# Study of the Induction Machine Unsymmetrical Condition Using *In Total Fluxes* Equations

Alecsandru SIMION
*Technical University of Iasi, Romania*
*asimion@ee.tuiasi.ro*

***Abstract*—On the basis of the mathematical model, called *in total fluxes* in a previous paper, and which is proper for the analysis of transient operation of the two-phase induction machine, one obtains the symmetrical steady-state equations, which are valid for three-phase machines, as well. The obtained mathematical expressions are much more simple and easier to use than the consecrated ones, which are generally applied in scientific literature. Moreover, considerations are to be made upon the *space-time rotational vectors*, emphasizing their importance in understanding the physical phenomena that characterize induction machines. The use of these space vectors is further tested out for the study of unsymmetrical supply, which gives a much faster method in obtaining the electromagnetic torque expression. Finally, the results are compared with the ones that come out from the traditional methods, more exactly, the *symmetric component* method.**

***Index Terms*—induction machine, representative rotational phase vector, symmetric components, unsymmetrical condition**

## I. Introduction

In support of our approach, defining of the space-time rotational vectors, which were presented under similar formulations in scientific literature [1-8], is necessary first. The superposing effect concerning the quantities of the electric field determined by the two-phase supply system and the corresponding magnetic field is also considered as a prior probability. The supply voltages are applied along the turns placed in the slots (collinear to *Oz* axis). If the winding is placed in the slots according to a sinusoidal law, *one decides that the applied voltage phase vector, u* to be represented as a segment in the *xOy* plane, orientated towards positive axis of the winding. The length of this segment is maximum when the applied voltage is maximum, too. Ideally, in default of magnetic leakages and drop voltages corresponding to winding resistance, the magnetic fields (more precisely, *the total fluxes* $\psi$, which are preponderantly effective for the real cylindrical machines) close in a radial pattern inside the *xOy* plane. It has a harmonic distribution on the periphery (on the circle that matches the middle of the air-gap), which means that the *flux density* has maximum values in the centrum of the supplied winding only when *the applied voltage reaches the 0 value* (according to induced voltage law: u=dψ/dt). For our demonstration, the stator phase vectors of the voltages and total fluxes will be represented in the *xOy* plane for different but consecutive moments according to Leblanc theorem (any magnetic flux of $\psi_m$ amplitude, which is created by a winding with cosine distribution and single-phase feeding, is equivalent with two rotating and equal fluxes with the amplitude of $(1/2)\psi_m$ but which rotates in opposite directions with coequal speed, *forward* and *backward* traveling waves respectively, [9-14].

## II. Hybrid Parameters of Induction Machine, Total Fluxes

Fig. 1a presents a cross section of an induction machine with a distributed single-phase stator winding. One starts with the voltage equation:

$$u = Ri + \frac{d\psi}{dt};\; u = Ri + \frac{d(Li)}{dt} = Ri + \frac{d(L)}{dt}i + \frac{d(i)}{dt}L;$$
$$u = \left[R + \frac{d(L)}{dt}\right]\cdot i + \left(L\right)\cdot\frac{d(i)}{dt} \tag{1}$$

which, by using the Ohm law for magnetic circuits ($Wi = R_m \cdot \frac{\psi}{W}$) becomes:

$$u = \frac{R}{W}(Wi) + \frac{d\psi}{dt} = \frac{R}{W}\left(R_m\frac{\psi}{W}\right) + \frac{d\psi}{dt} = \frac{R}{L}\psi + \frac{d\psi}{dt} \quad \text{or}$$

$$u = \nu\psi + \frac{d\psi}{dt} \tag{2}$$

One defines the hybrid parameter called “niutance”, which depends on machine geometry and material characteristics: resistivity-$\rho_{Cu}$, vacuum permeability-$\mu_0$, total cross section of the winding-$S_{Cu}$ , end winding factor-$k_{fr}$, polar pitch-$\tau$, polar pitch cofficient-$\alpha_i$ , ideal length-$l_i$ and global air-gap-$\delta k$.

$$\nu = \frac{R}{L} = \frac{RR_m}{W^2} = \left(\rho_{Cu}\frac{l_s}{W\cdot s_{Cu}}\right)\left(\frac{2\delta k}{\mu_0 S_p}\right) \; or$$
$$\nu = \left(4\frac{\rho_{Cu}}{\mu_0}\frac{k_{fr}}{\alpha_i}\frac{\delta k}{S_{Cu}}\right)\frac{\left(k_{fr}\tau + l_i\right)}{k_{fr}\tau\cdot l_i} \tag{3}$$

A preliminary analysis of (2) shows that the hybrid parameter, $\upsilon$, (unit of measurement - $s^{-1}$ that is the reverse of the time constant of the winding) has minimum value for certain geometrical dimensions:

$$\nu = \min \rightarrow k_{fr} = \frac{l_i}{\tau} \approx 1.6 \div 1.8 = \lambda \tag{4}$$

Equation (2) shows that a voltage (with harmonic variation in this case) applied to the terminals of a coil (Fig. 1b) determines a total flux, $\psi$, that represents the solution of the first order differential equation with hybrid parameter, $\upsilon$, which is considered constant. Eq. (1) is non-linear even if the parameters $R$ and $L$ have constant values.

Digital Object Identifier 10.4316/AECE.2010.01006

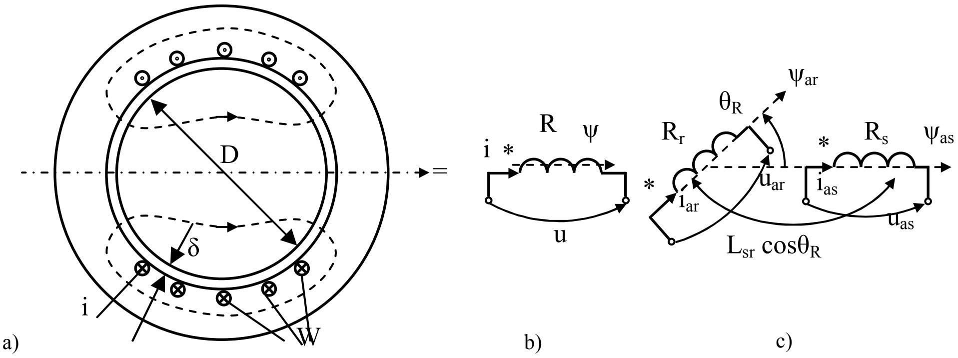


**Figure 1**. Annotation to hybrid parameters and total fluxes.

When the stator coil *as* is close to a rotor coil *ar* (Fig. 1c), there are mutual effective fluxes, $\psi_{sr} = L_{sr} i_{ar} \cdot \cos\theta_R$, and $\psi_{rs} = L_{rs} i_{as} \cdot \cos\theta_R$ respectively. (To simplify the approach one considers that the two coils have coequal numbers of turns, $L_{sr} = L_{rs}$; otherwise, a reducing to stator process, similar to the transformer study, is required). The total fluxes of the windings can be defined with $\psi_{as} = L_{ss} i_{as} + L_{sr} i_{ar} \cos\theta_R ; \psi_{ar} = L_{rr} i_{ar} + L_{sr} i_{as} \cos\theta_R$ or under matrix form:

$$[\psi] = [L] \cdot [i] \quad where:$$

$$[\psi] = \begin{bmatrix} \psi_{as} \\ \psi_{ar} \end{bmatrix}; [L] = \begin{bmatrix} L_{ss} & L_{sr}\cos\theta_R \\ L_{sr}\cos\theta_R & L_{rr} \end{bmatrix}; [i] = \begin{bmatrix} i_{as} \\ i_{ar} \end{bmatrix} \tag{5}$$

By multiplying to the left with inverse matrix of inductances

$$[L]^{-1} = \frac{1}{\Delta}\begin{bmatrix} L_{rr} & -L_{sr}\cos\theta_R \\ -L_{sr}\cos\theta_R & L_{ss} \end{bmatrix}; \quad where: \Delta = L_{ss}L_{rr} - L_{sr}^2 \cos^2\theta_R \tag{6}$$

one obtains the currents dependent on total fluxes:

$$[L]^{-1}[\psi] = [L]^{-1}[L] \cdot [i] \quad or: [i] = [L]^{-1}[\psi] \Leftrightarrow$$

$$i_{as} = \frac{L_{rr}}{\Delta}\psi_{as} - \frac{L_{sr}\cos\theta_R}{\Delta}\psi_{ar}; \quad i_{ar} = -\frac{L_{sr}\cos\theta_R}{\Delta}\psi_{as} + \frac{L_{ss}}{\Delta}\psi_{ar} \tag{7}$$

The voltage equations, similar to (1), for the two circuits, are:

$$\begin{aligned} u_{as} &= R_s i_{as} + \frac{d\psi_{as}}{dt}, \\ or: u_{as} &= \frac{R_s}{(\Delta / L_{rr})}\psi_{as} + \frac{d\psi_{as}}{dt} - \frac{R_s}{(\Delta / L_{sr})}\psi_{ar}\cos\theta_R, \\ u_{ar} &= R_r i_{ar} + \frac{d\psi_{ar}}{dt}, \\ or: u_{ar} &= -\frac{R_r}{(\Delta / L_{sr})}\psi_{as}\cos\theta_R + \frac{R_r}{(\Delta / L_{ss})}\psi_{ar} + \frac{d\psi_{ar}}{dt} \end{aligned} \tag{8}$$

One obtained a system of two second order differential equations with variable coefficients depending on $\theta_R$. Obviously, the cosine of the angle between the positions of the two windings is present since, for a general case, the windings stand or are in a relative rotating movement.

This reasoning can be applied, as well, for other coils that are magnetically coupled with *as,* as a consequence of superimposing effect principle and considering the linear averages first.

## III. The Representative Phase Vectors Of The Ideal Induction Machine

The case of *ideal machine with no leakage fluxes zero value winding resistances* is taken into discussion. The symbolic notations used for this approach are presented in Appendix A.

If one considers a cross section of a symmetrical two-phase machine (and more precisely a path along mid-stator cylinder) and the winding axes have the orientation *as-Ox* and *bs-Oy* respectively, then the phase vectors of the voltages and total fluxes, for different moments, have the positions indicated in Fig. 2 (where a two-phase symmetrical condition is assumed, $U_{as} = U_{bs}$).

Fig. 2 presents the voltage and total flux phase vectors corresponding to *forward* and *backward* order for three different moments: $\omega t = 0$ - Figures 2a1) and a3), $\omega t = \pi / 4$ - Figures 2b1) and b3), and $\omega t = \pi / 2$ - Figures 2c1) and c3). In contrast with the "classical" representation manner, where the *cross* and *dot* signs which are placed inside a turn, show the orientation of the flowing current, in our case, the symbols define the polarity of applied voltage across turns (to avoid any confusion, it has to be mentioned that the purely inductive circuits have a phase time alteration of $\pi / 2$ between current and applied voltage).

During the considered interval (a quarter period) the applied voltage on as phase starts from maximum value and decreases to zero value (see Figures 2 a1, b1and c1) while the voltage corresponding to *bs* phase starts from zero value and increases to maximum value (Figures 2 a2, b2 and c2). The resultant values of voltage and total flux phase vectors come from a geometrical summation of the forward components corresponding to the two phases of the machine. These components are coequal and collinear. On the contrary, the backward components are coequal but in opposite directions for every moment and consequently the sum is always zero. For example, the resultant phase vector of total stator flux has a constant absolute value and the rotation angle is of $\pi/2$ rad. (during a quarter period, the apex of the resultant phase vector covers a quarter of the circle inside the plane *xOy*). This is a coincidence that allows a reciprocal conversion of the temporal and spatial angles and the phase vectors (obtained by means of *analytical representation*) from the complex space (+1, +j) can be assimilated to phase vectors in *xOy* plane, which will be denoted *representative vectors*.

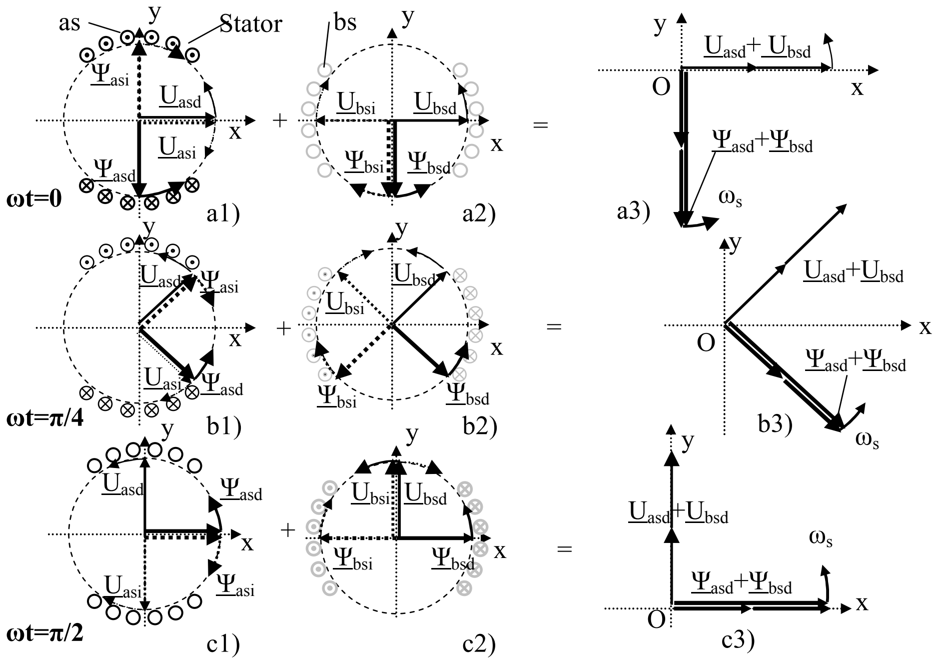


**Figure 2.** Representative rotational phase vectors of two-phase induction machine.Along *Ox* axis, the total flux is created by *as* winding with a cosine time variation and an initial phase of $-\pi/2$ . The same assumption is made for *bs* phase, but acting on *Oy* axis with an initial phase of $-\pi$ .

$$\overline{\Psi}_{sR} = \psi_{asx}\bar{i} + \psi_{bsy}\bar{j}$$
$$\overline{\Psi}_{sR} = \Psi_{as}\sqrt{2}\cos\left(\omega_s t - \frac{\pi}{2}\right)\bar{i} + \Psi_{as}\sqrt{2}\cos\left(\omega_s t - \frac{\pi}{2} - \frac{\pi}{2}\right)\bar{j}; \qquad (9)$$

This vector can be expressed in a different way (polar coordinates) as $\overline{\Psi}_{sR} = \Psi_{sR}\angle\theta_R$. The absolute value of this vector is $\Psi_{sR} = \sqrt{\psi_{asx}^2 + \psi_{bsy}^2} = \Psi_{as}\sqrt{2}$ -constant, and the argument $\theta_R = \omega_s t - \pi/2$, which is time dependent. The angular speed comes from phase derivative $\Omega_s = d\theta_R/dt = \omega_s$ and is equal to applied voltage pulsation. Notable is the fact that *the projections of the representative vector along Ox and Oy axes have coequal length with instantaneous values* of the total fluxes created by the *as* and *bs* windings. This is a strong reason in adopting the concept of *representative space-time vector* of the stator flux. It must be also mentioned that the representative vector of the stator flux shows the radial direction of the cross section plane at any moment, where the density of stator magnetic flux lines (with radial air-gap flux density) is maximum.

A different way to define the *representative space-time* vector of the stator flux is based on one-to-one correspondence between *xOy* plane ($\bar{i}$, $\bar{j}$ versors) and complex space *(+1,+j)*. The following statement is allowed: the $\bar{j}$ versor can be obtained by rotating in the same plane the versor $\bar{i}$ with $\pi/2$ which is equivalent to "multiplication by *j*" in "simplified complex" approach. One can define:

$$\overline{\Psi}_{sR} = \psi_{asx}\bar{i} + \psi_{bsy}\bar{j} \leftrightarrow \underline{\Psi}_{sR} = \underline{\Psi}_{as} + e^{j\delta}\underline{\Psi}_{bs}; \delta = \pi/2 \qquad (10)$$

The angle $\delta = \pi/2$ has the signification of a *spatial angle* among the machine windings. Taking into consideration the forward and backward components presented above, one obtains:

$$\underline{\Psi}_{sR} = \underline{\Psi}_{as} + j\underline{\Psi}_{bs} = \underline{\Psi}_{asd} + \underline{\Psi}_{asi} + j\underline{\Psi}_{bsd} + j\underline{\Psi}_{bsi}$$
$$\underline{\Psi}_{sR} = $$
$$= \frac{\Psi_{as}}{2}\sqrt{2}\left[e^{j(\omega_s t-\pi/2)} + e^{-j(\omega_s t-\pi/2)} + je^{j(\omega_s t-\pi/2-\pi/2)} + je^{-j(\omega_s t-\pi)}\right] = \qquad (11)$$
$$= \Psi_{as}\sqrt{2}\, e^{j(\omega_s t-\pi/2)}$$

which prove in an analytical way that there is a summation of the forward components and a subtraction of backward components which nullifies them.

*Observation*: The direction of rotation of the representative vector can be reversed if the polarity of the supply voltage for one winding is reversed as well. For example, if the reversed phase is *b-y*, then $\underline{\Psi}_{bs}$ is reversed and the representative vector becomes:

$$\underline{\Psi}'_{sR} = \underline{\Psi}_{as} + j(-\underline{\Psi}_{bs}) = \underline{\Psi}_{asd} + \underline{\Psi}_{asi} - j\underline{\Psi}_{bsd} - j\underline{\Psi}_{bsi}$$
$$\underline{\Psi}'_{sR} = \qquad (12)$$
$$= \frac{\Psi_{as}}{\sqrt{2}}\left[e^{j(\omega_s t-\pi/2)} + e^{-j(\omega_s t-\pi/2)} - je^{j(\omega_s t-\pi/2-\pi/2)} - je^{-j(\omega_s t-\pi)}\right]$$
$$= \Psi_{as}\sqrt{2}\, e^{-j(\omega_s t-\pi/2)}$$

In this case, the forward components annihilate each other and the backward components add up.

*In conclusion*, for the symmetrically fed two-phase induction machine, the representative vector of the resultant stator flux has the length coequal to the amplitude of total flux generated by one phase (*as* or *bs*), $\Psi_{sR} = U_{as}\sqrt{2}/\omega_s$ (its apex covers a circle), the rotation speed is constant and equal to pulsation of applied voltage $\omega_s$, and the direction of rotation is conditioned by the initial phase angle of the two applied voltages. In anticipation, it has to be revealed that the representative vector is given by the summation of two opposite rotating vectors, a forward component and a backward one.

## III. The Representative Phase Vectors For Unsymmetrical Two-Phase Condition

The study takes into discussion the unsymmetrical supply when $U_{as} \neq U_{bs}$. An analytical approach is possible if the expressions presented in Appendix B are used.

Using the above reasoning, one can define the space-time stator phase vector:

$$\overline{\Psi}_{sR} = \psi_{asx}\bar{i} + \psi_{bsy}\bar{j} \leftrightarrow \underline{\Psi}_{sR} = \underline{\Psi}_{as} + e^{j\delta}\underline{\Psi}_{bs}; \delta = \pi/2 \qquad (13)$$

Taking into consideration the forward and backward components above designated, one obtains:

$$\begin{aligned}
&\underline{\Psi}_{sR} = \underline{\Psi}_{as} + j\underline{\Psi}_{bs} = \\
&= \underline{\Psi}_{asd} + \underline{\Psi}_{asi} + j\underline{\Psi}_{b1d} + j\underline{\Psi}_{b1i} + j\underline{\Psi}_{b2d} + j\underline{\Psi}_{b2i} = \\
&= \frac{\Psi_{as}}{\sqrt{2}}\left[e^{j(\omega_s t - \pi/2)} + e^{-j(\omega_s t - \pi/2)}\right] + \\
&+ \frac{\Psi_{b1}}{\sqrt{2}}\left[e^{j(\omega_s t - \pi/2)} + e^{-j(\omega_s t + \pi/2)}\right] + \\
&+ \frac{\Psi_{b2}}{\sqrt{2}}\left[e^{j\omega_s t} + e^{-j(\omega_s t - \pi)}\right] = \\
&= \frac{\Psi_{as}}{\sqrt{2}}\left[\lambda\sin\varepsilon - j(1+\lambda\cos\varepsilon)\right]e^{j\omega_s t} + \\
&+ \frac{\Psi_{as}}{\sqrt{2}}\left[-\lambda\sin\varepsilon + j(1-\lambda\cos\varepsilon)\right]e^{-j\omega_s t} = \underline{\Psi}_{sRd} + \underline{\Psi}_{sRi}
\end{aligned} \qquad (14)$$

Thus, it is analytically proved that, in contrast to symmetrical condition that keeps nothing but one component (forward or backward), the unsymmetrical condition has two flux components: a forward one, more significant in amplitude and a weaker backward component (this fact is generally valid for the studied quantities). The two components can be expressed as follows:

$$\begin{aligned}
&\underline{\Psi}_{sRd} = Ae^{j(\omega_s t + \alpha_d)}; \\
&\quad A = \frac{q_1\Psi_{as}}{\sqrt{2}}; \\
&\quad q_1 = \sqrt{1+\lambda^2 + 2\lambda\cos\varepsilon}; \\
&\quad \alpha_d = \arctan\frac{1+\lambda\cos\varepsilon}{-\lambda\sin\varepsilon} \\
&\underline{\Psi}_{sRi} = Be^{-j(\omega_s t - \alpha_i)}; \\
&\quad B = \frac{q_2\Psi_{as}}{\sqrt{2}}; \\
&\quad q_2 = \sqrt{1+\lambda^2 - 2\lambda\cos\varepsilon}; \\
&\quad \alpha_i = 180^0 - \arctan\frac{1-\lambda\cos\varepsilon}{\lambda\sin\varepsilon}
\end{aligned} \qquad (15)$$

The next step is a rotation of the space-time phase vector inside the complex space with the angle $-\alpha = -(\alpha_i + \alpha_d)/2$. This operation is equivalent with a multiplication by $e^{-j(\alpha_d + \alpha_i)/2}$. It results:

$$\underline{\Psi}_{sR}e^{-j\alpha} = Ae^{j\left(\omega_s t - \frac{\alpha_i - \alpha_d}{2}\right)} + Be^{-j\left(\omega_s t - \frac{\alpha_i - \alpha_d}{2}\right)} \qquad (16)$$

The trigonometric form is:

$$\begin{aligned}
&\Psi_{sR}e^{j\varphi_{sR}} \cdot e^{-j\alpha} = \\
&= (A+B)\cos[\omega_s t - (\alpha_i - \alpha_d)/2] + \\
&+ j(A-B)\sin[\omega_s t - (\alpha_i - \alpha_d)/2]
\end{aligned} \qquad (17)$$

Dissociation of the real ($x$ subscript) and imaginary ($y$ subscript) parts leads to the equation system:

$$\begin{aligned}
&\Psi_{sRx}\cos\alpha + \Psi_{sRy}\sin\alpha = \\
&= (A+B)\cos[\omega_s t - (\alpha_i - \alpha_d)/2] = \Psi_{RX} \\
\\
&-\Psi_{sRx}\sin\alpha + \Psi_{sRy}\cos\alpha = \\
&= (A-B)\sin[\omega_s t + (\alpha_i - \alpha_d)/2] = \Psi_{RY}
\end{aligned} \qquad (18)$$

which, by eliminating the parameter $t$, gives:

$$\text{(E)} \quad \frac{\Psi_{RX}^2}{(A+B)^2} + \frac{\Psi_{RY}^2}{(A-B)^2} = 1 \qquad (19)$$

In conclusion, the apex coordinates of the **representative flux** vector which are related to the new axes (**XOY**) and rotated by the angle $\alpha = (\alpha_i + \alpha_d)/2$ towards the initial system (**xOy**) are accomplishing the characteristic equations of an ellipse (E) with *A*+*B* and *A*-*B* as semi-axes.

The apex of the *forward* vectors covers a circle ($C_d$), while the backward components cover the circle ($C_i$). Both vectors have coequal angular speeds, $\omega_s$, but to opposite directions.

Fig. 3 presents the representative phase vectors of the total fluxes corresponding to three consecutive moments: $\omega t = 0$, - the apex in A; $\omega t = \pi/4$, - the apex in B; $\omega t = \pi/2$, - the apex in C.

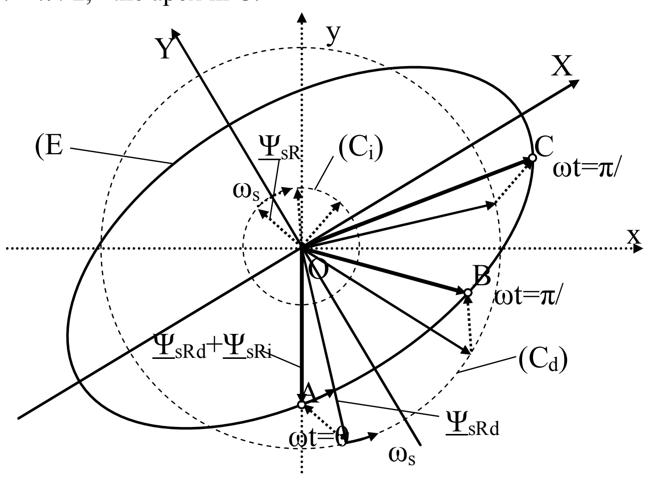


**Figure 3.** Representative phase vector for the unsymmetrical condition of the induction machine.

As an example, using the values $\lambda = 0.75$; $\varepsilon = \pi/6$; $\Psi_{as} = 6$, then:

$$\begin{aligned}
&\underline{\Psi}_{sR} = \\
&= 4.2\left[e^{j(\omega_s t - \pi/2)} + e^{-j(\omega_s t - \pi/2)}\right] + \\
&+ 2.8\left[e^{j(\omega_s t - \pi/2)} + e^{-j(\omega_s t + \pi/2)}\right] + \\
&+ 1.6\left[e^{j\omega_s t} + e^{-j(\omega_s t - \pi)}\right]
\end{aligned} \qquad (20)$$

Separation of the forward and backward terms gives:

$$\begin{aligned}
&\underline{\Psi}_{sR} = (1.6 - j7)e^{j\omega_s t} + (-1.6 + j1.4)e^{-j\omega_s t} = \\
&= 7.2e^{j(\omega_s t - 77^0)} + 2.1e^{-j(\omega_s t - 137^0)}; \alpha = 30^0
\end{aligned} \qquad (21)$$

The amplitude of total flux during a period covers the range (9.3 - 5.1) Wb. It is obvious that the instantaneous angular speed of the resultant flux is a variable quantity despite the assumption that asserts the mean value remains constant and coequal with the synchronism one. Practically, during a period, the torque has twice both maximum and minimum values. This may cause intolerable vibrations and noise or possible mechanical faults. These variable torques determine non-uniform rotor speed, increased frictions, higher temperature irradiated non-uniformly in the machine components, trepidations, premature ageing of the bearings, a global decrease of the output power and lifetime.

## IV. Analysis Of Unsymmetrical Condition Using Representative Space-Time Phase Vectors

A proper analysis can be achieved as follows: one starts with the equations of two-phase unsymmetrical induction machine written in the simplified complex manner, under matrix form, [15-20]

$$\begin{bmatrix} \underline{U}_{as} \\ \underline{U}_{bs} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \nu_{rs} + j\omega_s & 0 & 0 & -\nu_{hs} \\ 0 & \nu_{rs} + j\omega_s & -\nu_{hs} & 0 \\ 0 & -\nu_{hr} & \nu_{sr} + j\omega_s & -\omega_R \\ -\nu_{hr} & 0 & \omega_R & \nu_{sr} + j\omega_s \end{bmatrix} \cdot \begin{bmatrix} \underline{\psi}_{as} \\ \underline{\psi}_{bs} \\ \underline{\psi}_{qr} \\ \underline{\psi}_{dr} \end{bmatrix} \tag{22}$$

Assuming the superposing effect principle for voltages and total fluxes, one can formulate the connective expression between complex quantities and corresponsive space-time phase vectors as follows:

$$\underline{U}_{sR} = \underline{U}_{as} + j\underline{U}_{bs}; \underline{\psi}_{sR} = \underline{\psi}_{as} + j\underline{\psi}_{bs}; \underline{\psi}_{rR} = \underline{\psi}_{dr} + j\underline{\psi}_{qr} \tag{23}$$

In matrix equation (29), one multiplies the 2nd and 3rd rows by $j$, which has as consequence the modification of the terms, including $\omega_R$:

$$\begin{bmatrix} \underline{U}_{as} \\ j\underline{U}_{bs} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \nu_{rs} + j\omega_s & 0 & 0 & -\nu_{hs} \\ 0 & \nu_{rs} + j\omega_s & -\nu_{hs} & 0 \\ 0 & -\nu_{hr} & \nu_{sr} + j\omega_s & -j\omega_R \\ -\nu_{hr} & 0 & -j\omega_R & \nu_{sr} + j\omega_s \end{bmatrix} \cdot \begin{bmatrix} \underline{\psi}_{as} \\ j\underline{\psi}_{bs} \\ j\underline{\psi}_{qr} \\ \underline{\psi}_{dr} \end{bmatrix} \tag{24}$$

The system equation (24) leads to other two equations: one for the representative space-time phase vector of the stator total flux and the other for the similar rotor phase vector. The analysis implies two steps.

a) One considers a first machine, denoted with F.M., which includes nothing but rotational representative phase vectors that circulate forward (denoted with *d* subscript). The following notations are used:

$$\underline{N}_{rs} = \nu_{rs} + j\omega_s; \underline{N}_{sr} = \nu_{sr} + j\omega_s;$$

$$\nu_{tt}\omega_s = \nu_{sr}\nu_{rs} - \nu_{hs}\nu_{hr}; \omega_s - \omega_R = s\omega_s .$$

The first equation comes from summation of the first two rows and the second equation is the sum of the third and forth rows. The matrix form of the matrix is:

$$(\text{F.M.}) \begin{bmatrix} \underline{U}_{sRd} \\ 0 \end{bmatrix} = \begin{bmatrix} \underline{N}_{rs} & -\nu_{hs} \\ -\nu_{hr} & \underline{N}_{sr} - j\omega_R \end{bmatrix} \times \begin{bmatrix} \underline{\psi}_{sRd} \\ \underline{\psi}_{rRd} \end{bmatrix} \tag{25}$$

The right member determinant is a complex quantity:

$$\begin{aligned} \underline{\Delta} &= \omega_s\left[\left(\nu_{tt} - s\omega_s\right) + j\left(\nu_{sr} + s\,\nu_{rs}\right)\right]; \Delta^2 = \\ &= \omega_s^2\left[\left(\nu_{rs}^2 + \omega_s^2\right)s^2 + 2s\,\nu_{hs}\nu_{hr} + \nu_{sr}^2 + \nu_{tt}^2\right] \end{aligned} \tag{26}$$

The representative space-time phase vectors of the fluxes are:

$$\underline{\psi}_{sRd} = \frac{\underline{U}_{sRd}}{\Delta^2}\left(\nu_{sr} + js\omega_s\right)\underline{\Delta}^*; \underline{\psi}_{rRd} = \frac{\underline{U}_{sRd}}{\Delta^2}\nu_{hr}\underline{\Delta}^*; \tag{27}$$

The electromagnetic torque developed by the forward machine is:

$$\begin{aligned} M_{ed} &= -\frac{p}{L_{\sigma\sigma}}\mathrm{Re}\left(j\underline{\psi}_{sRd}\underline{\psi}_{rRd}^*\right) = \\ &= \frac{p\,\nu_{hr}}{\omega_s L_{\sigma\sigma}}U_{sRd}^2 \frac{s}{\left(\nu_{rs}^2 + \omega_s^2\right)s^2 + 2s\,\nu_{hs}\nu_{hr} + \nu_{sr}^2 + \nu_{tt}^2} \end{aligned} \tag{28}$$

*Observation*: A reversal of the supply phase order (or any polarity phase inversion) with preservation of rotor rotation direction determines the reversal of the rotation direction of the representative phase vector of applied voltages and fluxes. This fact implies the reversal of the sign of the pulsation, $\omega_s \to -\omega_s$. In (32), there act the new parameters $\underline{N}_{rs}^* = \nu_{rs} - j\omega_s; \underline{N}_{sr}^* = \nu_{sr} - j\omega_s$. The machine operates as brake and the new equations give the torque expression easily.

b) From the viewpoint of the backward components, the machine acts as a brake (B.M.), according to equations:

$$\begin{bmatrix} \underline{U}_{sRi} \\ 0 \end{bmatrix} = \begin{bmatrix} \underline{N}_{rs}^* & -\nu_{hs} \\ -\nu_{hr} & \underline{N}_{sr}^* - j\omega_R \end{bmatrix} \times \begin{bmatrix} \underline{\psi}_{sRi} \\ \underline{\psi}_{rRi} \end{bmatrix} \tag{29}$$

where $-\omega_s - \omega_R = -(2-s)\omega_s$.

The left member determinant is:

$$\begin{aligned} \underline{\Delta}_1 &= \omega_s\left[\left(\nu_{tt} - (2-s)\omega_s\right) + j\left(\nu_{sr} + (2-s)\nu_{rs}\right)\right]; \\ \Delta_1^2 &= \omega_s^2\left[A(2-s)^2 + 2B(2-s) + C\right] \end{aligned} \tag{30}$$

where: $A = \nu_{rs}^2 + \omega_s^2; B = \nu_{hs}\nu_{hr}; C = \nu_{sr}^2 + \nu_{tt}^2$

The representative space-time phase vectors of the fluxes are:

$$\underline{\psi}_{sRi} = \frac{\underline{U}_{sRi}}{\Delta_1^2}\left[\nu_{sr} - j(2-s)\omega_s\right]\underline{\Delta}_1^*; \underline{\psi}_{rRi} = \frac{\underline{U}_{sRi}}{\Delta_1^2}\nu_{hr}\underline{\Delta}_1^* \tag{31}$$

The electromagnetic torque developed by the "backward machine" is:

$$\begin{aligned} M_{ei} &= -\frac{p}{L_{\sigma\sigma}}\mathrm{Re}\left(j\underline{\psi}_{sRi}\underline{\psi}_{rRi}^*\right) = \\ &= -\frac{p\,\nu_{hr}}{\omega_s L_{\sigma\sigma}}U_{sRi}^2 \frac{2-s}{A(2-s)^2 + 2Bs + C} \end{aligned} \tag{32}$$

The resultant torque, obtained by means of superposing effect law, is:

$$\begin{aligned} M_{erez} &= M_{ed} + M_{ei} = \\ &= \frac{p\,\nu_{hr}}{\omega_s L_{\sigma\sigma}}\left[\frac{sU_{sRd}^2}{As^2 + 2Bs + C} - \frac{(2-s)U_{sRi}^2}{A(2-s)^2 + 2B(2-s) + C}\right] \end{aligned} \tag{33}$$

The components of the representative phase vector of the voltages can be deduced similarly:

$$\begin{aligned} \underline{U}_{sR} &= \underline{U}_{as} + j\underline{U}_{bs} = \\ &= \underline{U}_{asd} + \underline{U}_{asi} + j\left(\underline{U}_{bsd} + \underline{U}_{bsi}\right) = \\ &= \left(1 + \underline{\lambda}_d\right)\underline{U}_{asd} + \left(1 - \underline{\lambda}_i\right)\underline{U}_{asi} = \\ &= \left(1 + \lambda\cos\varepsilon + j\lambda\sin\varepsilon\right)\underline{U}_{asd} + \\ &+ \left(1 - \lambda\cos\varepsilon + j\lambda\sin\varepsilon\right)\underline{U}_{asi} = \\ &= \frac{U_{as}}{\sqrt{2}}\left[E_d e^{j\varepsilon_d}e^{j\omega_s t} + E_i e^{j\varepsilon_i}e^{-j\omega_s t}\right] = \underline{U}_{sRd} + \underline{U}_{sRi} \end{aligned} \tag{34}$$

where:

$$E_d = \sqrt{1+\lambda^2+2\lambda\cos\varepsilon};$$
$$E_i = \sqrt{1+\lambda^2-2\lambda\cos\varepsilon};$$
$$\varepsilon_d = arctg\frac{\lambda\sin\varepsilon}{1+\lambda\cos\varepsilon};$$
$$\varepsilon_i = arctg\frac{\lambda\sin\varepsilon}{1-\lambda\cos\varepsilon}$$

Hence, the resultant torque

$$M_{erez} = M_{ed} + M_{ei} = \\ = \frac{p\nu_{hr}U_{as}^2}{2\omega_s L_{\sigma\sigma}}\left[\frac{s\left(1+\lambda^2+2\lambda\cos\varepsilon\right)}{As^2+2Bs+C} - \frac{(2-s)\left(1+\lambda^2-2\lambda\cos\varepsilon\right)}{A(2-s)^2+2B(2-s)+C}\right] \quad (35)$$

No doubt, there is a confirmation of these results, which are similar to ones that are obtained by means of other methods, such as "symmetrical components method" [21-26].

Graphic inference of the amplitude of the two voltage components is presented in Fig. 4. The graphic construction is made for a specific case, and more precisely at *t=0*. After the calculus of the length of the two voltage components (forward and backward) and their difference of phase corresponding to *t=0*, one determines the track of the apices representing a circle. Each component length represents the circle radius, which runs to opposite directions with coequal speeds, $\omega_s$.

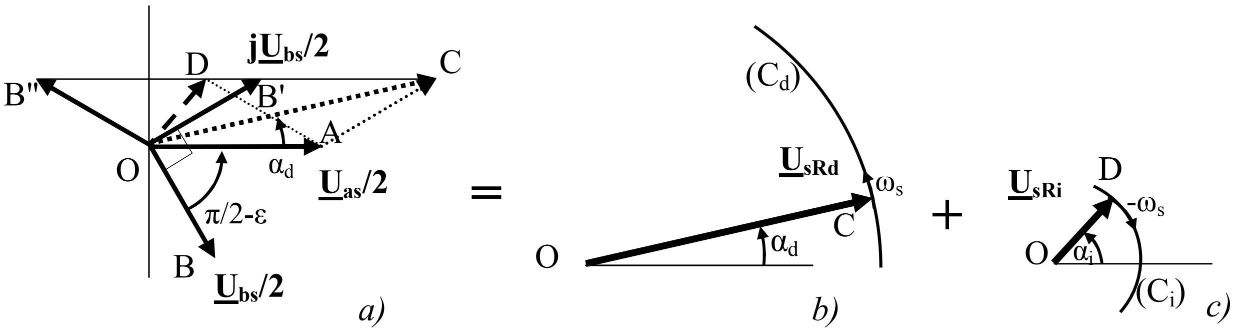


**Figure 4.** Graphic inference of the representative space-time phase vector component
a) Two-phase unsymmetrical voltage system, b) Forward rotational phase vector, c) Backward rotational phase vector.

The following methodology is used (Fig. 4): the voltage $\underline{U}_{as}$ is considered as phase reference, Fig. 4a. One plots the phase vector $\underline{OA}$, its value being half voltage amplitude, $u_{as}$, that is $|OA| = U_{as}/\sqrt{2}$. Similarly, one plots the phase vector $\underline{OB}$ with $|OB| = U_{bs}/\sqrt{2}$, but with a difference of phase of *ε-π/2* (which is negative in this case). Further, the phase vector $\underline{OB}$ is rotated towards positive direction with *π/2* resulting $\underline{OB'}$. The vectors $\underline{OA}$ cu $\underline{OB'}$ are geometrically summated, thus resulting $\underline{OC}$= $\underline{U}_{sRd}$, that is *the representative forward rotational phase vector*. Then, the vector $\underline{OB''}$ is obtained as the symmetric segment against vertical axis. The sum of $\underline{OB''}$ with $\underline{OA}$ gives $\underline{OD}$= $\underline{U}_{sRi}$, that is *the representative backward rotational phase vector*. The representative forward rotational phase vector covers the circle ($C_d$) towards positive trigonometric direction (Fig. 4b) and the representative backward rotational phase vector covers the circle ($C_i$) towards negative trigonometric direction (Fig. 4c). Obviously, the apex of the representative rotational phase vector covers an ellipse. This graphical construction is justified by the following reasoning. The cosine theorem applied in OAC triangle gives $|OC| = \left(U_{as}/\sqrt{2}\right)\sqrt{1+\lambda^2+2\lambda\cos\varepsilon}$ and for OAD triangle $|OD| = \left(U_{as}/\sqrt{2}\right)\sqrt{1+\lambda^2-2\lambda\cos\varepsilon}$. These are the lengths of the forward and backward phase vectors.

## V. CONCLUSION

The representative space-time rotational phase vectors of total fluxes represent a useful tool for understanding the phenomena that take place inside the induction machine (with stator-inductor, rotor-armature). They give a physical signification close to the image of the traveling waves. The equations have a reduced number of variables. Practically, there are only voltages (that characterize the electric field) and total fluxes (characterizing the magnetic field). The presence of current is no longer necessary.

The equations containing *nothing but fluxes* lead to simple analytical expressions for total fluxes of the stator and rotor. It is easy to handle these equations both for the analysis of symmetrical and unsymmetrical conditions.

For symmetrical conditions, the apex of the representative stator and rotor phase vectors (for flux) covers a circle and the rotation speeds are constant. For unsymmetrical conditions, the apices cover ellipses and the instantaneous speeds during a revolution vary between two limits. The analysis can be accomplished by using two representative phase vectors: a forward and a backward one, respectively. They have coequal but opposite directions.

For unsymmetrical conditions, it is possible to have a significant saturation of the magnetic circuit corresponding to major axis position. In this approach, this fact can be pointed out more easily, in comparison with classic formulations, where the presence of currents is mandatory.

## APPENDIX A

Symbolic notations for ideal induction machine

$$u_{as} = U_{as}\sqrt{2}\cos\omega_s t \leftrightarrow \underline{U}_{as} = U_{as}\sqrt{2}(e^{j\omega_s t} + e^{-j\omega_s t})/2 = \underline{U}_{asd} + \underline{U}_{asi};$$
$$\underline{U}_{asd} = \sqrt{2}\frac{U_{as}}{2}e^{j\omega_s t}; \quad \underline{U}_{asi} = \sqrt{2}\frac{U_{as}}{2}e^{-j\omega_s t}; \tag{A-1}$$

$$\psi_{as} = \frac{U_{as}}{\omega_s}\sqrt{2}\cos\left(\omega_s t - \frac{\pi}{2}\right) \leftrightarrow \underline{\Psi}_{as} = \frac{\Psi_{as}}{2}\sqrt{2}\left[e^{j(\omega_s t - \pi/2)} + e^{-j(\omega_s t - \pi/2)}\right] = \underline{\Psi}_{asd} + \underline{\Psi}_{asi}$$
$$\underline{\Psi}_{asd} = \sqrt{2}\frac{\Psi_{as}}{2}e^{j(\omega_s t - \pi/2)} = -\frac{j}{\omega_s}\underline{U}_{asd}; \underline{\Psi}_{asi} = \sqrt{2}\frac{\Psi_{as}}{2}e^{-j(\omega_s t - \pi/2)} = \frac{j}{\omega_s}\underline{U}_{asi}; \Psi_{as} = \frac{U_{as}}{\omega_s} \tag{A-2}$$

$$u_{bs} = U_{bs}\sqrt{2}\cos(\omega_s t - \pi/2) \leftrightarrow \underline{U}_{bs} = \frac{U_{bs}}{2}\sqrt{2}\left[e^{j(\omega_s t - \pi/2)} + e^{-j(\omega_s t - \pi/2)}\right] = \underline{U}_{bsd} + \underline{U}_{bsi};$$
$$\underline{U}_{bsd} = -j\frac{U_{bs}}{U_{as}}\underline{U}_{asd}; \underline{U}_{bsi} = j\frac{U_{bs}}{U_{as}}\underline{U}_{asi}, \tag{A-3}$$

$$\psi_{bs} = \frac{U_{bs}}{\omega_s}\sqrt{2}\cos(\omega_s t - \pi) \leftrightarrow \underline{\Psi}_{bs} = \frac{\Psi_{bs}}{2}\sqrt{2}\left[e^{j(\omega_s t - \pi)} + e^{-j(\omega_s t - \pi)}\right] = \underline{\Psi}_{bsd} + \underline{\Psi}_{bsi};$$
$$\underline{\Psi}_{bsd} = -\sqrt{2}\frac{\Psi_{bs}}{2}e^{j\omega_s t}; \quad \underline{\Psi}_{bsi} = -\sqrt{2}\frac{\Psi_{bs}}{2}e^{-j\omega_s t}; \Psi_{bs} = \frac{U_{bs}}{\omega_s}. \tag{A-4}$$

## APPENDIX B

Symbolic notations for unsymmetrical two-phase condition

$$u_{as} = U_{as}\sqrt{2}\cos\omega_s t \leftrightarrow \underline{U}_{as} = \frac{1}{2}U_{as}\sqrt{2}(e^{j\omega_s t} + e^{-j\omega_s t}) = \underline{U}_{asd} + \underline{U}_{asi};$$
$$\underline{U}_{asd} = \sqrt{2}\frac{U_{as}}{2}e^{j\omega_s t}; \quad \underline{U}_{asi} = \sqrt{2}\frac{U_{as}}{2}e^{-j\omega_s t};$$
$$\psi_{as} = \frac{U_{as}}{\omega_s}\sqrt{2}\cos\left(\omega_s t - \frac{\pi}{2}\right) \leftrightarrow \underline{\Psi}_{as} = \frac{\Psi_{as}}{2}\sqrt{2}\left[e^{j(\omega_s t - \pi/2)} + e^{-j(\omega_s t - \pi/2)}\right] = \underline{\Psi}_{asd} + \underline{\Psi}_{asi}$$
$$\underline{\Psi}_{asd} = \sqrt{2}\frac{\Psi_{as}}{2}e^{j(\omega_s t - \pi/2)} = -\frac{j\underline{U}_{asd}}{\omega_s}; \underline{\Psi}_{asi} = \sqrt{2}\frac{\Psi_{as}}{2}e^{-j(\omega_s t - \pi/2)} = \frac{j\underline{U}_{asi}}{\omega_s}; \Psi_{as} = \frac{U_{as}}{\omega_s} \tag{B-1}$$
$$u_{bs} = \lambda U_{as}\sqrt{2}\cos(\omega_s t - \pi/2 + \varepsilon) = U_{b1}\sqrt{2}\cos(\omega_s t - \pi/2) + U_{b2}\sqrt{2}\cos\omega_s t \leftrightarrow$$
$$\leftrightarrow \underline{U}_{b1d} + \underline{U}_{b1i} + \underline{U}_{b2d} + \underline{U}_{b2i} = \frac{U_{b1}}{\sqrt{2}}\left[e^{j(\omega_s t - \pi/2)} + e^{-j(\omega_s t - \pi/2)}\right] + \frac{U_{b2}}{\sqrt{2}}\left[e^{j\omega_s t} + e^{-j\omega_s t}\right],$$

where: $U_{b1} = \lambda U_{as}\cos\varepsilon$; $U_{b2} = \lambda U_{as}\sin\varepsilon$ . A different form to express the above equtions is

$$u_{bs} = \lambda U_{as}\sqrt{2}\cos(\omega_s t - \pi/2 + \varepsilon) \leftrightarrow \frac{\lambda U_{as}}{\sqrt{2}}\left[e^{j(\omega_s t)}e^{j(\varepsilon - \pi/2)} + e^{-j(\omega_s t)}e^{-j(\varepsilon - \pi/2)}\right] =$$
$$= \frac{\lambda U_{as}}{\sqrt{2}}\left[e^{j\omega_s t}e^{j\varepsilon}(-j) + e^{-j\omega_s t}e^{-j\varepsilon}j\right] = -j\underline{\lambda}_d\underline{U}_{asd} + j\underline{\lambda}_i\underline{U}_{asi}, \quad \underline{\lambda}_d = \lambda e^{j\varepsilon}, \underline{\lambda}_i = \lambda e^{-j\varepsilon} \tag{B-2}$$

$$\psi_{bs} = \frac{\lambda U_{as}}{\omega_s}\sqrt{2}\cos(\omega_s t - \pi + \varepsilon) \leftrightarrow \underline{\Psi}_{bs} = \frac{\Psi_{b1}}{\sqrt{2}}\left[e^{j(\omega_s t - \pi)} + e^{-j(\omega_s t - \pi)}\right] +$$
$$+ \frac{\Psi_{b2}}{\sqrt{2}}\left[e^{j(\omega_s t - \pi/2)} + e^{-j(\omega_s t - \pi/2)}\right] = \underline{\Psi}_{b1d} + \underline{\Psi}_{b1i} + \underline{\Psi}_{b2d} + \underline{\Psi}_{b2i}; \Psi_{b1} = \frac{U_{b1}}{\omega_s}; \Psi_{b2} = \frac{U_{b2}}{\omega_s}. \tag{B-3}$$

## APPENDIX C

### Particular cases

- Machine with symmetrical supply system - $\lambda = 1; \varepsilon = 0; E_d = 2; E_i = 0; \varepsilon_d = \varepsilon_i = 0;$

$$M_{erez} = M_{ed} = \frac{2 p \nu_{hr} U_{as}^2}{\omega_s L_{\sigma\sigma}} \frac{s}{As^2 + 2Bs + C} \quad \text{(C-1)}$$

- Machine with single-phase supply system (broken *b-y* phase) - $\lambda = 0; \varepsilon = 0; E_d = 1; E_i = 1; \varepsilon_d = \varepsilon_i = 0;$

$$M_{erez} = M_{ed} + M_{ei} = \\ = \frac{p \nu_{hr} U_{as}^2}{2\omega_s L_{\sigma\sigma}} \left[ \frac{s}{As^2 + 2Bs + C} - \frac{(2-s)}{A(2-s)^2 + 2B(2-s) + C} \right] \quad \text{(C-2)}$$

Obviously, for *s=1* (start-up) one obtain $M_{erez}=0$. At same the time, it is noticeable that the dependence $M_{erez}=f(s)$ is a symmetric curve around start-up point, $M_{erez}(1-x)=-M_{erez}(1+x)$.

- Machine with both windings connected to the same voltage (null difference of phase between the two applied voltages), $\lambda = 1; \varepsilon = \pi/2; E_d = E_i = \sqrt{2}; \varepsilon_d = \varepsilon_i = \pi/4$ . As a matter of fact, this is a *complete single phase or 1/1 machine*. The torque expression is:

$$M_{erez} = M_{ed} + M_{ei} = \\ \frac{p \nu_{hr} U_{as}^2}{\omega_s L_{\sigma\sigma}} \left[ \frac{s}{As^2 + 2Bs + C} - \frac{(2-s)}{A(2-s)^2 + 2B(2-s) + C} \right] \quad \text{(C-3)}$$

There is a duplication of the torque for a certain slip. This is a solid argument for the solution that uses for the single-phase motors only 2/3 slots and consequently the developed torque decrease to 8:9 value. As expected, for *s=1* then $M_{erez}=0$.

## REFERENCES


[1] A. Câmpeanu. Maşini electrice, Ed. Scrisul românesc, Craiova, 1988.

[2] S.E. Lyshevski. Nano - and Microelectromechanical Systems, CRC Press LLC USA, 2001.

[3] S. Enache, I. Vlad. Maşina de inducție – noțiuni fundamentale, procese dinamice, Ed. Universitaria, Craiova, 2002.

[4] S. Muşuroi, D. Popovici. Acționări electrice cu servomotoare, Ed. Politehnica, Timişoara, 2006.

[5] I. Boldea, Gh. Atanasiu. Analiza unitară a maşinilor electrice, Ed. Academiei Române, Bucureşti, 1983.

[6] I. Boldea. Transformatoare şi maşini electrice, Ed. Politehnica, Timişoara, 2001.

[7] I. Boldea, S. Nasar. The Induction Machine Handbook, CRC Press, LLC USA, 2002.

[8] J. Chatelain. Traité d'Électricité-vol X, Machines électriques, Ed. Giorgi, Presses polyt. rom., Lausanne, 1986.

[9] T. Dordea. Maşini electrice, Ed. Did. şi Ped., Bucureşti, 1977.

[10] I.A. Viorel, R. Ciorbă. Maşini electrice în sisteme de acționare, Ed. U.T. Pres, Cluj-N., 2002.

[11] I. Gheorghiu, Al. Fransua. Tratat de maşini electrice, vol. III, Maşini asincrone, Ed. Acad. Române, Bucureşti, 1971.

[12] Al. Simion. Maşini electrice speciale pentru automatizări, Ed. Universitas, Chişinău, 1993.

[13] Al. Simion. Maşini electrice, vol. II, Maşina sincronă, Editura Gh. Asachi, Iaşi, 2003.

[14] L. Livadaru, Al. Simion. Maşina asincronă bifazată, Ed. Junimea, Iaşi, 2003.

[15] Al. Simion, "Representative Phase Vectors Corresponding to the Mathematical Model in Total Fluxes of the Two-phase Induction Machine", Bul. Inst. Polit., Iaşi, LIV (LVIII), 4, pp. 657-666, 2008.

[16] Al. Simion, L. Livadaru, "The Mathematical Model with total fluxes of the Two-phase Induction Machine", Scientific Bulletin of the „Politehnica" University of Timisoara, T. 53(67) Sp. Issue - Proc. of the 14th National Conference on Electric Drives - Timisoara - Romania, pp. 237-242, 2008.

[17] Al. Simion, "Study of the steady state regime of the two-phase induction machine using the mathematical model in total fluxes", Scientific Bulletin of the „Politehnica" University of Timisoara, T. 53(67) Sp. Issue -Proc. of the 14th National Conference on Electric Drives- Timisoara - Romania, pp. 243-248, 2008.

[18] Al. Simion, L. Livadaru, D. Lucache, "Study of the Dynamic Behavior during Start-up at the Two-phase Induction Machine", Proc. of the National Symposium on Electrotechnics, Bucuresti, pp. 43-48, 2008.

[19] Al. Simion, "A Proper Mathematical Model for the Study of the Two-phase Induction Machine", Proc. of the National Symposium on Electrotechnics, Bucuresti, pp. 265-270, 2008.

[20] Al. Simion, L. Livadaru, M. Cojan, "On the Angular Characteristic Concept of an Induction Machine", Proc. of the SNET'07 Conference, Bucureşti, pp. 53/567-58/567, 2007.

[21] Al. Simion, L. Livadaru, D. Lucache, E. Romila, "Third Order Harmonics Evaluation by Finite Element Analysis of a Two-phase Induction Machine", Revue Roumaine des Sciences Techniques, Serie Electrotechnique et Energetique, 52, 1, pp. 23-32, 2007.

[22] Al. Simion, "Reversible Operation of the Drag-Cup A.C. – Servomotors", Proc. of the International Conference on Electrical Machines, München, pp. 562-565, 1986.

[23] Al. Simion, L. Livadaru, "Study of Unsymmetrical Regime of Induction Machine with the Mathematical Model in Total Fluxes", Proc. of the 7th International Conference of Electromechanical and Power Systems, Iaşi, Romania, pp. (II-245 – II-250), 2009.

[24] Gh. Atanasiu, S. Muşuroi, "Dynamic Analysis of Induction Motors using Simulation Technique", Proc. of the 14th National Conf. on Electrical drives, Timişoara, Romania, in Sci. Bulletin of Politechnica Univ. of Timişoara, Tom 53(67), pp. 19-24, 2008.

[25] X. Wang, D. Xie, "Analysis of Induction Motor Using Field-Circuit Coupled Time-Periodic Finite Element Method Taking Account of Hysteresis", IEEE Trans. Magn., Vol. 45, No. 3, pp. 1740-1743, March 2009.

[26] A.-K. Repo, P. Rasilo, A. Niemenmaa, A. Arkkio, "Identification of Electromagnetic Torque Model for Induction Machines with Numerical Magnetic Field Solution", IEEE Trans. Magn., Vol. 44, No. 6, pp. 1586-1589, June 2008.