Jaroslav Nešetřil, Patrice Ossona de Mendez
A model theory approach to structural limits

Comment.Math.Univ.Carolin. 53,4 (2012) 581 –603.

Abstract: The goal of this paper is to unify two lines in a particular area of graph limits. First, we generalize and provide unified treatment of various graph limit concepts by means of a combination of model theory and analysis. Then, as an example, we generalize limits of bounded degree graphs from subgraph testing to finite model testing.

Keywords: graph, graph limits, model theory, first-order logic
AMS Subject Classification: 05C99

References

[1] Adams S., *Trees and amenable equivalence relations*, Ergodic Theory Dynam. Systems **10** (1990), 1–14.
[2] Aldous D., Lyons R., *Processes on unimodular random networks*, arXiv:math/0603062 (2006).
[3] Benjamini I., Schramm O., *Recurrence of distibutional limits of finite planar graphs*, Electron. J. Probab. **6** (2001), no. 23, 13pp.
[4] Borgs C., Chayes J., Lovász L., Sós V., Szegedy B., Vesztergombi K., *Graph limits and parameter testing*, in Proc. 38th Annual ACM Symp. Principles of Dist. Comp., pp. 51–59, 2005.
[5] Conley C., Kechris A., Tucker-Drob R., *Ultraproducts of measure preserving actions and graph combinatorics*, Ergodic Theory and Dynamical Systems (2012), DOI 10.1017/S0143385711001143.
[6] Elek G., *Note on limits of finite graphs*, Combinatorica **27** (2007), 503–507, DOI 10.1007/s00493-007-2214-8.
[7] Elek G., Szegedy B., *Limits of hypergraphs, removal and regularity lemmas. A non-standard approach*, arXiv:0705.2179v1 [math.CO] (2007).
[8] Gaboriau D., *Invariant percolation and harmonic Dirichlet functions*, Geom. Funct. Anal. **15** (2005), 1004–1051, DOI 10.1007/s00039-005-0539-2.
[9] Gaifman H., *On local and non-local properties*, in Proceedings of the Herbrand Symposium, Logic Colloquium '81, 1982.
[10] Halmos P., Givant S., *Logic as Algebra*, Dolciani Mathematical Expositions, 21, The Mathematical Association of America, Washington DC, 1998.
[11] Hanf W., *Model-theoretic methods in the study of elementary logic*, in Theory of Models, J. Addison, L. Henkin, A. Tarski (eds.), pp. 132–145, North-Holland, Amsterdam, 1965.
[12] Hodges W., *A Shorter Model Theory*, Cambridge University Press, Cambridge, 1997.
[13] Lascar D., *La théorie des modèles en peu de maux*, Cassini, 2009.
[14] Łoś J., *Quelques remarques, théorèmes et problèmes sur les classes définissables d'algèbres*, in Mathematical Interpretation of Formal Systems, Studies in Logic and the Foundations of Mathematics, North-Holland, Amsterdam, 1955.
[15] Lovász L., Szegedy B., *Limits of dense graph sequences*, J. Combin. Theory Ser. B **96** (2006), 933–957.
[16] Martin D., Steel J., *A proof of projective determinacy*, J. Amer. Math. Soc. **2** (1989), no. 1, 71–125.
[17] Matoušek J., Nešetřil J., *Invitation to Discrete Mathematics*, Oxford University Press, New York, 1998 (second edition 2009)
[18] Mycielski J., Świerczkowski S., *On the Lebesgue measurability and the axiom of determinateness*, Fund. Math. **54** (1964), 67–71.
[19] Nešetřil J., Ossona de Mendez P., *Tree depth, subgraph coloring and homomorphism bounds*, European J. Combin. **27** (2006), no. 6, 1022–1041, DOI 10.1016/j.ejc.2005.01.010.
[20] Nešetřil J., Ossona de Mendez P., *From sparse graphs to nowhere dense structures: Decompositions, independence, dualities and limits*, in European Congress of Mathematics, pp. 135–165, European Mathematical Society, Zürich, 2010, DOI 10.4171/077-1/7.

[21] Nešetřil J., Ossona de Mendez P., *Sparsity. Graphs, Structures, and Algorithms*, Algorithms and Combinatorics, 28, Springer, Heidelberg, 2012, 465 pp.
[22] Nešetřil J., Ossona de Mendez P., *Graph limits: a unified approach with application to the study of limits of graphs with bounded diameter components*, manuscript, 2012.
[23] Rudin W., *Functional Analysis*, Mc-Graw Hill, New York, 1973.
[24] Stone M., *The theory of representations of Boolean algebras*, Trans. Amer. Math. Soc. **40** (1936), 37–111.