(1+\frac{k_2}{k_1}\right)\cdot e^{k_1\cdot y_1} + e^{-(k_1+k_2)\cdot h_1}\cdot\left(1-\frac{k_2}{k_1}\right)\cdot e^{-k_1\cdot y_1}}{2\cdot e^{k_2\cdot y_2}}\right| \tag{27}$$

Rewriting the relation (27), taking into consideration the notations for k_1 and k_2 and considering $y_1 = -h_1/2$ and $y_2 = -2h_1$, it results:

$$\left|\frac{\underline{J}_1(x,y_1)}{\underline{J}_2(x,y_2)}\right| \cong \frac{\sigma_1}{\sigma_2}\left|\frac{e^{(k_1-k_2)\cdot h_1}\cdot\left(1+\frac{k_2}{k_1}\right)\cdot e^{k_1\cdot\frac{h_1}{2}} + e^{-(k_1+k_2)\cdot h_1}\cdot\left(1-\frac{k_2}{k_1}\right)\cdot e^{-k_1\cdot\frac{h_1}{2}}}{2\cdot e^{-2k_2\cdot h_1}}\right| = \tag{28}$$

$$= \frac{\sigma_1}{\sigma_2}\cdot\left|e^{k_2 h_1}\right|\cdot\left|ch\left(\frac{k_1 h_1}{2}\right)+\frac{k_2}{k_1}\cdot sh\left(\frac{k_1 h_1}{2}\right)\right|$$

The expressions for the constants k_1 and k_2 are:

$k_1 = \sqrt{m^2+\gamma_1^2} = \sqrt{m^2 + j\omega\mu_0\sigma_1}$ and $\omega\mu_0\sigma_1 = 4\pi^2 10^{-5}\sigma_1$, thus:

$$|k_1| = \sqrt[4]{m^4+\omega^2\mu_0^2\sigma_1^2} = \sqrt[4]{m^4+16\pi^4 10^{-10}\sigma_1^2} \le \sqrt[4]{m^4+16\cdot 10^{-8}\sigma_1^2} \cong m$$

$\sigma_1 \approx 100[\Omega m]$, $\sigma_2 \approx 1000[\Omega m]$.

The ratio becomes:

$$\left|\frac{\underline{J}_1(x,y_1)}{\underline{J}_2(x,y_2)}\right| \cong \frac{\sigma_1}{\sigma_2}\cdot\left|e^{m\cdot h_1}\right|\cdot\left|e^{\frac{m\cdot h_1}{2}}\right| = \frac{\sigma_1}{\sigma_2}\cdot\frac{e^{\frac{3m\cdot h_1}{2}}}{2} \tag{29}$$

We conclude, from the deduced approximation between the induced current densities in the two soil layers, that if the first soil layer increases in depth, the induced current density in this layer is higher with respect to the current density induced in the latter layer.

III. PROPOSED NUMERICAL INTEGRATION SCHEME

Direct numerical integration is used for the calculation of the semi-infinite integrals in (26). The integrals are highly oscillatory, showing also an initial steep descent. For these reasons, the use of a single numerical integration method proved to be inefficient. To overcome these difficulties, a combination of numerical integration methods was implemented. More specifically, the Gauss-Legendre method [8], a highly accurate numerical integration method applicable in finite intervals of functions, is combined with two other methods: the Gauss-Laguerre method, which is best suited for infinite integrals, and the Lobatto rule, a very efficient method for oscillatory functions. The selective implementation of the different integration methods in the intervals between the roots of the cosine function of the integrals leads to a quick and very efficient integration scheme for the evaluation of the semi-infinite integrals (26). This integration scheme was tested in the numerical calculation of highly oscillatory infinite integral terms of the earth return impedances [6].

A choice of numerical integration may be given by the use of a family of orthogonal polynomials. These polynomials generate a Gaussian quadratic rule, according to the following theorem: let $w(x) \ge 0$ be a weight function on the interval [a;b], and let $\{\varphi_k(x)\}$ be the family of orthogonal polynomials with respect to this weight function and this interval.

The quadratic rule is defined by:

$$G_n(x) = \sum_{i=1}^{n} w_i^{(n)}\cdot\varphi\left(x_i^{(n)}\right) \tag{30)}$$

For $x_i^{(n)}$ the roots of φ_n and $w_i^{(n)}$ given by relation above:

$$w_i^{(n)} = \int_a^b w(x)\cdot\left(\prod_{k=1,\ k\neq i}^{n}\frac{x-x_k^{(n)}}{x_i^{(n)}-x_k^{(n)}}\right)dx \tag{31}$$

Then $G_n(p)$ is exact for all polynomials $p \in P_{2\cdot n-1}$, and there exists $\xi \in [a;b]$ such that is verified:

$$\int_a^b w(x)\cdot f(x)dx - G_n(f) = \frac{1}{(2\cdot n)!}\cdot\left(\int_a^b \Psi(x)dx\right)f^{(2\cdot n)}(\xi_n) \tag{32}$$

for all $f \in C^{2\cdot n}([a;b])$, where (33) expresses:

$$\Psi_n(x) = \prod_{k=1}^{n}\left(x - x_k^{(n)}\right)^2 \tag{33}$$

We applied this theorem to construct a Gaussian quadratic rule for semi-infinite integrals having the form:

$$I(f) = \int_0^{\infty} e^{-x}\cdot f(x)dx \tag{34}$$

In our case, the weight function is the reduced decaying exponential $w(x) = e^{-x}$, so the orthogonal polynomial family that we need to use is the Laguerre family. We choose the solutions of the fourth-order Laguerre polynomial:

$$L_4(x) = \frac{1}{24}\cdot\left(x^4 - 16\cdot x^3 + 72\cdot x^2 - 96\cdot x + 24\right) \tag{35}$$

The correspondent Gauss points are presented in Table I.

TABLE I.
NUMERICAL SOLUTIONS OF LAGUERRE FOURTH-ORDER POLYNOMIAL

x_1	x_2	x_3	x_4
0.32254769	1.74576094	4.53662056	9.39507082

Next, the weights are evaluated through the particular form of the relation above:

$$w_1^{(4)}(h)=\int_0^{\infty} e^{-x\cdot h}\cdot\prod_{k=1,\,k\neq i}^{4}\frac{\left(x-x_k^{(4)}\right)}{\left(x_i^{(4)}-x_k^{(4)}\right)} \tag{36}$$

The parameter *h* provides an extension in the numerical evaluation of the semi-infinite integral. These weight integrals are computed using a sort of numerical integration routine, such as the trapezoidal of Simpson rule. Their values, accordingly to the imposed parameter, are exposed in Table II:

TABLE II
NUMERICAL VALUES OF THE WEIGHT INTEGRALS

h [m]	1	2	2
w_1	0.693	0.441	0.339
w_2	0.357	0.056	-0.011
w_3	0.039	0.0024	0.0055
w_4	0.0005	-0.00019	-0.00047

In these conditions, we may apply the explained solving procedure in the case of a semi infinite integral involving the evaluation of a pipeline to ground impedance [9], and, indirectly, the induced potential or current density on the metallic pipeline due to the coexistence with a high voltage power grid, above the ground. The explicit integral which is to be calculated from relation (26), is presented below:

$$I=\int_0^{\infty} e^{y\cdot\sqrt{x^2+m^2}-x\cdot h}\cdot\frac{\cos(u\cdot x)}{x+\sqrt{x^2+m^2}}dx \tag{37}$$

We consider the evaluation of this integral at the surface level of the earth: *y=0*.

The involved function is:

$$f(x,u)=\frac{\cos(u,x)}{x+\sqrt{x^2+m^2}} \tag{38}$$

It remains only to apply and evaluate the quadratic rule:

$$\begin{aligned}I(u,h)=&\,w_1(h)f(x_1,u)+w_2(h)f(x_2,u)+w_3(h)f(x_3,u)+\\&+w_{41}(h)f(x_4,u)\end{aligned} \tag{39}$$

For an imposed test parameter *u=1,2,3*, in the conditions presented up to here, regarding the weight integrals and the parameter *h*, a matrix of results is achieved (Table III):

TABLE IIII
NUMERICAL VALUES OF THE SEMI INFINITE INTEGRAL – TEST CASE

0.868-1.681j	0.646-1.231j	0.049-0.946j	$\cdot 10^{-5}$
0.647-1.411j	0.531-1.036j	0.042-0.797j	
0.584-1.01j	0.039-0.736j	0.029-0.565j	

It is worthwhile to notice that the results are complex values, and their physical significance relates to mutual impedance, with the real part – the resistance and the imaginary part – the reactance.

Another possibility of solving this improper integral may be given by the use of Monte Carlo simulations. For a random variable *x,* with the probability density function *g(x)* (where this function has the primitive equal to 1 on the $(0;\infty)$ interval), we have the identity:

$$\int_0^{\infty}\frac{\cdot e^{-x}f(x)}{g(x)}dx=\int_0^{\infty}\cdot e^{-x}f(x)dx \tag{40}$$

Hence, to compute the semi-infinite integral, we have to generate *N* independent random variables, distributed accordingly to a probability density function *g(x)*. The sample mean below gives an estimate for the integral:

$$\overline{e^{-x}\cdot f(x)}=\frac{1}{N}\cdot\sum_{i=1}^{N}\frac{e^{-x_i}\cdot f(x_i)}{g(x_i)} \tag{41}$$

IV. INVERSE FORMULATION OF AN ELECTROMAGNETIC INTERFERENCE PROBLEM

A. Proposed approach for solving an electromagnetic interference problem

The input data for all this type of interference problems are (see figure 2): power line and pipelines geometrical configuration; conductor and pipeline physical characteristics (including insulating and coating characteristics); environmental parameters (air characteristics, soil structure and characteristics); power system terminal (or boundary) parameters (power source voltages, equivalent source impedances) [9].

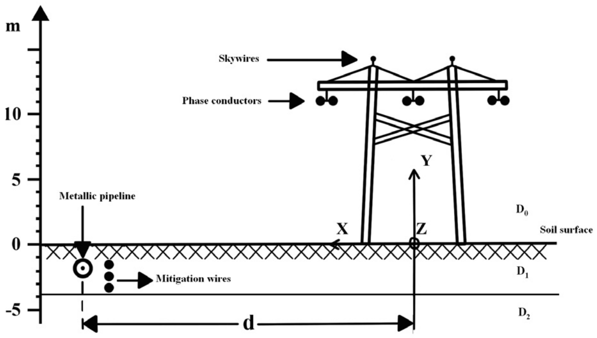


Figure 2. The layered structure AC power lines-metallic pipeline.

We take into consideration, also, the layered soil, with the formulas developed in the first part of the paper.

The results demonstrate the possibility to obtain a precise evaluation of the solicitations, if the resistance and the adduction current in the pipeline are known. After the determination, in each point, of the potentials due to the right and left side of the line, the superposition method is applied. [11]

What we propose is the computation of the voltages along an underground pipeline using an original algorithm of Monte Carlo numerical method. Having a starting base [12], the interference model can be introduced and tested.

In order to determine the place of the Monte Carlo method in the synthesis theory, the paper presents the steps that must be taken in solving any inverse problem associated with an electromagnetic field model. The first step defines the physical model, i.e. the known uniqueness conditions are stated, as well as the known (imposed) field, so that the unknown variables that are to be determined by synthesis are clearly defined. The second step determines the mathematical model, which may be differential or integral, depending on the form under which the field laws are used. The third step may be called the mathematical model processing. After the three steps above, we obtain the matrix equation: [A][z]=[u], where [z] is the vector including the unknown variables (source vector) and [A] is the operator matrix that reflects the relationship source-effect.

The fourth step refers to solving the matrix equation by methods specific to synthesis as numerical procedures of Tikhonov regularization or truncated singular value decomposition. [13]

The Monte Carlo synthesis method is derived from the analysis method [14] and it is used in its form under the variant of the fixed route, or random route.

B. *Electrostatic model*

The synthesis of the stationary electric field in a sub-domain of interest, such as a plane or a spatial domain, is reduced to the synthesis of the potential in the respective sub-domain. That means that some of the boundary conditions have to be determined, as well as some of the domain sources that assure fixed potential values in a sub-domain of interest.

Let us consider a plane domain D, limited by boundary Γ. The medium is linear homogeneous or inhomogeneous and isotropic. The electrostatic potential satisfies the Poisson or Laplace equation. The domain is meshed by means of a square network of step h, so that the curve passes through the nodes of this network (Fig. 3).

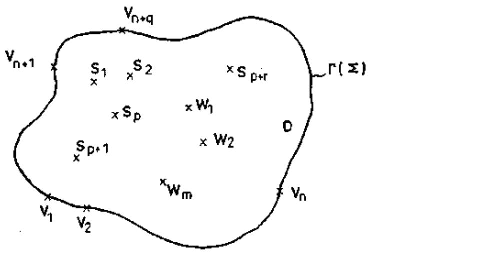


Figure 3. Physical model scheme for demonstrating the synthesis method.

We denote by S_j (j=1 to p+r) the values of the source function in the network nodes. These values are unknown for j=1 to p and known for j=p+1 to p+r. The boundary conditions are of the Dirichlet type, denoted by V_i, and are considered unknown from index 1 to n, and known from i=n+1 to n+q. The problem of synthesis is reduced to determine the unknown sources S_j (j=l to p) and unknown potentials on the boundary V_i (i=l to n) that result in the given potentials W_k (k=l to m) in the points of the sub-domain of interest.

The solution by means of the Monte Carlo synthesis method is done in several steps. Some $N^{(k)}$ aleatory routes are considered, having as starting node the point where the potential has the value W_k. The absorbent nodes will be those situated on the boundary, in order that the V_i potential node represents the absorbent state for $N_i^{(k)}$ routes (i=1 to n+q). We denote by $M_j^{(k)}$ the number of passing of the $N^{(k)}$ aleatory routes through the interior node of value S_j of the source function. We repeat the same procedures *m* times, for each interior node of given potential, and the result (after adequately grouping the unknown values in the left member) is the system of m equations with n+p unknowns, as shown in relation (42):

$$\sum_{i=1}^{n} N_i^{(k)} V_i - \frac{h^2}{\omega}\sum_{j=1}^{p} M_j^{(k)} S_j = N^{(k)} W_k - \sum_{i=n+1}^{n+q} N_i^{(k)} V_i + \frac{h^2}{4}\sum_{j=p+1}^{p+r} M_j^{(k)} S_j \qquad (42)$$

A reduced system [A][z]=[u] can then be written and regularization techniques applied, in order to achieve precise and stable solutions.

For a given geometry of an electrostatic problem, let the imposed potentials W to be in the interior of the domain (see figure 4). In order to synthesize these potentials, we must evaluate the upper boundary potentials, the other having the zero value.

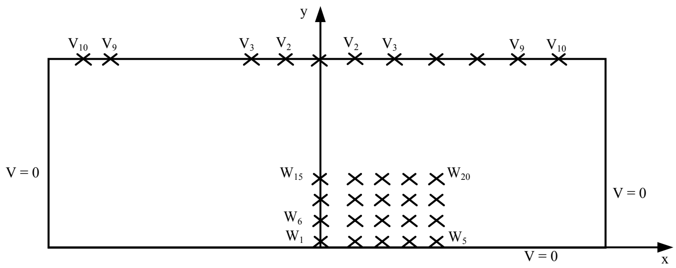


Figure 4. Imposed potentials W and unknown potentials V.

The previous Monte Carlo model applies to this geometry, considering a set of random routes starting from the imposed potentials, with the reach the boundary as stopping criteria.

A Monte Carlo numerical tool was created in order to generate the random routes and to count their destination in respect to each of the boundary nodes. As it is shown below, this Monte Carlo instrument gives the possibility to set the number of the random routes, to state the type of moving, to define the geometry, imposed potential locations, some sources location if they exist and the associated mesh. Before any calculation may start, the user has the opportunity to save the model. Afterwards, in a matter of up to two minutes, for a magnitude of 10^6 random routes, the program retrieves a matrix of results, namely the route counters of each boundary and source nodes. With this coefficient matrix, any solving method may be applied using dedicated software like MathCAD, Matlab or Mathematica.

The interface of the Monte Carlo instrument was captured in figure 5:

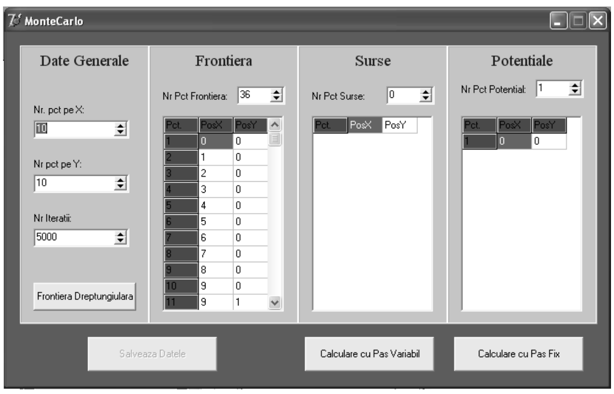
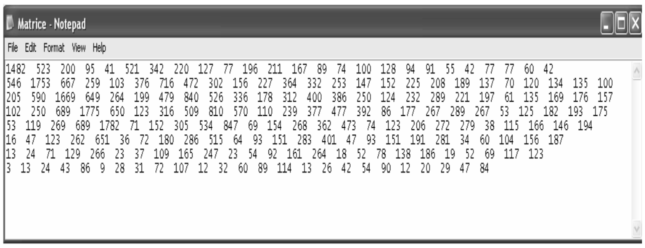


Figure 5. Monte Carlo instrument - developed software.

As it was highlighted above, there is an increased flexibility in handling numerical results given by the program. The output matrix is exported in .txt format, easily to import in any calculus program. For the earlier considered example and some 5000 random routes, the output Monte Carlo matrix looks like in figure 6: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1482 523 200 95 41 521 342 220 127 77 196 211 167 89 74 100 128 94 91 55 42 77 77 60 42
546 1753 667 259 103 376 716 472 302 156 227 364 332 253 147 152 225 208 189 137 70 120 134 135 100
205 590 1669 649 264 199 479 840 526 336 178 312 400 386 250 124 232 289 221 197 61 135 169 176 157
102 250 689 1775 650 123 316 509 810 570 110 239 377 477 392 86 177 267 289 267 53 125 182 193 175
53 119 269 689 1782 71 152 305 534 847 69 154 268 362 473 74 123 206 272 279 38 115 166 146 194
16 47 123 262 651 36 72 180 286 515 64 93 151 283 401 47 93 151 191 281 34 60 104 156 187
13 24 71 129 266 23 37 109 165 247 23 54 92 161 264 18 52 78 138 186 19 52 69 117 123
3 13 24 43 86 9 28 31 72 107 12 32 60 89 114 13 26 42 54 90 12 20 29 47 84
```

Figure 6. Numerical results from the Monte Carlo simulation.

C. Numerical aspects of the problem

In this stage, the matrix equation [A][z]=[u], yielded from the Monte Carlo simulation, may be extremely difficult to solve because it is an ill-posed problem. Such problem is characterized by the fact that arbitrarily small perturbations of the right hand side [u] may lead to arbitrarily large perturbations of the solution [z]. In other words, the solution is extremely sensitive to perturbations. An important tool for this kind of problems is the condition number of the coefficients matrix [A]. This mathematical instrument applies to the linear systems of equations, as suggested, and reflects the stability of the solution when a perturbation occurs.

A general definition of the condition number involves the use of the singular values corresponding to the studied matrix, as relation (43) defines:

$$K(A) = \frac{\sigma_{max}}{\sigma_{min}} \tag{43}$$

There is a wide choice in evaluating the singular values of a matrix. σ_{max} represents the first singular value and σ_{min} the last one. A large value of the condition number indicates instability in the solution and, therefore, an incorrectly formulated problem. [15]

When the condition number reflects a weak stability of the problem, special solving methods have to be applied, in order to accomplish the physical achievement and the stability of the solution. The numerical procedure was stated as regularization, and regards a reformulation of the initial problem, which consists in adding or losing some of the given information. [16]

By the regularization process, the condition number of the given coefficients matrix improves its value. Truncated singular value decomposition yields such a regularization procedure. The initial coefficients matrix, with an ill-conditioned character, suffers a decomposition of the following kind:

$$A_{TSVD} = U \cdot \Sigma \cdot V^T \tag{44}$$

The two left and right matrices U and V are orthogonal. The middle diagonal matrix includes all the singular values of the A matrix, that are higher than an imposed limit (see the relation below):

$$\Sigma = \begin{pmatrix} \sigma_1 & 0 & \dots & 0 \\ 0 & \sigma_1 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & nr \end{pmatrix} \tag{45}$$

Instead of those values that are lower than an imposed limit μ, one may consider a predefined number *nr*, with a value in the same range with the maximum singular value σ_1. Thus, the immediate effect in the computation of the coefficients matrix with truncated singular value decomposition appears in a better condition number:

$$K(A_{TSVD}) = \frac{\sigma_1}{nr} << \frac{\sigma_1}{\sigma_n} \tag{46}$$

But this improvement also brings an information loss in the coefficients matrix, when TSVD re-computation applies to the initial matrix.

The paper presents a TSVD calculation. It can be shown that the information loss is insignificantly, as related to the efficiency brought by the SVD stabilization process. It can be clearly seen that for a regularized solution, the impact of the perturbation has almost no significance at all, while for the initial system of equations, no physical solution can be achieved.

D. Monte Carlo synthesis method applied in electromagnetic interference problems

Let us now consider the simplified model of an interference problem between an AC power line and a buried metallic structure sharing a common corridor with the power line, as shown in figure 2. From the analysis problem, we assume as known the magnetic vector potential on the boundary of the considered domain, as Dirichlet type, and let us impose homogeneous Neumann conditions on the vertical and bottom sides.

A similar procedure as in the electrostatic case may be performed by Monte Carlo synthesis method. Here the unknown sources are represented by the induced voltages and eddy current densities on the buried pipeline and the known sources are the power line wires (see figure 7).

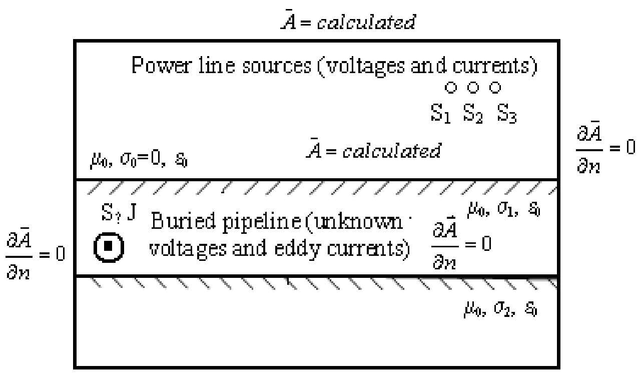

Figure 7. The model of the electromagnetic interference problem.

The authors' contribution relates to the exposure and implementation of a robust Monte Carlo simulation method, in an original approach, for solving ill-posed synthesis magnetic or electric field problems.

From this ground base model, we wish to demonstrate that the algorithm may be applied in an inverse approach to the interference problem of the power line – underground pipeline coexistent structures. Further evaluations of the present paper tend to offer an alternative solution for the inductive interference problem of AC power lines and affected underground metallic structures.

V. CONCLUSIONS

In the present paper are exposed some analytical and numerical approaches of how to determine the level of influence that soil layers with different resistivity have on the induced current densities level. New analytical formulas for the induced current densities in the two-layer soil case are derived, based on some mathematical analytical developments.

The determined formulas contain semi-infinite integral terms that are calculated by a stable and efficient numerical integration scheme, in order to overcome the problems arising from the oscillatory form of the infinite integrals. A choice of numerical integration may be given, by the use of a family of orthogonal polynomials that generates a Gaussian quadratic rule. In our case, we choose the weight function, the reduced decaying exponential, so the orthogonal polynomial family that we need to use is the fourth-order Laguerre family. The weight integrals are computed using a numerical integration routine, such as the Simpson rule. For an imposed test parameter, regarding the weight integrals and the parameter *h*, a matrix of results is achieved. We presented, also, another possibility of solving this improper integral by the use of Monte Carlo simulations.

In the second part of the paper, the authors' contribution relates to the exposure and implementation of a robust Monte Carlo simulation method, in an original approach, for solving ill-posed synthesis magnetic or electric field problems. Further evaluations of the present paper tend to offer an alternative solution method for the inductive and conductive interference problems of an AC power lines and underground metallic pipelines placed in a homogeneous or layered soil structure.

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