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On preimages of ultrafilters in ZF

Abstract: We show that given infinite sets X, Y and a function $f : X \to Y$ which is onto and n-to-one for some $n \in \mathbb{N}$, the preimage of any ultrafilter $\mathcal{F}$ of Y under f extends to an ultrafilter. We prove that the latter result is, in some sense, the best possible by constructing a permutation model $\mathcal{M}$ with a set of atoms A and a finite-to-one onto function $f : A \to \omega$ such that for each free ultrafilter of ω its preimage under f does not extend to an ultrafilter. In addition, we show that in $\mathcal{M}$ there exists an ultrafilter compact pseudometric space $\mathbf{X}$ such that its metric reflection $\mathbf{X}^*$ is not ultrafilter compact.

References

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