



### Keywords

Isotropic Coordinates,  
Super-Dense Stars,  
Neutron Stars,  
Pressure Anisotropy

Received: January 12, 2015

Revised: January 26, 2015

Accepted: January 27, 2015

# Modeling of Anisotropic Fluid Stars in Isotropic Coordinates

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### Citation

Neeraj Pant, Narendra Pradhan, Mohammad Hassan Murad, Manuel Malaver. Modeling of Anisotropic Fluid Stars in Isotropic Coordinates. *American Journal of Science and Technology*. Vol. 2, No. 2, 2015, pp. 43-48.

### Abstract

In this paper, interior solutions of the Einstein field equations for a anisotropic fluid by considering Hajj-Boutros (1986) type metric potential and a specific choice of pressure anisotropy ( $\Delta$ ) which involves a parameter  $\alpha$  are obtained. The new solutions are utilized to construct the models for super-dense star models as neutron stars. This class of solutions are well behaved within a wide range of values of the parameter  $\alpha$ . In the absence of pressure anisotropy ( $\alpha = 0$ ) present model reduces to the isotropic model Murad and Pant (2014). It is found that with the increase of  $\alpha$ , maximum mass of fluid configuration decreases.

## 1. Introduction

Relativistic modeling of compact astrophysical objects with locally anisotropic fluid in spherically symmetric space-time has continued to attract the attention of researchers since the pioneering work of Bowers and Liang [1]. Further, contemporarily the theoretical investigations of Hajj-Boutros [2], Murad and Pant [3], Ruderman [4], Canuto [5], Canuto and Lodenquai [6] about more realistic stellar models show that the nuclear matter may be locally anisotropic at least in a very high density ranges ( $\rho > 10^{15} \text{gcm}^{-3}$ ), where the relativistic behavior of nuclear interactions in the stellar matter is significant. According to such views, the radial pressure in such massive stellar objects may not necessarily be equal to the tangential pressure. Some interesting work in this regard include Mak and Harko [7], Mak et al. [8], Ivanov [9], Chaisi and Maharaj [10], Komathiraj and Maharaj [11], Feroze and Siddiqui [12], Maharaj and Takisa [13], Thirukkanesh and Ragel [14], Takisa and Maharaj [15,16], Maurya and Gupta [17,18], Cosenza et al. [19], Malaver [20, 21] and Pant et al. [22].

Being motivated by the above mentioned works, in this paper we intend to study a class of exact well behaved stellar models by solving Einstein's gravitational field equations in isotropic coordinates for anisotropic fluid distribution by assuming particular forms of one of the metric potentials and pressure anisotropy. In the absence of pressure anisotropy the present model reduces to the isotropic neutral model of Murad and Pant [3].

## 2. Anisotropic Stellar Models

### 2.1. Field Equations in Isotropic Coordinates

For a static and spherically symmetric matter distribution the interior metric in isotropic coordinates may be taken as,

$$ds^2 = c^2 e^{\nu(r)} dt^2 - e^{\omega(r)} (dr^2 + r^2 (d\theta^2 + r^2 \sin^2 \theta d\varphi^2)) \quad (1)$$

$\nu(r)$  and  $\omega(r)$  are the two arbitrary functions. For the metric (1) the Einstein's field equations reduce to the following set of relevant equations (Stewart [23]),

$$\kappa p_r = e^{-\omega} \left( \frac{\omega'^2}{4} + \frac{\omega'}{r} + \frac{\omega' \nu'}{2} + \frac{\nu'}{r} \right) \quad (2)$$

$$\kappa p_\perp = e^{-\omega} \left( \frac{\omega''}{2} + \frac{\nu''}{2} + \frac{\nu'^2}{4} + \frac{\omega'}{2r} + \frac{\nu'}{2r} \right) \quad (3)$$

$$\kappa c^2 \rho = e^{-\omega} \left( \omega'' + \frac{\omega'^2}{4} + \frac{2\omega'}{r} \right) \quad (4)$$

where  $p_r, p_\perp$  and  $\rho$  are radial pressure, tangential pressure and energy density respectively, prime (') denotes differentiation with respect to  $r$  and  $\kappa = 8\pi G/c^4$  where  $c$  is the speed of light and  $G$  is the gravitational constant. From the eqs. (2) and (3) we obtain following differential equation in  $\omega d\nu$ ,

$$e^{-\omega} \left( \frac{\omega''}{2} + \frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\omega'^2}{4} - \frac{\omega' \nu'}{2} - \frac{\omega' + \nu'}{2r} \right) = \kappa (p_\perp - p_r) \quad (5)$$

The quantity  $p_\perp - p_r$  is the measure of anisotropy in this model. In the next subsection we explore the solutions of eq. (5) under some particular assumptions on one of the metric potentials and pressure anisotropy.

### 2.2. Choices of Metric Potential and Pressure Anisotropy

For the locally isotropic matter distribution,  $p_\perp = p_r$  and eq. (5) has been solved in Murad and Pant [3], Pant *et al.* [24] by

$$\frac{\kappa}{c} p_r = \frac{1}{n^2 B^2 f^{4n-4} (1+A f^{2S})} [x [A f^{2S} (2n^2 - 12n + 12 + 2nS - 4S) + (2n^2 - 12n + 12 + 2nS + 4S)] + 2n [A f^{2S} (n - 4 + S) + (n - 4 - S)]] \quad (11)$$

$$\frac{\kappa}{c} p_\perp = \frac{\kappa}{c} p_r + \frac{\Delta}{c} \quad (12)$$

$$\frac{\kappa}{c} c^2 \rho = \frac{1}{n^2 B^2 f^{4n-4}} (12n + 4(n-1)x) \quad (13)$$

Where  $f = (1+x)^{\frac{1}{n}}$

## 3. Physical Properties of the Solution

The central values of radial pressure, tangential pressure and energy density are given by,

$$\left( \frac{\kappa}{c} p_r \right)_{r=0} = \left( \frac{\kappa}{c} p_\perp \right)_{r=0} = \frac{2[(n-4)(1+A)+S(A-1)]}{nB^2(1+A)} \quad (14)$$

$$\left( \frac{\kappa}{c} c^2 \rho \right)_{r=0} = \frac{12}{nB^2 f^{4n-4}} \quad (15)$$

The central values of the pressures and energy density will

considering the following metric potential,

$$e^{\frac{\omega}{2}} = B(1+x)^{-\frac{1}{n}} \quad (6)$$

with  $n > 0$

We make the specific choice

$$\kappa (p_\perp - p_r) = \Delta = \frac{2\alpha C x (1+x)^{\frac{2-2n}{n}}}{B^2} \quad (7)$$

where  $x = Cr^2$ ,  $B$  is an arbitrary constant and  $\Delta$  is the measure of pressure anisotropy which is zero at the center and increases towards the boundary of the fluid sphere.  $\alpha \geq 0$  is defined as anisotropy parameter.

Substituting the eqs. (6)–(7) into the eq. (5), we get the following Riccati differential equation,

$$\frac{dy}{dx} + \frac{2}{n(1+x)} y + \frac{y^2}{2} = -\frac{2n-2}{n^2(1+x)^2} + \frac{\alpha}{(1+x)^2} \quad (8)$$

$y = dv/dx$  and the eq. (8) yields the following solution,

$$e^{\frac{\nu}{2}} = \frac{[1+A(1+x)^{\frac{2S}{n}}] (1+x)^{\frac{n-2-S}{2n}}}{B^2} \quad (9)$$

$A, B$  are determined by imposing applicable boundary conditions and

$$S = \sqrt{2n^2 \alpha + n^2 - 8n + 8} \quad (10)$$

which is real for

$$\alpha > \frac{8n - n^2 - 8}{2n^2}$$

The expressions for radial pressure, tangential pressure and energy density now become,

be non zero positive definite, if the following conditions will be satisfied

$$A > \frac{S-n+4}{S+n-4} \quad (16)$$

The ratios of radial and tangential pressure-density are given by eqs. (11)–(13)

Subjecting the condition  $0 \leq p_0/c^2 \rho_0 \leq 1$  leads to the following inequality,

$$\left( \frac{p_r}{c^2 \rho} \right)_{r=0} = \left( \frac{p_\perp}{c^2 \rho} \right)_{r=0} = \frac{(n-4)}{6} - \frac{S(1-A)}{6(1+A)} \leq 1 \quad (17)$$

All the values of  $A$  which satisfy eq. (17), will also lead to the condition  $p_0/c^2 \rho_0 \leq 1$ .

Differentiating eqs. (11)–(13) with respect to  $r$ , we get

$$\frac{\kappa}{c} \frac{dp_r}{dx} = \frac{1}{n^3 B^2 f^{6n-4} (1+Af^{2S})^2} [-x(n-2)[A^2 f^{4S}(2n^2 - 12n + 12 + (2n-4)S) + Af^{2S}(2n^2 - 12n + 12 - 2S^2)] + n[A^2 f^{4S}(-2n^2 + 8n - 4 - 2nS) + (-2n^2 + 8n - 4 + 2nS) + Af^{2S}(-2n^2 + 8n - 4 + 2S^2)] \quad (18)$$

$$\frac{\kappa}{c} \frac{dp_{\perp}}{dx} = \frac{\kappa}{c} \frac{dp_r}{dx} + \frac{1}{c} \frac{d\Delta}{dx} \quad (19) \qquad R_b = r_b e^{\frac{\omega_b}{2}} \quad (22b)$$

$$\frac{\kappa}{c} c^2 \frac{d\rho}{dx} = \frac{1}{n^3 B^2 f^{6n-4}} [-20n^2 + 20n - 4(n^2 - 3n + 2)x] \quad (20) \qquad \frac{1}{2} \left(\omega' + \frac{2}{r}\right)_b r_b = \left(1 - \frac{2GM}{c^2 R_b}\right)^{1/2} \quad (22c)$$

where,

$$\frac{d\Delta}{dx} = \frac{2\alpha C(1+x)^{\frac{2-3n}{n}}}{B^2} \left[1 + \left(\frac{2-n}{n}\right)x\right] \qquad \frac{1}{2} (v')_b r_b = \frac{GM}{c^2 R_b} \left(1 - \frac{2GM}{c^2 R_b}\right)^{-1/2} \quad (22d)$$

Thus the expressions of right side of eqs. (18)–(20) is negative for all values of  $A$  satisfying condition (16), showing thereby that the quantities  $(p_r, p_{\perp}, \rho)$  is maximum at the center and monotonically decreasing.

The square of radial and tangential adiabatic sound speeds at the center are given by

$$\frac{1}{c^2} \left(\frac{dp_r}{d\rho}\right)_{r=0} = \frac{(-2n^2+8n-4)(1+A)^2+2nS+4AS^2-2nSA^2}{20(1-n)(1+A)^2}$$

$$\frac{1}{c^2} \left(\frac{dp_{\perp}}{d\rho}\right)_{r=0} = \frac{(2n^2\alpha-2n^2+8n-4)(1+A)^2+2nS+4AS^2-2nSA^2}{20(1-n)(1+A)^2}$$

Due to cumbersome expressions the trend of pressure-density ratios and adiabatic sound speeds are studied numerically after applying the boundary conditions.

### 4. Boundary Conditions in Isotropic Coordinates

For exploring the boundary conditions, we use the principle that the metric coefficients  $g_{ij}$  and their first derivatives  $g_{ij,k}$  in interior solution as well as in exterior solution are continuous up to and on the boundary. The continuity of interior metric coefficients  $g_{ij}$  on the boundary is known *first fundamental form*. The continuity of derivatives of interior metric coefficients on the boundary is known *second fundamental form*.

The exterior field of a spherically symmetric static fluid distribution is described by Schwarzschild metric given by

$$ds^2 = \left(1 - \frac{2GM}{c^2 R}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 R}\right)^{-1} dR^2 - R^2 d\theta^2 - R^2 \sin^2 \theta d\phi^2, R \geq R_b \quad (21)$$

where  $M$  is the mass of the sphere as determined by the external observer and  $R$  is the radial coordinate of the exterior region and  $R_b$  is the radius of the fluid distribution determined by the condition  $p_r(R = R_b) = 0$ .

Since the Schwarzschild metric (21) is considered as the exterior solution, thus we arrive at the following conclusions by matching first and second fundamental forms:

$$e^{\nu_b} = \left(1 - \frac{2GM}{c^2 R}\right) \quad (22a)$$

Equations (22a)–(22d) are four conditions, known as boundary conditions in isotropic coordinates. Moreover, (22b) and (22d) are equivalent to zero radial pressure of the interior solution on the boundary.

Applying the boundary conditions, eq. (22), we get the values of the arbitrary constants in terms Schwarzschild parameter  $u = GM/c^2 R_b$  and radius of the star  $R_b$ .

$$A = \frac{nu(1+x)^{\frac{n-2-S}{2n}} - (n-2-S)X\sqrt{1-2u}(1+x)^{\frac{-n-2-S}{2n}}}{(n-2-S)\sqrt{1-2u}X(1+x)^{\frac{-n-2-S}{2n}} - nu(1+x)^{\frac{n-2+S}{2n}}} \quad (23)$$

$$B = \sqrt{\frac{(1+x)^{\frac{n-2-S}{2n}} + A(1+x)^{\frac{n-2+S}{2n}}}{\sqrt{1-2u}}} \quad (24)$$

$$X = Cr_b^2 = \frac{n(1-\sqrt{1-2u})}{(n\sqrt{1-2u}-n+2)} \quad (25)$$

and for  $X > 0$

$$u \leq \frac{(2n-2)}{n^2}$$

The stellar surface density  $\rho_b$  may be calculated by,

$$\kappa c^2 \rho_b R_b^2 = \frac{X(12n+4(n-1)X-n^2KX)}{n^2(1+X)^2} \quad (26)$$

The central redshift  $Z_0$  and the surface redshift  $Z_b$  respectively are given by,

$$Z_0 = \frac{B^2}{1+A} - 1, \quad Z_b = e^{-\frac{\nu_b}{2}} - 1$$

### 5. Physical Analysis

For well behaved nature of the solution in isotropic coordinates, the following conditions should be satisfied:

- (i) The solution should be free from physical and geometrical singularities i.e., finite and positive values of central pressure, central density and non zero positive values of  $e^{\omega}$  and  $e^{\nu}$ .
- (ii) The density  $\rho$  and pressures  $p_r, p_{\perp}$  should be positive inside the star.
- (iii) The radial pressure  $p_r$  must be vanishing but the tangential pressure  $p_{\perp}$  may not necessarily vanish at the boundary  $r = r_b$  of the sphere. However, the radial pressure is equal to the tangential pressure at the center of the fluid sphere.

- (iv)  $(dp_r/dr)_{r=0} = 0$  and  $(d^2p_r/dr^2)_{r=0} < 0$  so that radial pressure gradient  $dp_r/dr < 0$  for  $0 \leq r \leq r_b$ .
- (v)  $(dp_\perp/dr)_{r=0} = 0$  and  $(d^2p_\perp/dr^2)_{r=0} < 0$  so that tangential pressure gradient  $dp_\perp/dr < 0$  for  $0 \leq r \leq r_b$ .
- (vi)  $(d\rho/dr)_{r=0} = 0$  and  $(d^2\rho/dr^2)_{r=0} < 0$  so that energy density gradient  $d\rho/dr < 0$  for  $0 \leq r \leq r_b$ .

Conditions (iv)–(vi) imply that pressure and density should be maximum at the center and monotonically decreasing towards the surface.

- (vii) Inside the static configuration the causality condition should be obeyed i.e. the speed of sound should be less than the speed of light, i.e.,

$$0 \leq \sqrt{\frac{dp_r}{c^2 d\rho}} \leq 1, \quad 0 \leq \sqrt{\frac{dp_\perp}{c^2 d\rho}} \leq 1$$

- (viii) In addition to the above the velocity of sound should be decreasing towards the surface i.e.,

$$\frac{d}{dr} \left( \frac{dp_r}{d\rho} \right) < 0, \quad \frac{d^2 p_r}{d\rho^2} > 0 \quad \text{and} \quad \frac{d}{dr} \left( \frac{dp_\perp}{d\rho} \right) < 0, \quad \frac{d^2 p_\perp}{d\rho^2} > 0$$

for  $0 \leq r \leq r_b$  which implies that the velocity of sound is increasing with the increase of density. In this context it is worth mentioning that for different equations of state at ultra-high densities available in the literature (Canuto [4], Baym and Pethick [25] and Arnett and Bowers [27]) it is found that the speed of sound is decreasing outwards from the center of fluid

sphere.

- (ix) A physically reasonable energy–momentum tensor has to obey the energy conditions

$$\rho \geq p_r + 2p_\perp, \quad \rho + p_r + 2p_\perp \geq 0.$$

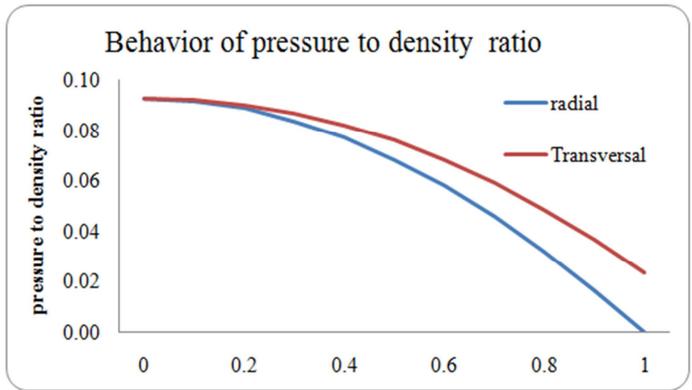
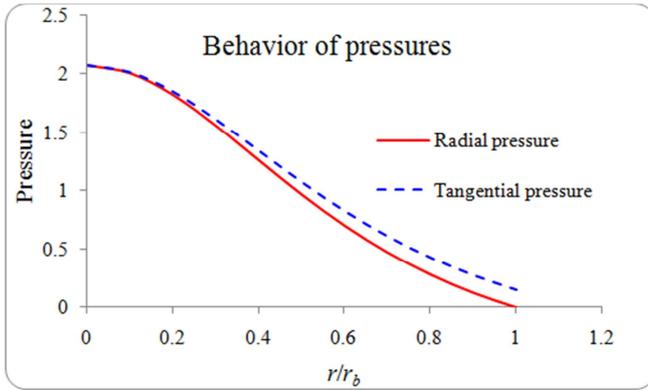
- (x) The central and surface redshifts  $Z_0, Z_b$  should be positive and finite.
- (xi) The anisotropy factor  $\Delta$  should be zero at the center and increasing towards the surface.
- (xii) The relativistic adiabatic index defined by

$$\gamma = \frac{(p_r + c^2\rho)}{p_r} \frac{dp_r}{c^2 d\rho}$$

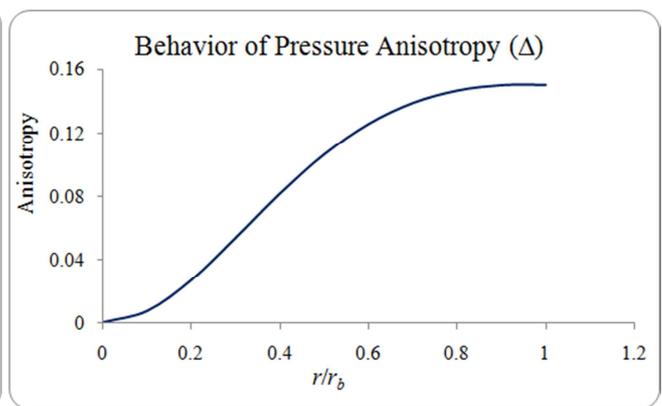
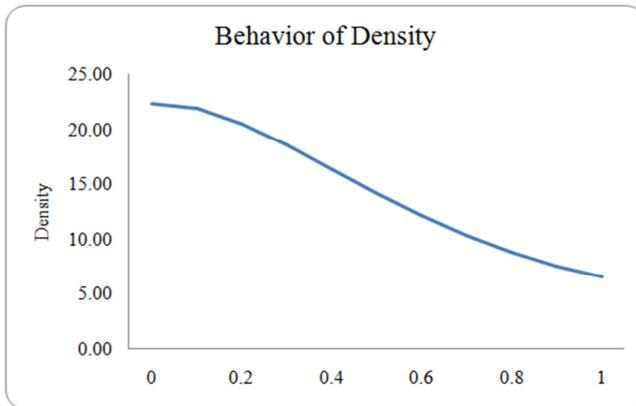
should be increasing from its lowest value 4/3 at the center to infinity at the surface.

**Table 1.** The range of  $n$  for well-behaved solution, corresponding anisotropy parameter  $\alpha$ , Schwarzschild parameter  $u$  and maximum mass of neutron star.

$n$	$\alpha_{\min}$	$\alpha$	$u_{\max}$	$M_{\max}(M_\odot)$
$n_{\min} = 5.83$	0.0685	0.0685	0.002	0.0024
5.9	0.0631	0.0631	0.03	0.15
6.5	0.021	0.021	0.1679	1.474
6.52	0.0195	0.0195	0.1703	1.497
6.55	0.0175	0.0175	0.1700	1.49
6.82	0.00051	0.01	0.1630	1.40
6.83	0	0.01	0.1622	1.393
7	0	0.02	0.1311	1.073
16.4	0	0.001	0.01	0.026
$n_{\max} = 21.6$	0	0	0.001	0.001



**Fig. 1.** Variations of  $p_r, p_\perp, p_r/c^2\rho, p_\perp/c^2\rho$  from center to surface for  $n = 6.52, \alpha = 0.0195$ .



**Fig. 2.** Variations of  $\rho$  and  $\Delta$  from center to surface for  $n = 6.52, \alpha = 0.0195$ .

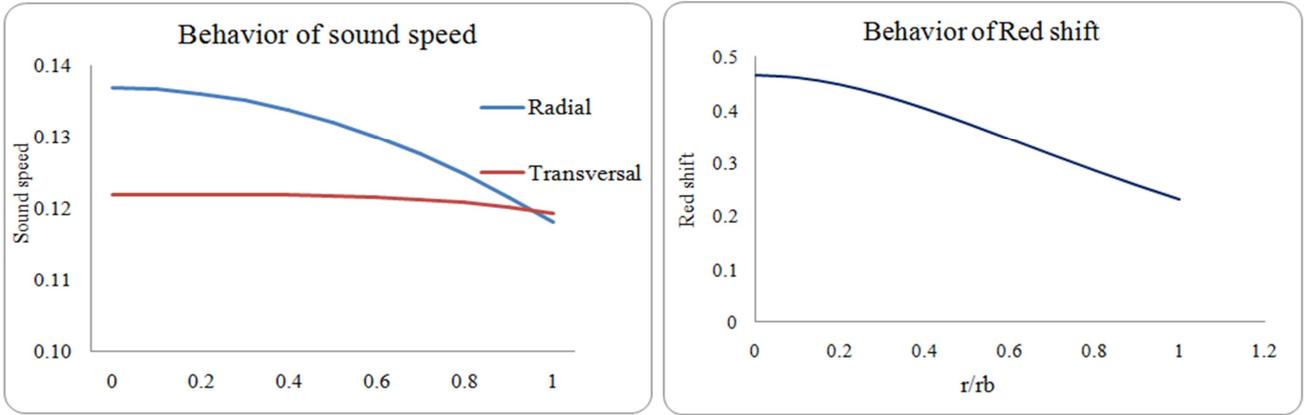


Fig. 3. Variations of  $dp_r/c^2d\rho$  and Redshift ( $Z$ ) from center to surface for  $n = 6.52, \alpha = 0.0195$ .

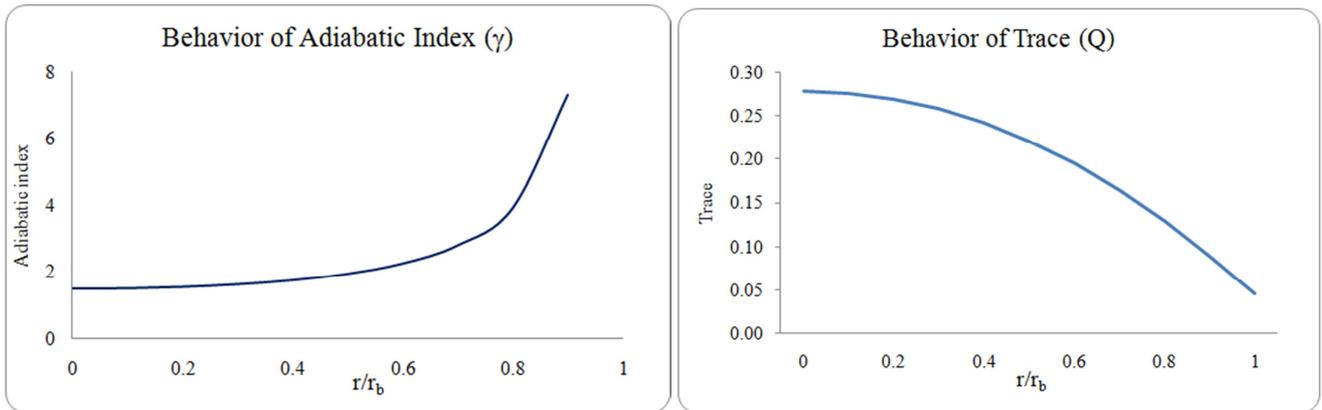


Fig. 4. Variations of  $\gamma$  and  $Q$  from center to surface for  $n = 6.52, \alpha = 0.0195$ .

Table 2. The variation of maximum mass of neutron star and corresponding radius  $R_b$  and surface anisotropy and surface red shift with  $u$  for  $n = 6.52, \alpha = 0.0195$  for which  $u_{max} = 0.1703$ .

$u$	$\frac{8\pi G}{c^2} \rho_b r_b^2$	$\rho_b = 2 \times 10^{14} \text{ g cm}^{-3}$		$\Delta_b r_b^2$	$Z_b$
		$R_b$ (km)	$M_{max} (M_\odot)$		
0.010	0.82691	3.96705	0.02663	0.000585	0.0101
0.020	1.58977	5.54263	0.07404	0.002304	0.0204
0.040	2.99120	7.72858	0.20940	0.009371	0.0428
0.080	5.03618	10.38141	0.55720	0.035835	0.0905
0.100	5.76587	11.33338	0.76754	0.056147	0.1186
0.120	6.23083	12.02021	0.97159	0.078652	0.1468
0.140	6.49392	12.54399	1.18143	0.104751	0.1779
0.160	6.54801	12.90441	1.39026	0.134053	0.2121
0.170	6.49157	13.02606	1.49688	0.150524	0.2315

## 6. Discussions and Conclusions

We have generated a new class of exact solutions for the Einstein-Maxwell system with a particular form for the measure of anisotropy. The new models presented in this paper may be used to model relativistic compact objects in astrophysics as neutron stars.

From table 1 it is observed that with the presence of anisotropy the range of  $n$  increases to  $5.83 \leq n \leq 21.6$  from  $4 + 2\sqrt{2} \leq n \leq 21.6$  (isotropic case of Murad and Pant [3])

but decreases from  $4 < n \leq 21.6$  (anisotropic charged case of Pant et al. [26]).

From the figure 1 it is observed that the physical quantities  $(p_r, p_\perp, p_r/c^2\rho, p_\perp/c^2\rho, dp_r/c^2d\rho, dp_\perp/c^2d\rho, Z)$  are positive at the center and within the limit of realistic state equation and monotonically decreasing while the quantities  $\gamma$  and  $\Delta$  are increasing towards the boundary.

Our solutions satisfy all the conditions mentioned in Sec. 3, and hence are *well-behaved*, within the following ranges of different parameters:  $5.83 \leq n \leq 21.6, 0 \leq \alpha \leq 0.0685$  and  $0 \leq u \leq 0.1703$ . With the increase of  $\alpha$  from  $\alpha=0.0631$ , maximum mass of the super-dense star decreases because of pressure diversion away from radial direction. This is because of the fact that in pressure isotropy there is only one component of pressure which is along radial direction that can support more masses. However, in a pressure anisotropic star some of the pressure gives rise to tangential pressure and hence the effective pressure in radial direction reduces and thus mass that can support by the radial pressure also decreases. With  $\alpha = 0$  we recover the isotropic model of Pant et al. [24]).

In table 2 we present models of super-dense neutron star based on the particular solution taking  $n = 6.52, \alpha = 0.0195$  for which  $u_{max} = 0.1703$ . By assuming surface density  $\rho_b = 2 \times 10^{14} \text{ g cm}^{-3}$  the resulting well behaved solution generates fluid sphere having maximum mass  $M = 1.497M_\odot$  and radius  $R = 13.03 \text{ km}$ . For an increase of  $u$ , the mass of neutron star also increases.

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