

# Fractional-Order Derivative and Time-Dependent Viscoelastic Behaviour of Rocks and Minerals

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## Abstract

A general constitutive equation for viscoelastic behaviour of rocks and minerals with fractional-order derivative is investigated. This constitutive law is derived based on differential geometry and thermodynamics of rheology, and the fractional order of derivative represents the degree of time delay. Analyzing some laboratory experimental data of high temperature deformation of rocks and minerals such as halite, marble and orthopyroxene, we propose how to determine the orders of fractional derivative for viscoelastic behaviours of rocks and minerals. The order is related to the exponents for the temporal scaling in the relaxation modulus and the stress power-law of strain rate, *i.e.*, the non-Newtonian flow law, and considered as an indicator representing the macroscopic behaviour and microscopic dynamics of rocks.

**Key words:** fractional-order derivative, differential geometry, memory effect, viscoelasticity, rock rheology.