A Novel Chaotic System for Random Pulse Generation

Victor GRIGORAS[1], Carmen GRIGORAS[2,3]
[1]*Gheorghe Asachi Technical University of Iasi, 700506, Romania*
[2]*Gr.T. Popa University of Medicine and Pharmacy of Iaşi, 700115, Romania*
[3]*Institute of Computer Science, Romanian Academy, Iaşi, 700481, Romania*
grigoras@etti.tuiasi.ro

Abstract—We introduce a novel third order analogue nonlinear system, using three multipliers as nonlinear functions. The proposed system exhibits rich nonlinear dynamics, with periodic and chaotic behaviors, depending on the system parameters. The strange attractor of the system, under chaotic parameter choice, is characterized by a limited domain occupied in the state space. The analysis of the nonlinear dynamics of the system is presented and in depth simulation results confirm the desired properties. Suggested applications of the proposed system include analogue noise generators and spread spectrum clock generators.

***Index Terms*—chaos, noise generators, nonlinear attractors, nonlinear dynamical systems, pulse generation.**

I. Introduction

The present paper presents a new analogue nonlinear system, based on multiplication as a nonlinear algebraic function, which has complex dynamics and can be applied for generating random-sign pulses or spread-spectrum clock signals.

Some well studied analogue nonlinear systems, which exhibit complex dynamics, are using multiplication as a nonlinear function in their state equations. The Rossler system [1-2] is based on a single multiplier structure and the Lorenz [3] and Chen [4-6] systems use two such building blocks. The nonlinear dynamics of such systems is based on chaotically switching the state space trajectory between two branches of the strange attractor (Lorenz or Chen systems) or impulsive interrupting an increasing oscillation, in the case of the Rossler system. In both cases, large value transients occur, making signal maximum values estimation and circuit implementation difficult.

Proposed applications of these analog chaotic systems include chaotic modulation and encryption [7, 8] and chaos control [9, 10]. In order to achieve such goals, circuit implementations were desired, and typical implementations regard Chua's circuit [11] and power circuits with nonlinear reactive elements [12], all difficult to integrate on silicon chips.

In order to keep state variables confined to a limited state-space domain, we propose an alternative system, with multipliers included in all three state equations.

Possible applications of the proposed system are noise generation and spread spectrum clock generation. Noise generation is one of the first applications of chaotic dynamics [13] and has evolved on both analog [13-15] and discrete-time [16-20] tracks due to promising applications in communication security and test and measurement equipment.

The following section is devoted to the development of the system design, with dynamic analysis based on a linear prototype and product type nonlinearities added to it. Simulation results, which highlight the desired properties of the proposed system, are included in the third section, showing possible applications of the proposed system. The final section draws resulting conclusions and suggests further research.

II. System Analysis

The proposed nonlinear system is designed using a linear part added to a nonlinear algebraic function, leading to the structure:

$$\mathbf{x}' = \mathbf{A}\cdot\mathbf{x} + \mathbf{f}(\mathbf{x}) \tag{1}$$

The linear part of the system is designed starting from a state transition matrix of the form (2):

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & a \\ -b & -a & 1 \end{bmatrix} \tag{2}$$

resulting from an initial eigenvalue set intended for oscillatory dynamics (3):

$$\mathbf{\Lambda} = \begin{bmatrix} -1 \\ j\sqrt{a^2-1} \\ -j\sqrt{a^2-1} \end{bmatrix} \tag{3}$$

where the parameter, a, is always considered to be larger than unit.

This leads to the scalar state equations system (4):

$$\begin{cases} x_1' = -x_1 \\ x_2' = -x_2 + a\cdot x_3 \\ x_3' = -b\cdot x_1 - a\cdot x_2 + x_3 \end{cases} \tag{4}$$

The nonlinear part is based on three state variables multiplications, as shown in the following equation (5):

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} -\alpha\cdot x_2\cdot x_3 \\ -\beta\cdot x_1\cdot x_3 \\ \gamma\cdot x_1\cdot x_2 \end{bmatrix} \tag{5}$$

The final form of the state equations of the proposed system is (6):

$$\begin{cases} x_1' = -x_1 - \alpha\cdot x_2\cdot x_3 \\ x_2' = -x_2 + a\cdot x_3 - \beta\cdot x_1\cdot x_3 \\ x_3' = -b\cdot x_1 - a\cdot x_2 + x_3 + \gamma\cdot x_1\cdot x_2 \end{cases} \tag{6}$$

The equilibrium points of the nonlinear system result in the form given in equations (7) and (8), as detailed in

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Appendix A:

$$\begin{cases} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{cases} \tag{7}$$

and

$$\begin{cases} x_1 = -\dfrac{\alpha \cdot a \cdot \rho^2}{1-\alpha \cdot \beta \cdot \rho^2} \\ x_2 = \dfrac{a \cdot \rho}{1-\alpha \cdot \beta \cdot \rho^2} \\ x_3 = \rho \end{cases} \tag{8}$$

where ρ is a root of the polynomial (9):

$$P(z) = z^4 - \frac{a \cdot b}{\beta} \cdot z^3 + \frac{a^2(\beta-\gamma)-2\beta}{\alpha\beta^2} \cdot z^2 + + \frac{a \cdot b}{\alpha\beta^2} \cdot z - \frac{(a^2-1)}{\alpha^2\beta^2} \tag{9}$$

The fact that the proposed nonlinear system is dissipative can be easily verified from the form of the state equations, as resulting from (10):

$$\nabla f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} = -1 \tag{10}$$

Moreover, this property does not depend on the system parameters, as defined in the proposed state equations. On the contrary, the ergodicity and sensitivity to initial conditions are dependent on the coefficients in the system state equations and, as such, will be treated in the following section, by numerical simulation.

III. Simulation Results

In order to verify the nonlinear dynamics of the proposed system, in depth simulations were performed. To choose the proper system parameters values, a parametric analysis was performed. We non-uniformly sampled the state variables x_2 and x_3 at the time moments when $x_1 = 5$. This way we obtained a second order discrete-time system, with the state variables denoted y_1 and y_2. Increasing slowly the system coefficients, one at a time, keeping the other ones constant, we obtained graphs such as the one in Fig. 1, where chaotic intervals alternate with periodic ones. This analysis suggests the value ranges for the system parameters in order to obtain different dynamical behaviors.

The ergodic character of the third order nonlinear system can be viewed in the 3D state space trajectory depicted in Fig. 2, for a choice of system parameters (11), leading to chaotic dynamics:

$$a = 5; \quad b = 9; \quad \alpha = 50; \quad \beta = 20; \quad \gamma = 4.1; \tag{11}$$

If a similar simulation is performed using a set of parameters suited for periodic behavior, e.g. by modifying the values $\beta = 10$ and $\gamma = 7.1$ and maintaining all others as in (11), a phase portrait such as the one in Fig. 3 can be obtained, highlighting a period multiplication with a factor of two.

In order to check the sensitive behavior with respect to the initial conditions of the nonlinear system, for the choice of system parameters leading to chaotic dynamics (11), the RMS value of the difference between the state vectors of two identical systems, starting from initial conditions differing with 0.05% was computed and the simulation result depicted in Fig. 4 confirms the chaotic hypothesis.

In order to properly understand the chaotic dynamics of the analyzed system, Poincare sections were made, by intersecting the phase portrait of the system with a half plane chosen by imposing a null value to a state variable. If the third state variable is equaled to zero and the first two state variables are sampled at the intersection moment, a second order discrete-time phase portrait as the one depicted in Fig. 5 is obtained. In the case of switching roles between x_2 and x_3 state variables, we obtained obtained the results in Fig. 6. The dense clustering of the sample values on a thin domain confirms the ergodic character of the analog system.

Simulations on the time evolution of the state variables were also performed, for the case when the system parameters take the standard values in (11), leading to the results depicted in the following figures.

Under the presented conditions, the first state variable, x_1, shows the waveform, shown in Fig. 7, is suggesting applications in random pulse generation. The wideband frequency spectrum of the same state variable, depicted in Fig. 8, confirms the possibility of using the proposed system as an analogue noise generator.

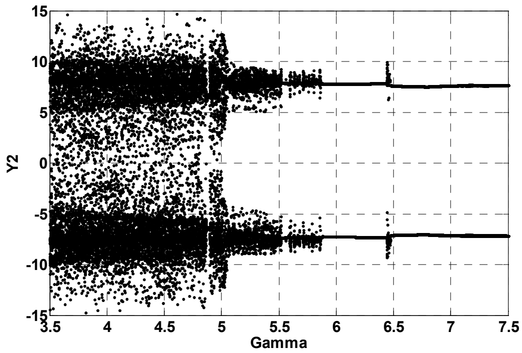

Figure 1. Parametric analysis at the variation of the gamma coefficient

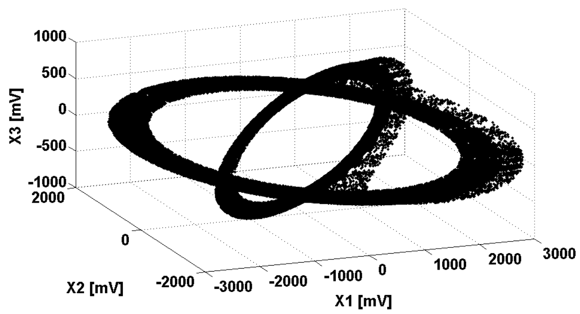

Figure 2. State space portrait for the chaotic dynamics parameter choice

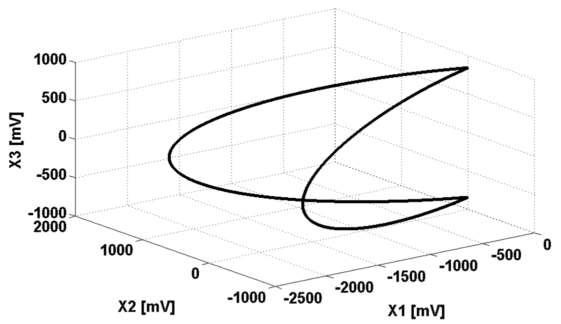

Figure 3. State space portrait for the periodic dynamics parameter choice

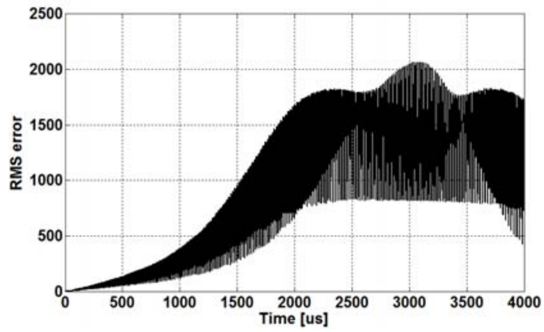

Figure 4. RMS value of the difference of the two state vectors

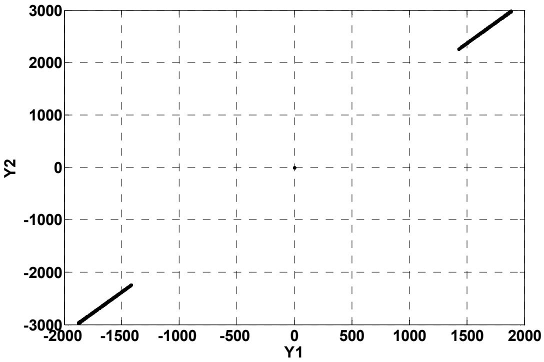

Figure 5. Poincare section in the case of sampling x_1 and x_2 state variables

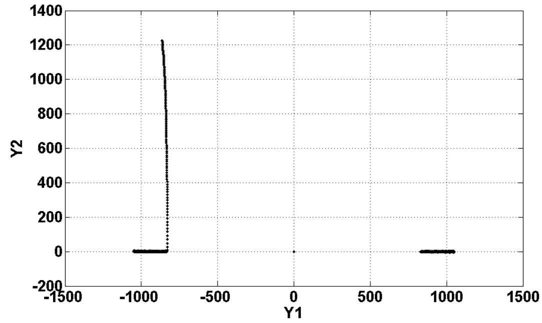

Figure 6. Poincare section in the case of sampling x_1 and x_3 state variables

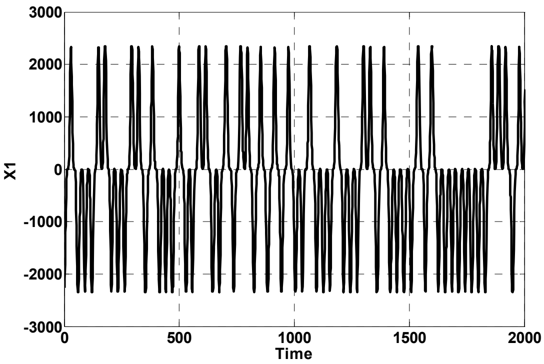

Figure7. Time evolution of the first state variable

The same simulation shows, for the third state variable, x_3, the waveform presented in Fig. 9. Such simulations highlight chaotic variation of x_3 increasing and decreasing fronts. The time domain observation is confirmed by the frequency domain spectrum of the third state variable, depicted in Fig. 10, which looks similar to a PWM spectrum with low frequency, random modulating signal.

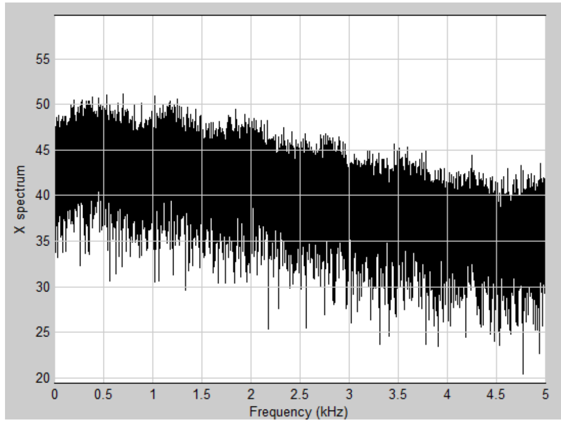

Figure 8. Frequency spectrum of the first state variable

The presented results suggest that the proposed system may be used for spread spectrum clock generation, by using a comparator to form the desired digital signal from the x_3 state variable, as depicted in Fig. 11. Due to the small front jitter, confirmation of the spread spectrum characteristic for the clock signal is more obvious in the frequency spectrum, depicted in Fig. 12.

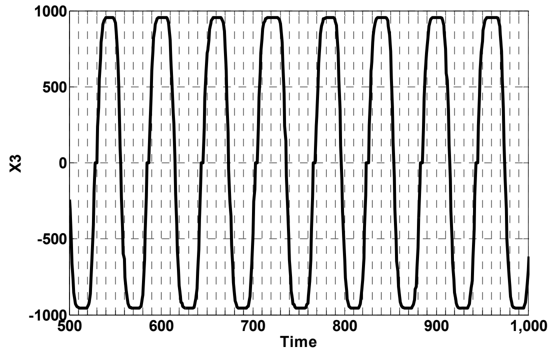

Figure 9. Time evolution for the third state variable

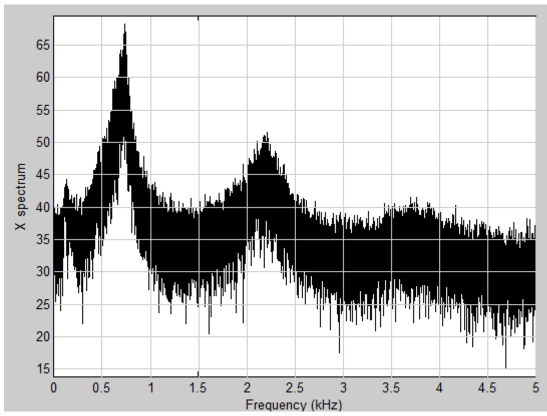

Figure 10. Frequency spectrum of the third state variable

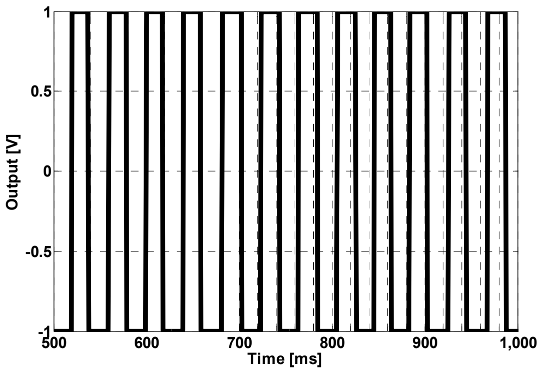

Figure 11. The rectangular spread spectrum clock signal

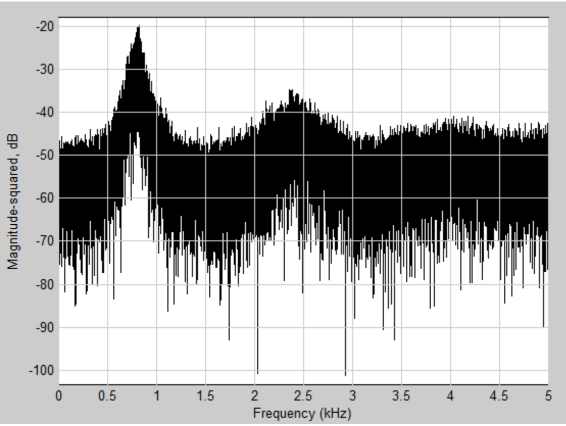

Figure 12. Frequency spectrum of the spread spectrum digital signal

IV. Conclusion

The present paper introduces a novel analogue nonlinear system, which exhibits complex dynamics with limited dynamic range of the generated signals. The fact that the first two state variables of the proposed system have chaotic impulsive waveforms and the third one exhibits jitter on its fronts lead to applications in the fields of random pulse generation and spread spectrum clock generation. Further research is needed for electronic circuit implementation of the nonlinear system developed here, to overcome technological restrictions and device non-idealities.

Appendix A

In order to compute the equilibrium points of the proposed system we impose the equilibrium condition (A1) to the state equations (6):

$$\mathbf{x}' = 0 \tag{A1}$$

This leads to the nonlinear algebraic system of equations:

$$\begin{cases} x_1 + \alpha \cdot x_2 \cdot x_3 = 0 \\ x_2 - a \cdot x_3 + \beta \cdot x_1 \cdot x_3 = 0 \\ -b \cdot x_1 - a \cdot x_2 + x_3 + \gamma \cdot x_1 \cdot x_2 = 0 \end{cases} \tag{A2}$$

The system can be solved iteratively, as follows:

$$\begin{cases} x_1 = -\alpha \cdot x_2 \cdot x_3 \\ x_2 - a \cdot x_3 - \beta \cdot \alpha \cdot x_2 \cdot x_3^2 = 0 \\ b \cdot \alpha \cdot x_2 \cdot x_3 - a \cdot x_2 + x_3 - \gamma \cdot \alpha \cdot x_2^2 \cdot x_3 = 0 \end{cases} \tag{A3}$$

For $x_3^2 \neq 1/\alpha \cdot \beta$

$$\begin{cases} x_1 = -\dfrac{\alpha \cdot a \cdot x_3^2}{1 - \alpha \cdot \beta \cdot x_3^2} \\ x_2 = \dfrac{a \cdot x_3}{1 - \alpha \cdot \beta \cdot x_3^2} \\ \dfrac{\alpha \cdot a \cdot b \cdot x_3^2 - a^2 \cdot x_3}{1 - \alpha \cdot \beta \cdot x_3^2} + x_3 - \dfrac{\gamma \cdot \alpha \cdot a^2 \cdot x_3^3}{\left(1 - \alpha \cdot \beta \cdot x_3^2\right)^2} = 0 \end{cases} \tag{A4}$$

The last equation in the system can be brought to the polynomial form:

$$\begin{aligned} &\left(\alpha \cdot a \cdot b \cdot x_3^2 - a^2 \cdot x_3\right) \cdot \left(1 - \alpha \cdot \beta \cdot x_3^2\right) + \\ &+ x_3 \cdot \left(1 - \alpha \cdot \beta \cdot x_3^2\right)^2 - \alpha \cdot \gamma \cdot a^2 \cdot x_3^3 = 0 \end{aligned} \tag{A5}$$

The polynomial equation has the obvious root:

$$x_3 = 0 \tag{A6}$$

For this root, we obtain the null system solution (7). By dividing with x_3, we obtain:

$$\begin{aligned} &\alpha^2 \cdot \beta^2 \cdot x_3^4 + \alpha^2 \cdot \beta \cdot a \cdot b \cdot x_3^3 + \\ &+ \left(\alpha \cdot \beta \cdot a^2 - \alpha \cdot \gamma \cdot a^2 - 2\alpha \cdot \beta\right) \cdot x_3^2 - \\ &- \alpha \cdot a \cdot b \cdot x_3 + 1 - a^2 = 0 \end{aligned} \tag{A7}$$

This can be further processed to obtain the monic polynomial (9) and the non-null solutions (8).

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