Experimental Modelling of the Breakdown Voltage of Air Using Design of Experiments

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***Abstract*—Many experimental and numerical studies were devoted to the electric discharge of air, and some mathematical models were proposed for the critical breakdown voltage. As this latter depends on several parameters, it is difficult to find a formula, theoretical or experimental, which considers many factors. The aim of this paper is to model the critical breakdown voltage in a "Sphere-Sphere" electrodes system by using the methodology of experimental designs. Several factors were considered, such as geometrical factors (inter-electrodes interval, diameter of the electrodes) and climatic factors (temperature, humidity). Two factorial centred faces experimental designs (CCF) were carried out, a first one for the geometrical factors and a second one for the climatic factors. The obtained results made it possible to propose mathematical models and to study the interactions between the various factors.**

***Index Terms*—High voltage, electrical breakdown, modelling, design of experiments**

I. INTRODUCTION

The breakdown of air was the subject of many experimental, theoretical and simulation studies. The physical phenomenon is nowadays well known and researchers such as Townsend, Meek and Raether contributed to the comprehension and the explanation of the breakdown mechanism [1-4].

Many factors affect the breakdown, such as electrical, geometrical and climatic parameters. Nowadays, the influence of each one of them is well-known, but we do not appreciate the interactions existing between these factors. When the relative humidity and the temperature vary simultaneously for example, which one influences more than the other? Thus, we make use of the experimental designs methodology, a powerful tool which proved to be very helpful for modelling and analysing the interactions between factors [5-6]. We examine in this paper several factors: relative humidity, temperature, radius of electrodes and inter-electrodes interval.

II. DESCRIPTION OF THE EXPERIMENTAL DEVICE

All the experiments were carried out in a "sphere-sphere" configuration of electrodes (Fig.1). A multimeter (FLUKE 867B, U_{max}=1000V) and a high voltage probe (Metrix-HT212-typeB) are used to measure the applied high voltage. The high voltage is delivered by a reversible Direct Current supply (U_{max}= 50 kV, I_{max}=10 mA). Experiments carried out according to climatic conditions were done in a climatic room where temperature and humidity can be varied and controlled.

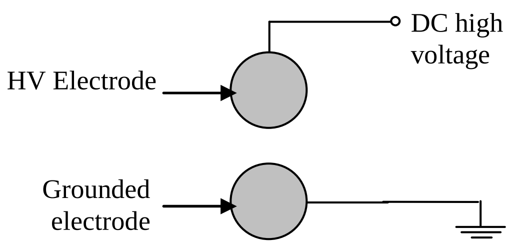

Figure 1. Schematic representation of the Experimental device.

III. EXPERIMENTAL DESIGN METHODOLOGY

Methodology of the experimental designs makes it possible to determine the number of experiments to be achieved according to a well defined objective, to study several factors simultaneously, to reduce dispersion related to measurements, to appreciate the effects of coupling between factors, to evaluate the respective influence of the factors and their interactions [7-8]. Many papers were written about the application of this methodology in electrical and electrostatic processes [9-18].

A. Development of the Method

Finding mathematical models of good quality with minimum efforts depends on the way in which intervals of input factors are selected. This method can be used as follows [19-20]:

- Selection of the most interesting and influent factors;
- Determination of variation interval of each factor, i.e. maximal, minimal and central values;
- Carry out a matrix of experiments with all the possible states and corresponding responses.

Before starting the experiments, it is necessary to opt for the best and suitable design which can model the process with the most possible precision. In this paper, we opted for the centred facess composite design (CCF) which allows using Response Surfaces Modelling (RSM). It is possible to determine a quadratic dependence between the output function to optimize (response) and the input variables u_i (i = 1,.....,k):

$$y = f(u_i) = c_0 + \sum c_i u_i + \sum c_{ij} u_i u_j + \sum c_{ii} u_i^2 \qquad (1)$$

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Knowing that Δu_i and u_{i0} are respectively the step of variation and the central value of factor i, reduced centred values of input factors are defined by the following relation:

$$x_i = (u_i - u_{i0}) / \Delta u_i \qquad (2)$$

With these new variables, the output function (response) becomes:

$$y = f(x_i) = a_0 + \sum a_i x_i + \sum a_{ij} x_i x_j + \sum a_{ii} x_i^2 \qquad (3)$$

The coefficients can be calculated or estimated by a data-processing program, in such way to have a minimal variance between the predictive mathematical model and the experimental results.

B. *Planning of the Experiments*

The main advantage of CCF composite designs is to carry out the experiments sequentially, i.e. to try initially modelling the process with a polynomial of first order [21]. The first step consists thus in a full factorial design. If the 1st order model is validated we stop modelling, if not we continue modelling with a polynomial of second order using a CCF composite design.

As figure 2 shows it, in the case of a design with 3 factors, the full factorial design (first order) corresponds to the experiments located at the tops of the cube (square points A,B…H) and 3 identical experiments done in the central point M (star point). The CCF composite design (second order) corresponds to the 11 experiments mentioned before, i.e. the preceding factorial design, and 6 experiments located in the centres of the cube faces (round points a,b…f). Thus, a CCF composite design with 3 factors includes 17 experiments.

C. *Software Modde.05*

We used software MODDE 5.0 (Umetrics AB, Umea, Sweden) which is a Windows program for the creation and the evaluation of experimental designs [22], the program assists the user for interpretation of the results and prediction of the responses. It calculates the coefficients of the mathematical model, draws surfaces of response (RSM) and identifies best adjustments of the parameters for optimizing the process.

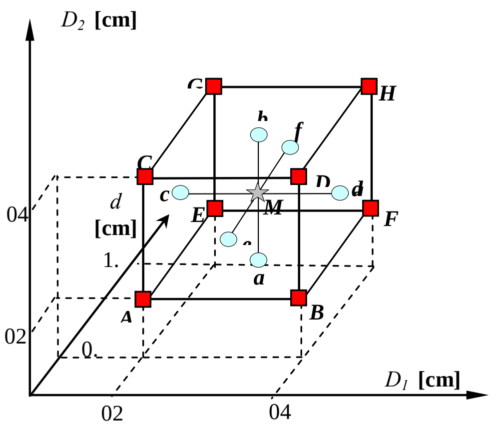

Figure 2. Diagram of experiments of a CCF design with 3 factors:
D_1: HV sphere diameter (D_{1min} = 2 cm & D_{1max} = 4 cm)
D_2: grounded sphere diameter (D_{2min} = 2 cm & D_{2max} = 4 cm)
d: inter-electrodes interval (d_{min} = 0.5 cm & d_{max}=1.5 cm)

Moreover, the program calculates two significant statistical criteria which make it possible to validate or not the mathematical model:

- The predictive power is given by Q^2. This is a measure of how well the model will predict the responses for new experimental condition.
- The goodness of fit parameter given by R^2.

A good mathematical model must have criteria Q^2 and R^2 which the numerical value closes to the unit.

IV. RESULTS

A. *Experimental Design of Geometrical Factors*

As the insulation in high voltage remains the major problem of dielectrics, optimization of breakdown means the maximization of equation (3), expressing the response which is in our case the critical breakdown voltage U_c. The analysis concerning geometrical parameters was made using a CCF experimental design with 3 factors. According to the development of the method described below, we determine limits of variation of each factor:

- Diameter of the high voltage spherical electrode: D_{1min} = 2 cm & D_{1max} = 4 cm;
- Diameter of the grounded spherical electrode:
- D_{2min} = 2 cm & D_{2max} = 4 cm;
- Inter-electrodes interval:
- d_{min} = 0.5 cm & d_{max} = 1.5 cm.

Measurements of voltage U_c obtained in positive polarity (U_{cp}) and negative polarity (U_{cn}) are deferred in table I. We represented in the same table the results of both factorial and composite designs.

Once experimental values of voltage U_c are measured, software MODDE.05 checks first if obtained experimental results are "reasonable" and detects any "doubtful" measurement result. Graph represented on figure 3 shows that all the experiments are located inside the validation limits of results and makes it possible to validate experiment's results.

TABLE I. RESULTS OF THE 1ST EXPERIMENTAL DESIGN (GEOMETRICAL FACTORS)

Exp.	*d* [cm]	D_1 [cm]	D_2 [cm]	U_{cp} [kV]	U_{cn} [kV]
1	0.5	2	4	15.5	16.8
2	0.5	4	4	16.3	16.1
3	0.5	2	8	15.3	16.5
4	0.5	4	8	15.9	15.9
5	1.5	2	4	39.2	40.4
6	1.5	4	4	41.7	38.5
7	1.5	2	8	38.5	39.0
8	1.5	4	8	41.3	38.0
9	1.0	3	6	31.5	31.4
10	1.0	3	6	31.4	31.3
11	1.0	3	6	31.5	31.5
12	1.0	2	6	30.9	30.2
13	1.0	4	6	31.9	30.0
14	1.0	3	4	31.7	31.0
15	1.0	3	8	31.2	30.9
16	0.5	3	6	15.6	16.3
17	1.5	3	6	40.6	38.7

Factorial design (Exp. 1–11); Composite design CCF (Exp. 1–17)

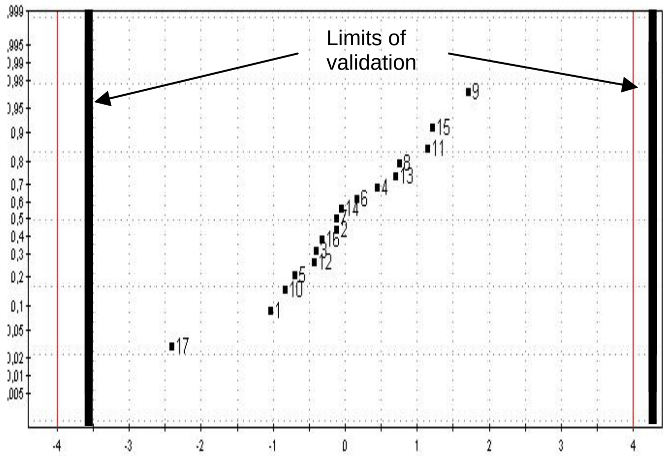

Figure 3. Graph for validation of measurements

MODDE.05 gives values of criteria R^2 and Q^2 lower than 0.6 for the factorial design. Consequently, mathematical models of first order for voltages U_{cp} and U_{cn} can not be validated. Thus we must carry out six additional experiments to accomplish a CCF composite design (grey lines of table I). The statistical tests led this time to a valid mathematical model since criteria R^2 and Q^2 reached high values ranging between 0.996 and 0.999.

The models suggested by MODDE.05 are:

$$U_{cp} = 31.5 + 0.77D_1^* - 0.22D_2^* + 12.3d^* - 0.08\,(D_1^*)^2 - 0.04(D_2^*)^2 - 3.38\,d^{*2} + 0.01\,(D_1^*\,D_2^*) - 0.49\,(D_1^*d^*) - 0.06\,(D_2^*d^*) \quad (4)$$

for positive polarity, and

$$U_{cn} = 31.0 - 0.44\,D_1^* - 0.25\,D_2^* + 11.3\,d^* - 0.55\,(D_1^*)^2 + 0.3\,(D_2^*)^2 - 3.1\,d^{*2} + 0.12\,(D_1^*\,D_2^*) - 0.2\,(D_1^*d^*) - 0.17\,(D_2^*d^*) \quad (5)$$

for negative polarity.

B. *Experimental Design of Climatic Factors*

The influence of climatic parameters, i.e. temperature and humidity, was also studied using a CCF composite experimental design. The experiments were carried out in a climatic room where temperature and humidity can be varied and controlled. According to the development of the method described below, we determine limits of variation of each factor:

- Inter-électrodes interval d : d_{min} =0.5 cm & d_{max} = 1.5 cm ;
- Temperature T : T_{min}=30° & T_{max}=50° ;
- Humidity : H_{min}=40 % & H_{max}=60 %.

Measurements of voltage U_c obtained in positive polarity (U_{cp}) and negative polarity (U_{cn}) are deferred in table II. We represented in the same table the results of both factorial and composite designs. For this second design, software MODDE.05 gives also values of criteria R^2 and Q^2 lower than 0.6 for the factorial design. Consequently, mathematical models of first order have not been validated, and we carry out six additional experiments (grey lines of table II) to accomplish a CCF design. The statistical tests made by the software led this time to a valid mathematical model since criteria R^2 and Q^2 reached high values ranging between 0.968 and 0.989.

The models suggested by MODDE.05 are:

$$U_{cp} = 31.6 + 11.4\,d^* - 1.01\,T^* + 0.22\,H^* - 4.6\,d^{*2} + 0.31\,T^{*2} - 0.34\,H^{*2} - 0.33\,d^*T^* + 1.1.10^{-6}\,d^*H^* - 0.02\,T^*H^* \quad (6)$$

$$U_{cn} = 30.7 + 12.06\,d^* - 0.8\,T^* + 0.32\,H^* - 1.5\,d^{*2} + 0.04\,T^{*2} + 0.3\,H^{*2} - 0.12\,d^*T^* - 0.17\,d^*H^* - 0.2\,T^*H^* \quad (7)$$

TABLE II. RESULTS OF THE 2ND EXPERIMENTAL DESIGN (CLIMATIC FACTORS)

Exp.	*d* [cm]	*T* [°C]	*H* [%]	U_{cp} [kV]	U_{cn} [kV]
1	0.5	30	40	16.0	16.8
2	1.5	30	40	39.5	41.7
3	0.5	50	40	14.7	16.3
4	1.5	50	40	36.9	40.0
5	0.5	30	60	16.5	18.5
6	1.5	30	60	40.0	42.0
7	0.5	50	60	15.1	16.5
8	1.5	50	60	37.3	40.2
9	1.0	40	50	31.7	31.0
10	1.0	40	50	31.6	31.0
11	1.0	40	50	31.5	31.1
12	0.5	40	50	15.7	16.6
13	1.5	40	50	38.4	41.4
14	1.0	30	50	33.0	31.6
15	1.0	50	50	30.9	29.5
16	1.0	40	40	31.1	29.8
17	1.0	40	60	31.5	30.6

Factorial design (rows 1–11)

Composite design CCF (rows 1–17)

V. DISCUSSIONS

Values of the coefficients associated with the factors show the degree of influence of each factor. It arises from mathematical models given by equations 4 and 5 for geometrical factors, that within the variation limits of the selected intervals, the inter-electrodes interval d is the one which has the most effect. Moreover, the response surfaces plots of figure 4 show that in positive polarity, diameter D_1 of the high voltage electrode is more influent on the values of breakdown voltage compared to diameter D_2 of the grounded electrode. Moreover, in negative polarity, obtained results illustrated by response surfaces of figure5 show that the effect of one diameter compared to the other depends on their values. Indeed, for small spheres (both diameters < 3cm), it is rather diameter D_2 (i.e. grounded electrode) which has more effect on voltage U_c. This tendency is reversed for the great values of diameters D_1 and D_2.

Concerning the climatic factors and as we could expect it, the increase in the temperature makes decrease the breakdown voltage U_c (negative coefficients: -1.01 for U_{cp} and – 0.8 for U_{cn}), while the humidity makes it increase (positive coefficients: +0.22 for U_{cp} and +0.32 for U_{cn}). Moreover, obtained mathematical models did not reveal a great interaction between these two climatic factors, the interaction coefficients being weak (-0.02 for U_{cp} and -0.2 for U_{cn}). Moreover, by analyzing the response surfaces illustrated on figures 6 and 7, we notice that within the variation limits of intervals, i.e. 30°. 50°C for temperature and 40%.. 60% for humidity, the temperature has undeniably more effect than the humidity.

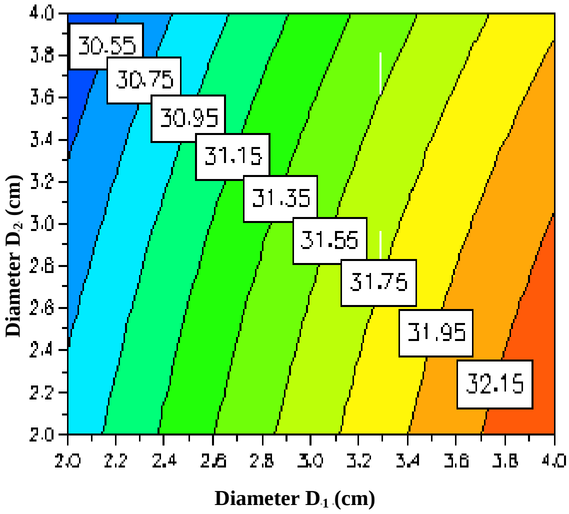

Figure 4. Response surfaces of voltage U_{cp} according to sphere's diameters (inter-electrodes interval d=1cm).

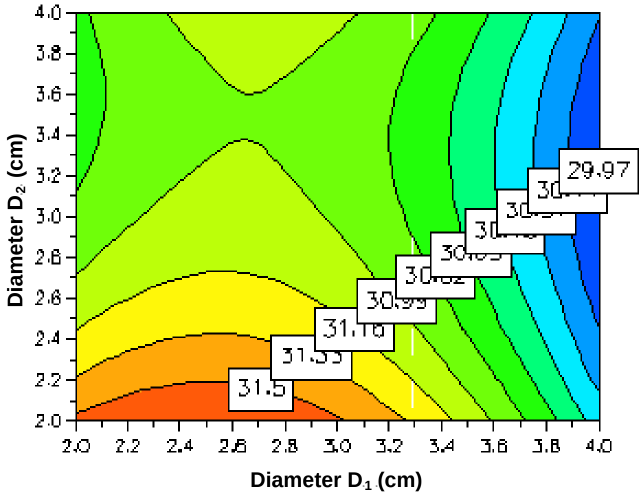

Figure 5. Response surfaces of voltage U_{cn} according to sphere's diameters (inter-electrodes interval d=1cm).

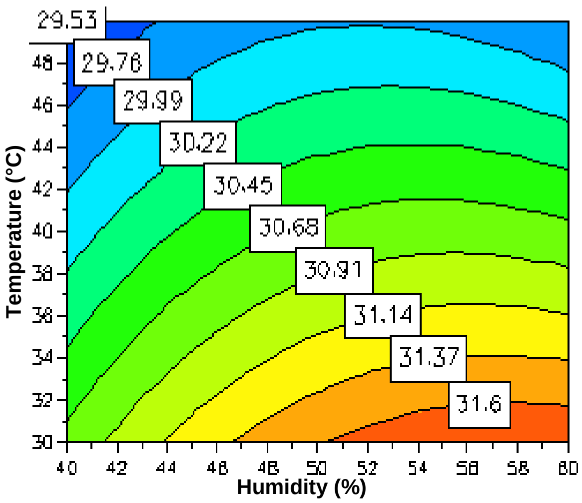

Figure 6. Response surfaces of voltage U_{cp} according to climatic factors (inter-electrodes interval d=1cm).

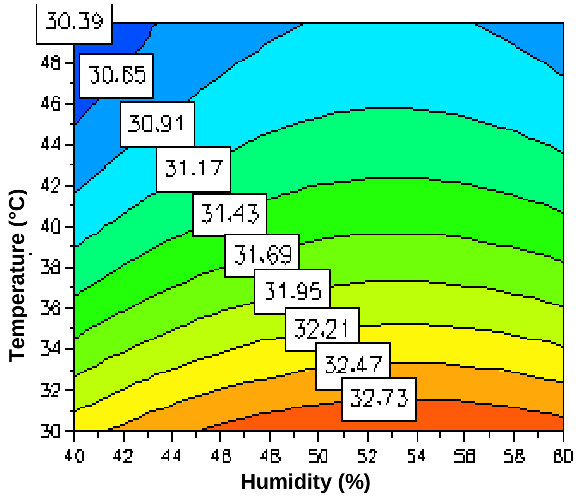

Figure 7. Response surfaces of voltage U_{cn} according to climatic factors (inter-electrodes interval d=1cm).

VI. CONCLUSION

Breakdown voltage U_c of gaseous dielectrics remains the subject of several research tasks in the world, because it depends on numerous factors. As it is difficult to find a formula, theoretical or experimental, which considers the various parameters, the aim of this paper consisted in modelling voltage U_c using the methodology of experimental designs. Several factors were considered in this study: geometrical factors (such as inter-electrodes interval and radius of spherical electrodes) and climatic factors (such as temperature and humidity). Obtained results in the two polarities made it possible to propose mathematical models and to analyze the various interactions between these factors.

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REFERENCES

[1] L.B.Loeb, "Electrical coronas", University of California Press, Berkeley and Los Angeles 1965.

[2] L.B.Loeb, "The mecha&nism of the electrical spark", Stanford University Press (1941)

[3] J.M.Meek and J.D.Draggs, "Electrical breakdown of gases". Oxford, 1953.

[4] J.M.Meek and J.D.Draggs, "Electrical breakdown of gases". John Wiley and sons, 1978, p473.

[5] N.L. Frigon, and D. Mathews, "Practical Guide to Experimental Design", New York: Wiley, 1996.

[6] G. Taguchi, "System of Experimental Designs", New York: Kraus International Publications, 1987.

[7] L. Eriksson, E. Johansson, N. Kettaneh-Wold, C. Wikström, and S. Wold, "Design of Experiments. Principles and Applications". Learnways AB, Stockholm, 2000.

[8] L ; Eriksson, E. Johansson, N. Kettaneh, C. Wikström et S. Wold, "Design of experiments". *Umetrics Academy, Sweden*. 2000.

[9] L.Dascalescu, A.Tilmatine, F.Aman and M. Mihailescu. « Optimisation of electrostatic separation processes using response surface modeling ». *IEEE Trans. Ind. Appl.*, VOL. 40, N°.1, JANUARY/FEBRUARY 2004.

[10] L. Dascalescu, A. Samuila, A. Mihalcioiu, S. Bente, and A. Tilmatine. « Robust Design of Electrostatic Separation Processes ». IEEE Transactions On Industry Applications, VOL. 41, N°. 3, MAY/JUNE 2005.

[11] L. Dascalescu, A. Mihalcioiu, A. Tilmatine, M. Mihailescu, A. Iuga, & A. Samuila. « A linear-interaction optimization model using Taguchi's experimental design technique ». IEEE Industry Applications Magazine • Nov|Dec 2004.

[12] N. Kadous F.Miloua F.Z Rahou A. Tilmatine. « Optimization of the electrostatic separation process using design of experiments methodology ». *Journal of Materials Technology.* Lancaschire, England. Volume 19-4. December 2004.

[13] M.Rezouga, A.Tilmatine, R.Gouri, K.Medles. "Experimental modelling of corona discharge in point-plane configuration" Front. Electr. Electron. Eng. China 2007, 2(2): 139–143. Higher Education Press and Springer-Verlag 2007.

[14] K. Medles, A. Tilmatine, A. Bendaoud, M. Rahli, L. Dascalescu. « Set Point Identification and Robustness Testing of Electrostatic Separation Processes ». *IEEE Trans. Ind. Appl*, Vol.3, MAY/JUNE 2007. ISSN: 0197-2618.

[15] Dascalescu. L.; Mihalcioiu. A.; Tilmatine, A.; Medles. K, Samuila, A : « Effect of ambient humidity on the outcome of electrostatic separation processes». IEEE Transactions on Industry Applications Society; Publication date: Oct. 2004, pp.1959- 1966, vol.3.

[16] K. Medles, L Dascalescu, A. Tilmatine, A.bendaoud and M Younes "Experimental Modeling of the Electrostatic Separation of Granular Materials" Particulate Science and Technology, Volume 25, Issue 2 March 2007 , pages 163 - 171.

[17] F. Miloua, A. Tilmatine, R. Gouri, N. Kadous and L. Dascalescu, Eur. Phys. J. Appl. Phys. (2007), DOI: 10.1051/epjap:2007175, Experimental modelling of high-voltage corona discharge using design of experiments

[18] Rezzouga M, Tilmatine A, Gouri R, Medles K and Dascalescu L, Frontiers of Electrical and Electronic Engineering in China. Springer-Verlag GmbH, Volume 2, Number 2 / avril 2007.

[19] Hallouche, A, Tilmatine, A. Structure for Improving Short-Circuit Capability and the Method for Protecting the IGBT Devices Advances in Electrical and Computer Engineering, Suceava, Romania ISSN 1582-7445, No 2/2008, volume 8 (15), pp. 11-14

[20] C.R. Hicks, and K.V. Turner Jr., "Fundamental Concepts in the Design of Experiments", Oxford: Oxford University Press, 1999.

[21] D.C. Montgomery, "Design and Analysis of Experiments", 6th Ed., New York: Wiley, 2004.

[22] L. Dascalescu, A. Tilmatine, F. Aman, M. Mihailescu. "Optimization of electrostatic separation Processes using response surface modelling". Industry Applications, IEEE Transactions. Volume 40, Issue 1, Jan.-Feb. 2004 Page(s):53 - 59

[23] MODDE.05, "User guide and tutorial". *Umetrics*, 1999.