

**SPECTRAL RAY TRACER:
A CLASS OF ACCURATE TWO-POINT RAY TRACERS**

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A b s t r a c t

The recent development of high resolution seismic tomography and the increasing necessity for high precision seismic (acoustic) source locations calls for robust and very precise numerical methods of estimating of seismic (acoustic) wave travel times and propagation ray paths. This paper deals with two issues.

First of all we present a ray path tracing algorithm based on the parameterization of ray paths by a series of Chebyshev polynomials. This pseudo-spectral method combined with the accurate Gauss-Lobatto integration procedure allows to reach a very high relative accuracy of travel time calculation, of the order of $\Delta t/t \approx 10^{-7}$. The pseudo-spectral representation of sought ray paths turns the ray tracing problem into a numerical optimization task which, depending on the requirements, can be solved by a properly chosen optimizer. The used numerical representation designates the developed algorithm for tomography/location applications when no sharp interfaces occur.

Secondly, we consider the question of the accuracy of the ray path tracing and travel time calculations. Achieving the highest tracing accuracy in terms of both accurate travel time estimation and precise spatial tracing (ray path geometry) requires a very careful analysis of all classes of errors. Some of them are caused by the numerical approximation (discretization) of the (continuous) physical law underlying a chosen computational algorithm. We demonstrate that

these errors can degrade the ray tracing accuracy even by two orders of magnitude. To suppress this type of inaccuracy we propose to use a damping mechanism which introduces a kind of “tension” to the numerically generated ray paths. When properly applied, it suppresses the artificial spatial oscillations caused by the approximation errors and improves the ray tracing accuracy. The damping mechanism can also be regarded as the *a priori* requirement to keep the physical ray path trajectory as simple (smooth) as possible.

Key words: ray tracing, spectral methods, Fermat’s principle.